



Book Review: Holographic Entanglement Entropy

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A Book Review on

Holographic Entanglement Entropy

Mukund Rangamani, Tadashi Takayanagi, Lecture Notes in Physics Vol. 931 (Berlin: Springer Verlag), 2017, 246 pages, ISBN: hardback 978-3-319-52571-6, ebook 978-3-319-52573-0, doi: 10.1007/978-3-319-52573-0

The entanglement entropy is a quantity to measure the correlation between two entangled quantum many-body subsystems in quantum field theories (QFT). The divergence of the entanglement entropy of an entangled many-body subsystem is proportional to its area, which is analogous to the Bekenstein-Hawking black hole entropy formula [1–3], where the entropy of a black hole is proportional to its event horizon area. The holographic correspondence between $d+1$ -dimensional Anti-de Sitter (AdS) spacetime and conformal field theory in d dimensions (AdS _{$d+1$} /CFT _{d} correspondence) [4, 5] points to the geometric counterpart of the entanglement entropy, the so-called *holographic entanglement entropy* [6–8], which is the topic of the new book by Rangamani and Takayanagi [9].

This textbook is part of the book series Lecture Notes in Physics published by Springer Nature aimed at providing up-to-date research and teaching materials for postgraduate students and researchers. The book covers the holographic view of the entanglement entropy in the AdS/CFT correspondence. The book is divided into four parts: quantum entanglement, holography and entanglement, entanglement and quantum dynamics, and quantum gravity. The basic formalism of entanglement in quantum mechanics was briefly introduced in the first part, which is necessary for understanding the entanglement entropy in the holographic dictionary. The second part of the book focuses on the entanglement entropy in holographic field theories, followed by their applications and recent developments in the third part of the book. The final part of the book focuses on the holographic map between quantum entanglement and geometric concepts such as the Einstein-Rosen bridge (ER), which could be associated with the underlying quantum system of gravity.

The most distinctive feature of quantum mechanics is the presence of *quantum entanglement* in such a way that the quantum state of groups of entangled particles are not independent, even over an unprecedented distance. This phenomenon, which is not present in classical physics, was referred to as “spooky action at a distance” by Einstein in his letter to Born in 1947 [10], which was first speculated in the Einstein-Podolsky-Rosen (EPR) thought experiment in 1935 [11–13]. The von Neumann entropy of the reduced density matrix [14, 15], referred to as the entanglement entropy, is used as a quantitative measure of entanglement in quantum mechanics. There is another quantity, called the Rényi entropy [16], capturing the moments of the reduced density matrix, which provides a measure of quantum purity of the system. In the first part of the book, the authors briefly reviewed the von Neumann and Rényi entanglement entropies and discussed them in the

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simplest quantum many-body system, i.e., two-dimensional conformal field theory (CFT₂).

The AdS_{d+1}/CFT_d correspondence, which was proposed by Maldacena in 1997 [4, 5], relates quantum gravity in $d + 1$ -dimensional AdS spacetime to CFT in d dimensions. Accordingly, type IIB supergravity in AdS₅ with 5-D compact sphere is equivalent to $\mathcal{N} = 4$ super Yang-Mills theory in 4 dimensions [17]. The AdS/CFT correspondence is one of the examples of the holographic principle, which states that the degrees of freedom in $d + 1$ -dimensional quantum gravity are comparable to those of d -dimensional quantum many-body systems [18, 19]. Thus, one would expect to geometrically imagine the gravitational picture of the QFT entanglement entropy according to the AdS/CFT correspondence. This idea was elaborated in the Ryu-Takayanagi (RT) proposal in 2006 [6, 7], which was generalized to time-dependent states in the Hubeny-Rangamani-Takayanagi (HRT) prescription in 2007 [8]. A comprehensive review on the formulations and proprieties of the RT/HRT proposals, referred to as the *holographic entanglement entropy*, is presented in the second part of the book.

In the view point of the AdS/CFT correspondence, a black hole in AdS_{d+1} is dual to a CFT_d at a finite temperature. Accordingly, a holographic counterpart of a quantum quench, which is an important class of non-equilibrium physics in QFT, corresponds to the black hole creation in AdS. Moreover, the eternal Schwarzschild-AdS_{d+1} black hole could be equivalent to the thermofield double state of the entanglement entropy in CFT_d [20]. These are some applications of the RT/HRT prescriptions, which were considered in the third part of the book.

The holographic principle implies that quantum entanglement is somehow associated with the building block of

the gravitational spacetime. Accordingly, the ER bridge, which describes an internal wormhole connection between two distant, entangled black holes, is related to quantum entanglement prescribed by the EPR thought experiment, the so called “ER = EPR” principle [21]. The connection between spacetime geometry and quantum entanglement was discussed in the final part of the book. The final chapter of the book summarized a class of tensor networks for ground states in a many-body system, called the multi-scale entanglement renormalization ansatz (MERA) [22] and the continuous MERA (cMERA) [23, 24]. These tensor networks might help us develop a better understanding of the quantum nature of gravity.

The RT/HRT proposals prescribed by Hubeny, Rangamani, Ryu, and Takayanagi [6–8] over a decade ago are one of outcomes of the gauge/gravity or AdS/CFT correspondence proposed by Maldacena nearly two decades ago [4, 5], which generalizes the QFT entanglement entropy to the geometric configuration in gravitational field theories, and could be the base for the ER = EPR principle recently proposed by Susskind and Maldacena in 2013 [21]. The authors of this book, among those who introduced the RT/HRT prescriptions [6–8], have provided a comprehensive guide to the recent developments in the holographic entanglement entropy and its applications to gravitational theories. This book is therefore an excellent resource for getting acquainted with the relatively new topic of the holographic dual of the entanglement entropy under the gauge/gravity duality.

AUTHOR CONTRIBUTIONS

The author confirms being the sole contributor of this work and has approved it for publication.

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