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Numerical Simulation of Magnetohydrodynamic Nanofluids Under the Influence of Shape Factor and Thermal Transport in a Porous Media Using CVFEM

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In this article, the migration of nanomaterials through a permeable domain was modeled numerically. Aluminum oxide was dispersed into testing fluid which was selected water in the current paper. Utilizing Darcy LAW for a porous medium helps us to find simpler form of equations. Influences of shape factor and radiation on the thermal conduct of nanoparticles within a porous region were scrutinized. Nanomaterial within such region is applied under the Lorentz force. CVFEM approach for simulation goals has been applied. This approach provides the advantages of two common CFD methods. Impacts of radiation, magnetic, buoyancy parameters on the treatment of nanomaterials were demonstrated. Outcomes showed that greater amounts of shape factor cause stronger convection. Reverse relationships exist between the Hartmann number and temperature gradient.

Keywords: nanoparticle's shape, porous space, magnetic force, darcy LAW, radiation, nanofluid, CVFEM

INTRODUCTION

Nanotechnology is one of the most interesting fields nowadays. It is interesting due to its vast applications in solar cells, food, fuel cells, batteries, and fuel, etc. In simple, nanotechnology has made its way to each and every branch. Investigators started interest in this field and developed a new sub-branch of nanotechnology, nanofluids. Nanofluids were utilized by Choi [1] for the first time. In real-world fluids exist in abundance, among all, nanopowders can be offered as the most applicable fluids both from its use and its unique nature. Nanofluids are two-phase nanometer-size fluids in which base fluid ranging up to 100 nm. Nanofluids are used in metal oxides, oxides ceramics, and allotropes of carbon and in other chemical stable elements. Nanoparticles nowadays play a key role in thermal analysis. Pak and Cho [2] used titanium dioxide and, and found an improvement in the heat flux. Nanofluids in which the nanoparticles size range less than are considered more ideal [3]. Radiation impact on nanomaterial flow was performed by Zeeshan et al. [4] and they added the impact of MHD on titanium dioxide transportation. Impose of nanomaterial into usual carrier fluid leads to greater conductivity [5–10]. Copper oxide migration within an absorptive medium with the use of Lorentz force in the actuality of magnetic force has been demonstrated by Sheikholeslami [11]. A numerical survey is performed by Sheikholeslami [12] for CuO-H_2O nanofluid in a penetrable medium with the help of a microscopic technique. Shah et al. [13] have worked on the 3-D nanofluid flow of third-grade fluid with physical properties inside a rotating frame. An analytical investigation is performed by Dawar et al. [14] for Casson fluid with MHD carbon nanotubes (CNT's) inside a rotating channel. A numerical survey is presented by Sheikholeslami and Shehzad [15] by analyzing Fe_3O_4–H_2O nanofluid flow with inside a permeable channel. To depict the changes in flow style in the appearance of Kelvin forces, Sheikholeslami and Vajravelu [16] examined the FHD impact on nanomaterial flow. CNT migration in a time-dependent problem has been analyzed by Ahmed et al. [17] and they supposed the plate is porous and Lorentz force was added in momentum equations. The transfer of heat due to convection of ferrofluid is described by Yimin et al. [18]. In recent years, Thermal irreversibility in nanofluid through a pipe with a turbulator by means of FVM was analyzed by Sheikholeslami et al. [19]. For a detailed survey, interested readers are referred to Sheikholeslami et al. [20], Dat et al. [21], Bhatti et al. [22], Sheikholeslami [23], Cattaneo [24], Sheikholeslami and Shehzad [25] for more detail and related study of nanofluids flow. Cattaneo [24] made a modification in the thermal relaxation time to improve the heat transfer effects. Cattaneo attempt made for a specific material and obtained some interesting results in the heat transmission investigation by presenting an innovative flux approach. A Maxwell fluid was realized to this model by Mustafa [26] for the study of upper convection. A numerical investigation is performed by Ai and Sandeep [27] by considering this model for MHD Casson-ferrofluid for heat transfer analysis. Previous articles on Nanomaterials for dissimilar phenomena and their usages can be found [28–33]. Sheikholeslami et al. [34] recently presented the application of electric and magnetic field of nanofluid and ferrofluid and with transfer in an enclosure walls. Jawad et al. [35, 36] studied nanofluid thin film and their applications. Nasir et al. [37, 38] have studied 3-D nanomaterial flow CNTs and thermal analysis along a stretching surface. Entropy generation in nanofluid flow can be studied in Alharbi et al. [39]. The studied of nanofluids are further extended to liquid film due to its abundant uses in various sciences [40–48]. Nanomaterial transportation over a wedge was scrutinized by Hassan et al. [49]. An experimental approach was performed by Sheikholeslami et al. [50] to study the boiling of refrigerant with the use of nanoparticles.

