



Creative Problem Solving as Overcoming a Misunderstanding

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Solving or attempting to solve problems is the typical and, hence, general function of thought. A theory of problem solving must first explain how the problem is constituted, and then how the solution happens, but also how it happens that it is not solved; it must explain the correct answer and with the same means the failure. The identification of the way in which the problem is *formatted* should help to understand how the solution of the problems happens, but even before that, the source of the difficulty. Sometimes the difficulty lies in the calculation, the number of operations to be performed, and the quantity of data to be processed and remembered. There are, however, other problems – the insight problems – in which the difficulty does not lie so much in the complexity of the calculations, but in one or more critical points that are susceptible to *misinterpretation*, incompatible with the solution. In our view, the way of thinking involved in insight problem solving is very close to the process involved in the understanding of an utterance, when a misunderstanding occurs. In this case, a more appropriate meaning has to be selected to resolve the misunderstanding (the “impasse”), the default interpretation (the “fixation”) has to be dropped in order to “restructure.” to grasp another meaning which appears more relevant to the context and the speaker’s intention (the “aim of the task”). In this article we support our view with experimental evidence, focusing on how a misunderstanding is formed. We have studied a paradigmatic insight problem, an apparent trivial arithmetical task, the Ties problem. We also reviewed other classical insight problems, reconsidering in particular one of the most intriguing one, which at first sight appears impossible to solve, the Study Window problem. By identifying the problem knots that alter the aim of the task, the reformulation technique has made it possible to eliminate misunderstanding, without changing the mathematical nature of the problem. With the experimental versions of the problems exposed we have obtained a significant increase in correct answers. Studying how an insight problem is formed, and not just how it is solved, may well become an important topic in education. We focus on undergraduate students’ strategies and their errors while solving problems, and the specific cognitive processes involved in misunderstanding, which are crucial to better exploit what could be beneficial to reach the solution and to teach how to improve the ability to solve problems.

Keywords: creative problem solving, insight, misunderstanding, pragmatics, language and thought

INTRODUCTION

“A problem arises when a living creature has a goal but does not know how this goal is to be reached. Whenever one cannot go from the given situation to the desired situation simply by action, then there has to be recourse to thinking. (...) Such thinking has the task of devising some action which may mediate between the existing and the desired situations.” (Duncker, 1945, p. 1). We agree with Duncker’s general description of every situation we call a problem: the problem solving activity takes a central role in the general function of thought, if not even identifies with it.

So far, psychologists have been mainly interested in the solution and the solvers. But the *formation* of the problem remained in the shadows.

Let’s consider for example the two fundamental theoretical approaches to the study of problem solving. “What questions should a theory of problem solving answer? First, it should predict the performance of a problem solver handling specified tasks. It should explain how human problem solving takes place: what processes are used, and what mechanisms perform these processes.” (Newell et al., 1958, p. 151). In turn, authors of different orientations indicate as central in their research “How does the solution arise from the problem situation? In what ways is the solution of a problem attained?” (Duncker, 1945, p. 1) or that of what happens when you solve a problem, when you suddenly *see the point* (Wertheimer, 1959). It is obvious, and it was inevitable, that the *formation* of the problem would remain in the shadows.

A theory of problem solving must first explain how the problem is constituted, and then how the solution happens, but also how it happens that it is not solved; it must explain the correct answer and with the same means the failure. The identification of the way in which the problem is constituted – the *formation* of the problem – and the awareness that this moment is decisive for everything that follows imply that failures are considered in a new way, the study of which should help to understand how the solution of the problems happens, but even before that, the source of the difficulty.

Sometimes the difficulty lies in the calculation, the number of operations to be performed, and the quantity of data to be processed and remembered. Take the well-known problems studied by Simon, *Crypto-arithmetic* task, for example, or the *Cannibals and Missionaries* problem (Simon, 1979). The difficulty in these problems lies in the complexity of the calculation which characterizes them. But, the text and the request of the problem is univocally understood by the experimenter and by the participant in both the explicit (*said*) and implicit (*implied*) parts.¹ As Simon says, “Subjects do not initially choose deliberately among problem representations, but almost always adopt the representation suggested by the verbal problem statement” (Kaplan and Simon, 1990, p. 376). The verbal problem statement determines

a problem representation, implicit presuppositions of which are shared by both.

There are, however, other problems where the usual (generalized) interpretation of the text of the problem (and/or the associated figure) prevents and does not allow a solution to be found, so that we are soon faced with an impasse. We’ll call this kind of problems *insight problems*. “In these cases, where the complexity of the calculations does not play a relevant part in the difficulty of the problem, a *misunderstanding* would appear to be a more appropriate abstract model than the labyrinth” (Mosconi, 2016, p. 356). Insight problems do not arise from a fortuitous misunderstanding, but from a deliberate violation of Gricean conversational rules, since the implicit layer of the discourse (the *implied*) is not shared both by experimenter and participant. Take for example the problem of how to remove a one-hundred dollar bill without causing a pyramid balanced atop the bill to topple: “A giant inverted steel pyramid is perfectly balanced on its point. Any movement of the pyramid will cause it to topple over. Underneath the pyramid is a \$100 bill. How would you remove the bill without disturbing the pyramid?” (Schooler et al., 1993, p. 183). The solution is burn or tear the dollar bill but people assume that the 100 dollar bill must not be damaged, but contrary to his assumption, this is in fact the solution. Obviously this is not a trivial error of understanding between the two parties, but rather a misunderstanding due to social conventions, and dictated by conversational rules. It is the essential condition for the forming of the problem and the experimenter has played on the very fact that the condition was not explicitly stated (see also Bulbrook, 1932).

