



Progress in Global Helioseismology: A New Light on the Solar Modeling Problem and Its Implications for Solar-Like Stars

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Since the first observations of solar oscillations in 1960, helioseismology has probably been one of the most successful fields of astrophysics. Data of unprecedented quality were obtained through the implementation of networks of ground-based observatories such as the GONG project or the BiSON network, coupled with space-based telescopes such as SOHO and SDO missions and more data is expected from the Solar Orbiter mission. Besides the improvement of observational data, solar seismologists developed sophisticated techniques to infer the internal structure of the Sun from its eigenfrequencies. These methods, then already extensively used in the field of Geophysics, are called inversion techniques. They allowed to precisely determine the position of the solar convective envelope, the helium abundance in this region and the internal radial profiles of given thermodynamic quantities. Back in 1990s these comparisons showed a very high agreement between solar models and the Sun. However, the downward revision of the CNO surface abundances in the Sun in 2005, confirmed in 2009, induced a drastic reduction of this agreement leading to the so-called solar modeling problem. More than 10 years later, in the era of the space-based photometry missions which have established asteroseismology of solar-like stars as a standard approach to obtain their masses, radii and ages, the solar modeling problem still awaits a solution. In this paper, we will present the results of new helioseismic inversions, discuss the current uncertainties of solar models as well as some possible solutions to the solar modeling problem. We will show how helioseismology can help us grasp what is amiss in our solar models. We will also show that, far from being an argument about details of solar models, the solar problem has significant implications for seismology of solar-like stars, on the main sequence and beyond, impacting asteroseismology as a whole as well as the fields requiring precise and accurate knowledge of stellar masses, radii and ages, such as Galactic archaeology and exoplanetology.

Keywords: the Sun, helioseismology, asteroseismology, solar abundances, stellar structure and evolution, solar-like stars

1. INTRODUCTION

For the past decades, helioseismology has been a thriving field, enjoying numerous successes and paving the way for asteroseismology of solar-like oscillators. These achievements are a consequence of the very high-quality seismic data obtained thanks to ground-based observation networks (Brookes et al., 1978; Harvey et al., 1988; Isaak et al., 1989) and space based observatories such as the SOHO satellite (Domingo et al., 1995).

These excellent data enabled the precise determination of the position of the base of the convective envelope (Christensen-Dalsgaard et al., 1991; Kosovichev and Fedorova, 1991; Basu and Antia, 1997), the determination of the solar rotation (Brown and Morrow, 1987; Kosovichev, 1988; Schou et al., 1998; Christensen-Dalsgaard and Thompson, 2007; García et al., 2007), density and sound speed profile (Antia and Basu, 1994) as well as an estimation of the helium abundance in the convective envelope through the use of sophisticated seismic analysis techniques (Vorontsov et al., 1991; Antia and Basu, 1994; Basu and Antia, 1995; Richard et al., 1998). The importance of helioseismology as a test of fundamental physics was also highlighted with the so-called solar neutrino problem, which was first thought to stem from inaccurate modeling of the solar core but was ultimately solved with the discovery of neutrino oscillations (Fukuda et al., 1999; Ahmad et al., 2002; Eguchi et al., 2003) and its impact for solar models (Turck-Chièze et al., 1988; Elsworth et al., 1990; Bahcall and Peña-Garay, 2004; Turck-Chièze, 2005; Turck-Chièze and Couvidat, 2011; Haxton et al., 2013). These successes led to the elaboration of stellar models well-suited for helioseismic studies and validated by seismic inversions (see for example Christensen-Dalsgaard et al., 1996) and served as a validation of the depiction of the solar structure and evolution to an excellent degree of accuracy.

However, the downward revision by about 30% of the abundances of carbon, nitrogen and oxygen determined in Asplund et al. (2004), Asplund et al. (2005b) revealed a new problem for standard models, the so-called solar metallicity, or solar modeling problem. This revision stems from the use of 3D atmospheric models instead of outdated 1D empirical models, from the inclusion of NLTE effects and from a careful selection of spectral lines. As C, N, and O are key contributors to the opacity in solar conditions, standard solar models built using the revised abundances from Asplund et al. (2005a) strongly disagreed with helioseismology (e.g., Turck-Chièze et al., 2004; Guzik, 2008). Further studies were performed in 2009, 2011, and 2015 (Asplund et al., 2009; Caffau et al., 2011; Grevesse et al., 2015; Scott et al., 2015a,b), showing that the 3D models agreed with each other and that the remaining differences were due to line selection effects. Recent re-investigations using spectroscopy further confirmed the results of 2009 and recent helioseismic determinations of the solar metallicity also agreed with a rather “low” value (Vorontsov et al., 2013; Buldgen et al., 2017d).

