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Research Article

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Extrinsic aberrations in optical imaging systems

Abstract: This paper discusses extrinsic aberrations in optical systems and provides formulas for their computation. Extrinsic aberrations result when two optical systems are combined, and there is a mismatch between the exit pupil of the first system and the entrance pupil of the second system. Some significant aberration corrections can take place as a result of extrinsic aberrations.

Keywords: aberration theory; geometrical optics; lens design; optical aberrations.

OCIS codes: 080.1005; 080.6755; 080.1010; 080.0080.

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1 Introduction

The aberration contributed by a surface, or a system, can be divided as intrinsic and extrinsic (also known as induced [1]). The intrinsic aberration is the aberration that the surface or system contributes when an incoming beam has no aberration. The extrinsic part results from beam aberration from a previous surface or system.

Let us consider two optical systems A and B with aberration functions to the sixth order given by

$$W_A(\vec{H},\vec{\rho})=W_A^2(\vec{H},\vec{\rho})+W_A^4(\vec{H},\vec{\rho})+W_A^6(\vec{H},\vec{\rho})$$

and

$$W_B(\vec{H},\vec{\rho})=W_B^2(\vec{H},\vec{\rho})+W_B^4(\vec{H},\vec{\rho})+W_B^6(\vec{H},\vec{\rho}),$$

where $\vec{H}$ and $\vec{\rho}$ are the normalized field and aperture vectors. Both aberration functions are described with the aperture vector at the exit pupil of each system. That is, a given ray through a system requires two points to be defined (see Figure 1); one point is given by the tip of the field vector at the object plane and the other point by the tip of the aperture vector at the exit pupil plane. However, the exit pupil of system A connects with the entrance pupil of system B. In the presence of aberration in system B, the point $\vec{\rho}$ in the exit pupil of system B corresponds to the point $\vec{\rho}+\Delta\vec{\rho}_B$ in the exit pupil of system A. The term $\Delta\vec{\rho}_B$ is the normalized transverse ray error at the entrance pupil of system B,

$$\Delta\vec{\rho}_B=-\frac{1}{Ж}\nabla_H\bar{W}_B(\vec{H},\vec{\rho}),$$

where $\bar{W}_B(\vec{H},\vec{\rho})$ is the pupil aberration function of system B, $Ж$ is the Lagrange invariant, and ∇_H stands for the gradient with respect to the field vector.

Figure 2 shows how the entrance pupil of system B (solid line) can be distorted [2, 3] in relation to the exit pupil of system A (broken line) according to $\Delta\vec{\rho}_B$.

By substitution of $\vec{\rho}+\Delta\vec{\rho}_B$ in the aberration function of system A, we obtain

$$\begin{aligned}W_A(\vec{H},\vec{\rho}+\Delta\vec{\rho}_B)&=W_A(\vec{H},\vec{\rho}+\Delta\vec{\rho}_B)-W_A(\vec{H},\vec{\rho})+W_A(\vec{H},\vec{\rho})\\&\cong\nabla W_A(\vec{H},\vec{\rho})\cdot\Delta\vec{\rho}_B+W_A(\vec{H},\vec{\rho})\end{aligned}$$

where the term $\nabla W_A(\vec{H},\vec{\rho})\cdot\Delta\vec{\rho}_B$ represents the extrinsic aberration in the combination of systems A and B.

The extrinsic aberrations can be written in terms of the gradient of the aberration function and the gradient of the pupil aberration function as

$$W_E(\vec{H},\vec{\rho})=-\frac{1}{Ж}\vec{\nabla}_\rho W_A(\vec{H},\vec{\rho})\cdot\vec{\nabla}_H\bar{W}_B(\vec{H},\vec{\rho}).$$

This relationship has been presented previously in reference [4].

We can now consider the extrinsic aberrations that result from errors in the field vector for system B. In the aberration theory, the wavefront deformation is measured with respect to a reference sphere centered at the ideal image point. Therefore, changing the field point of interest from $\vec{H}$ to $\vec{H}+\Delta\vec{H}_A$ for system B changes the image point and the corresponding reference sphere. Thus, it is not correct to calculate additional extrinsic aberrations by the substitution of $\vec{H}+\Delta\vec{H}_A$ in $W_B(\vec{H},\vec{\rho})$ where

$$\Delta\vec{H}_A=-\frac{1}{Ж}\vec{\nabla}_\rho W_A(\vec{H},\vec{\rho})$$

is the transverse ray error at the image plane of system A.