When insight problems are used in research, it could be said that the researcher sets a trap, more or less intentionally, inducing an interpretation that appears to be pertinent to the data and to the text; this interpretation is adopted more or less automatically because it has been validated by use but the default interpretation does not support understanding, and misunderstanding is inevitable; as a result, sooner or later we come up against an impasse. The theory of misunderstanding is supported by experimental evidence obtained by Mosconi in his research on insight problem solving (Mosconi, 1990), and by Bagassi and Macchi on problem solving, decision making and probabilistic reasoning (Bagassi and Macchi, 2006, 2016; Macchi and Bagassi, 2012, 2014, 2015, 2020; Macchi, 1995, 2000; Mosconi and Macchi, 2001; Politzer and Macchi, 2000).

The implication of the focus on problem forming for education is remarkable: everything we say generates a communicative and therefore interpretative context, which is given by cultural and social assumptions, default interpretations, and attribution of intention to the speaker. Since the text of the problem is expressed in natural language, it is affected, it shares the characteristics of the language itself. Natural language is ambiguous in itself, differently from specialized languages (i.e., logical and statistical ones), which presuppose a univocal, unambiguous interpretation. The understanding of what a speaker means requires a disambiguation process centered on the intention attribution.

¹The theoretical framework assumed here is Paul Grice’s theory of communication (1975) based on the existence in communication of the explicit layer (*said*) and of the implicit (*implied*), so that the recognition of the communicative intention of the speaker by the interlocutor is crucial for comprehension.

RESTRUCTURING AS REINTERPRETING

Traditionally, according to the Gestaltists, finding the solution to an insight problem is an example of “productive thought.” In addition to the reproductive activities of thought, there are processes which create, “produce” that which does not yet exist. It is characterized by a switch in direction which occurs together with the transformation of the problem or a change in our understanding of an essential relationship. The famous “aha!” experience of genuine insight accompanies this change in representation, or restructuring. As Wertheimer says: “... Solution becomes possible only when the central features of the problem are clearly recognized, and paths to a possible approach emerge. Irrelevant features must be stripped away, core features must become salient, and some representation must be developed that accurately reflects how various parts of the problem fit together; relevant relations among parts, and between parts and whole, must be understood, must make sense” (Wertheimer, 1985, p. 23).

The restructuring process circumscribed by the Gestaltists to the representation of the perceptual stimulus is actually a general feature of every human cognitive activity and, in particular, of communicative interaction, which allows the understanding, the attribution of meaning, thus extending to the solution of verbal insight problems. In this sense, restructuring becomes a process of reinterpretation.

We are able to get out of the impasse by neglecting the default interpretation and looking for another one that is more pertinent to the situation and which helps us grasp the meaning that matches both the context and the speaker’s intention; this requires continuous adjustments until all makes sense.

In our perspective, this *interpretative function* is a characteristic inherent to all reasoning processes and is an adaptive characteristic of the human cognitive system in general (Levinson, 1995, 2013; Macchi and Bagassi, 2019; Mercier and Sperber, 2011; Sperber and Wilson, 1986/1995; Tomasello, 2009). It guarantees cognitive economy when meanings and relations are familiar, permitting recognition in a “blink of an eye.” This same process becomes much more arduous when meanings and relations are unfamiliar, obliging us to face the novel. When this happens, we have to come to terms with the fact that the usual, default interpretation will not work, and this is a necessary condition for exploring other ways of interpreting the situation. A restless, conscious and unconscious search for other possible relations between the parts and the whole ensues until everything falls into place and nothing is left unexplained, with an interpretative heuristic-type process. Indeed, the solution *restructuring* – is a *re-interpretation* of the relationship between the data and the aim of the task, a search for the appropriate meaning carried out at a deeper level, not by automaticity. If this is true, then a disambiguated reformulation of the problem that eliminates the trap into which the subject has fallen, should produce restructuring and the way to the solution.

INSIGHT PROBLEM SOLVING AS THE OVERCOMING OF A MISUNDERSTANDING: THE EFFECT OF REFORMULATION

In this article we support our view with experimental evidence, focusing on how a *misunderstanding* is formed, and how a pragmatic reformulation of the problem, more relevant to the aim of the task, allows the text of the problem to be interpreted in accordance with the solution.