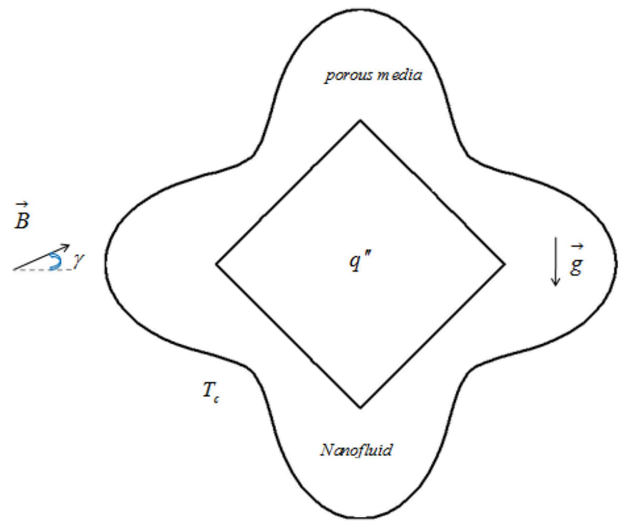

FIGURE 1 | Present tank and sample element.

TABLE 1 | Coefficient of carrier fluid.

Coefficient values	Al_2O_3-H_2O
α_1	52.813488759
α_2	6.115637295
α_3	0.6955745084
α_4	4.17455552786
α_5	0.176919300241
α_6	−298.19819084
α_7	−34.532716906
α_8	−3.9225289283
α_9	−0.2354329626
α_{10}	−0.999063481

Though there is intense research in the literature about nanofluid modeling and the MHD effect in different categories, there is still limited information about the complex geometries and Darcy model. Additionally, the radiative effect is an important source. The determination of this article is to study the migration of nanopowder within a porous space. The effects of shape factor and radiation on the thermal conduct of nanomaterials within a porous space were scrutinized.

PROBLEM EXPLANATION

In this modeling and simulation, water-based nanofluid exists through permeable geometry has considered. Impact of Lorentz force and thermal behavior are taken on nanofluid. Sketch of the porous tank is depicted in **Figure 1**. Nanofluid is thermally conducting and impact of Lorentz force was involved. Control Volume finite element technique with a triangular element has been used (see **Figure 1**). Needed boundary constraints were established in **Figure 1**. The Darcy LAW [15] is involved for porous terms.

TABLE 2 | Some physical thermal features.

	C_p $(jkg^{-1}K^{-1})$	C_p $(jkg^{-1}K^{-1})$	$\beta \times 10^5$ (K^{-1})	K $(Wm^{-1}.K^{-1})$	$\sigma\,(\Omega \cdot m)^{-1}$
H_2O	4,179	4,179	21	0.613	0.05
Al_2O_3	765	765	0.85×10^{-5}	25	1×10^{-10}

TABLE 3 | Structure of m at dissimilar values.

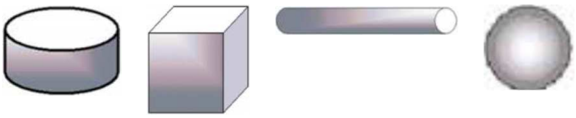				Shape
5.7	3.7	4.8	3	m

TABLE 4 | Deviation of Nu_{ave} at different mesh size when $Ra = 600$, $Ha = 0, Rd = 0.8$ and $\phi = 0.04$.

