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Synchronization of Chaos in Neural Systems

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Multiple non-linear systems demonstrate the phenomenon where fluctuations enhance the synchronization and periodic behaviors of the system. In the phenomenon induced by stochastic additive noise, stochastic resonance, noise enhances the synchronization of system behaviors against weak input signals. Along with stochastic noise, deterministic chaos induces a phenomenon like stochastic resonance called chaotic resonance. This review summarizes the progress of studies on chaotic resonance over the most recent decade. First, the fundamental characteristics of chaotic resonance are reviewed. Second, chaotic resonance in brain informatics, including cerebellar learning and deterministic fluctuations observed in electroencephalography/magnetoencephalography examinations, are reviewed. Third, the "reduced region of orbit" method is reviewed for the potential application of chaotic resonance. This review emphasizes the potential importance of recapturing neural fluctuation functionality, previously considered in the framework of stochastic resonance (also called stochastic facilitation), through chaotic resonance and assesses the effectiveness of applying chaotic resonance.

Keywords: chaotic resonance, stochastic resonance, neural system, spiking neural network, chaos control, synchronization

1. INTRODUCTION

Various non-linear systems reportedly demonstrate a phenomenon where fluctuations enhance synchronization and periodic behaviors (reviewed in [1–5]) [6, 7]. Phenomena induced by stochastic additive noise, stochastic resonance [8–10] and coherence resonance/noise-induced order [11–17] are well-known. In stochastic resonance, the synchronization of system behaviors against weak input signal is enhanced by noise [8–10]. In coherence resonance/noise-induced order, the intrinsic periodic system behaviors are induced by noise [11–17]. In addition to stochastic noise, deterministic chaos induces stochastic resonance, also called chaotic resonance (reviewed in [3, 4]). These phenomena are summarized in **Table 1**.

Studies have assessed these phenomena and have identified various functionalities of fluctuations in neural activity (reviewed in [38–41]). These studies have primarily focused on two kinds of phenomena: sensitivity enhancement of sensory neural systems [38] and functionality enhancement of the central nervous system [39–42]. A study on sensory neural systems reported that crayfish and paddlefish use background noise to detect slight movements in the water made by predators and prey [38]. Another study on the central nervous system reported that the chaotic dynamics of electroencephalography (EEG) in the olfactory bulb of rabbits contributes to efficient memory search [43]. Moreover, studies on the cerebellar learning process (i.e., the inferior olive nucleus) showed transmission of teacher signals with rich error information in the form

Phenomenon	Brief explanation	Objective systems
Stochastic resonance	Enhancement of the degree of synchronization against weak input signal by stochastic noise in non-linear systems with a barrier or threshold.	Climate system [8], neural systems [18–20], social systems [6, 7, 21], non-linear circuits [9, 10].
Coherence resonance	Emergence of inherent periodic behaviors induced by stochastic noise in non-linear systems with a barrier or threshold.	Neural systems [11, 12, 16], social systems [7, 17], non-linear circuits [13].
Noise-induced order	Stabilizing chaotic behaviors to periodic states by stochastic noise in non-linear systems.	Neural systems [14], chemical reaction system [15].
Chaotic resonance	Synchronization of chaotic behaviors against weak input signals by optimizing chaotic state in a non-linear system.	Neural systems [22–25] (in particular, cerebellar learning system [26–30]).
Chaos synchronization	Synchronization of chaotic behaviors in the assembly of non-linear systems, which are classified as complete, phase, and general synchronizations. Additionally, the intermittent transition between synchronous and asynchronous states is known as chaotic itinerancy.	Non-linear circuits [31, 32], social systems [33], neural systems [34–37].

of a low-rate chaotic spiking pattern (∼1.0 Hz) [26–30]. Furthermore, neuroimaging modalities, such as functional magnetic resonance imaging (fMRI) and EEG/magnetoencephalography (MEG) revealed the degrees of neural fluctuations are related to cognitive functions, aging, development, and psychiatric disorders [44–48] (reviewed in [40, 41]).