We consider two paradigmatic insight problems, the intriguing *Study Window* problem, which at first sight appears impossible to solve, and an apparent trivial arithmetical task, the *Ties* problem (Mosconi and D’Urso, 1974).

The Study Window problem

The study window measures 1 m in height and 1 m wide. The owner decides to enlarge it and calls in a workman. He instructs the man to double the area of the window without changing its shape and so that it still measures 1 m by 1 m. The workman carried out the commission. How did he do it?

This problem was investigated in a previous study (Macchi and Bagassi, 2015). For all the participants the problem appeared impossible to solve, and nobody actually solved it. The explanation we gave for the difficulty was the following: “The information provided regarding the dimensions brings a square form to mind. The problem solver interprets the window to be a square 1 m high by 1 m wide, resting on one side. Furthermore, the problem states “without changing its shape,” intending *geometric* shape of the two windows (square, independently of the orientation of the window), while the problem solver interprets this as meaning the *phenomenic* shape of the two windows (two squares with the same orthogonal orientation)” (Macchi and Bagassi, 2015, p. 156). And this is where the difficulty of the problem lies, in the mental representation of the window and the concurrent interpretation of the text of the problem. Actually, spatial orientation is a decisive factor in the perception of forms. “Two identical shapes seen from different orientations take on a different phenomenic identity” (Mach, 1914).

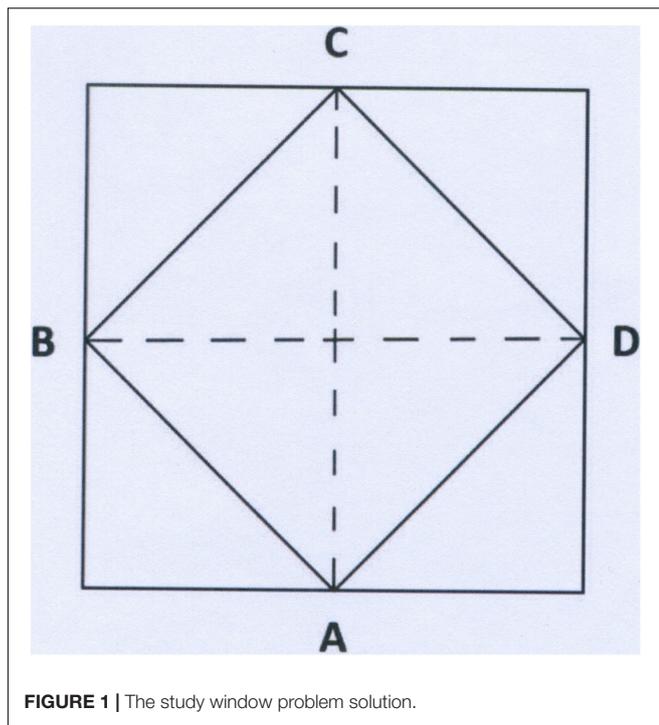
The solution is to be found in a square (geometric form) that “rests” on one of its angles, thus becoming a rhombus (phenomenic form). Now the dimensions given are those of the two diagonals of the represented rhombus (ABCD).

Figure 1

The “inverted” version of the problem gave less trouble: [...] The owner decides to make it smaller and calls in a workman. He instructs the man to halve the area of the window [...].

Figure 2

With this version, 30% of the participants solved the problem ($n = 30$). They started from the representation of the orthogonal square (ABCD) and looked for the solution within the square, trying to respect the required height and width of the window, and inevitably changing the orientation of the internal square. This time the height and width are the diagonals, rather than the side (base and height) of the square.



Eventually, in another version (the “orientation” version) it was explicit that orientation was not a mandatory attribute of the shape, and this time 66% of the participants found the solution immediately ($n = 30$). This confirms the hypothesis that an inappropriate representation of the relation between the orthogonal orientation of the square and its geometric shape is the origin of the *misunderstanding*.

The “orientation” version:

A study window measures 1 m in height and 1 m wide. The owner decides to make it smaller and calls in a workman. He instructs the man to halve the area of the window: the workman can change the orientation of the window, but not its shape and in such a way that it still measures one meter by one meter. The workman carries out the commission. How did he do it?

While with the *Study window* problem the subjects who do not arrive at the solution, and who are the totality, know they are wrong, with the problem we are now going to examine, the *Ties* problem, those who are wrong do not realize it at all and the solution they propose is experienced as the correct solution.

The Ties Problem (Mosconi and D’Urso, 1974)

Peter and John have the same number of ties.

Peter gives John five of his ties.

How many ties does John have now more than Peter?

We believe that the seemingly trivial problem is actually the result of the simultaneous activation and mutual interference of complex cognitive processes that prevent its solution.

The problem has been submitted to 50 undergraduate students of the Humanities Faculty of the University of Milano-Bicocca. The participants were tested individually and were randomly

assigned to three groups: control version ($n = 50$), experimental version 2 ($n = 20$), and experimental version 3 ($n = 23$). All groups were tested in Italian. Each participant was randomly assigned to one of the conditions and received a form containing only one version of the two assigned problems. There was no time limit. They were invited to think aloud and their spontaneous justifications were recorded and then transcribed.