Quickly, it became clear that the solution to the solar metallicity problem was not purely a question of chemical abundances but could also be linked to other ingredients of the models. Investigations on various possible modifications to the solar models were rapidly performed following the publication of the revised abundances (see e.g., Bahcall et al., 2005a,b,c,

2006; Guzik et al., 2005, 2006; Delahaye and Pinsonneault, 2006; Montalbán et al., 2006; Basu and Antia, 2008; Pinsonneault and Delahaye, 2009; Serenelli et al., 2009). These studies showed that a higher opacity could help solving the current discrepancies between solar models and helioseismology. This hypothesis gained some credence with the first experimental measurements of iron opacity in conditions close to those of the base of the solar convective envelope, showing strong disagreement with theoretical opacity computations (Bailey et al., 2015).

At the same time, new theoretical opacity computations became available for solar and stellar modeling (Le Pennec et al., 2015; Mondet et al., 2015; Colgan et al., 2016). Various groups showed that the modifications stemming from these recomputations were insufficient to solve the solar modeling problem and could even lead to larger discrepancies than before. Recently, Pradhan and Nahar (2018) and Zhao et al. (2018) presented new computations of iron opacity showing an increase compatible with experimental measurements. Yet, an opacity increase for a specific element will not necessarily be sufficient to solve the solar problem (Iglesias and Hansen, 2017) and other sources of uncertainties are present in the models.

Ultimately, the solar problem encompasses a wide variety of uncertain physical processes and key ingredients whose impacts are often neglected in standard stellar modeling. In this study, we will list some of the key contributors to the solar issue in section 2 and illustrate their impact on helioseismic constraints in section 3. We discuss the usual suspects of the micro- and macrophysics of the solar models but also briefly present some non-standard scenarios including accretion of material during the early stages of solar evolution as well as the hypothesis of an initial solar mass higher than the currently measured value. In addition, the solar modeling problem, despite its very specific nature, cannot be easily overlooked by stellar modelers. In the current era of high-quality asteroseismic data, stellar modelers are asked to provide very precise and accurate fundamental parameters for solar-like stars. This race to precision is however meaningless if the accuracy of the stellar models is not ensured. Currently, it is well known that the main limitations of stellar physics are the shortcomings of the theoretical models. Consequently, the solar modeling problem is still a very timely issue, as the recipe applied for the Sun is replicated for most of the solar-like oscillators. To illustrate the relevance of the solar modeling problem in asteroseismology, we briefly discuss in section 4 the impact of a potential solution to the solar problem on the seismic parameters of the 16Cyg binary system, one of the most observationally constrained solar-like oscillator, and discuss the potential use of seismic inversions to further constrain this system using asteroseismology. In section 5, we discuss some future prospects for solar models and further tracks for improving the physical accuracy of solar and stellar models. This discussion is then followed by a brief conclusion in section 6.

2. THE SOLAR MODELING PROBLEM AND ITS VARIOUS CONTRIBUTORS

While the solar modeling problem has at first been linked to the revision of the solar metallicity, its clear origin is still disputed

and could well be the result of multiple small contributions from various micro- and macrophysical ingredients of the solar models. As such, the definition of the standard solar models derived by Bahcall et al. (1982) 30 years ago imposes a strict framework which does not take into account all the information we have on the solar structure.

A standard solar model is a one solar mass model, evolved to the solar age, taking microscopic diffusion into account and reproducing the current photospheric ratio of heavy elements over hydrogen, the current solar luminosity (or effective temperature) and the solar photospheric radius. To fulfill these constraints, the models are built using the initial abundance ratio of the heavy elements to hydrogen and the mixing length parameter for convection as free parameters of a minimization process. With this definition, the mathematical problem of reproducing the Sun is well-posed for a given set of constraints.

While this methodology leads to a simple approach for producing solar models using standard stellar evolution codes, it does not take into account all observational constraints. For example, standard solar models do not reproduce neither the rotation profile inside the Sun, nor the photospheric lithium abundance. Both constraints are well determined and point toward the absence or inaccurate implementation of various transport processes of both angular momentum and chemical elements in the current solar models.

Moreover, it is also clear that the mixing-length formalism of convection is inherently flawed and leads to an inaccurate depiction of both the upper layer of the solar convective envelope and its lower boundary, where additional chemical mixing is supposed to occur.

Finally, it should also be pointed out that standard solar model properties are strongly dependent on fundamental physical ingredients such as nuclear reaction rates, radiative opacities, chemical abundances, and the equation of state used for the stellar material.

Consequently, when discussing the inadequacy between standard solar models and helioseismic constraints, various contributors can be listed and could be held responsible for the observed disagreements. In the next sections, we will briefly discuss some of these contributors and their potential impact.

2.1. Chemical Abundances

The chemical abundances are the first and perhaps most important contributors to the solar modeling problem. The determination of the photospheric abundances of most elements heavier than helium is performed using spectroscopic data. For decades, spectroscopists used 1D empirical models of the solar atmosphere to determine the solar metallicity (e.g., Holweger and Mueller, 1974; Vernazza et al., 1976). These abundance tables are the so-called GN93 abundances from Grevesse and Noels (1993), which were used in the standard solar models of the 90s and led to the tremendous successes of helioseismology. They were slightly revised a couple of years later and recompiled in the so-called GS98 abundance tables (Grevesse and Sauval, 1998) still used today in helio- and asteroseismology.