		Mesh size		
41 × 121	51 × 151	61 × 181	71 × 211	81 × 241
5.0591	5.0688	5.0715	5.0767	5.0793

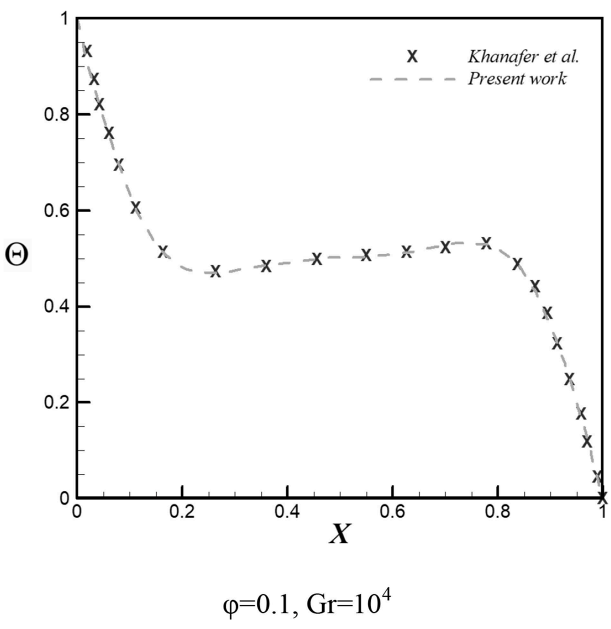

FIGURE 2 | Justification for nanofluid [5].

GOVERNING EQUATIONS, FORMULATION, AND CVFEM

Nanopowder migration through a permeable domain with the help of Darcy model was considered in the current article and involving single-phase model results in below equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\mu_{nf}}{K} u = -\frac{\partial P}{\partial x} + \sigma_{nf} B_0^2 \left[(\sin\gamma)\, v\, (\cos\gamma) - u(\sin\gamma)^2\right] \tag{2}$$

$$\frac{\mu_{nf}}{K} v = -\frac{\partial P}{\partial y} + (T - T_c)\, g\rho_{nf}\beta_{nf} + \sigma_{nf} B_0^2 (\cos\gamma)\left[(\sin\gamma)\, u - (\cos\gamma)\, v\right] \tag{3}$$

$$\frac{\partial q_r}{\partial y} \left(\rho C_p\right)_{nf}^{-1} + \left(\frac{\partial T}{\partial y} v + u\frac{\partial T}{\partial x}\right) = k_{nf}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2}\right)\left(\rho C_p\right)_{nf}^{-1},$$

$$,\quad \left[T^4 \cong 4T_c^3 T - 3T_c^4, q_r = -\frac{4\sigma_e}{3\beta_R}\frac{\partial T^4}{\partial y}\right] \tag{4}$$

The fundamental characteristics of nanofluid are estimated as:

$$\Re_{nf} = \Re_f + \left(\Re_s - \Re_f\right)\phi \tag{5}$$

$$\Re = C_p \rho$$

$$(\rho\beta)_{nf} + (\rho\beta)_f(\phi - 1) = \phi(\rho\beta)_s \tag{6}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{7}$$

$$\frac{\sigma_{nf}}{\sigma_f} - 1 = \frac{\left(-1 + \frac{\sigma_s}{\sigma_f}\right)(3\phi)}{\left(2 + \frac{\sigma_s}{\sigma_f}\right) + \phi\left(1 - \frac{\sigma_s}{\sigma_f}\right)} \tag{8}$$

To apply the effects of shape factor and $k_{Brownian}$, the following correlations were examined:

$$\mu_{eff} = \mu_{static} + \frac{k_{Brownian}}{k_f} \times \frac{\mu_f}{\Pr_f}$$

$$k_{Brownian} = 5\times10^4 c_{p,f}\rho_f g'(d_p,\phi,T)\phi\sqrt{\frac{\kappa_b T}{\rho_p d_p}} \tag{9}$$

$$\begin{aligned} g'\left(d_p,\phi,T\right) = {} & \Big(a_1 + a_5 Ln\big(d_p\big)^2 + Ln\,(\phi)\, a_4 Ln\,(d_p) + a_2 Ln\,(d_p) \\ & + a_3 Ln\,(\phi)\Big)\, Ln\,(T) + \Big(a_6 + a_{10} Ln\big(d_p\big)^2 + a_8 Ln\,(\phi) \\ & + a_7 Ln\,(d_p) + a_9 Ln\,(d_p)\, Ln\,(\phi)\Big) \end{aligned}$$

$$\frac{k_{nf}}{k_f} = \frac{-\Im m\phi + k_p + \Im\phi + k_f + mk_f + k_f}{k_f m + k_p + \Im\phi + k_f},$$

$$\Im = k_p - k_f \tag{10}$$

Equation (11) presents a dimensionless form:

$$\Psi = \psi/\alpha_{nf},\ \theta = \frac{T - T_c}{\Delta T},\ \Delta T = L\frac{q''}{k_f},\ (X, Y) = L^{-1}\left(x, y\right) \tag{11}$$