The correct answer is obviously “ten,” but it must not be so obvious if it is given by only one third of the subjects (32%), while the remaining two thirds give the wrong answer “five,” which is so dominant.

If we consider the text of the problem from the point of view of the information explicitly transmitted (*said*), we have that it only theoretically provides the necessary information to reach the solution and precisely that: (a) the number of ties initially owned by P. and J. is equal, (b) P. gives J. five of his ties. However, the subjects are wrong. What emerges, however, from the spontaneous justifications given by the subjects who give the wrong answer is that they see only the increase of J. and not the consequent loss of P. by five ties. We report two typical justifications: “P. gives five of his to J., J. has five more ties than P., the five P. gave him” and also “They started from the same number of ties, so if P. gives J. five ties, J. should have five more than P.”

Slightly different from the previous ones is the following recurrent answer, in which the participants also consider the decrease of P. as well as the increase of J.: “I see five ties at stake, which are the ones that move,” or also “There are these five ties that go from one to the other, so one has five ties less and the other has five more,” reaching however the conclusion similar to the previous one that “J. has five ties more, because the other gave them to him.”²

Almost always the participants who answer “five” use a numerical example to justify the answer given or to find a solution to the problem, after some unsuccessful attempts. It is paradoxical how many of these participants accept that the problem has two solutions, one “five ties” obtained by reasoning without considering a concrete number of initial ties, owned by P. and J., the other “ten ties” obtained by using a numerical example. So, for example, we read in the protocol of a participant who, after having answered “five more ties,” using a numerical example, finds “ten” of difference between the ties of P. and those of J.: “Well! I think the “five” is still more and more exact; for me this one has five more, period and that’s it.” “Making the concrete example: “ten” – he chases another subject on an abstract level. I would be more inclined to another formula, to five.”

About half of the subjects who give the answer “five,” in fact, at first refuse to answer because “we don’t know the initial number and therefore we can’t know how many ties J. has more than P.,” or at the most they answer: “J. has five ties more, P. five less, more we can’t know, because a data is missing.”

²A participant who after having given the solution “five” corrects himself in “ten” explains the first answer as follows: “it is more immediate, in my opinion, to see the real five ties that are moved, because they are five things that are moved; then as a more immediate answer is ‘five,’ because it is something more real, less mathematical.”

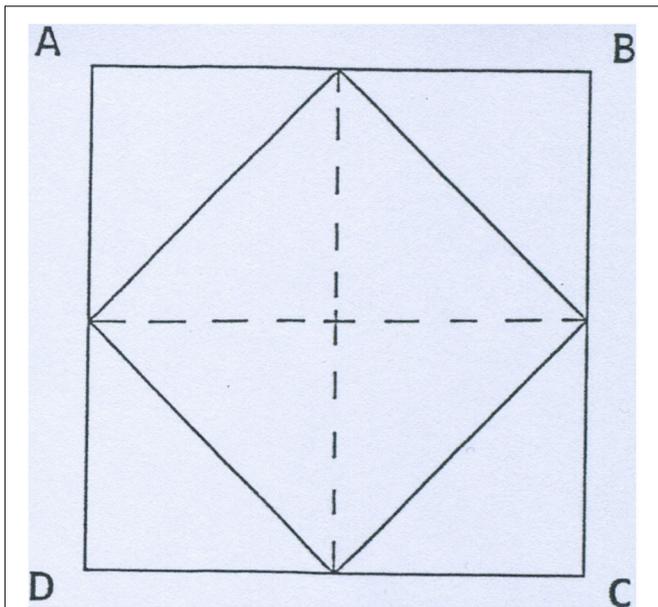


FIGURE 2 | The inverted version.

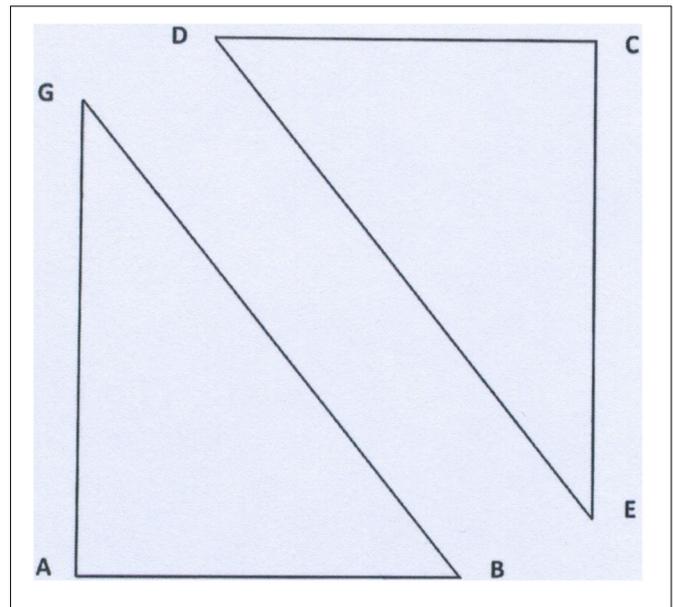


FIGURE 4 | Solution.