These resonance phenomena induced by fluctuations, especially stochastic resonance, have recently been applied to biomedical engineering [49–52]. Kurita et al. developed a wearable device that enhances the tactile sensitivity of surgeons' hands by applying appropriate vibrations [50, 51]. Enders et al. and Seo et al. further utilized this method by applying vibrotactile noise to study stochastic resonance in the human sensory systems and proposed a rehabilitation method to improve haptic sensations in patients with paralysis [49, 53]. In addition to the sensory neural systems at the cognitive levels of brain function, Van der Groen et al. developed a method to enhance perceptual decision-making using the stochastic resonance effect [52]. Specifically, applying the optimal amount of noise by transcranial random noise stimulation to the visual cortex can be used as a non-invasive brain stimulation technique to enhance the accuracy of perceptual decisions. Regarding chaotic resonance, although several studies reported that its sensitivity to a weak input signal is higher than that of stochastic resonance [23, 54], no study has reported any application of chaotic resonance. One possible reason for this might be the strength of additive noise, which can be easily controlled from the outside during stochastic resonance. Conversely, controlling the chaotic states to elicit chaotic resonance is difficult in many cases, especially in biological systems. To tackle this difficulty, we previously proposed a new chaos control method called the "reduced region of orbit (RRO)" method, where the chaotic signals are shifted to the appropriate chaotic state to elicit a

chaotic resonance by external feedback signals [55]. First, the RRO-based method was adopted for a simple discrete chaotic system, such as the cubic map and its assembly [55, 56], as well as to a continuous chaotic system termed Chua's circuit [57]. Studies have now started applying the RRO-based method to neural systems [58, 59]. The RRO-based method is expected to open an avenue for utilizing chaotic resonance in biomedical engineering in addition to its application in stochastic resonance.

In this review, we summarize the progress of studies on chaotic resonance over the decade. First, the fundamental characteristics of chaotic resonance were reviewed. Second, chaotic resonance in brain informatics, including cerebellar learning and deterministic fluctuations observed in EEG/MEG, are reviewed. Finally, studies on the RRO-based method for controlling chaotic resonance are also reviewed. Through chaotic resonance, this review emphasizes the potential importance of recapturing neural fluctuation functionality, previously considered in the framework of stochastic resonance (also called stochastic facilitation), and assesses the effectiveness of applying chaotic resonance.

2. MECHANISM AND FUNDAMENTAL CHARACTERISTICS OF CHAOTIC RESONANCE

First, we review the mechanism of chaotic resonance in comparison with stochastic resonance (see the overview of these resonance phenomena in **Figure 1**). We assumed a condition in which the input signal is too weak for the system state to surpass the barrier or threshold. Under this condition, in case of stochastic resonance, the additive stochastic noise is applied to the system with appropriate strength; the system state surpasses the barrier or threshold due to the noise, especially

when the input signal exhibits a peak. Consequently, the weak input signal and the system output synchronize with one another. Initially, the concept of stochastic resonance was devised to explain the mechanism of periodically recurrent ice ages, called the Milankovitch cycle [8]. Presently, it is widely recognized that stochastic resonance emerges in various kinds of systems with the following three factors: a barrier/threshold, a source of noise, and a weak input signal [8–10]. Moreover, the noise source is not restricted to additive stochastic noise; chaos also causes a phenomenon of stochastic resonance, i.e., chaotic resonance (reviewed in [3, 4]).

