



Fractional View Analysis of Third Order Korteweg-De Vries Equations, Using a New Analytical Technique

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In the present article, fractional view of third order Korteweg-De Vries equations is presented by a sophisticated analytical technique called Mohand decomposition method. The Caputo fractional derivative operator is used to express fractional derivatives, containing in the targeted problems. Some numerical examples are presented to show the effectiveness of the method for both fractional and integer order problems. From the table, it is investigated that the proposed method has the same rate of convergence as compare to homotopy perturbation transform method. The solution graphs have confirmed the best agreement with the exact solutions of the problems and also revealed that if the sequence of fractional-orders is approaches to integer order, then the fractional order solutions of the problems are converge to an integer order solution. Moreover, the proposed method is straight forward and easy to implement and therefore can be used for other non-linear fractional-order partial differential equations.

Keywords: analytical solution, Mohand transform, Adomian decomposition, caputo derivatives, third order Korteweg-De Vries equations

1. INTRODUCTION

The class of partial differential equations known as Korteweg-De Vries (KDV) equation which play a vital role in the diverse field of physics such as fluid mechanics, signal processing, hydrology, viscoelasticity and fractional kinetics [1, 2]. The KDV equation was first time derived by Korteweg and Vries in 1895. The KDV equation used to model long waves, tides, solitary waves, and wave propagating in a shallow canal. A partial differential Korteweg-De Vries equation of third order is also applied to study the non-linear model of water waves in superficial canal certain namely canal [3], in the time when wave in water was of important concentration in applications in navigational design and also for the awareness of flood and tides [4, 5]. The applications in numerous areas of physics, applied science and other scientific applications, therefore the excessive amount of investigation as a research work has been capitalized in the study of KDV equations [6–10]. We considered the third order time fractional KDV equation in the form [1]

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} + \kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} + \psi(\chi, \mathfrak{S}), \quad 0 < \gamma \leq 1, \quad (1)$$

with initial source

$$u(\chi, 0) = u(\chi),$$

where, κ and λ are real numbers.

The KdV equations of fractional order can be applied to examine the influence of the higher-order wave dispersion. The KdV-Burgers equation defines the waves on lower water surfaces. The strength of fractional KdV equation is the non-local property [11–21]. For a higher order Korteweg-de Vries equation, which is a natural extension of the Korteweg-de Vries equation written in a bilinear form, a Bcklund conversion in bilinear forms is provided. For this higher-order equation the Bcklund transition is given in ordinary forms and the inverse scattering scheme [22], Korteweg-de Vries type of equations 3rd order coefficient variable [23] and Solution of the third order Korteweg-De Vries homotopy perturbation approach using elzaki transform [24].

In few decades, integral transform of various types such as Fourier transform, Laplace transform, Hankel transform, Mellin transform, Z-transform, Wavelet transform, Elzaki transform, Kamal transform, Mahgoub transform, Aboodh transform, Mohand transform, Sumudu transform, Hermite transform etc., gained a enormous importance in solving advanced model in the field mathematics, physics and engineering [25–36].

In the current article, we have applied the Mohand transform with decomposition procedure for the analytical treatment of time fractional KDV equation. The Mohand Transform is one of the new integral transform use for the analytical treatment of different physical phenomena are molded by Differential Equations (DEs) of integer order or Fractional Partial Differential Equations (FPDEs). Recently, Kumar and Viswanathan used Mohand transform and solved the mechanics and electrical circuit problems [37]. Aggarwal have Comparatively Studied Mohand and Aboodh transforms for the solution of differential equations. The numerical applications reflect that both the transforms (Mohand and Aboodh transforms) are closely related to each other [38]. Sudhanshu Aggarwal have also discussed the comparative study of Mohand and Laplace transforms, Mohand and Sumudu transforms, Mohand and Mahgoub transforms [39–41]. Sudhanshu Aggarwal have successfully discussed the Mohand transform of Bessels functions of zero, one and two orders, which is very useful for solving many equations in cylindrical or spherical coordinates such as heat equation, wave equation etc. [42]. The exact solution of second kinds of linear Volterra integral equations get by using Mohand transform. It is claimed that Mohand transform take very little time and has no large computational work [43]. Mohand transform have also used the for solution of Abel’s integral equation. The obtained results show that Mohand transform is a powerful integral transform for handling Abel’s integral equation [44]. The remaining section of the paper are managed as follows. In the second section, we present some related definitions of fractional calculus and basic concepts of Mohand transform. The third section presents the implementation the proposed methodology. The four section represent different models of KDV equation are examined separately and plotted. Finally, we depict our conclusions.

PRELIMINARIES CONCEPTS

In this section, we present some basic necessary definitions and preliminaries concepts related to fractional calculus and Mohand transform.