So, the last format of equations is:

$$\frac{\partial^2\Psi}{\partial Y^2} + \frac{\partial^2\Psi}{\partial X^2} = -Ha\frac{A_6}{A_5}\left[2(\sin\gamma)\frac{\partial^2\Psi}{\partial X\,\partial Y}(\cos\gamma) + \frac{\partial^2\Psi}{\partial Y^2}\left(\sin^2\gamma\right) + (\cos\gamma)\frac{\partial^2\Psi}{\partial X^2}(\cos\gamma)\right] - \frac{A_3}{A_4}\frac{A_2}{A_5}\frac{\partial\theta}{\partial X}Ra \tag{12}$$

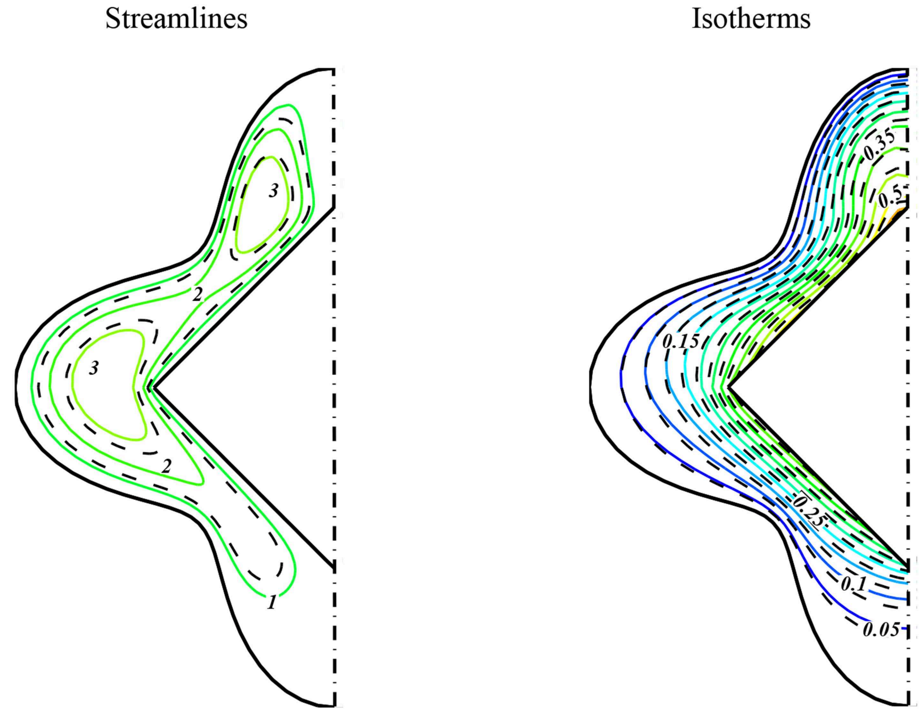

FIGURE 3 | Changing in behavior of nanofluid with rise of ϕ on nanofluid behavior. ($\phi = 0.04$ (—) and $\phi = 0$ (– – –)) when $m = 5.7, Rd = 0.8, Ra = 600, Ha = 0$.

$$\left(1 + \frac{4}{3} Rd\, A_4\right) \frac{\partial^2 \theta}{\partial Y^2} + \left(\frac{\partial^2 \theta}{\partial X^2}\right) = \frac{\partial \theta}{\partial X}\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} \tag{13}$$

The mentioned variables in Equation (13) are:

$$\begin{aligned} & Ha = \frac{\sigma_f K B_0^2}{\mu_f},\ Ra = \frac{g\,K\,(\rho\beta)_f L\,\Delta T}{\mu_f\,\alpha_f},\ \ Rd = 4\sigma_e T_c^3 / \left(\beta_R k_f\right) \\ & A_1 = \frac{\rho_{nf}}{\rho_f}, \quad A_2 = \frac{(\rho C_P)_{nf}}{(\rho C_P)_f}, \ \ A_5 = \frac{\mu_{nf}}{\mu_f}, \\ & A_3 = \frac{(\rho\beta)_{nf}}{(\rho\beta)_f}, \quad A_6 = \frac{\sigma_{nf}}{\sigma_f}, \ \ A_4 = \frac{k_{nf}}{k_f} \end{aligned} \tag{14}$$

