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Complex and Real Optical Soliton Properties of the Paraxial Non-linear Schrödinger Equation in Kerr Media With M-Fractional

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In this paper, we use the modified exponential function method in terms of $K^{f(x)}$ instead of $e^{f(x)}$and the extended sinh-Gordon method to find some new family solution of the M-fractional paraxial non-linear Schrödinger equation. The novel complex and real optical soliton solutions are plotted in 2-D, 3-D with a contour plot. Moreover, the dark exact solutions, singular soliton solutions, kink-type soliton solution, and periodic dark-singular soliton solutions for M-fractional paraxial non-linear Schrödinger equation are constructed. We guarantee that all solutions are new and verified the main equation of the M-fractional paraxial wave equation. For existence, the constraint condition is also added.

Keywords: paraxial wave equation, complex soliton, extended sinh-Gordon method, soliton structures, contour surfaces

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INTRODUCTION

The breaking up and moving away from ultrashort pulses of a field related to electricity-producing magnetic fields or radiation into a medium is a multidimensional important physical phenomenon. The interaction between different physical procedures such as breaking up/spreading out, material breaking up or spreading out, diffraction, and non-linear response affects the pulse patterns of relationships, movement, or sound. According to the interaction of breaking up or spreading out, diffraction and non-linearity, a non-dispersive, and non-diffractive wave packet called soliton is created. Solitons have many uses in optical microscopy, optical information storage, laser caused particle increasing speed, Bose-Einstein (a liquid that forms from a gas/change from gas to liquid), and bright and sharp signal transmission.

In the research papers, researchers have been noted several computational methods for solving NPDEs, building separate solitons, and other alternatives for distinct types of NPDEs such as, the Haar wavelet method [1], the homotopy perturbation method [2], the Adomian decomposition method [3, 4], the shooting method [5–8], the sine-Gordon expansion method [9–12], the inverse scattering method [13], the sinh-Gordon expansion method [14–16], the $\tan(\phi(\xi)/2)$-expansion method [17, 18], the inverse mapping method [19], modified $\exp(-\varphi(\xi))$-expansion function method [20–23], the decomposition-Sumudu-like-integral-transform method [24], a functional variable method [25], the Bernoulli sub-equation function method [26–28], modified exponential function method [29], the modified auxiliary expansion method [30], the Riccati-Bernoulli sub-ODE method [31], the extended trial equation method [32, 33], and tanh function method [34, 35].

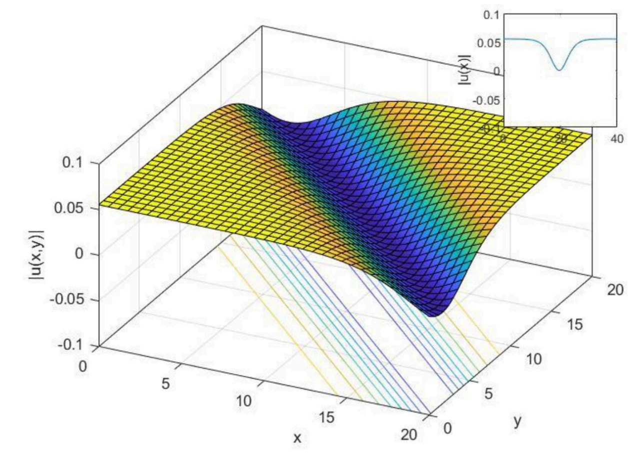

FIGURE 1 | 2-D, 3-D, and contour plot of dark soliton solution Equation (20) when $\lambda = 3, \mu = 2, \beta = 0.6, \alpha = 0.9, \varepsilon = 0.2, c = 0.3, t = 2, \gamma = 3$ and $z = 2$ for 2-D.

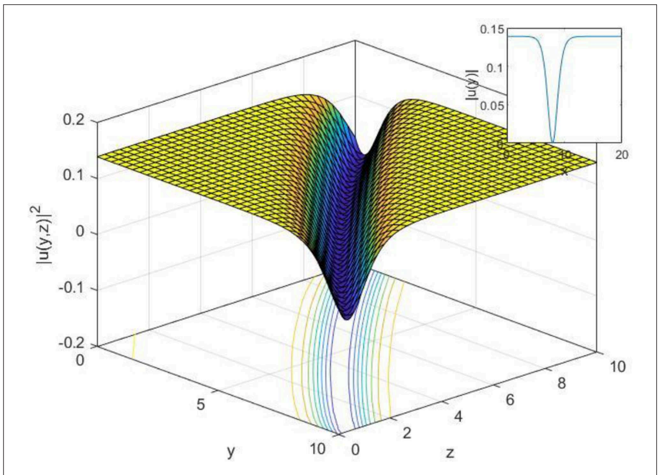

FIGURE 3 | 2-D, 3-D, and contour plot of dark soliton solution Equation (22) when $\lambda = 3, \mu = 1, \beta = 0.1, \alpha = 0.9, \varepsilon = 0.2, c = 0.3, t = 2, \gamma = 3,$ $a_0 = 1, b_0 = 2$ and $z = 2$ for 2-D.