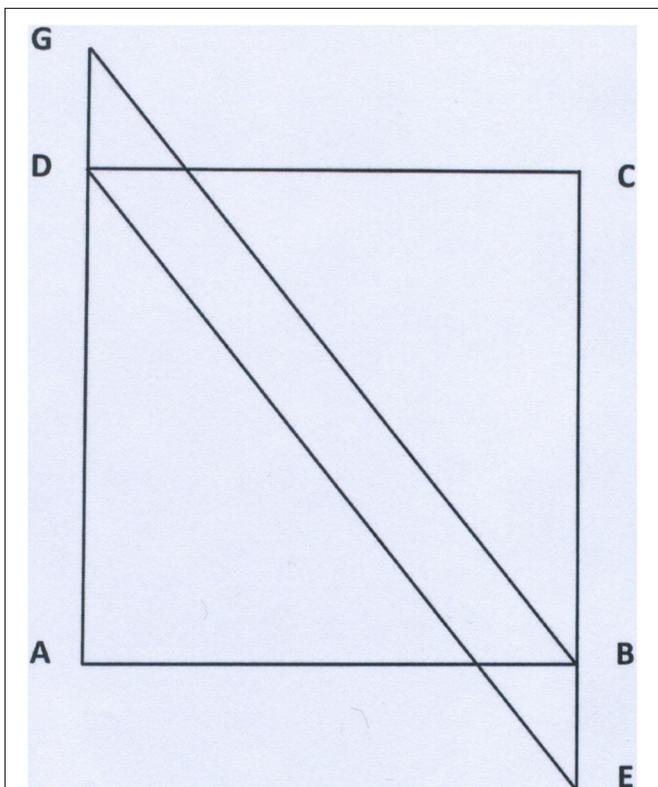


FIGURE 3 | The square and parallelogram problem.

Even before this difficulty, so to speak, operational, the text of the problem is difficult because in it the quantity relative to the decrease of P. remains implicit (-5). The resulting

misunderstanding is that if the quantity transferred is five ties, the resulting difference is only five ties: if the ties that P. gives to J. are five, how can J. have 10 ties more than P.?

So the difficulty of the problem lies in the discrepancy between the quantity transferred and the bidirectional effect that this quantity determines with its displacement. Resolving implies a restructuring of the sentence: “Peter gives John five of his ties (and therefore he loses five).” And this is precisely the reasoning carried out by those subjects who give the right answer “ten.”

We have therefore formulated a new version in which a pair of verbs should make explicit the loss of P.:

Version 2

Peter loses five of his ties and John takes them.

However, the results obtained with this version, submitted to 20 other subjects, substantially confirm the results obtained with the original version: the correct answers are 17% (3/20) and the wrong ones 75% (15/20). From a Chi-square test ($\chi^2 = 2,088$ $p = 0.148$) it results no significant difference between the two versions.

If we go to read the spontaneous justifications, we find that the subjects who give the answer “five” motivate it in a similar way to the subjects of the original version. So, for example: “P. loses five, J. gets them, so J. has five ties more than P.”

The decrease of P. is still not perceived, and the discrepancy between the lost amount of ties and the double effect that this quantity determines with its displacement persists.

Therefore, a new version has been realized in which the amount of ties lost by P. has nothing to do with J’s acquisition of five ties, the two amounts of ties are different and then they are perceived as decoupled, so as to neutralize the perceptual-conceptual factor underlying it.

Version 3

Peter loses five of his ties and John buys five new ones.

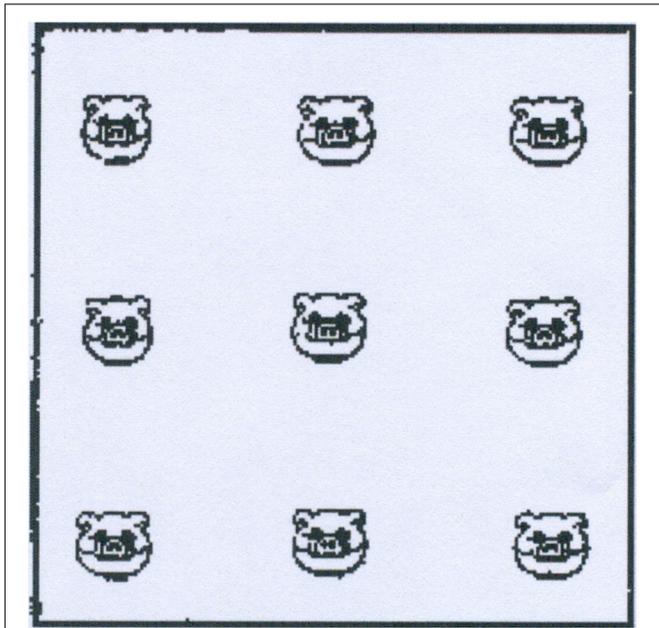


FIGURE 5 | The pigs in a pen problem.