In chaotic resonance, two forms have been proposed to utilize chaotic dynamical fluctuation (reviewed in [3, 4]). In one form, the external additive chaotic signal is applied to the system instead of the stochastic noise [60, 61]; in the other form, an alternative intrinsic chaotic dynamics is utilized. Many of the recently published studies were classified as the latter form (reviewed in [3, 4]). In the following sections, we review this type of chaotic resonance. Initially, chaotic resonance was found in chaotic systems with chaos-chaos intermittency, in which the chaotic orbit hops among the several separated chaotic attractor regions, such as the cubic map and Chua's circuits [62–64] (reviewed in [3]). In these chaotic systems, an attractor merging crisis arises according to the adjustment of internal order parameters; through this crisis, chaos-chaos intermittency is induced. The degree of synchronization in chaotic resonance approximately maximizes, which is the bifurcation point for this attractor merging crisis. The underlying reason for this phenomenon is that, approximately at this bifurcation, the intrinsic hopping of chaotic attractor regions seldomly occurs under the absence of weak external signals. In this situation, the application of the external signal acts as a perturbation that switches the orbit among the attractors. Consequently, the induced chaos-chaos intermittency synchronizes with the external signal. Under conditions without chaos-chaos intermittency, the external signal is too weak to induce the chaos-chaos intermittency. Conversely, because of the inherent chaotic dynamics with a large disturbance, under high-frequency conditions of chaos-chaos intermittency, the hopping does not synchronize with a weak external signal. Another fundamental characteristic of chaotic resonance is that the degree of synchronization exhibits a unimodal maximum peak around an appropriate signal strength at the frequency of the input signal [54, 64, 65]. These characteristics are interpreted such that chaotic resonance has resonance frequency as the resonance phenomenon, and the stabilization of chaotic states by large signal strength degrades the degree of synchronization [65].
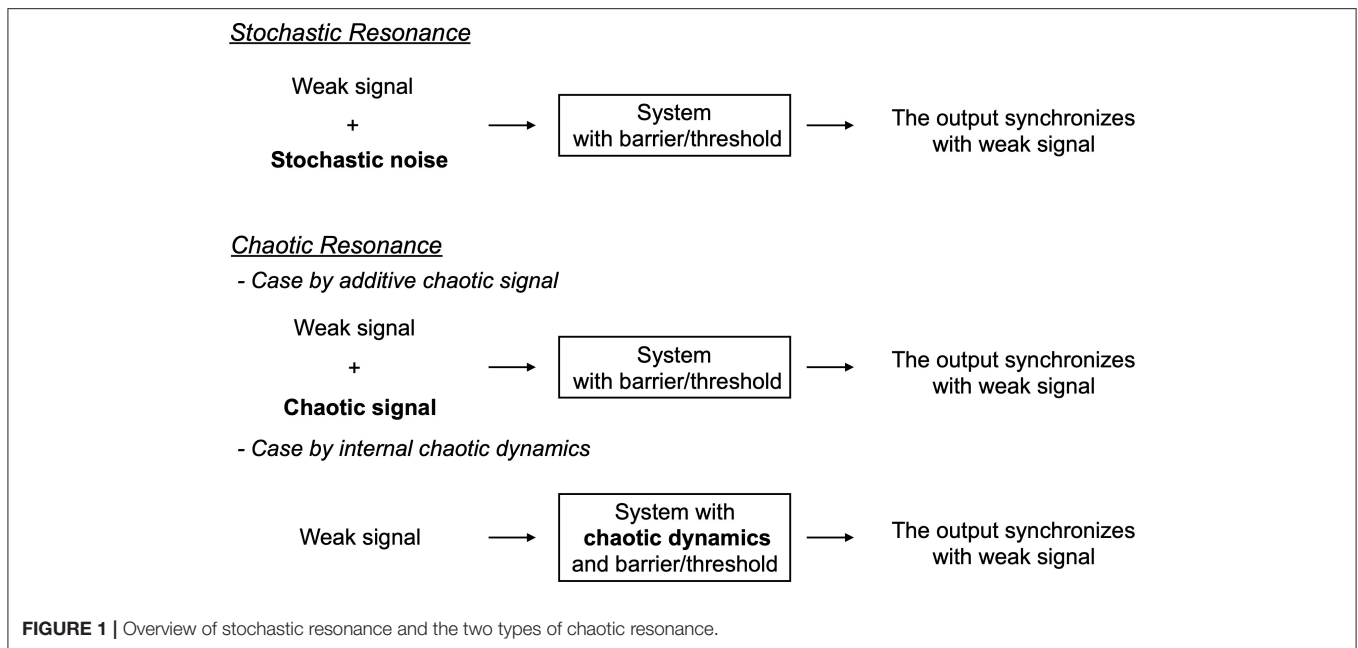
3. CHAOTIC RESONANCE IN NEURAL SYSTEMS