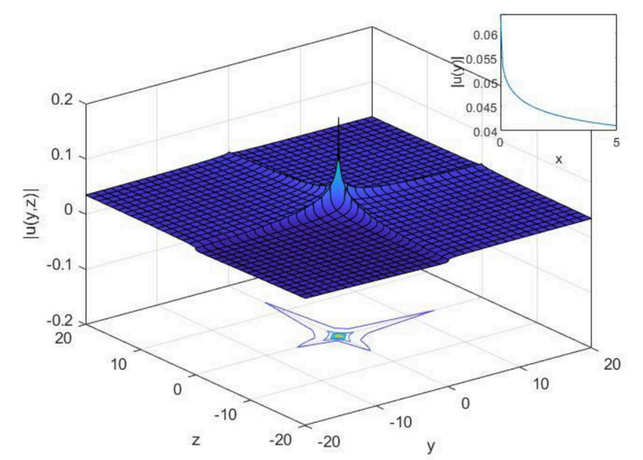

FIGURE 2 | 2-D, 3-D, and contour plot of singular soliton solution Equation (21) when $\lambda = 0.3, \mu = 0, \beta = 0.6, \alpha = 1/3, \varepsilon = 2, c = -0.3, t = 2, \gamma = 3$ and $z = 2$ for 2-D.

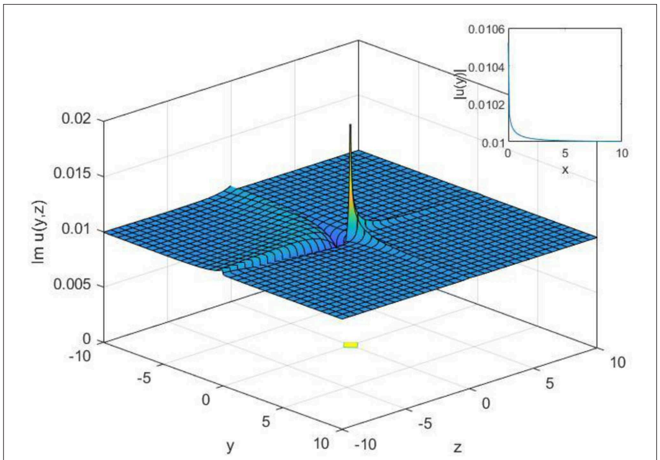

FIGURE 4 | 2-D, 3-D, and contour plot of singular soliton solution Equation (23) when $\lambda = 1, \mu = 0, \beta = 0.6, \alpha = \frac{1}{3}, \varepsilon = 2, c = 0.3, t = 2, \gamma = 0.3,$ $a_0 = 0.1, b_0 = 1$ and $z = 2$ for 2-D.

Also, different methods have been used to solve fractional differential equation such as, the finite difference method [36], the improved Adams–Bashforth algorithm [37, 38], Adams-Bashforth-Moulton method [39], the extended fractional sinh-Gordon expansion method [40], the Laplace transforms [41], the q-homotopy analysis transform method [42], local fractional series expansion method [43], the wavelets method [44], Local fractional homotopy perturbation method [45], and many other techniques [46, 47].

In this paper, we will construct some new complex and real soliton solutions of M-fractional paraxial non-linear Schrödinger equation in Kerr media by using a modified expansion function method as well as by the extended sinh-Gordon method. Over the previous two centuries, the field of fractional calculus has drawn many researchers' attention. They are used for modeling multiple non-linear features such as biological procedures, fluid mechanics, chemical processes, etc. Fractional order partial differential equations serve as the generalization of partial differential equations in the classical integer-order. The literature contains several definitions of fractional derivatives, such as the Hadamard derivative (1892) [48], the Weyl derivative [49], Caputo, Riesz derivative [50], Riemann-Liouville, Grunwald-Letnikov definitions, Atangana-Baleanu derivative in the context of Caputo, Atangana-Baleanu fractional derivative in the context of Riemann-Liouville [51, 52], Erdelyi-Kober [53], and the conformable fractional derivative [54]. Atangana et al. provided the conformable fractional derivative with some new characteristics [55]. Sousa and Oliveira in [56] have recently been created the new truncated M-fractional derivative.