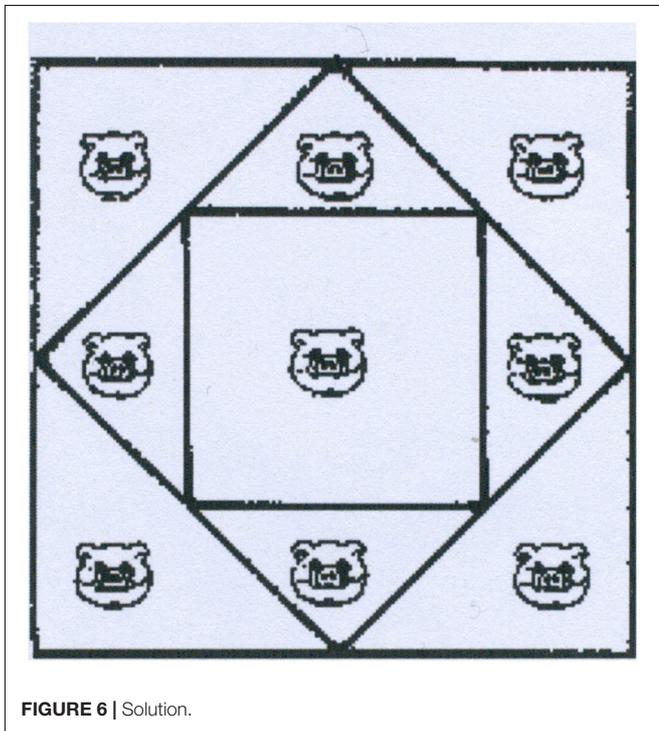


FIGURE 6 | Solution.

It was submitted to 23 participants. Of them, 17 (74%) gave the answer “ten” and only 3 (13%) the answer “five.” There was a significant difference ($\chi^2 = 16,104 p = 0.000$) between the results obtained using the present experimental version and the results from the control version. The participants who give the correct solution “ten” mostly motivate their answer as follows: “P. loses five and therefore J. has also those five that P. lost; he buys another

five, there are ten,” declaring that he “added to the five that P. had lost the five that J. had bought.” The effectiveness of the experimental manipulation adopted is confirmed.³

The satisfactory results obtained with this version cannot be attributed to the use of two different verbs, which proved to be ineffective (see version 2), but to the splitting, and consequent differentiation (J. has in addition five new ties), of the two quantities.

This time, the increase of J. and the decrease of P. are grasped as simultaneous and distinct and their combined effect is not identified with one or the other, but is equal to the sum of +5 and -5 in absolute terms.

The hypothesis regarding the effect of *reformulation* has also been confirmed in classical insight problems such as the *Square and the Parallelogram* (Wertheimer, 1925), the *Pigs in a Pen* (Schooler et al., 1993), the *Bat & Ball* (Frederick, 2005) in recent studies (Macchi and Bagassi, 2012, 2015) which showed a dramatic increase in the number of solutions.

In their original version these problems are true brain teasers, and the majority of participants in these studies needed them to be reformulated in order to reach the solution. In Appendix B we present in detail the results obtained (see Table 1). Below we report, for each problem, the text of the original version in comparison with the reformulated experimental version.

Square and Parallelogram Problem (Wertheimer, 1925)

Given that $AB = a$ and $AG = b$, find the sum of the areas of *square ABCD* and *parallelogram EBGD* (Figures 3, 4).

Experimental Version

Given that $AB = a$ and $AG = b$, find the sum of the areas of the two partially overlapping figures.

Pigs in a Pen Problem (Schooler et al., 1993)

Nine pigs are kept in a square pen. Build two more square enclosures that would put each pig in a pen by itself (Figures 5, 6).

Experimental Version

Nine pigs are kept in a square pen. Build two more squares that would put each pig in a by itself.

³The factor indicated is certainly the main responsible for the answer “five,” but not the only one (see the Appendix for a pragmatic analysis of the text).

TABLE 1 | Percentages of correct solutions with reformulated experimental versions.

Problems	Control Version	Experimental Version
<i>Square and parallelogram</i>	9 (19%) $n = 47$	28 (80%) $n = 35$
<i>Pigs in a pen</i>	8 (38%) $n = 25$	20 (87%) $n = 23$
<i>Bat and ball</i>	2 (10%) $n = 20$	28 (90%) $n = 31$
<i>Study window</i>	0 (0%) $n = 30$	21 (66%) $n = 30$

Bat and Ball Problem (Frederick, 2005)

A bat and a ball cost \$1.10 in total. The bat costs \$ 1.00 more than the ball. How much does the ball cost? ___ cents.

Experimental Version

A bat and a ball cost \$1.10 in total. The bat costs \$ 1.00 more than the ball. Find the cost of the bat and of the ball.

Once the problem knots that alter the aim of the task have been identified, the reformulation technique can be a valid didactic tool, as it allows to reveal the misunderstanding and to eliminate it without changing the mathematical nature of the problem. The training to creativity would consist in this sense in training to have interpretative keys different from the usual, when the difficulty cannot be addressed through computational techniques.