Previous studies on the dynamic behaviors of neural activity have revealed that chaos exists at several hierarchical levels in neural systems, ranging from the electrical response of a single neuron to brain activity produced by neural assemblies [66–68]. To identify the functionalities of chaos in neural systems,

previous studies evaluated chaotic resonance within them [22, 23, 54, 65]. Sinha constructed a neural population model composed of excitatory and inhibitory neural populations, and the neural activity of this model was indicated by the mean spiking rate [22]. This neural model exhibits chaos-chaos intermittency, which synchronizes with the weak external signal, i.e., it induces a chaotic resonance [22]. Moreover, we constructed chaos neural networks as per the mean spiking rate model with several embedded memory patterns; these networks exhibit chaos-chaos intermittency among the stored memory patterns [23, 54, 65]. The degree of synchronization of the period for recalling the memory against a weak input stimulus is maximized around the emergence of the attractor merging crises [23, 54, 65]. Additionally, comparing the sensitivity of chaotic resonance with that of stochastic resonance, chaotic resonance exhibits higher sensitivity than stochastic resonance [23, 54, 65].

In actual neural systems, various types of neural coding forms exist as well as the mean spiking rate coding, such as spike timing coding and spike population coding (reviewed in [69]). Spiking neural models have been widely utilized to describe the dynamic behaviors of these neural coding forms and to reveal the brain informatics mechanism [28, 70–72] (reviewed in [69, 73]). Previously, several studies have identified chaotic resonance in spiking neurons and spiking neural networks, such as in the inferior olive neural systems for cerebellar learning and in the Izhikevich neuron model [24–27, 30]. Spiking neural systems do not exhibit chaos-chaos intermittency but rather exhibit threshold characteristics for spike generation (reviewed in [74]). During chaotic resonance in spiking neural systems, the chaotic behavior against input signals induces spike-responses that do not occur at specific times but rather vary for each trial. In this manner, the distribution frequency of these spike timings against the input signal becomes congruent with the shape of the input signal [24–27, 30]. In addition to the mean spiking rate models, the chaotic resonance in spiking neural systems exhibits a unimodal maximum peak of signal response at the appropriate input frequency. The signal response of chaotic resonance is maintained under a weak strength of the input signal where the chaotic state is present [25, 30]. Moreover, chaotic resonance responses and their sensitivity maximize around the edge between the periodic spiking state and chaotic spiking state (called edge of chaos [75]) [25].

With large-scale neural fluctuations, observed as EEG, MEG, and fMRI signals, the complexity of temporal behaviors of brain activity reflects the ability of cognitive functions [76], development [47], aging [44, 77, 78], and the pathology of psychiatric disorders [45, 48, 79, 80] (reviewed in [40, 41, 81]). Previous studies considered the temporal complexity of brain activity under the framework of stochastic resonance (also called stochastic facilitation) (reviewed in [39]). Several studies reported that these complex brain activities involve the deterministic dynamical processes that reflect the internal brain informatics process rather than the stochastic process [47, 77, 82, 83]. In addition to these studies based on physiological data analysis, several model-based studies with spiking neural networks demonstrated that network structures (e.g., synaptic weight distribution and network topology) induce complex

FIGURE 1 | Overview of stochastic resonance and the two types of chaotic resonance.

deterministic dynamical neural activities [72, 84]. These studies may indicate the existence of chaotic resonance in a wide range of brain activities and cognitive processes. Although it is difficult to examine chaos with experimental time-series of brain activity involving stochastic noise in high-dimensional dynamical systems [85], future studies that assess the complexity of neural activity will validate this speculation.

4. CONTROLLING CHAOTIC RESONANCE BY EXTERNAL FEEDBACK SIGNALS

Although the internal system parameters cannot be adjusted from outside the system, our proposed RRO-based method induces chaotic resonance by adjusting the attractor merging crisis [55]. This RRO-based method is expected to facilitate the biomedical application of chaotic resonance [58, 59]. In this section, we review the mechanisms for eliciting and controlling chaotic resonance using the RRO-based method and the recent progress in its application to neural systems.