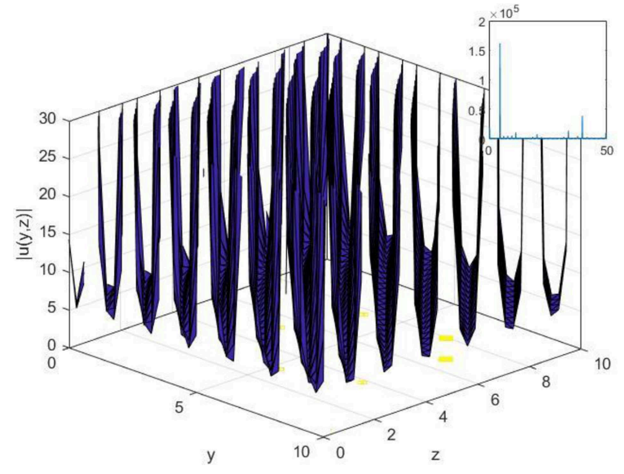

FIGURE 5 | 2-D, 3-D, and contour plot of periodic singular soliton solution Equation (24) when $\lambda = 0.1, \mu = 0.3, \beta = 0.6, \alpha = 0.9, \varepsilon = 0.1, c = 0.3, t = 2, \gamma = 0.3, a_0 = 0.5, b_1 = 0.2$ and $z = 2$ for 2-D.

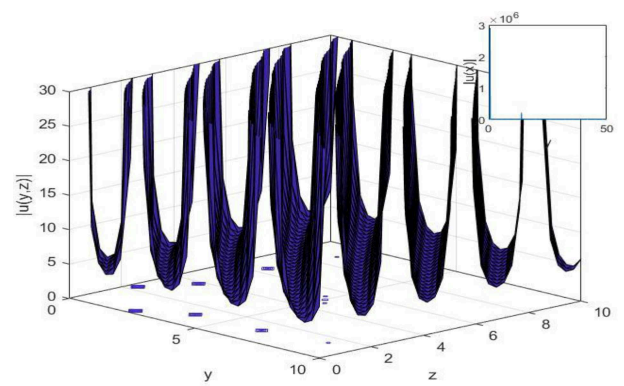

FIGURE 6 | 2-D, 3-D, and contour plot of periodic singular soliton solution Equation (25) when $\lambda = 1, \mu = 1, \beta = 0.6, \alpha = 0.9, \varepsilon = 0.1, c = 3, t = 2, \gamma = 3a_0 = 0.5, b_0 = 0.2$ and $z = 2$ for 2-D.

THE TRUNCATED M-FRACTIONAL DERIVATIVE

In this section, we give some definitions, theorems, and properties of the truncated M-fractional derivative of order α.

Definition 1. If the function $f : (0, \infty) \rightarrow \mathbb{R}$, then, the new truncated M-fractional derivative of function of order α is defined as,

$$D_M^{\alpha,\beta} f(t) = \lim_{\varepsilon \to 0} \frac{f\left(t\epsilon_\beta\left(\varepsilon t^{1-\alpha}\right)\right) - f(t)}{\varepsilon}, \quad \text{for all } t > 0,\ 0 < \alpha \leq 1,\ \beta > 0,$$

where $\epsilon_\beta(.)$ is a truncated Mittag-Leffler function of one parameter [56].

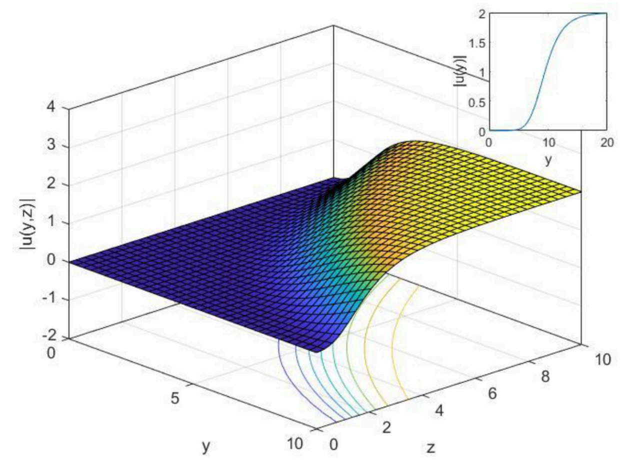

FIGURE 7 | 2-D, 3-D, and contour plot of Equation (27), when $t = 2, c = 3, \gamma = 2, \alpha = 0.5, \beta = 0.6$ and $z = 2$ for 2-D.