DEFINITION

Mohand transform first time was define by Mohand and Mahgoub of the function $u(\mathfrak{S})$ for $\mathfrak{S} \geq 0$ in the year 2017. The Mohand transform which is represented by $M(\cdot)$ for a function $u(\tau)$ is define as [45]

$$M\{u(\mathfrak{S})\} = R(v) = v^2 \int_0^\infty u(\mathfrak{S})e^{-v\mathfrak{S}}d\mathfrak{S}, \quad k_1 \leq v \leq k_2, \quad (2)$$

The Mohand transform of a function $u(\mathfrak{S})$ is $R(v)$ then $u(\mathfrak{S})$ is called the inverse of $R(v)$ which is expressed as.

$$M^{-1}\{R(v)\} = u(\mathfrak{S}), \quad M^{-1} \text{ is inverse Mohand operator.} \quad (3)$$

DEFINITION

Mohand transform for nth derivatives [46]

$$M\{u^n(\mathfrak{S})\} = v^n R(v) - v^{n+1}u(0) - v^n u'(0) - \dots - v^2 u^{n-1}(0), \quad (4)$$

DEFINITION

Mohand transform for fractional order derivatives [46]

$$R\{u^\gamma(\mathfrak{S})\} = v^\gamma R(v) - \sum_{k=0}^{n-1} \frac{u^k(0)}{v^{k-(\gamma+1)}}, \quad 0 < \gamma \leq n, \quad (5)$$

DEFINITION

Caputo operator of fractional partial derivative [47]

$$D_\tau^\gamma g(\chi, \mathfrak{S}) = \begin{cases} \frac{\partial^n g(\chi, \mathfrak{S})}{\partial \mathfrak{S}^n}, & \gamma = n \in N, \\ \frac{1}{\Gamma(n-\gamma)} \int_0^\mathfrak{S} (\mathfrak{S}-\phi)^{n-\gamma-1} g^n(\phi) \partial \phi, & n-1 < \gamma < n \end{cases} \quad (6)$$

2. IMPLEMENTATION OF MOHAND TRANSFORM

In this section we have considered the time fractional KDV model in the form

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} + \kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} = \psi(\chi, \mathfrak{S}), \quad 0 < \gamma \leq 1, \quad (7)$$

with initial source

$$u(\chi, 0) = u(\chi).$$

where, κ and λ are real numbers.

Applying Mohand transform [45]

$$M \left\{ \frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} + \kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} = M \{ \psi(\chi, \mathfrak{S}) \}, \quad 0 < \gamma \leq 1, \tag{8}$$

by using the transform property, we can simplify as

$$v^\gamma \{R(v) - vu(0)\} + M \left\{ \kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} = M \{ \psi(\chi, \mathfrak{S}) \}, \tag{9}$$

after some evaluation, Equation (8) simplified as

$$R(v) = vu(0) + \frac{1}{v^\gamma} M \left\{ -\kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} + \frac{1}{v^\gamma} M \{ \psi(\chi, \mathfrak{S}) \}, \tag{10}$$

by applying inverse Mohand transform

$$u(\chi, \mathfrak{S}) = vu(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -\kappa u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \lambda \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} + \frac{1}{v^\gamma} M \{ \psi(\chi, \mathfrak{S}) \} \right\}. \tag{11}$$

Finally we obtain the recursive general relation as

$$u_0(\chi, \mathfrak{S}) = u(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \{ \psi(\chi, \mathfrak{S}) \} \right\} \quad m = 0$$

$$u_{m+1}(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -\kappa u_m(\chi, \mathfrak{S}) \frac{\partial u_m(\chi, \mathfrak{S})}{\partial \chi} - \lambda \frac{\partial^3 u_m(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \quad m \geq 0. \tag{12}$$

THEOREM

Let χ and \mathcal{Y} be two Banach spaces and $T: \chi \rightarrow \mathcal{Y}$ be a contractive nonlinear operator, such that for all $u; u^* \in \chi, \|T(u) - T(u^*)\| \leq K\|u - u^*\|, \quad 0 < K < 1$ [48].

Then, in view of Banach contraction theorem, T has a unique fixed point u , such that $Tu = u$: Let us write the generated series (12), by the Mohand decomposition method as

$$\chi_m = T(\chi_{m-1}), \quad \chi_{m-1} = \sum_{j=1}^{m-1} u_j, \quad j = 0, 1, 2, \dots$$

and supposed that $\chi_0 = u_0 \in S_p(u)$, where $S_p(u) = \{u^* \in \chi : \|u - u^*\| < p\}$ then, we have

- (B₁) $\chi_m \in S_p(u)$
- (B₂) $\lim_{m \rightarrow \infty} \chi_n = u$.

Proof

(B₁) In view of mathematical induction for $m = 1$, we have

$$\|\chi_1 - u\| = \|T(\chi_0 - T(u))\| \leq K\|u_0 - u\|.$$

Let the result is true for $m - 1$, then

$$\|\chi_{m-1} - u\| \leq K^{m-1}\|u_0 - u\|.$$

We have

$$\|\chi_m - u\| = \|T(\chi_{m-1} - T(u))\| \leq K\|\chi_{m-1} - u\| \leq K^m\|u_0 - u\|.$$

Hence, using (B₁), we have

$$\|\chi_m - u\| \leq K^m\|u_0 - u\| \leq K^m p < p,$$

which implies that $\chi_m \in S_p(u)$.

(B₂): Since $\|\chi_m - u\| \leq K^m\|u_0 - u\|$ and as a $\lim_{m \rightarrow \infty} K^m = 0$.

Therefore; we have $\lim_{m \rightarrow \infty} \|u_n - u\| = 0 \Rightarrow \lim_{m \rightarrow \infty} u_n = u$.

3. APPLICATIONS AND DISCUSSION

Here, we have implemented the Mohand transform on some time fractional KVD equations.