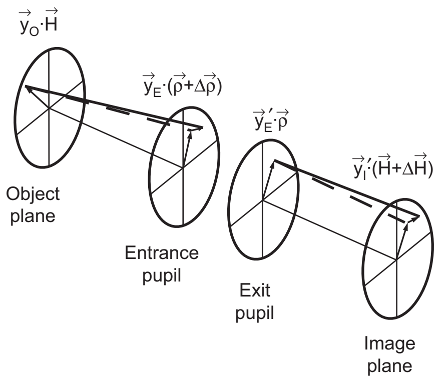

Figure 1 Representation of an optical system showing the optical axis, the object and image planes, the entrance and exit pupil planes, an ideal ray as a broken line, a real ray as a solid line, the field vector $\vec{H}$, the aperture vector $\vec{\rho}$, the transverse ray error vector $\Delta\vec{\rho}$, and the transverse ray error vector $\Delta\vec{H}$.

It can be shown that third-order transverse ray errors in the position of a ray in the object plane result in eighth-order aberration terms. These eighth-order terms are not treated in this paper. However, second-order errors in the position of the object for system *B* result in second-, fourth-, and higher-order wavefront errors for the system combination.

In this paper, we find the extrinsic aberrations of other optical system cases not considered in reference [4]. Specifically, we treat, here, the extrinsic chromatic aberrations and extrinsic aberrations in plane symmetric systems. The analysis presented here is novel and is, in part, made possible by using R. Shack's formulation of the wave aberration function that uses the field and aperture vectors, rather than using scalar parameters. The wave aberration approach was pioneered by H. H. Hopkins and has been further developed by researchers at the College of Optical Sciences at the University of Arizona.

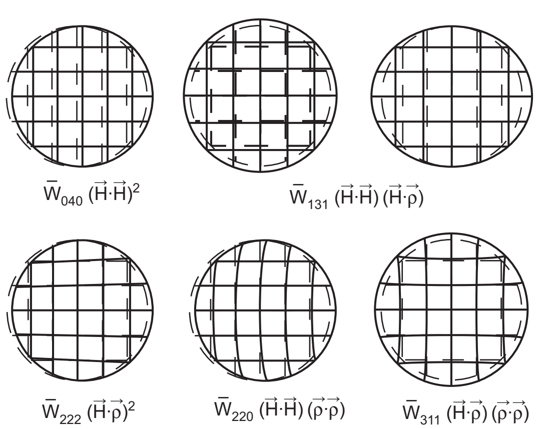

Figure 2 Pupil distortion due to different pupil aberrations.

2 Chromatic extrinsic aberrations

In standard aberration theory, the reference sphere is centered at the ideal Gaussian image point. Therefore, there are no second-order terms in the aberration function of a monochromatic system. In this case, the aberration function is

$$W_{\lambda_1}\left(\vec{H},\vec{\rho}\right)=W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right),$$

where we have neglected the piston terms.

For a second wavelength, the reference sphere is no longer centered at the ideal image point, and the aberration function has additional second-order terms. These are chromatic change of magnification and chromatic change of focus:

$$W_{\lambda_2}\left(\vec{H},\vec{\rho}\right)=W_{111}\left(\vec{H}\cdot\vec{\rho}\right)+W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$$
$$+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right).$$

The pupil aberration function for the second wavelength likewise has second-order terms:

$$\bar{W}_{\lambda_2}\left(\vec{H},\vec{\rho}\right)=\bar{W}_{111}\left(\vec{H}\cdot\vec{\rho}\right)+\bar{W}_{020}\left(\vec{H}\cdot\vec{H}\right)+\bar{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^2$$
$$+\bar{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\bar{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+\bar{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\bar{W}_{311}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right).$$