Besides, summarizations of boundaries are:

$$\begin{aligned} \theta &= 0.0 \ \text{ on outer surfaces} \\ \Psi &= 0.0 \ \text{ on all walls} \\ \frac{\partial \theta}{\partial n} &= 1.0 \ \text{ on inner wall} \end{aligned} \tag{15}$$

Nu_{loc} and Nu_{ave} are:

$$Nu_{loc} = \frac{1}{\theta}\left(1 + \frac{4}{3}\left(\frac{k_{nf}}{k_f}\right)^{-1} Rd\right)\left(\frac{k_{nf}}{k_f}\right) \tag{16}$$

$$Nu_{ave} = \frac{1}{S}\int_0^s Nu_{loc}\, ds \tag{17}$$

Simulation Technique, Grid and Verification

Sheikholeslami [29] has been discovered a new approach namely CVFEM for analyzing thermal problems. This technique utilizes a triangular element and the Gauss-Seidel approach uses for the final step of calculating scalars. **Tables 1–3** illustrate the properties of carrier fluid. Grid size must be independent of outcomes and we present special cases in **Table 4**. Validation for presents study for nanofluid [5] are presented in **Figure 2** and provide nice accuracy.

RESULTS AND DISCUSSION

In this article transportation of electrically and thermally conducting nanomaterial with different shapes were modeled numerically. Aluminum oxide was dispersed into testing fluid which was selected water in current paper. Utilizing Darcy low for porous medium helps us to find simpler form of equations. Impacts of shape factor and the radiation on thermal conduct of nanoparticles inside a permeable space were investigated. Impacts of Radiation parameter, shape factor, magnetic force, and fraction of alumina have been demonstrated. The Darcy Law is involved for a permeable term in geometry.

Impacts of imposing nanopowders into H_2O by selecting other parameters are shown in **Figure 3**. Actually this is the nanofluid scattering rule. It is observed that nanofluid motion augmented with the imposing of nanoparticles. The impacts of Hartmann for different cases were plotted in **Figures 4**, **5**. Impose of the Lorentz effect declines the motion of nanoparticles.