Example 4.1: Consider the third order time fractional KVD equation [49]

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} + 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} = 0, \quad 0 < \gamma \leq 1, \tag{13}$$

with initial source

$$u(\chi, 0) = \chi.$$

Taking Mohand transform of Equation (12), we get

$$v^\gamma \{R(v) - vu(0)\} = M \left\{ -6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\}, \tag{14}$$

after some evaluation, Equation (13) is simplified as

$$R(v) = vu(0) + \frac{1}{v^\gamma} \left\{ M \left\{ -6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{15}$$

by applying inverse Mohand transform, we get

$$u(\chi, \mathfrak{S}) = u(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{16}$$

thus, by using recursive scheme of Equation (11), we get

$$u_0(\chi, \mathfrak{S}) = u(0) = \chi, \tag{17}$$

$$u_{m+1}(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -6u_m(\chi, \mathfrak{S}) \frac{\partial u_m(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_m(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \quad m = 0, 1, \dots \tag{18}$$

From the recursive formula (17),

for $m = 0$

$$u_1(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -6u_0(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_0(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_1(\chi, \mathfrak{S}) = -6\chi \frac{\mathfrak{S}^\gamma}{\gamma!}, \tag{19}$$

for $m = 1$

$$u_2(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -6u_0(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} - 6u_1(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_1(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_2(\chi, \mathfrak{S}) = 72\chi \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!}, \tag{20}$$

for $m = 2$

$$u_3(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -6u_0(\chi, \mathfrak{S}) \frac{\partial u_2(\chi, \mathfrak{S})}{\partial \chi} - 6u_1(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} - 6u_2(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_2(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_3(\chi, \mathfrak{S}) = -864\chi \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!} - 216\chi(2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!}, \tag{21}$$

Similarly for $m = 3$, we can get

$$u_4(\chi, \mathfrak{S}) = 10368\chi \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)!} + 2592\chi(2\gamma)! \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)! \gamma!}$$

$$+ 5184\chi(3\gamma)! \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)! (2\gamma)! \gamma!}, \tag{22}$$

$$\vdots$$

The Mohand transform solution for example 4.1 is

$$u(\chi, \mathfrak{S}) = u_0(\chi, \mathfrak{S}) + u_1(\chi, \mathfrak{S}) + u_2(\chi, \mathfrak{S}) + u_3(\chi, \mathfrak{S}) + u_4(\chi, \mathfrak{S}) + \dots \tag{23}$$

$$u(\chi, \mathfrak{S}) = \chi - 6\chi \frac{\mathfrak{S}^\gamma}{\gamma!} + 72\chi \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!} - 864\chi \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!}$$

$$- 216\chi(2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!} + 10368\chi \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)!}$$

$$+ 2592\chi(2\gamma)! \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)! \gamma!} + 5184\chi(3\gamma)! \frac{\mathfrak{S}^{4\gamma}}{(4\gamma)! (2\gamma)! \gamma!} + \dots \tag{24}$$

For particular case $\gamma = 1$, the Mohand transform solution become as

$$u(x, \tau) = \chi(1 - 6\mathfrak{S} + 36\mathfrak{S}^2 - 216\mathfrak{S}^3 + 1296\mathfrak{S}^4 + \dots). \tag{25}$$

The calculated result provide the exact solution in the close form

$$u(\chi, \mathfrak{S}) = \frac{\chi}{1 + 6\mathfrak{S}}. \tag{26}$$

Example 4.2: Consider the third order time fractional KVD equation [50]

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} + u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} = 0, \quad 0 < \gamma \leq 1, \tag{27}$$

with initial source

$$u(\chi, 0) = 1 - \chi.$$

Taking Mohand transform of Equation (26)

$$v^\gamma \{R(v) - v u(0)\} = M \left\{ -u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\}, \tag{28}$$

after some evaluation, Equation (27) is simplified as

$$R(v) = v u(0) + \frac{1}{v^\gamma} \left\{ M \left\{ -u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{29}$$

taking inverse Mohand transform of Equation (28)

$$u(\chi, \mathfrak{S}) = u(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{30}$$

by using the recursive scheme Equation (11), we get

$$u_0(\chi, \mathfrak{S}) = u(0) = 1 - \chi, \tag{31}$$

$$u_{m+1}(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -u_m(\chi, \mathfrak{S}) \frac{\partial u_m(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_m(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{32}$$

From the recursive formula (31),

for $m = 0$

$$u_1(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -u_0(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_0(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_1(\chi, \mathfrak{S}) = (1 - \chi) \frac{\mathfrak{S}^\gamma}{\gamma!}, \tag{33}$$

for $m = 1$

$$u_2(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -u_0(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} - u_1(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_1(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_2(\chi, \mathfrak{S}) = 2(1 - \chi) \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!}, \tag{34}$$

for $m = 2$

$$u_3(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ -u_0(\chi, \mathfrak{S}) \frac{\partial u_2(\chi, \mathfrak{S})}{\partial \chi} - u_1(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} - u_2(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_2(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_3(\chi, \mathfrak{S}) = 6(1 - \chi) \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!}. \tag{35}$$

⋮

The Mohand transform solution for example 3.2 is

$$u(\chi, \mathfrak{S}) = u_0(\chi, \mathfrak{S}) + u_1(\chi, \mathfrak{S}) + u_2(\chi, \mathfrak{S}) + u_3(\chi, \mathfrak{S}) + \dots \tag{36}$$

$$u(\chi, \mathfrak{S}) = 1 - \chi + (1 - \chi) \frac{\mathfrak{S}^\gamma}{\gamma!} + 2(1 - \chi) \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!} + 6(1 - \chi) \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!} + \dots \tag{37}$$