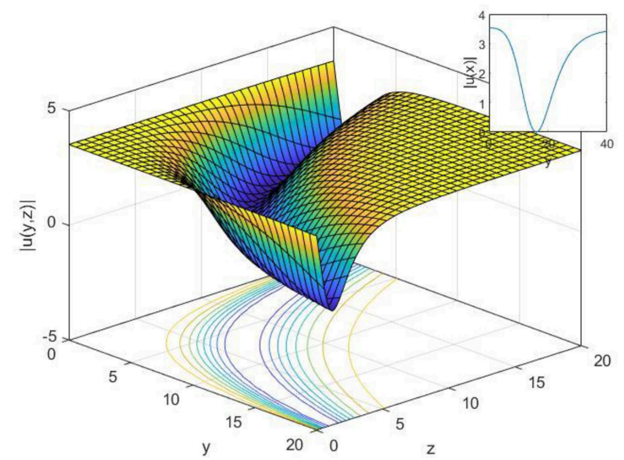

FIGURE 8 | 2-D, 3-D, and contour plot of Equation (28), when $t = 2, c = 3, \gamma = 0.2, \alpha = \frac{1}{3}, \beta = 0.6$ and $z = 2$ for 2-D.

Theorem 1. Let $\alpha \in (0, 1],\ \beta > 0$ and $f = f(t),\ g = g(t)$ be α-differentiable at a point $t > 0$, then:

I $\quad D_M^{\alpha,\beta}\left(af + bg\right) = aD_M^{\alpha,\beta} f + bD_M^{\alpha,\beta} g, \quad$ for all $a, b \in \mathbb{R}$.

II $\quad D_M^{\alpha,\beta}(c) = 0, \quad$ for all $c \in \mathbb{R}$.

III $\quad D_M^{\alpha,\beta}\left(f.g\right) = gD_M^{\alpha,\beta}\left(f\right) + fD_M^{\alpha,\beta}\left(g\right)$.

IV $\quad D_M^{\alpha,\beta}\left(\frac{f}{g}\right) = \frac{gD_M^{\alpha,\beta}(f) - fD_M^{\alpha,\beta}(g)}{g^2}$.

Furthermore; if the function f is a differentiable function; then $D_M^{\alpha,\beta}\left(f(t)\right) = \frac{t^{1-\alpha}}{\Gamma(\beta+1)} \frac{df}{dt}$.

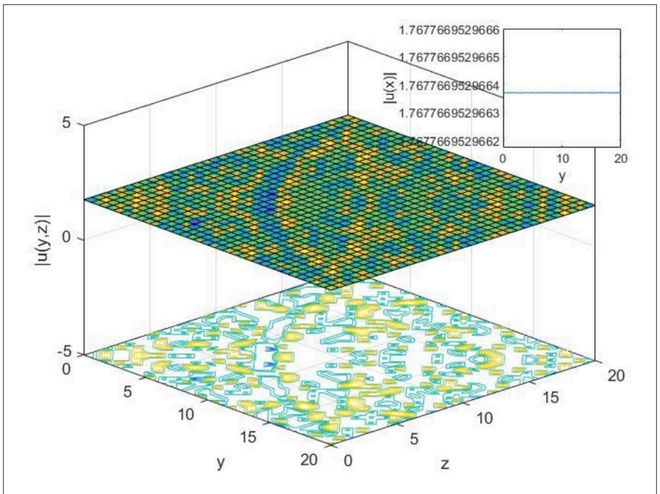

FIGURE 9 | 2-D, 3-D, and contour plot of Equation (29), when $t = 2, c = 0.3, \gamma = 0.2, \alpha = 0.5, \beta = 0.6$ and $z = 2$ for 2-D.