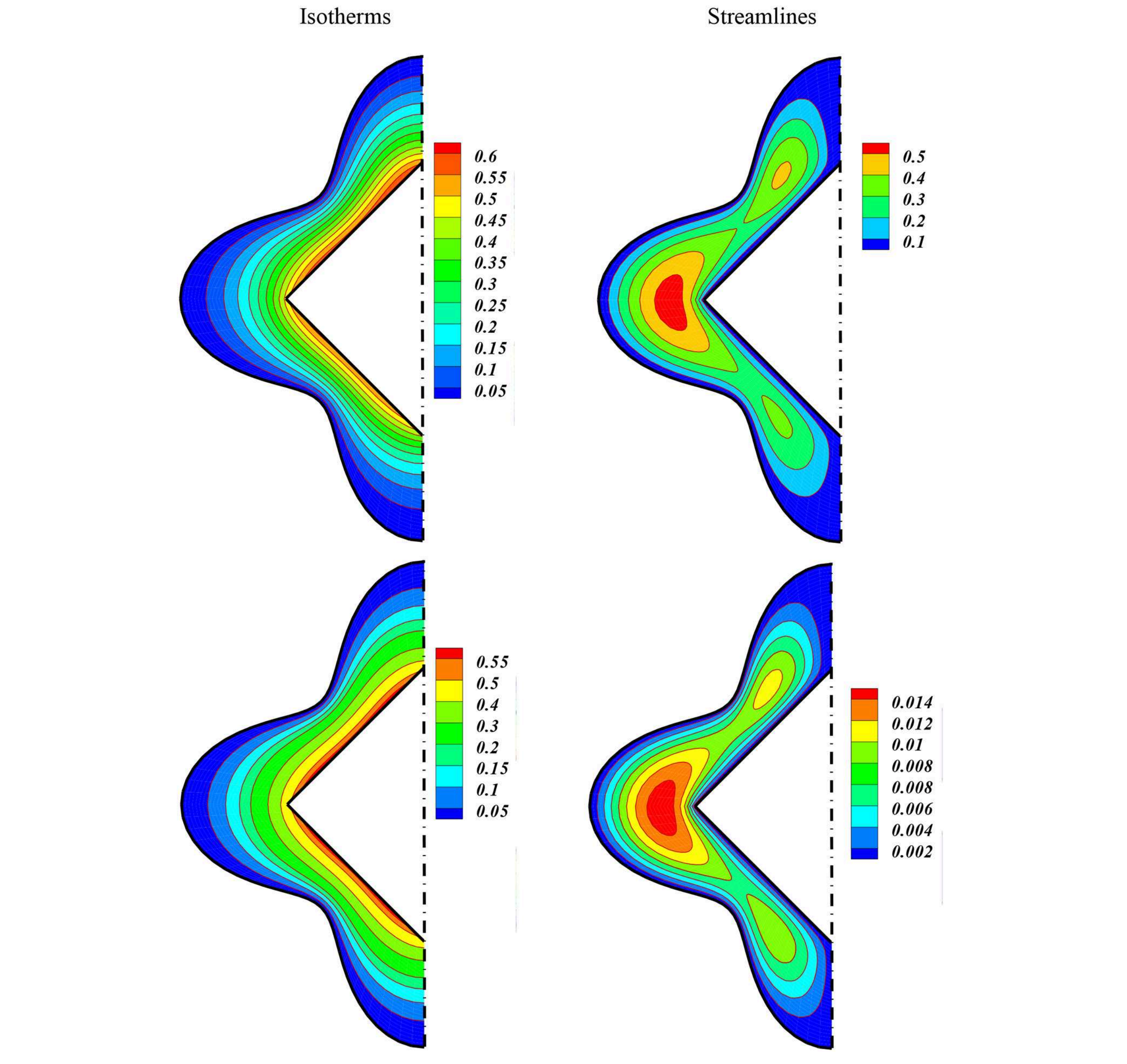

FIGURE 4 | Showing variation of Ha when $\phi = 0.04$, $m = 5.7$, $Rd = 0.8$, $Ra = 100$.

Actually, with the augmentation of the magnetic parameter, the top two eddies were amalgamate together and the thermal spiral disappear. It is observed that adding magnetic impact, stronger conduction occurs. Reverse relationships exist between the Hartmann number and temperature gradient. Impacts of scrutinized variables on Nusselt number were displayed in **Figure 6**. Variations for different cases are presented here.

Distortion of isotherms augments in consequence of augment in buoyancy and makes stronger vortex which indicates the growth of free convection. With the domination of convective mode, isotherms become more complex with generating plume. Therefore, increasing permeability and buoyancy term makes the Nusselt number to augment. Resistance against the nanomaterial migration reduces with augment of Lorentz forces and in turn, Nusselt number can reduce. Temperature distribution becomes less complex with involving magnetic field and higher Lorentz force can eliminate the plumes. Shear stress among nanoparticles declines with augment of permeability of the region. So, the power of the flow augments with rise of permeability which indicates greater convective flow. The influence of permeability on the style of nanofluid flow reduces with decreasing buoyancy forces. Greater nanofluid mixing occurs within the domain with