For particular case $\gamma = 1$, the Mohand transform solution become as

$$u(\chi, \mathfrak{S}) = 1 - \chi(1 + \mathfrak{S} + \mathfrak{S}^2 + \mathfrak{S}^3 + \dots). \tag{38}$$

The calculated result provide the exact solution in the close form

$$u(\chi, \mathfrak{S}) = \frac{1 - \chi}{1 - \mathfrak{S}}. \tag{39}$$

Example 4.3 Consider the third order time fractional KVD equation [6]

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} - 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} = 0, \quad 0 < \gamma \leq 1, \tag{40}$$

with initial source

$$u(\chi, 0) = 6\chi.$$

Taking Mohand transform of Equation (39)

$$v^\gamma \{R(v) - v u(0)\} = M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\}, \tag{41}$$

after some evaluation, Equation (40) is simplified as

$$R(v) = v u(0) + \frac{1}{v^\gamma} \left\{ M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{42}$$

by applying inverse Mohand transform, we get

$$u(\chi, \mathfrak{S}) = u(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{43}$$

thus, by using recursive scheme of Equation (11), we get

$$u_0(\chi, \mathfrak{S}) = u(0) = 6\chi \tag{44}$$

$$u_{m+1}(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_m(\chi, \mathfrak{S}) \frac{\partial u_m(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_m(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \quad m = 0, 1, \dots \tag{45}$$

From the recursive formula (44),

for $m = 0$

$$u_1(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_0(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_1(\chi, \mathfrak{S}) = 216\chi \frac{\mathfrak{S}^\gamma}{\gamma!}, \tag{46}$$

for $m = 1$

$$u_2(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} + 6u_1(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_1(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_2(\chi, \mathfrak{S}) = 15552\chi \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!}, \tag{47}$$

for $m = 2$

$$u_3(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_2(\chi, \mathfrak{S})}{\partial \chi} + 6u_1(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} + 6u_2(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_2(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_3(\chi, \mathfrak{S}) = 1119744\chi \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!} + 279936\chi(2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!}, \tag{48}$$

⋮

The Mohand transform solution for example 4.3 is

$$u(\chi, \mathfrak{S}) = u_0(\chi, \mathfrak{S}) + u_1(\chi, \mathfrak{S}) + u_2(\chi, \mathfrak{S}) + u_3(\chi, \mathfrak{S}) + \dots \tag{49}$$

$$u(\chi, \mathfrak{S}) = 6\chi + 216\chi \frac{\mathfrak{S}^\gamma}{\gamma!} + 15552\chi \frac{\mathfrak{S}^{2\gamma}}{(2\gamma)!} + 1119744\chi \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)!} + 279936\chi(2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!} + \dots \tag{50}$$

For particular case $\gamma = 1$, the Mohand transform solution become as

$$u(\chi, \mathfrak{S}) = 6\chi(1 + 36\mathfrak{S} + 1296\mathfrak{S}^2 + 46656\mathfrak{S}^3 + \dots). \tag{51}$$

The calculated result provide the exact solution in the close form

$$u(\chi, \mathfrak{S}) = \frac{6\chi}{1 - 36\mathfrak{S}}. \tag{52}$$

Example 4.4 Consider the third order time fractional KVD equation [6]

$$\frac{\partial^\gamma u(\chi, \mathfrak{S})}{\partial \mathfrak{S}^\gamma} - 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} + \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} = 0, \quad 0 < \gamma \leq 1, \tag{53}$$

with initial source

$$u(\chi, 0) = \frac{6}{\chi^2}.$$

Taking Mohand transform of Equation (52)

$$v^\gamma \{R(v) - vu(0)\} = M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\}, \tag{54}$$

after some evaluation, Equation (53) is simplified as

$$R(v) = vu(0) + \frac{1}{v^\gamma} \left\{ M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \tag{55}$$

by applying inverse Mohand transform, we get

$$u(\chi, \mathfrak{S}) = u(0) + M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u(\chi, \mathfrak{S}) \frac{\partial u(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}. \tag{56}$$

Thus, by using recursive scheme of Equation (11), we get

$$u_0(\chi, \mathfrak{S}) = u(0) = \frac{6}{\chi^2}, \tag{57}$$

$$u_{m+1}(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_m(\chi, \mathfrak{S}) \frac{\partial u_m(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_m(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\}, \quad m = 0, 1, \dots \tag{58}$$