If we combine two systems *A* and *B*, then, the extrinsic aberrations are given by

$$W_E\left(\vec{H},\vec{\rho}\right)=-\frac{1}{\text{Ж}}\vec{\nabla}_\rho W_A\left(\vec{H},\vec{\rho}\right)\cdot\vec{\nabla}_H\bar{W}_B\left(\vec{H},\vec{\rho}\right).$$

For the gradient of the aberration function of system *A*, we have

$$\nabla_\rho W_A\left(\vec{H},\vec{\rho}\right)$$
$$=\left\{\begin{array}{l}2W_{020A}\vec{\rho}+W_{111}\vec{H}+4W_{040A}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}\\+W_{131A}\left(\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H}+2\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho}\right)+2W_{222A}\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\\+2\cdot W_{220A}\left(\vec{H}\cdot\vec{H}\right)\vec{\rho}+W_{311A}\left(\vec{H}\cdot\vec{H}\right)\vec{H}\end{array}\right\}.$$

For the gradient of the pupil aberration function of system *B*, we have

$$\nabla_H\bar{W}_B\left(\vec{H},\vec{\rho}\right)$$
$$=\left\{\begin{array}{l}2\bar{W}_{020B}\vec{H}+\bar{W}_{111B}\vec{\rho}+4\bar{W}_{040B}\left(\vec{H}\cdot\vec{H}\right)\vec{H}\\+\bar{W}_{131B}\left(\left(\vec{H}\cdot\vec{H}\right)\vec{\rho}+2\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right)+2\bar{W}_{222B}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho}\\+2\cdot\bar{W}_{220B}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H}+\bar{W}_{311B}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}\end{array}\right\}.$$

Table 1 provides the extrinsic aberration coefficients up to the fourth order for the combination of system *A* and system *B*. The subscript *E* indicates that the coefficient is extrinsic.

In Table 1, the lower index *A* or *B* indicates that the coefficient refers to system *A* or system *B*. For a system of three surfaces, the aberration coefficients are obtained by treating the system formed by the first two surfaces as system *A* and combining it with the third surface as system *B*. This process is repeated to obtain the aberration coefficients for a system of several surfaces or of a system.

Thus, the presence of the second-order terms in the image and pupil aberration functions in the combination of the two systems gives place to the extrinsic second-, fourth-, and higher-order aberrations. The second-order terms in Table 1 represent the extrinsic chromatic change of focus and magnification. The extrinsic chromatic change of focus,

$$W_{020E} = -\frac{2}{\text{Ж}} W_{020A}\bar{W}_{111B},$$

depends on the chromatic change of focus of system *A* and on the pupil chromatic change of magnification of system *B*. This extrinsic term can be used to control the secondary spectrum.

The fourth-order terms in Table 1 represent the extrinsic chromatic aberrations. For example, the coefficient

$$W_{040E} = -\frac{1}{\text{Ж}}\left(2W_{020A}\bar{W}_{311B} + 4W_{040A}\bar{W}_{111B}\right)$$

is extrinsic spherochromatism.

Table 1 Extrinsic coefficients from the combination of system *A* and system *B*.

Second order

$$W_{020E} = -\frac{2}{\text{Ж}} W_{020A}\bar{W}_{111B}$$

$$W_{111E} = -\frac{1}{\text{Ж}}\left(4W_{020A}\bar{W}_{020B} + W_{111A}\bar{W}_{111B}\right)$$

$$W_{200E} = -\frac{2}{\text{Ж}} W_{111A}\bar{W}_{020B}$$

Fourth order

$$W_{040E} = -\frac{1}{\text{Ж}}\left(2W_{020A}\bar{W}_{311B} + 4W_{040A}\bar{W}_{111B}\right)$$

$$W_{131E} = -\frac{1}{\text{Ж}}\begin{pmatrix} 4W_{020A}\bar{W}_{222B} + 4W_{020A}\bar{W}_{220B} \\ +W_{111A}\bar{W}_{311B} + 8W_{040A}\bar{W}_{020B} \\ +3W_{131A}\bar{W}_{111B} \end{pmatrix}$$