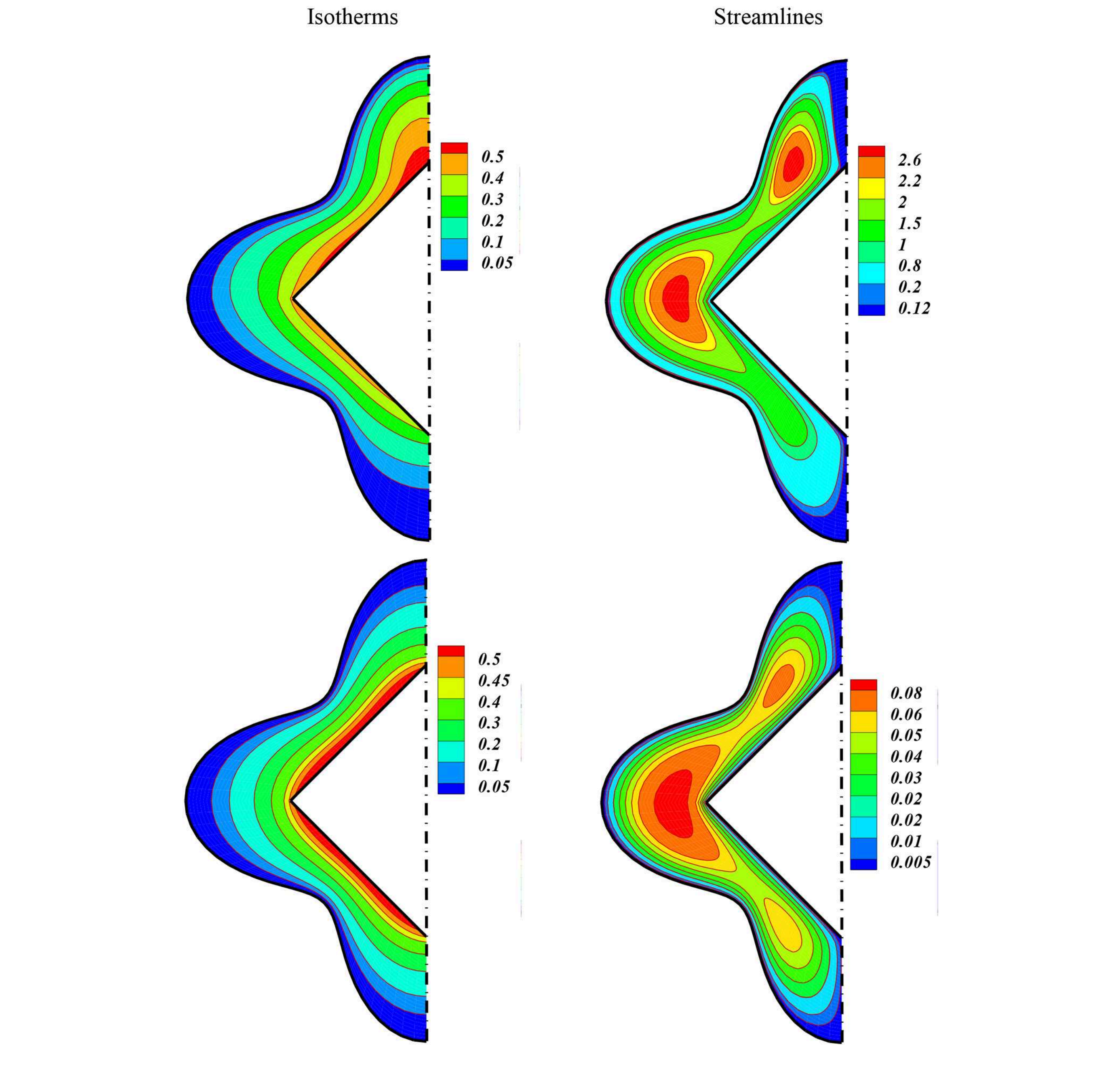

FIGURE 5 | Showing variation of Ha when $\phi = 0.04$, $m = 5.7$, $Rd = 0.8$, $Ra = 600$.

the rise of buoyancy forces and this influence reduces with imposing magnetic field. Resistance against the nanomaterial migration declines with augment of Darcy number but opposite phenomena appear with augment of the Hartmann number. Magnetic forces work against buoyancy forces, which can reduce the strength of streamline and imposing greater magnetic force, leads to conduction domination. The temperature gradient becomes independent on the Lorentz forces again, owing to the weakening of the buoyancy.

Changes of Nusselt number respect to variables are presented in **Figure 6**. The mathematical relationship has presented in Equation (18).

$$\begin{aligned} Nu_{ave} = {} & 3.34 + 0.087m + 1.04Rd + 0.19Ra - 0.14Ha \\ & + 1.1 \times 10^{-2} m\,Ha - 0.092Rd\,Ha - 0.19RaHa \\ & + 1.359 \times 10^{-4} m^2 \end{aligned} \tag{18}$$

It is concluded that the augment in distortion of temperature with buoyancy terms and permeability enhances the gradient of temperature. Moreover, transmission mode improves with a boost of the Lorentz force. Thus, convection diminishes with the