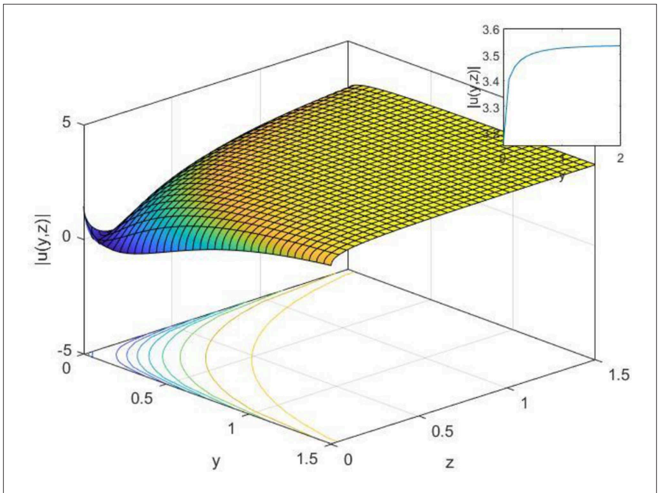

FIGURE 10 | 2-D, 3-D, and contour plot of Equation (30), when $t = 2, c = 0.3, \gamma = 0.2, \alpha = 0.5, \beta = 0.6$ and $z = 2$ for 2-D.

GENERAL FORM OF METHODS

Modified Expansion Function Method

Step 1. Suppose that, we have the following non-linear partial differential equation (NLPDE)

$$P\left(u, D_{M,x}^{\alpha,\beta}u, u^2 D_{M,x}^{\alpha,\beta}u, D_{M,t}^{\alpha,\beta}u, D_{M,t}^{2\alpha,\beta}u, \ldots\right) = 0. \quad (1)$$

To find explicit exact solutions of Equation (1), we use the following transformation

$$u\left(x,y,t\right) = U\left(\xi\right), \xi = \frac{\Gamma\left(\beta+1\right)}{\alpha}\left(x^{\alpha} - \nu\, t^{\alpha}\right), \quad (2)$$

where ν is arbitrary constant and ξ is the symbol of the wave variable. Substituting Equation (2) to Equation (1), the result is a non-linear ordinary differential equation (NLODE) as follow

$$N\left(U, U^2, U', U'', \ldots\right) = 0. \quad (3)$$

Step 2. Now the trial equation of solution for Equation (3) is defined a

$$U\left(\xi\right) = \frac{\sum_{i=1}^{n} a_i\left(K^{-i\Phi(\xi)}\right)^i}{\sum_{j=1}^{m} b_i\left(K^{-\Phi(\xi)}\right)^j} = \frac{a_0 + a_1K^{-\phi(\xi)} + a_2K^{-2\phi(\xi)} + \ldots + a_nK^{-n\phi(\xi)}}{b_0 + b_1K^{-\phi(\xi)} + b_2K^{-2\phi(\xi)} + \ldots + b_nK^{-m\phi(\xi)}}, \quad (4)$$

where a_i and b_i, $\left(0 \leq i \leq n,\ 0 \leq j \leq m\right)$ are non-zero constants and $\Phi\left(\xi\right)$ is the auxiliary ODE given by

$$\Phi'\left(\xi\right) = \frac{K^{-\Phi(\xi)} + \mu\, K^{\Phi(\xi)} + \lambda}{\ln\left(K\right)}, \quad (5)$$

where μ, λ are constants and $K > 0, K \neq 1$. The auxiliary ODE has the general solution as follows:

I When $\lambda^2 - 4\mu > 0$, then $f\left(\xi\right) = \log_K\left(-\lambda - \sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\left(\xi + \varepsilon\right)\right)\right)$.

II When $\lambda^2 - 4\mu < 0$, then $f\left(\xi\right) = \log_K\left(-\lambda + \sqrt{-\lambda^2 + 4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2 + 4\mu}\left(\xi + \varepsilon\right)\right)\right)$.

III When $\lambda^2 - 4\mu > 0$ and $\mu = 0$, then $f\left(\xi\right) = \log_K\left(\frac{\lambda}{-1+\cosh(\lambda(\xi+\varepsilon))+\sinh(\lambda(\xi+\varepsilon))}\right)$.

IV When $\lambda^2 - 4\mu = 0, \lambda \neq 0$ and $\mu \neq 0$, then $f\left(\xi\right) = \log_K\left(\frac{-2-\lambda(\xi+\varepsilon)}{2\mu(\xi+\varepsilon)}\right)$.

V When $\lambda^2 - 4\mu = 0, \lambda = 0$ and $\mu = 0$, then $f\left(\xi\right) = \log_K\left(\xi + \varepsilon\right)$.

Extended Sinh-Gordon Expansion Method

Step 1. The same structure of step 1 of MEFM is valid.