$$W_{222E} = -\frac{1}{\text{Ж}}\begin{pmatrix} 4W_{020A}\bar{W}_{131B} + 2W_{111A}\bar{W}_{222B} \\ +4W_{131A}\bar{W}_{020B} + 2W_{222A}\bar{W}_{111B} \end{pmatrix}$$

$$W_{220E} = -\frac{1}{\text{Ж}}\begin{pmatrix} 2W_{111A}\bar{W}_{220B} \\ +2W_{131A}\bar{W}_{020B} \\ +2W_{220A}\bar{W}_{111B} \end{pmatrix}$$

$$W_{311E} = -\frac{1}{\text{Ж}}\begin{pmatrix} 8W_{020A}\bar{W}_{040B} + 2W_{020A}\bar{W}_{131B} + 4W_{222A}\bar{W}_{020B} \\ +W_{111A}\bar{W}_{131B} + 2W_{111A}\bar{W}_{131B} + 4W_{220A}\bar{W}_{020B} \\ +W_{311A}\bar{W}_{111B} \end{pmatrix}$$

$$W_{400E} = -\frac{1}{\text{Ж}}\left(4W_{111A}\bar{W}_{040B} + 2W_{311A}\bar{W}_{020B}\right)$$

3 Sixth-order extrinsic aberrations

The absence of second-order terms in the aberration function of two axially symmetric systems results in no second- or fourth-order extrinsic terms. Specifically, when the aberration function of system *A* is

$$W_A\left(\vec{H},\vec{\rho}\right) = W_{040A}\left(\vec{\rho}\cdot\vec{\rho}\right)^2 + W_{131A}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) + W_{222A}\left(\vec{H}\cdot\vec{\rho}\right)^2 + W_{220A}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) + W_{311A}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right) + W_{400A}\left(\vec{H}\cdot\vec{H}\right)^2,$$

and the pupil aberration function of system *B* is

$$\bar{W}_B\left(\vec{H},\vec{\rho}\right) = \bar{W}_{040B}\left(\vec{H}\cdot\vec{H}\right)^2 + \bar{W}_{131B}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right) + \bar{W}_{222B}\left(\vec{H}\cdot\vec{\rho}\right)^2 + \bar{W}_{220B}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) + \bar{W}_{311B}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) + \bar{W}_{400B}\left(\vec{\rho}\cdot\vec{\rho}\right)^2,$$

the extrinsic aberration coefficients that result are of the sixth order as shown [4] in Table 2.

An equivalent table for the transverse third-order aberrations is provided by Buchdahl [5]. Researchers interested in higher-order aberration theory and tolerancing have been aware and calculated extrinsic aberrations [6–12].

4 Extrinsic aberrations in plane symmetric systems

The aberration function for a plane symmetric system [13] can be written as

$$W\left(\vec{i},\vec{H},\vec{\rho}\right) = \sum_{k,m,n,p,q}^{\infty} W_{\substack{2k+n+p,\\ 2m+n+q,\\ n,p,q}}\left(\vec{H}\cdot\vec{H}\right)^k\left(\vec{\rho}\cdot\vec{\rho}\right)^m\left(\vec{H}\cdot\vec{\rho}\right)^n\left(\vec{i}\cdot\vec{H}\right)^p\left(\vec{i}\cdot\vec{\rho}\right)^q,$$

where $W_{2k+n+p,2m+n+q,n,p,q}$ is the coefficient of a particular aberration form defined by the integers *k*, *m*, *n*, *p*, and *q*. The lower indices in the coefficient indicate the algebraic powers of *H*, ρ, $\cos(\phi)$, $\cos(\chi)$, and $\cos(\chi+\phi)$ in a given aberration term. The angle χ is between the vectors $\vec{i}$ and $\vec{H}$, and the angle $\chi+\phi$ is between the vectors $\vec{i}$ and $\vec{\rho}$.