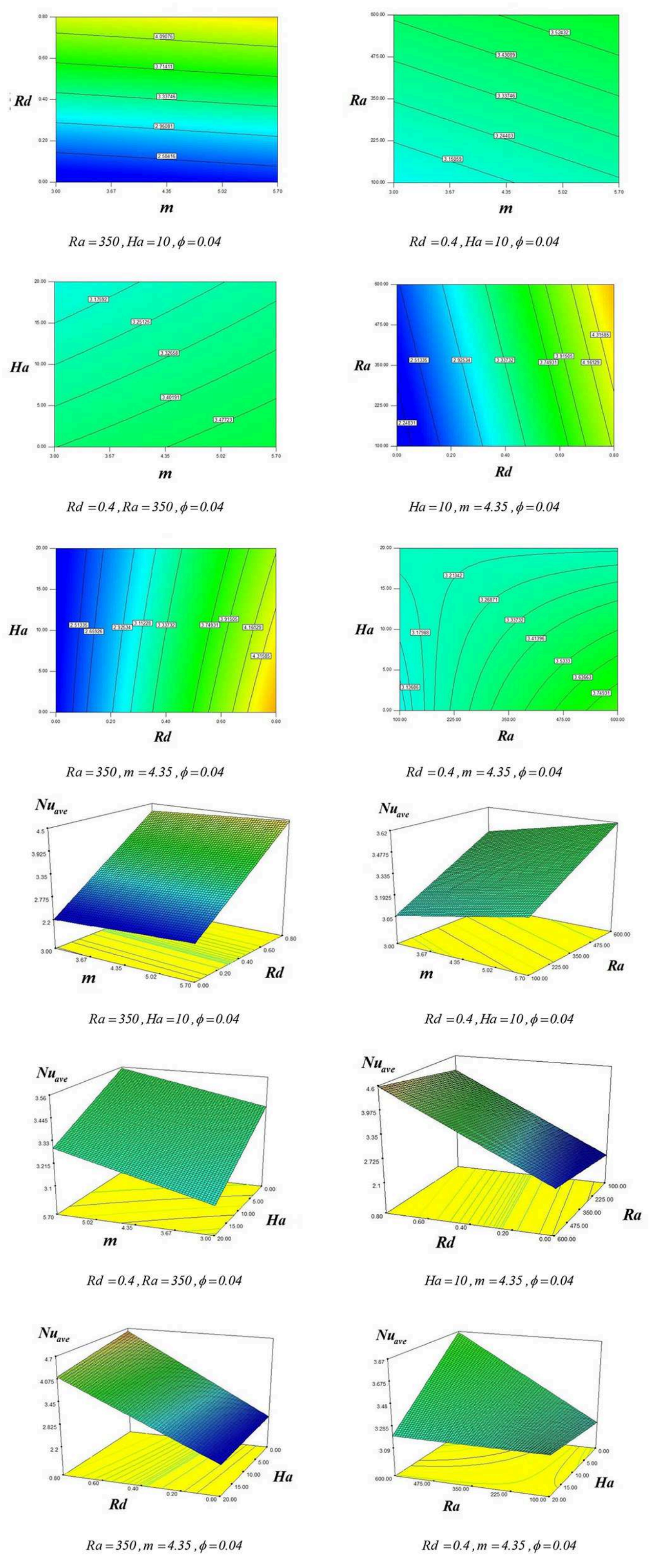

FIGURE 6 | How *Rd*, *m*, ϕ, *Rd*, *Ha* affect the Nu_{ave}.

escalation of the magnetic field. It is found from **Figure 6** that Nusselt number is augmenting function for radiation parameter.

CONCLUSIONS

In current CVFEM simulation, nanomaterial was offered as a feasible way to more augmentation of convection in permeable tank and various shapes of powder ware involved. To manage the migration of particles, magnetic forces was employed, and the influence of radiation has been imposed in the energy equation. Outcomes prove that augmenting Lorentz force declines the convection and make isotherms to lower dense near the wall. An indirect relationship was reported for temperature gradient and Lorentz forces. Furthered distortion was observed in isotherms with the rise of buoyancy force.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

ZS developed the numerical method and led the manuscript preparation. HB and PK contributed to the code development and to the article preparation. AS and PT contributed to the analysis and discussion of the results.

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NOMENCLATURE

L_f	Latent heat of solidification
C_p	Heat capacity
d_p	Diameter of alumina
NEPCM	Alumina-enhanced PCM
$T^{\cdot}$	Temperature
k	Thermal conductivity
CVFEM	Control volume based finite Element method
E_{total}	Energy saving
Greek symbols	
ϕ	Concentration of alumina
κ_b	Boltzmann constant
α	Diffusivity
Subscripts	
nf	Nano enriched PCM
f	fluid
P	solid