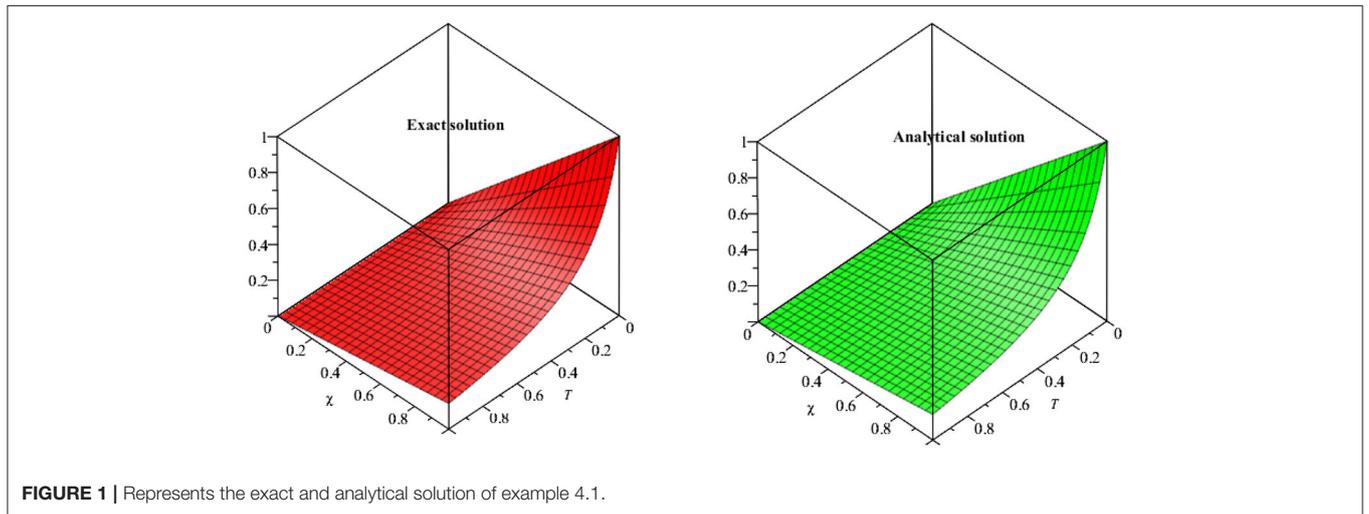


FIGURE 1 | Represents the exact and analytical solution of example 4.1.

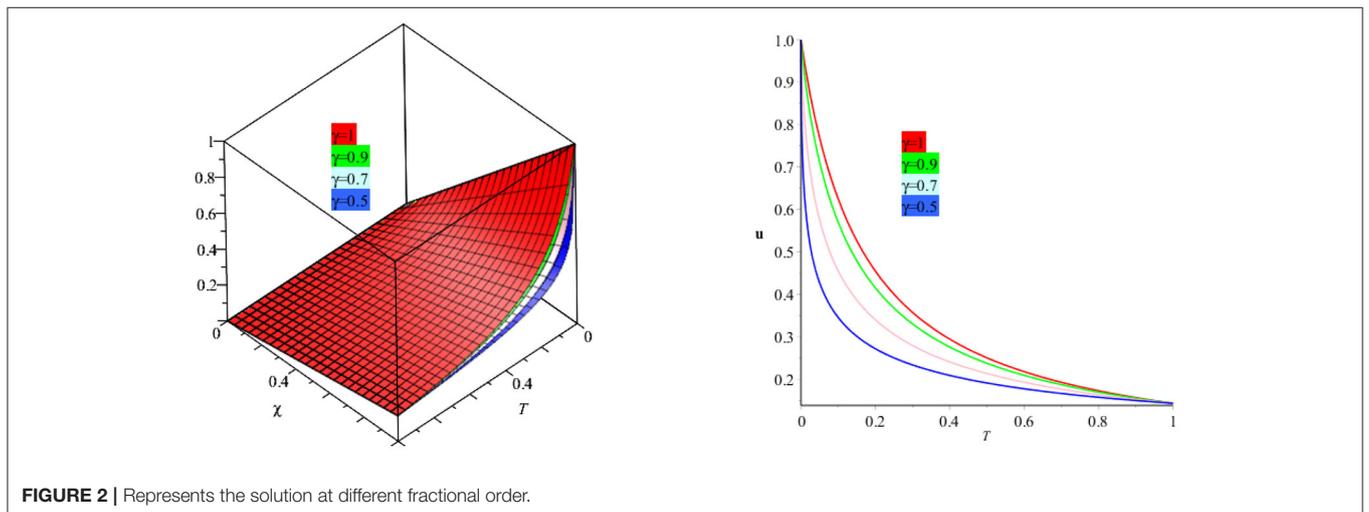


FIGURE 2 | Represents the solution at different fractional order.

From the recursive formula (44),
for $m = 0$

$$u_1(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_0(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_1(\chi, \mathfrak{S}) = \frac{-288 \mathfrak{S}^\gamma}{\chi^5 \gamma!}, \tag{59}$$

for $m = 1$

$$u_2(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} + 6u_1(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_1(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_2(\chi, \mathfrak{S}) = \frac{12096 \mathfrak{S}^{2\gamma}}{\chi^8 (2\gamma)!}, \tag{60}$$

for $m = 2$

$$u_3(\chi, \mathfrak{S}) = M^{-1} \left\{ \frac{1}{v^\gamma} M \left\{ 6u_0(\chi, \mathfrak{S}) \frac{\partial u_2(\chi, \mathfrak{S})}{\partial \chi} + 6u_1(\chi, \mathfrak{S}) \frac{\partial u_1(\chi, \mathfrak{S})}{\partial \chi} + 6u_2(\chi, \mathfrak{S}) \frac{\partial u_0(\chi, \mathfrak{S})}{\partial \chi} - \frac{\partial^3 u_2(\chi, \mathfrak{S})}{\partial \chi^3} \right\} \right\},$$

$$u_3(\chi, \mathfrak{S}) = \frac{4354560 \mathfrak{S}^{3\gamma}}{\chi^{11} (3\gamma)!} - \frac{2488320}{\chi^{11}} (2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!}, \tag{61}$$