Table 2 Extrinsic coefficients from the combination of system *A* and system *B*.

$$W_{060E}=-\frac{1}{\text{ж}}\left(4W_{040}^{A}\overline{W}_{311}^{B}\right)$$

$$W_{151E}=-\frac{1}{\text{ж}}\begin{pmatrix}3W_{131}^{A}\overline{W}_{311}^{B}+8W_{040}^{A}\overline{W}_{220}^{B}\\+8W_{040}^{A}\overline{W}_{222}^{B}\end{pmatrix}$$

$$W_{242E}=-\frac{1}{\text{ж}}\begin{pmatrix}2W_{222}^{A}\overline{W}_{311}^{B}+4W_{131}^{A}\overline{W}_{220}^{B}\\+6W_{131}^{A}\overline{W}_{222}^{B}+8W_{040}^{A}\overline{W}_{131}^{B}\end{pmatrix}$$

$$W_{333E}=-\frac{1}{\text{ж}}\left(4W_{131}^{A}\overline{W}_{131}^{B}+4W_{222}^{A}\overline{W}_{222}^{B}\right)$$

$$W_{240E}=-\frac{1}{\text{ж}}\begin{pmatrix}2W_{131}^{A}\overline{W}_{220}^{B}+2W_{220}^{A}\overline{W}_{311}^{B}\\+4W_{040}^{A}\overline{W}_{131}^{B}\end{pmatrix}$$

$$W_{331E}=-\frac{1}{\text{ж}}\begin{pmatrix}5W_{131}^{A}\overline{W}_{131}^{B}+4W_{220}^{A}\overline{W}_{220}^{B}\\+4W_{220}^{A}\overline{W}_{222}^{B}+4W_{222}^{A}\overline{W}_{220}^{B}\\+W_{311}^{A}\overline{W}_{311}^{B}+16W_{040}^{A}\overline{W}_{040}^{B}\end{pmatrix}$$

$$W_{422E}=-\frac{1}{\text{ж}}\begin{pmatrix}2W_{311}^{A}\overline{W}_{222}^{B}+4W_{220}^{A}\overline{W}_{131}^{B}\\+6W_{222}^{A}\overline{W}_{131}^{B}+8W_{131}^{A}\overline{W}_{040}^{B}\end{pmatrix}$$

$$W_{420E}=-\frac{1}{\text{ж}}\begin{pmatrix}2W_{220}^{A}\overline{W}_{131}^{B}+2W_{311}^{A}\overline{W}_{220}^{B}\\+4W_{131}^{A}\overline{W}_{040}^{B}\end{pmatrix}$$

$$W_{511E}=-\frac{1}{\text{ж}}\begin{pmatrix}3W_{311}^{A}\overline{W}_{131}^{B}+8W_{220}^{A}\overline{W}_{040}^{B}\\+8W_{222}^{A}\overline{W}_{040}^{B}\end{pmatrix}$$

$$W_{600E}=-\frac{1}{\text{ж}}\left(4W_{311}^{A}\overline{W}_{040}^{B}\right)$$

By setting the sum of the integers to 0, 1, 2..., groups of aberrations are defined as shown in Table 3. The vector $\vec{i}$ defines the direction of plane symmetry.

Considering aberration terms of the fourth order in $\vec{i}$, $\vec{H}$, and $\vec{\rho}$, we can write the aberration function as

$$\begin{aligned}W\left(\vec{i},\vec{H},\vec{\rho}\right)=&W_{02002}\left(\vec{i}\cdot\vec{\rho}\right)^2+W_{11011}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{i}\cdot\vec{\rho}\right)+W_{20020}\left(\vec{i}\cdot\vec{H}\right)^2\\&+W_{03001}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{12101}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{12010}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)\\&+W_{21001}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{H}\right)+W_{21110}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{30010}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{H}\right)\\&+W_{04000}\left(\vec{\rho}\cdot\vec{\rho}\right)^2+W_{13100}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{22200}\left(\vec{H}\cdot\vec{\rho}\right)^2\\&+W_{22000}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{31100}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{40000}\left(\vec{H}\cdot\vec{H}\right)^2.\end{aligned}$$