Closing Thoughts

By identifying the misunderstanding in problem solving, the reformulation technique has made it possible to eliminate the problem knots, without changing the mathematical nature of the problem. With the experimental reformulated versions of paradigmatic problems, both apparent trivial tasks or brain teasers have obtained a significant increase in correct answers.

Studying how an insight problem is formed, and not just how it is solved, may well become an important topic in education. We focus on undergraduate students' strategies and their errors while solving problems, and the specific cognitive processes involved in misunderstanding, which are crucial to better exploit what could be beneficial to reach the solution and to teach how to improve the ability to solve problems.

Without violating the need for the necessary rigor of a demonstration, for example, it is possible to organize the problem-demonstration discourse according to a different criterion, precisely by favoring the psychological needs of the subject to whom the explanation discourse is addressed, taking care to organize the explanation with regard to the way his mind works, to what can favor its comprehension and facilitate its memory.

On the other hand, one of the criteria traditionally followed by mathematicians in constructing, for example, demonstrations, or at least in explaining them, is to never make any statement that is not supported by the elements provided above. In essence, in the course of the demonstration nothing is anticipated, and indeed it happens frequently that the propositions directly relevant and relevant to the development of the reasoning (for example, the steps of a geometric demonstration) are preceded by digressions intended to introduce and deal with the elements that legitimize

them. As a consequence of such an expositive formalism, the recipient of the speech (the student) often finds himself in the situation of being led to the final conclusion a bit like a blind man who, even though he knows the goal, does not see the way, but can only control step by step the road he is walking along and with difficulty becomes aware of the itinerary.

The text of every problem, if formulated in natural language, has a *psychorhetoric* dimension, in the sense that in every speech, that is in the production and reception of every speech, there are aspects related to the way the mind works – and therefore psychological and rhetorical – that are decisive for comprehensibility, expressive adequacy and communicative effectiveness. It is precisely to these aspects that we refer to when we talk about the *psychorhetoric* dimension. Rhetoric, from the point of view of the broadcaster, has studied discourse in relation to the recipient, and therefore to its acceptability, comprehensibility and effectiveness, so that we can say that rhetoric has studied discourse “psychologically.”

Adopting this perspective, the commonplace that the rhetorical dimension only concerns the common discourse, i.e., the discourse that concerns debatable issues, and not the scientific discourse (logical-mathematical-demonstrative), which would be exempt from it, is falling away. The matter dealt with, the truth of what is actually said, is not sufficient to guarantee comprehension.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

ETHICS STATEMENT

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. Written informed consent for participation was not required for this study in accordance with the national legislation and the institutional requirements.

AUTHOR CONTRIBUTIONS

LM and MB devised the project, developed the theory, carried out the experiment and wrote the manuscript. Both authors contributed to the article and approved the submitted version.

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APPENDIX A

Pragmatic analysis of the problematic loci of the *Ties* problem, which emerged from the spontaneous verbalizations of the participants:

- “the same number of ties”

This expression is understood as a neutral information, a kind of base or sliding plane on which the transfer of the five ties takes place and, in fact, these subjects motivate their answer “five” with: “there is this transfer of five ties from P. to J. . . .”

- “5 more, 5 less”

We frequently resort to similar expressions in situations where, if I have *five* units more than another, the other has five less than me and the difference between us is five.

Consider, for example, the case of the years: say that J. is five years older than P. means to say that P. is five years younger than J. and that the difference in years between the two is five, not ten.

In comparisons, we evaluate the difference with something used as a term of reference, for example the age of P., which serves as a basis, the benchmark, precisely.

- “he gives”

The verb “to give” conveys the concept of the growth of the recipient, not the decrease of the giver, therefore, contributes to the crystallization of the “same number,” preventing to grasp the decrease of P.

APPENDIX B⁴

Square and Parallelogram Problem (Wertheimer, 1925)

Given that $AB = a$ and $AG = b$, find the sum of the areas of *square ABCD* and *parallelogram EBGD*.

Typically, problem solvers find the problem difficult and fail to see that a is also the altitude of parallelogram EBGD. They tend to calculate its area with onerous and futile methods, while the solution can be reached with a smart method, consisting of restructuring the entire given shape into two partially overlapping triangles ABG and ECD. The sum of their areas is $2 \times a \cdot b/2 = a \cdot b$. Moreover, by shifting one of the triangles so that DE coincides with GB, the answer is “ $a \cdot b$,” which is the area of the resultant rectangle. Referring to a square and a parallelogram fixes a favored interpretation of the perceptive stimuli, according to those principles of perceptive organization thoroughly studied by the Gestalt Theory. It firmly sets the calculation of the area on the sum of the two specific shapes dealt with in the text, while, the problem actually requires calculation of the area of the shape, however organized, as the sum of two triangles rectangles, or the area of only one rectangle, as well as the sum of square and parallelogram. Hence, the process of restructuring is quite difficult.