⋮

The Mohand transform solution for example 4.3 is

$$u(\chi, \mathfrak{S}) = u_0(\chi, \mathfrak{S}) + u_1(\chi, \mathfrak{S}) + u_2(\chi, \mathfrak{S}) + u_3(\chi, \mathfrak{S}) + \dots \tag{62}$$

$$u(\chi, \mathfrak{S}) = \frac{6}{\chi^2} - \frac{288 \mathfrak{S}^\gamma}{\chi^5 \gamma!} + \frac{12096 \mathfrak{S}^{2\gamma}}{\chi^8 (2\gamma)!} + \frac{4354560 \mathfrak{S}^{3\gamma}}{\chi^{11} (3\gamma)!} - \frac{2488320}{\chi^{11}} (2\gamma)! \frac{\mathfrak{S}^{3\gamma}}{(3\gamma)! \gamma! \gamma!}. \tag{63}$$

For particular case $\gamma = 1$, the Mohand transform solution become as

$$u(\chi, \mathfrak{S}) = \frac{6}{\chi^2} - \frac{288}{\chi^5} \mathfrak{S} + \frac{6048}{\chi^8} \mathfrak{S}^2 - \frac{103680}{\chi^{11}} \mathfrak{S}^3 + \dots \tag{64}$$

The calculated result converge to the exact solution in the close form

$$u(\chi, \mathfrak{S}) = \frac{6\chi(\chi^3 - 24\mathfrak{S})}{(\chi^3 + 12\mathfrak{S})^2}. \tag{65}$$

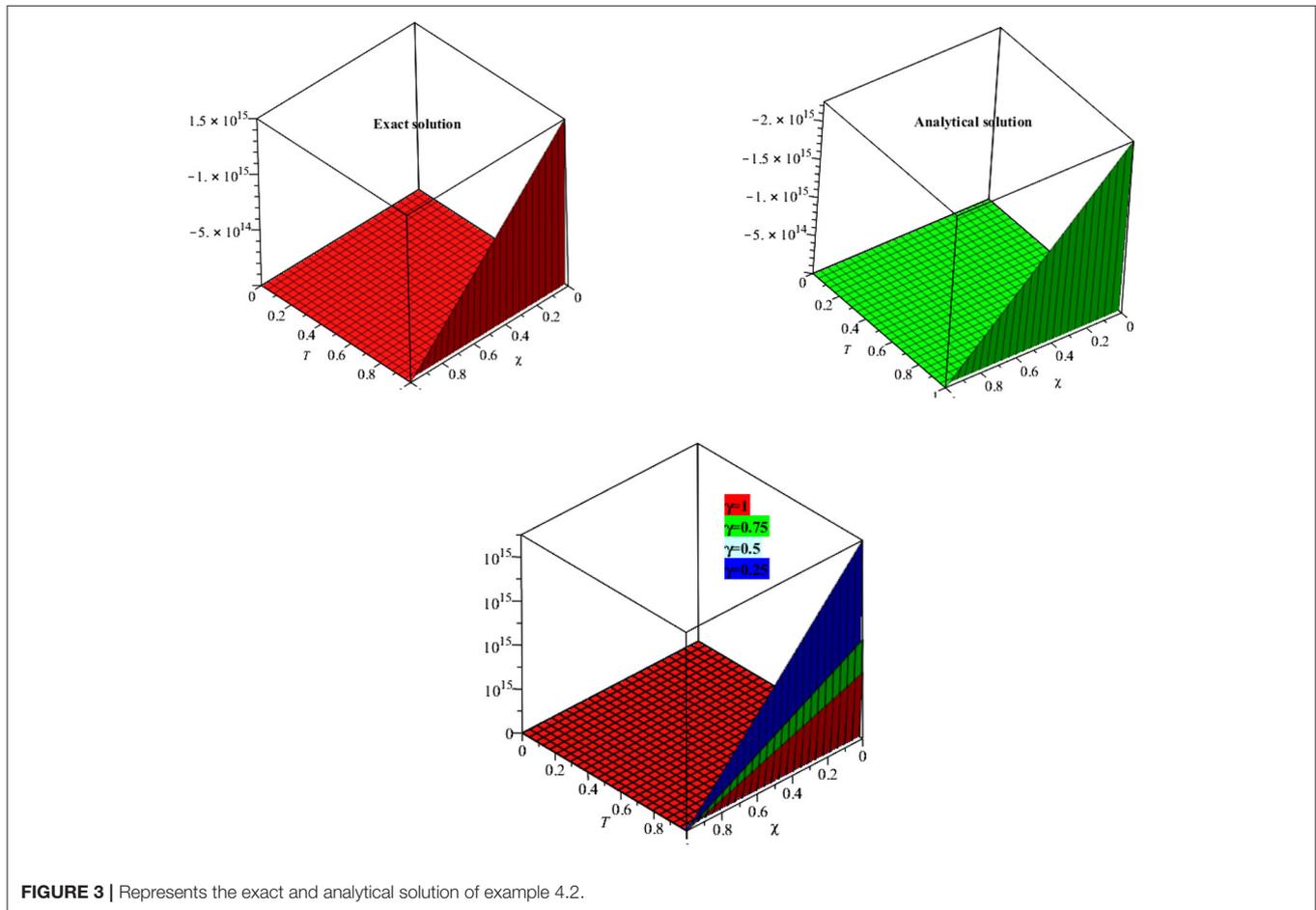


FIGURE 3 | Represents the exact and analytical solution of example 4.2.