Step 2. The trial solution of Equation (3) is expressed in the form [19],

$$U\left(w\right) = \sum_{i=1}^{n}\left[b_i \sinh\left(w\right) + a_i \cosh\left(w\right)\right]^i + a_0, \quad (6)$$

where $a_0, a_i, b_i\ (i = 1, 2, \cdots, n)$ are constants and to find it's value later, w is a function of ξ that satisfies the following equation

$$w' = \sinh\left(w\right). \quad (7)$$

The solution of Equation (7) possess the following solutions

$$\sinh\left(w\left(\xi\right)\right) = \pm\operatorname{csch}\left(\xi\right) \text{ or } \sinh\left(w\left(\xi\right)\right) = \pm i\operatorname{sech}\left(\xi\right), \quad (8)$$

and

$$\cosh\left(w\left(\xi\right)\right) = \pm\coth\left(\xi\right) \text{ or } \cosh\left(w\left(\xi\right)\right) = \pm\tanh\left(\xi\right), \quad (9)$$

where $i = \sqrt{-1}$.

Step 3. By putting Equation (7) and the derivatives of Equation (6) into Equation (3), we obtain a polynomial equation in $w'^{l}\sinh^i\left(w\right)\cosh^j\left(w\right)\left(l = 0, 1 \text{ and } i, j = 0, 1, 2, \ldots\right)$. As the result the obtained non-linear algebraic equations by equating the coefficients of $w'^l \sinh^i(w)\cosh^j(w)$ to zero, we can find the coefficients.

Step 4. Using Equation (9) and Equation (10), we get the following solutions of Equation (1)

$$U(\xi) = \sum_{i=1}^{n}\left[\pm b_i \operatorname{sech}(\xi) \pm a_i \tanh(\xi)\right]^i + a_0, \tag{10}$$

$$U(\xi) = \sum_{i=1}^{n}\left[\pm i b_i \operatorname{csch}(\xi) \pm a_i \coth(\xi)\right]^i + a_0, \tag{11}$$

where the value of n will finds by using the principal homogeneous balance.

GOVERNING EQUATION AND ITS APPLICATIONS

Application on MEFM

The paraxial NLSE in Kerr media is given by [57]

$$iD_{M,z}^{\alpha,\beta}u + \frac{a}{2}D_{M,t}^{2\alpha,\beta}u + \frac{b}{2}D_{M,y}^{2\alpha,\beta}u + \gamma|u|^2 u = 0, \tag{12}$$

where $u = u(y,z,t)$ is the complex wave envelope function. The constants a, b and γ are the symbols of the dispersion, diffraction, and Kerr non-linearity, respectively. In Equation (12) if $ab > 0$ we get elliptic non-linear Schrödinger equation and if $ab < 0$, Equation (12) becomes hyperbolic non-linear Schrödinger equation. Now assume the following wave transformations:

$$u(x,y,t) = U(\xi)e^{i\theta}, \xi = \frac{\Gamma(\beta+1)}{\alpha}(y + z - ct),$$
$$\theta = \frac{\Gamma(\beta+1)}{\alpha}\kappa(y + z - ct). \tag{13}$$

Inserting Equation (13) into Equation (12), and separate the result into the real and imaginary part, we get

$$-(c^2a + b)U'' + (b\kappa^2 + a\kappa^2c^2 + 2\kappa)U - 2\gamma U^3 = 0, \tag{14}$$
$$(1 + b\kappa + a\kappa c^2)U' = 0. \tag{15}$$

Now, we know that $U' \neq 0$, therefore

$$b = \frac{-1 - a\kappa c^2}{\kappa}. \tag{16}$$

Putting Equation (16) into Equation (14) to get the closed solution, we get

$$U'' + \kappa^2 U - 2\gamma U^3 = 0. \tag{17}$$

Finding the principal balance between U'' and U^3, we find the following relation between n and m

$$n = m + 1. \tag{18}$$

Let $m = 1$, then $n = 2$. Putting the value of $m = 1$ and $n = 2$ into Equation (4), the Equation (4) can be written as the following

$$U(\xi) = \frac{\sum_{i=1}^{2} a_i\left(K^{-i\Phi(\xi)}\right)^i}{\sum_{j=1}^{1} b_i\left(K^{-\Phi(\xi)}\right)^j} = \frac{a_0 + a_1K^{-\phi(\xi)} + a_2K^{-2\phi(\xi)}}{b_0 + b_1K^{-\phi(\xi)}}. \tag{19}$$

Where a_0, a_1, a_2, b_0, b_1 are constants and $b_2 \neq 0$ & $a_1 \neq 0$. Using Equation (19) and its second derivative with Equation (17), we analyze the following cases and solutions:

Case 1. When $a_0 = 0, a_1 = \frac{ib_1\lambda(\lambda^2-4\mu)^{1/4}}{2^{3/4}\sqrt{\gamma(\lambda^2-4\mu)}}, a_2 = \frac{i2^{1/4}b_1(\lambda^2-4\mu)^{1/4}}{\sqrt{\gamma(\lambda^2-4\mu)}}, \kappa = -\frac{\sqrt{\lambda^2-4\mu}}{\sqrt{2}}, b_0 = 0$, we get the following solutions:

Solution 1. When $\lambda^2 - 4\mu > 0, \lambda \neq 0, \mu \neq 0$, then

$$u(y,z,t) = \frac{ie^{-\frac{i\sqrt{\lambda^2-4\mu}\xi}{\sqrt{2}}}(\lambda^2-4\mu)^{1/4}\left(\lambda^2 - 4\mu + \lambda\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\varepsilon+\xi)\right)\right)}{2^{3/4}\sqrt{\gamma(\lambda^2-4\mu)}\left(\lambda + \sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\varepsilon+\xi)\right)\right)}. \tag{20}$$

Solution 2. When $\lambda^2 - 4\mu > 0, \mu = 0$, then

$$u(y,z,t) = \frac{ie^{-\frac{i\sqrt{\lambda^2}\xi}{\sqrt{2}}}\lambda(\lambda^2)^{1/4}\coth\left(\frac{1}{2}\lambda(\epsilon+\xi)\right)}{2^{3/4}\sqrt{\gamma\lambda^2}}. \tag{21}$$

Case 2. When $a_1 = a_0\left(\frac{b_1}{b_0} + \frac{2}{\lambda}\right), a_2 = \frac{2a_0b_1}{b_0\lambda}, \kappa = \frac{\sqrt{\lambda^2-4\mu}}{\sqrt{2}}, \gamma = \frac{{b_0}^2\lambda^2}{2\sqrt{2}{a_0}^2\sqrt{\lambda^2-4\mu}}$, then we get the following solutions

Solution 1. When $\lambda^2 - 4\mu > 0, \lambda \neq 0, \mu \neq 0$, then

$$u(y,z,t) = \frac{a_0e^{\frac{i\sqrt{\lambda^2-4\mu}\xi}{\sqrt{2}}}\left(\lambda^2 - 4\mu + \lambda\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\varepsilon+\xi)\right)\right)}{b_0\lambda\left(\lambda + \sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}\sqrt{\lambda^2-4\mu}(\varepsilon+\xi)\right)\right)}. \tag{22}$$

Solution 2. When $\lambda^2 - 4\mu > 0, \mu = 0$, then

$$u(y,z,t) = \frac{a_0e^{\frac{i\sqrt{\lambda^2}\xi}{\sqrt{2}}}\coth\left(\frac{1}{2}\lambda(\epsilon+\xi)\right)}{b_0}. \tag{23}$$

Case 3. When $a_1 = \frac{a_0\lambda}{\mu}, a_2 = \frac{a_0}{\mu}, b_0 = \frac{b_1\lambda}{2}, \kappa = -\sqrt{-\lambda^2+4\mu}, \gamma = -\frac{{b_1}^2\mu^2}{{a_0}^2\sqrt{-\lambda^2+4\mu}}$, we get the following solution

$$u\left(y,z,t\right) = -\frac{2a_0 e^{-i\sqrt{-\lambda^2+4\mu}\xi}\left(\lambda^2-4\mu\right)\sec^2\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)}{b_1\left(\lambda-\sqrt{-\lambda^2+4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)\right)\left(\lambda^2-4\mu-\lambda\sqrt{-\lambda^2+4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)\right)}, \quad (24)$$

where $\lambda^2 - 4\mu < 0$.

Case 4. When $a_1 = \frac{a_0\lambda}{\mu}, a_2 = \frac{a_0}{\mu}$, $b_1 = \frac{2b_0}{\lambda}, \kappa = -\sqrt{-\lambda^2+4\mu}, \gamma = -\frac{4{b_0}^2\mu^2}{{a_0}^2\lambda^2\sqrt{-\lambda^2+4\mu}}$ we get the following solutions

$$u\left(y,z,t\right) = -\frac{a_0 e^{-i\sqrt{-\lambda^2+4\mu}\xi}\lambda\left(\lambda^2-4\mu\right)\sec^2\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)}{b_0\left(\lambda-\sqrt{-\lambda^2+4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)\right)\left(\lambda^2-4\mu-\lambda\sqrt{-\lambda^2+4\mu}\tan\left(\frac{1}{2}\sqrt{-\lambda^2+4\mu}\left(\varepsilon+\xi\right)\right)\right)}, \quad (25)$$

where $\lambda^2 - 4\mu < 0$.