The pupil aberration function can also be written as

$$\begin{aligned}\overline{W}\left(\vec{i},\vec{H},\vec{\rho}\right)=&\overline{W}_{02002}\left(\vec{i}\cdot\vec{H}\right)^2+\overline{W}_{11011}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{i}\cdot\vec{\rho}\right)+\overline{W}_{20020}\left(\vec{i}\cdot\vec{\rho}\right)^2\\&+\overline{W}_{03001}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{H}\right)+\overline{W}_{12101}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{12010}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{H}\right)\\&+\overline{W}_{21001}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{21110}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{30010}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)\\&+\overline{W}_{04000}\left(\vec{H}\cdot\vec{H}\right)^2+\overline{W}_{13100}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{22200}\left(\vec{H}\cdot\vec{\rho}\right)^2\\&+\overline{W}_{22000}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{31100}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{40000}\left(\vec{\rho}\cdot\vec{\rho}\right)^2.\end{aligned}$$

Table 3 Aberration terms of a plane symmetric system.

First group	
W_{00000}	Piston
Second group	
$W_{01001}\vec{i}\cdot\vec{\rho}$	Field displacement
$W_{10010}\vec{i}\cdot\vec{H}$	Linear piston
$W_{02000}\vec{\rho}\cdot\vec{\rho}$	Defocus
$W_{11100}\vec{H}\cdot\vec{\rho}$	Magnification
$W_{20000}\vec{H}\cdot\vec{H}$	Quadratic piston
Third group	
$W_{02002}\left(\vec{i}\cdot\vec{\rho}\right)^2$	Uniform astigmatism
$W_{11011}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{i}\cdot\vec{\rho}\right)$	Anamorphic distortion
$W_{20020}\left(\vec{i}\cdot\vec{H}\right)^2$	Quadratic piston
$W_{03001}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$	Uniform coma
$W_{12101}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)$	Linear astigmatism
$W_{12010}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$	Field tilt
$W_{21001}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{H}\right)$	Quadratic distortion
$W_{21110}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)$	Quadratic distortion
$W_{30010}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{H}\right)$	Cubic piston
$W_{04000}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$	Spherical aberration
$W_{13100}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$	Linear coma
$W_{22200}\left(\vec{H}\cdot\vec{\rho}\right)^2$	Quadratic astigmatism
$W_{22000}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$	Field curvature
$W_{31100}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)$	Cubic distortion
$W_{40000}\left(\vec{H}\cdot\vec{H}\right)^2$	Quadratic piston

The gradient of the aberration function is

$$\begin{aligned}\nabla_{\rho}W\left(\vec{i},\vec{H},\vec{\rho}\right)=&2W_{02002}\left(\vec{i}\cdot\vec{\rho}\right)\vec{i}+W_{11011}\left(\vec{i}\cdot\vec{H}\right)\vec{i}\\&+W_{03001}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{i}+2W_{03001}\left(\vec{i}\cdot\vec{\rho}\right)\vec{\rho}+W_{12101}\left(\vec{H}\cdot\vec{\rho}\right)\vec{i}\\&+W_{12101}\left(\vec{i}\cdot\vec{\rho}\right)\vec{H}+2W_{12010}\left(\vec{i}\cdot\vec{H}\right)\vec{\rho}+W_{21001}\left(\vec{H}\cdot\vec{H}\right)\vec{i}\\&+W_{21110}\left(\vec{i}\cdot\vec{H}\right)\vec{H}+4W_{04000}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}+W_{13100}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H}\\&+2W_{13100}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho}+2W_{22200}\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}+2W_{22000}\left(\vec{H}\cdot\vec{H}\right)\vec{\rho}\\&+W_{31100}\left(\vec{H}\cdot\vec{H}\right)\vec{H}.\end{aligned}$$