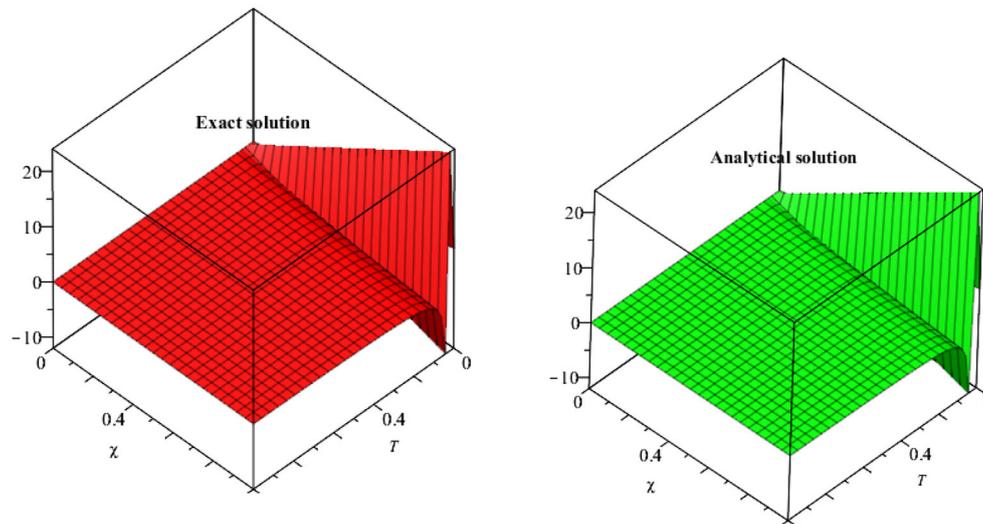


FIGURE 4 | Represents the exact and analytical solution of example 4.3.

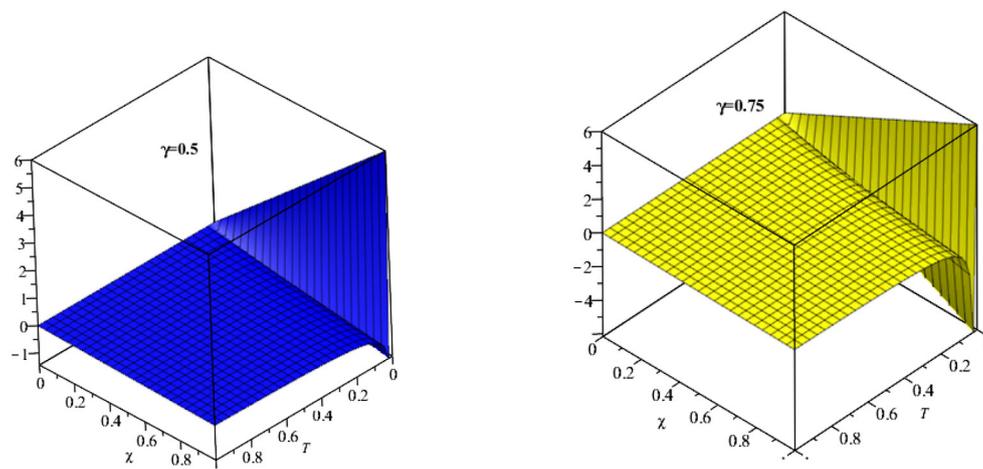


FIGURE 5 | Represents the solution of example 4.3 at $\gamma = 0.75$, $\gamma = 0.5$.

4. RESULTS AND DISCUSSION

In **Figure 1**, the exact and analytical solutions of example 4.1 are presented. The solution-graph have confirmed that the obtained results are in good contact with the exact solutions of example 4.1. In **Figure 2**, the fractional-order solutions are calculated at fractional-order $\gamma = 1, 0.9, 0.7$, and 0.5 . The solutions graphs are expressed in both two and three dimensions. The convergence phenomena can be observed from **Figure 2**. The similar implementation and results can be seen in **Figures 3–7** for example 4.3 and 4.4 also. In **Table 1**, the results of MDM are compared with the results of HPTM which provide identical results. It is observed that the proposed method has the sufficient accuracy and rate of convergence to the exact solutions of the problems. It is also investigated that the proposed method

provided the simple and straightforward implementation for all examples 1, 2, 3, and 4. These investigations of results have confirmed that the present method can be extended to other fractional-order problems arising in science and engineering.

5. CONCLUSION

The proposed method is considered to be one of the pre-eminent and new analytical technique, to solve fractional order partial differential equation. In current research article, the proposed method is applied to solve fractional-order Korteweg-De Vries equations. The current method is constructed by using Mohand transformation along with Adomian decomposition method. The new hybrid method is very useful to handle the analytical