Systems with chaos-chaos intermittency possess cubic map structures in their dynamics [86, 87]. RRO feedback signals control the conditions for the development of attractor merging crisis by adjusting the local maximum and minimum values of the map function [55, 59]. In particular, under the condition of high-frequency chaos-chaos intermittency, RRO feedback signals with positive feedback strength, including the effect of reducing the absolute values of local maximum and minimum of the map function, reduce the frequency of chaos-chaos intermittency. Without the chaos-chaos intermittency, the RRO feedback signals with negative feedback strength, including those with the effect of increasing the local maximum/minimum of the map function, induce chaos-chaos intermittency and enhance the frequency of chaos-chaos intermittency. Consequently, in both

cases, the appropriate frequency of chaos-chaos intermittency for chaotic resonance is achieved. Previously, various methods were proposed for controlling the chaotic state by using external perturbations, such as the Ott-Grebogi-Yorke method [88], delayed feedback method [89, 90], and H_∞ method [91] (reviewed in [92]). These conventional chaos control methods provide a transition for the conversion of chaotic states to stable periodic states or stable fixed points. The RRO-based method not only eliminates the chaotic state but also optimizes the chaotic state for chaotic resonance. Initially, the RRO-based method was adopted for a discrete cubic map system [55]; subsequently, the RRO-based method has been applied to coupled chaotic systems [56]. Moreover, in continuous chaotic systems, RRO feedback signals based on dynamical behavior on the Poincaré section have been developed, which can induce chaotic resonance [57].

As a potential application of the RRO-based method to neural systems, we applied RRO feedback signals to a discrete neural population model composed of excitatory and inhibitory neural populations [55]. This neural model, proposed by Sinha, possesses the cubic map structure in its dynamics and can elicit chaos-chaos intermittency behaviors [22]. Through this RRO mechanism, the frequency of chaos-chaos intermittency is adjusted, and a chaotic resonance is induced [55, 59]. Compared with conventional stochastic resonance induced by additive noise, chaotic resonance induced by RRO feedback signals exhibits a relatively high degree of synchronization against the input signal and a high sensitivity against weak input signals [59]. Moreover, additive noise for inducing stochastic resonance only causes the development of the attractor merging crisis in the non-chaos-chaos intermittency states, whereas the RRO feedback signals can either induce or prevent the attractor merging crisis by applying a negative or positive feedback strength, respectively [59]. For these reasons, the RRO-based method has higher adaptability to more varied attractor conditions than does

additive noise in stochastic resonance. These advantages of the RRO-based method might facilitate the development of chaotic resonance instead of stochastic resonance in systems in which the internal system parameters cannot be adjusted, especially biological systems.

5. CONCLUSIONS

Based on the accumulation of studies over the past decades, we reviewed the fundamental characteristics of chaotic resonance. The recent studies, dealing with the complex neural activity caused by the deterministic process and multiple complex network structures, indicated the possibility that chaotic resonance, rather than stochastic resonance/stochastic facilitation, will become the framework for other studies to understand the functionality of neural fluctuations. Moreover, with regard to the application of chaotic resonance in biomedical regions, our proposed RRO-based method might be utilized for the enhancement of neural functionality, which, to date, has restricted the application of stochastic resonance. Although judging and controlling chaotic states is difficult in many systems, including actual neural systems, recently proposed analysis methods for predicting and judging chaos [93–95] and future studies will bring new insights to the functionality of neural fluctuation and biomedical application of chaotic resonance. Moreover, in future works, robust noise evaluation methods for chaos under noisy environments and the development of

RRO-based methods against more physiological neural systems are needed. Furthermore, as the extension of the RRO-based method, the development of new methods for controlling intermittent chaotic behaviors in systems with more complex attractors, such as multiple structures for co-existing chaotic attractors and limit cycles [96] will be important topics for future studies. Not restricted to biomedical applications, chaotic neural oscillations have been widely used in bio-inspired informatics and robotics systems [97, 98]. In these systems, by not removing the chaotic states but rather transiting the chaotic states to the appropriate chaotic state according to each objective, the RRO-based method might be a candidate for these controlling methods. Therefore, as part of future studies, these aspects are also important.

AUTHOR CONTRIBUTIONS

SN and HN discussed the progress of studies on chaotic resonance. SN drafted the main manuscript text and prepared all figures. All authors contributed to the article and approved the submitted version.

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