The gradient of the pupil aberration function is

$$\begin{aligned}\nabla_{H}\overline{W}\left(\vec{i},\vec{H},\vec{\rho}\right)=&2\overline{W}_{02002}\left(\vec{i}\cdot\vec{H}\right)\vec{i}+\overline{W}_{11011}\left(\vec{i}\cdot\vec{\rho}\right)\vec{i}\\&+\overline{W}_{03001}\left(\vec{H}\cdot\vec{H}\right)\vec{i}+2\overline{W}_{03001}\left(\vec{i}\cdot\vec{H}\right)\vec{H}+\overline{W}_{12101}\left(\vec{H}\cdot\vec{\rho}\right)\vec{i}\\&+\overline{W}_{12101}\left(\vec{i}\cdot\vec{H}\right)\vec{\rho}+2\overline{W}_{12010}\left(\vec{i}\cdot\vec{\rho}\right)\vec{H}+\overline{W}_{21001}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{i}\end{aligned}$$

$$+\bar{W}_{21110}\left(\vec{i}\cdot\vec{\rho}\right)\vec{\rho}+4\bar{W}_{04000}\left(\vec{H}\cdot\vec{H}\right)\vec{H}+\bar{W}_{13100}\left(\vec{H}\cdot\vec{H}\right)\vec{\rho}$$
$$+2\bar{W}_{13100}\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}+2\bar{W}_{22200}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho}+2\bar{W}_{22000}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H}$$
$$+\bar{W}_{31100}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}.$$

The extrinsic aberration terms from the combination of systems A and B are given by

$$W_E\left(\vec{H},\vec{\rho}\right)=-\frac{1}{\text{Ж}}\vec{\nabla}_\rho W_A\left(\vec{H},\vec{\rho}\right)\cdot\vec{\nabla}_H\bar{W}_B\left(\vec{H},\vec{\rho}\right).$$

Retaining the fourth-order terms in $\vec{i}$, $\vec{H}$, and $\vec{\rho}$, we obtain the extrinsic aberration coefficients as shown in Table 4.

Table 4 shows that in a non-axially symmetric system, the aberration terms of a given order give place to the aberration terms of the same order. In this case, the order is a function of the vectors $\vec{i}$, $\vec{H}$, and $\vec{\rho}$.

It is interesting to observe that quadratic distortion of the pupil $\bar{W}_{21001B}\left(\vec{i}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$ and uniform coma $W_{03001A}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$ result in spherical aberration $W_{04000E}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$. This can be seen by the substitution of $\vec{\rho}+\Delta\vec{\rho}_B=\vec{\rho}+\bar{W}_{21001B}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{i}$ for $\vec{\rho}$ in $W_{03001A}\left(\vec{i}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$.

Table 4 Extrinsic aberration coefficients from the combination of systems A and B that are plane symmetric.

$W_{02002E}=-\frac{1}{\text{Ж}}\left(2W_{02002A}\bar{W}_{11011B}\right)$	Uniform astigmatism
$W_{11011E}=-\frac{1}{\text{Ж}}\left(4W_{02002A}\bar{W}_{02002B}+W_{11011A}\bar{W}_{11011B}\right)$	Anamorphic distortion
$W_{20020E}=-\frac{1}{\text{Ж}}\left(2W_{11011A}\bar{W}_{02002B}\right)$	Quadratic piston
$W_{03001E}=-\frac{1}{\text{Ж}}\left(2W_{02002A}\bar{W}_{21001B}+W_{03001A}\bar{W}_{11011B}\right)$	Uniform coma
$W_{12101E}=-\frac{1}{\text{Ж}}\left(2W_{02002A}\bar{W}_{12101B}+W_{12101A}\bar{W}_{11011B}\right)$	Linear astigmatism
$W_{12001E}=-\frac{1}{\text{Ж}}\left(W_{11011A}\bar{W}_{21001B}+2W_{03001A}\bar{W}_{02002B}\right)$	Field tilt
$W_{21001E}=-\frac{1}{\text{Ж}}\left(W_{21001A}\bar{W}_{11011B}+W_{21001A}\bar{W}_{11011B}+2W_{02002A}\bar{W}_{03001B}\right)$	Quadratic distortion
$W_{21110E}=-\frac{1}{\text{Ж}}\left(2W_{12101A}\bar{W}_{02002B}+W_{11011A}\bar{W}_{12101B}\right)$	Quadratic distortion
$W_{30010E}=-\frac{1}{\text{Ж}}\left(W_{11011A}\bar{W}_{03001B}+2W_{21001A}\bar{W}_{02002B}\right)$	Cubic piston
$W_{04000E}=-\frac{1}{\text{Ж}}\left(W_{03001A}\bar{W}_{21001B}\right)$	Spherical aberration
$W_{13100E}=-\frac{1}{\text{Ж}}\left(W_{03001A}\bar{W}_{12101B}+W_{12101A}\bar{W}_{21001B}\right)$	Linear coma
$W_{22200E}=-\frac{1}{\text{Ж}}\left(W_{12101A}\bar{W}_{12101B}\right)$	Quadratic astigmatism
$W_{22000E}=-\frac{1}{\text{Ж}}\left(W_{03001A}\bar{W}_{03001B}+W_{21001A}\bar{W}_{21001B}\right)$	Field curvature
$W_{31100E}=-\frac{1}{\text{Ж}}\left(W_{21001A}\bar{W}_{12101B}+W_{12101A}\bar{W}_{03001B}\right)$	Cubic distortion
$W_{40000E}=-\frac{1}{\text{Ж}}\left(W_{21001A}\bar{W}_{03001B}\right)$	Quadratic piston