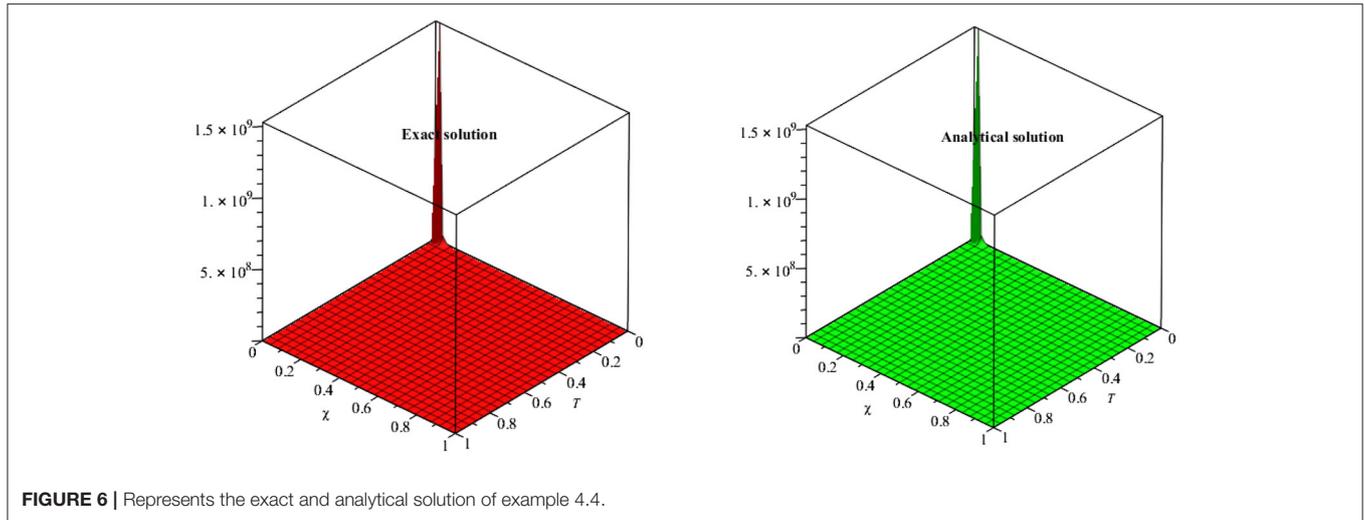


FIGURE 6 | Represents the exact and analytical solution of example 4.4.

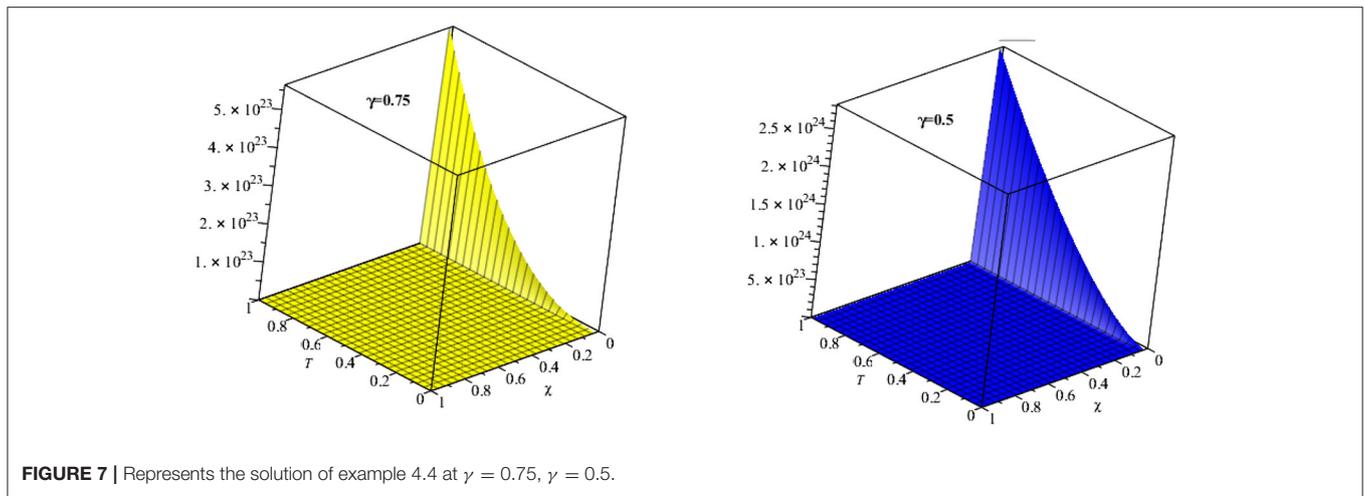


FIGURE 7 | Represents the solution of example 4.4 at $\gamma = 0.75$, $\gamma = 0.5$.

TABLE 1 | Comparison of MDM and HPTM [49] of example 1 at $\gamma = 0.5$.

x	MDM	MDM	MDM	Absolute error	Absolute error
	$\gamma = 0.55$	$\gamma = 0.75$	$\gamma = 1$	HPTM ($\gamma = 1$)	MDM ($\gamma = 1$)
0.1	0.0712628292	0.0893256192	0.0970873	7.86E-08	7.85E-08
0.2	0.1425256585	0.1786512385	0.1941746	1.57E-07	1.56E-07
0.3	0.2137884877	0.2679768577	0.2912619	2.35E-07	2.35E-07
0.4	0.2850513169	0.3573024770	0.3883492	3.14E-07	3.14E-07
0.5	0.3563141462	0.4466280962	0.4854365	3.93E-07	3.93E-07
0.6	0.4275769754	0.5359537154	0.5825238	4.71E-07	4.71E-07
0.7	0.4988398046	0.6252793347	0.6796111	5.50E-07	5.50E-07
0.8	0.5701026338	0.7146049539	0.7766984	6.29E-07	6.29E-07
0.9	0.6413654631	0.8039305732	0.8737857	7.07E-07	7.07E-07
1	0.7126282923	0.8932561924	0.9708730	7.86E-07	7.86E-07

solutions of fractional-order partial differential equations. To verify, the validity of the suggested method some numerical examples of time fractional third order KdV equations are

considered to solve it analytically. The solution graphs have confirmed the validity and reliability of the suggested method toward the solutions of other fractional-order non-linear partial differential equations.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

HK, RS, and UF has the primary contribution to produce this manuscript. PK has provided the financial support to publish this article. DB and MA have provided their expert opinion and writing draft of the paper.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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