To test our hypotheses we formulated an experimental version:

⁴ Versions and results of the problems exposed are already published in Macchi e Bagassi 2012, 2014, 2015.

Experimental Version

Given that $AB = a$ and $AG = b$, find the sum of the areas of the two partially overlapping figures.

In this formulation of the problem, the text does not impose constraints on the interpretation/organization of the figure, and the spontaneous, default interpretation is no longer fixed. Instead of asking for “the areas of square and parallelogram,” the problem asks for the areas of “the two partially overlapping figures.” We predicted that the experimental version would allow the subjects to see and consider the two triangles also.

Actually, we found that 80% of the participants (28 out of 35) gave a correct answer, and most of them (21 out of 28) gave the smart “two triangles” solution. In the control version, on the other hand, only 19% (9 out of 47) gave the correct response, and of these only two gave the “two triangles” solution.

The findings were replicated in the “Pigs in a pen” problem:

Pigs in a Pen Problem (Schooler et al., 1993)

Nine pigs are kept in a square pen. Build two more square enclosures that would put each pig in a pen by itself.

The difficulty of this problem lies in the interpretation of the request, nine pigs each individually enclosed in a square pen, having only two more square enclosures. This interpretation is supported by the favored, orthogonal reference scheme, with which we represent the square. This privileged organization, according to our hypothesis, is fixed by the text which transmits the implicature that the pens in which the piglets are individually isolated must be square in shape too. The function of enclosure wrongfully implies the concept of a square. The task, on the contrary, only requires to pen each pig.

Once again, we created an experimental version by reformulating the problem, eliminating the word “enclosure” and the phrase “in a pen.” The implicit inference that *the pen is necessarily square* is not drawn.

Experimental Version

Nine pigs are kept in a square pen. Build two more squares that would put each pig in a by itself.

The experimental version yielded 87% correct answers (20 out of 23), while the control version yielded only 38% correct answers (8 out of 25).

The formulation of the experimental versions was more relevant to the aim of the task, and allowed the perceptual stimuli to be interpreted in accordance with the solution.

The relevance of text and the re-interpretation of perceptual stimuli, goal oriented to the aim of the task, were worked out in unison in an interrelated interpretative “game.”

We further investigated the interpretative activity of thinking, by studying the “Bat and ball” problem, which is part of the CRT. Correct performance is usually considered to be evidence of reflective cognitive ability (correlated with high IQ scores), versus intuitive, erroneous answers to the problem (Frederick, 2005).

Bat and Ball problem

A bat and a ball cost \$1.10 in total. The bat costs \$ 1.00 more than the ball. How much does the ball cost? ____cents

Of course the answer which immediately comes to mind is 10 cents, which is incorrect as, in this case, the difference between \$ 1.00 and 10 cents is only 90 cents, not \$1.00 as the problem stipulates. The correct response is 5 cents.

Number physiognomics and the plausibility of the cost are traditionally considered responsible for this kind of error (Frederick, 2005; Kahneman, 2003).

These factors aside, we argue that if the rhetoric structure of the text is analyzed, the question as formulated concerns only the ball, implying that the cost of the bat is already known. The question gives the key to the interpretation of what has been said in each problem and, generally speaking, in every discourse. Given data, therefore, is interpreted in the light of the question. Hence, “The bat costs \$ 1.00 more than” becomes “The bat costs \$ 1.00,” by leaving out “more than.”

According to our hypothesis, independently of the different cognitive styles, erroneous responses could be the effect of the rhetorical structure of the text, where the question is not adequate to the aim of the task. Consequently, we predicted that if the question were to be reformulated to become more relevant, the subjects would find it easier to grasp the correct response. In the light of our perspective, the cognitive abilities involved in the correct response were also reinterpreted. Consequently, we reformulated the text as follows in order to eliminate this misleading inference:

Experimental Version

A bat and a ball cost \$1.10 in total. The bat costs \$ 1.00 more than the ball. Find the cost of the bat and of the ball.

This time we predicted an increase in the number of correct answers. The difference in the percentages of correct solutions was significant: in the experimental version 90% of the participants gave a correct answer (28 out of 31), and only 10% (2 out of 20) answered correctly in the control condition.

The simple reformulation of the question, which expresses the real aim of the task (to find the cost of both items), does not favor the “short circuit” of considering the cost of the bat as already known (“\$1,” by leaving out part of the phrase “more than”).

It still remains to be verified if those subjects who gave the correct response in the control version have a higher level of cognitive reflexive ability compared to the “intuitive” respondents. This has been the general interpretation given in the literature to the difference in performance.

We think it is a matter of a particular kind of reflexive ability, due to which the task is interpreted in the light of the context and not abstracting from it. The difficulty which the problem implicates does not so much involve a high level of abstract reasoning ability as high levels of pragmatic competence, which disambiguates the text. So much so that, intervening only on the pragmatic level, keeping numbers physiognomics and maintaining the plausible costs identical, the problem becomes a trivial arithmetical task.