5 Application of extrinsic aberrations

There are several examples of optical systems where extrinsic aberrations play a significant role. For example, extrinsic chromatic aberration has been used to design apochromatic lens systems using normal glasses [14–16]. The field lens in the Offner Null corrector [17, 18] effectively controls pupil distortion, and this results in the effective control of higher-order spherical aberration. The field lens in the Schupmann medial telescope [19], in addition to controlling chromatic change of magnification, also controls spherochromatism. Field lenses, or lenses at a beam constriction, can control pupil coma, pupil astigmatism, and pupil distortion, and therefore, they provide an effective way to control oblique spherical aberration of the image.

The Schupmann medial telescope is a single glass achromatic system that uses a Mangin mirror to create a real image and correct chromatic change of focus. The Mangin mirror introduces spherical aberration $W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$. Without a field lens, the marginal ray height at the Mangin mirror is different for two colors by $\Delta\vec{\rho}$. For the second color, the spherical aberration is,

$$W_{040}\left(\left(\vec{\rho}+\Delta\vec{\rho}\right)\cdot\left(\vec{\rho}+\Delta\vec{\rho}\right)\right)^2\cong W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2+4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\Delta\vec{\rho}\right).$$

The term $4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\Delta\vec{\rho}\right)$ represents spherochromatism. If a field lens is introduced at the focus of the objective lens, the difference in marginal ray height difference $\Delta\vec{\rho}$ at the Mangin mirror can be changed according with the optical power of the field lens. The aperture stop is at the objective lens, and the Mangin mirror is located at the image of the objective lens by the field lens; that is, at a pupil. Then, the difference in the marginal ray height $\Delta\vec{\rho}$ is given by the chromatic change of magnification of the pupil. That is,

$$\Delta\vec{\rho}=-\frac{1}{\text{Ж}}\nabla_H\left(\partial_\lambda\bar{W}_{111}\left(\vec{H}\cdot\vec{\rho}\right)\right)=-\frac{1}{\text{Ж}}\partial_\lambda\bar{W}_{111}\vec{\rho}.$$

Therefore, the spherochromatism can be written as,

$$\partial_\lambda W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2=-\frac{4}{\text{Ж}}W_{040}\bar{W}_{111}\left(\vec{\rho}\cdot\vec{\rho}\right)^2.$$

This result is in agreement with our earlier prediction. Thus, the field lens in the Schupmann medial telescope not only controls the chromatic change of magnification by making the chief ray zero at the Mangin mirror, but it also controls spherochromatism. This theoretical result on the behavior of aberration in a Schupmann medial telescope is easy to verify in a lens design program.

6 Conclusions

Extrinsic aberrations are a common phenomenon in optical systems. The claim often given in the literature [20, 21] that the aberration coefficients add from surface to surface in a system is correct only in the absence of extrinsic aberrations. This is not often the case, and extrinsic terms must be accounted for.

One noteworthy case where the aberration coefficients from surface to surface add is the Seidel sum contribution for a system that has no second-order terms in the aberration function of each surface.

Understanding the subject of extrinsic aberrations has historically permitted us to design some useful optical systems. This paper provides a theoretical foundation on the subject.

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