Application on Extended Sinh-Gordon Method

In this subsection, we apply the extended sinh-Gordon method to the M-fractional paraxial wave equation that labeled Equation (12). Consider the Equation (17) and applying the principal homogeneous balance between the between U'' and U^3, we find $n = 1$. Using the value of $n = 1$and substituting it into Equation (6), we get

$$U\left(w\right) = b_1\sinh\left(w\right) + a_1\cosh\left(w\right) + a_0 \quad (26)$$

Putting Equation (26) and its derivatives into Equation (17), we get the polynomial equation includes for $\left(i, j = 0, 1, 2, ...\right)$. Equating its coefficients to zero, and using Mathematica package, one can investigate the following cases.

Case 5. When $A_0 = 0, A_1 = 0, B_1 = \frac{(-1)^{1/4}}{\sqrt{\gamma}}, \kappa = -i$, we get

$$u\left(y,z,t\right) = \frac{(-1)^{3/4}}{\sqrt{\gamma}}e^{\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}} sech\left(\frac{\Gamma\left(1+\beta\right)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right) \quad (27)$$

or

$$u\left(y,z,t\right) = \frac{(-1)^{1/4}}{\sqrt{\gamma}}e^{\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}} csch\left(\frac{\Gamma\left(1+\beta\right)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right),$$

providing that $\gamma > 0$.

Case 6. When $A_0 = 0, A_1 = -\frac{i}{2^{1/4}\sqrt{\gamma}}, B_1 = 0, \kappa = -\sqrt{2}$, we get

$$u\left(y,z,t\right) = -\frac{i}{2^{1/4}\sqrt{\gamma}}e^{-\frac{i\sqrt{2}\left(-ct^\alpha+y^\alpha+z^\alpha\right)\Gamma(1+\beta)}{\alpha}} \tanh\left(\frac{\Gamma\left(1+\beta\right)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right) \quad (28)$$

or

$$u\left(y,z,t\right) = -\frac{i}{2^{1/4}\sqrt{\gamma}}e^{-\frac{i\sqrt{2}\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}} \coth\left(\frac{\Gamma\left(1+\beta\right)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right)$$

providing that $\gamma > 0$.

Case 7. When $A_0 = 0, A_1 = 0, B_1 = -\frac{(-1)^{1/4}}{\sqrt{\gamma}}, \kappa = -i$, we get

$$u\left(y,z,t\right) = -\frac{(-1)^{3/4}}{\sqrt{\gamma}}e^{\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}} sech\left(\frac{\Gamma\left(1+\beta\right)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right) \quad (29)$$

or

$$u\left(y,z,t\right) = e^{\frac{i\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\sqrt{2}\alpha}}\left(\frac{\coth\left(\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right)}{2^{3/4}\sqrt{\gamma}} + \frac{csch\left(\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right)}{2^{3/4}\sqrt{\gamma}}\right),$$

providing that $\gamma > 0$.

Case 8. When $A_0 = 0, A_1 = \frac{1}{2^{1/4}\sqrt{\gamma}}, B_1 = 0, \kappa = \sqrt{2}$, we get

$$u\left(y,z,t\right) = \frac{e^{\frac{i\sqrt{2}\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}}\tanh\left(\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right)}{2^{1/4}\sqrt{\gamma}},$$

$$u\left(y,z,t\right) = \frac{e^{\frac{i\sqrt{2}\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}}\coth\left(\frac{\Gamma(1+\beta)\left(-ct^\alpha+y^\alpha+z^\alpha\right)}{\alpha}\right)}{2^{1/4}\sqrt{\gamma}} \quad (30)$$

providing that $\gamma > 0$.

CONCLUSION

In this article, the modified exponential function method in a new trial solution and the extended sinh-Gordon expansion method are used to construct some new soliton solutions of M-fractional paraxial non-linear Schrödinger equation. The new exact solutions are included in the hyperbolic function and trigonometric function. **Figures 1**, **3**, **8**, **10** are expressing dark wave solutions, **Figures 2**, **4** are expressing the singular wave, **Figure 7** is the kink-type soliton solution, **Figure 9** is a surface solution and **Figures 5**, **6** are the periodic dark-singular soliton solutions as well as 2D, 3D with a contour plot of all new solutions are plotted. We guarantee that all solutions are new and verified the main equation of M-fractional paraxial wave equation after it substituted to the main equation labeled Equation (6). All our new solutions of (2+1)-dimensional M-fractional paraxial wave equation might be useful and applicable in the optical fiber industry.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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