The first solar abundance tables using 3D atmospheric models were the so-called AGS05 abundances (Asplund et al., 2005a)

which initiated the solar modeling problem. These tables were revised in 2009 and became the AGSS09 abundance tables. Further determinations were made in 2011 and 2015 (Caffau et al., 2011; Grevesse et al., 2015; Scott et al., 2015a,b), one leading to an intermediate value between the GS98 and AGSS09 and the most recent confirming the results of 2009. Ultimately the remaining differences are related to the important aspects of line selection and blends (Allende Prieto et al., 2001), which can lead to different values for key chemical elements.

In a series of paper Bahcall et al. (2005c, 2006), Serenelli et al. (2009), and Vinyoles et al. (2017) discussed comparisons between standard solar models using photospheric and meteoritic abundances for the refractory elements. Comparisons between photospheric and meteoritic values for these elements have shown slight differences. Vinyoles et al. (2017) suggest that the meteoritic scale could be used as a higher precision substitute to the solar photospheric values. The main argument is that in recent revisions of solar abundances by Scott et al. (2015a,b) and Grevesse et al. (2015), the differences between photospheric and meteoritic values have been further reduced. However, this approach makes the assumption that the CI chondrites used to infer the meteoritic scale have not undergone any differentiation and represent a realistic sample of mean solar system materials. Recent investigations seem to indicate that this is not the case and that meteoritic abundances cannot be used as such substitutes for solar materials (N. Grevesse, private communication).

While spectroscopy is the most famous approach to determine the solar metallicity, helioseismology has also been used to derive this key ingredient of solar models. The first of such studies was performed by Takata and Shibahashi (2001), who favored a low value for the metallicity, in agreement with the results of AGS05. However, the precision of these results did not allow them to conclude, as the uncertainties were large enough to agree with all abundance tables. Antia and Basu (2006) used a different seismic technique and found an agreement with the GS98 abundance tables whereas Houdek and Gough (2011) found an intermediate value. Recently, Vorontsov et al. (2013) and Buldgen et al. (2017d) used different techniques and concluded that helioseismic methods favored a low metallicity in the solar envelope, more in agreement with the AGSS09 determination. Both studies stressed the strong dependency of these inferences on the equation of state, which dominates the uncertainties.

It is also worth noticing that the abundance of some elements cannot be directly inferred from spectroscopy of the solar photosphere. One of such elements is neon, which is derived from quiet regions of the solar corona (see Young, 2018, and references therein). Varying the neon abundance has a significant impact on opacity. Quickly after the revision of the solar abundances, Antia and Basu (2005) and Bahcall et al. (2005b) investigated the impact of changing the neon abundances to reconcile the AGS05 models with helioseismology. They found that a large increase was required. Recently, two independent studies (Landi and Testa, 2015; Young, 2018) have demonstrated that the abundance ratio of neon over oxygen should be increased by 40%, which leads to significant changes in solar models, but still well below the values found by previous studies.

The reason for this large impact of the abundances of elements heavier than helium is due to their large contribution to the radiative opacity inside the Sun (Blancard et al., 2012; Mondet et al., 2015). Despite their low abundance, they significantly shape the transport of energy in the radiative layers of the Sun, which represent most of its structure. This implies that they have a significant impact on the stratification of solar models and therefore on their (dis)agreement with observational constraints.

2.2. Opacity Tables

Since the transport of energy in most of the solar structure is carried out by radiation, it is unsurprising that the radiative opacities have a large impact on solar models. As the solar modeling problem was unveiled in 2004, the opacities were quickly pointed out as one of the potential causes of the discrepancies between the models and helioseismology (Basu and Antia, 2004).

Today, they remain one of the most uncertain elements of the solar models. Indeed, various tables disagree with each other and lead to significantly different solar models at the level of precision of helioseismic constraints. Moreover, none of the current tables provides a satisfactory agreement with helioseismic constraints with recent abundances. In this paper, we will present results using the OPAL (Iglesias and Rogers, 1996), OP (Badnell et al., 2005), OPAS (Mondet et al., 2015), and OPLIB (Colgan et al., 2016) tables which have been computed by different groups at different times. Moreover, purely numerical considerations are also relevant, related to the various approaches chosen for the interpolation procedure of the opacity tables (Houdek and Rogl, 1996).

These disagreements have motivated attempts to measure experimentally the opacity of key elements in physical conditions as close to solar as possible. The first of such measurements using a Z machine at the Sandia National Laboratories have been recently published for iron (Bailey et al., 2015) and showed large discrepancies with theoretical calculations of iron spectral opacities, between 30 and 400%. The origin of these discrepancies is still unclear and these experimental results still await independent confirmation. Nevertheless, various studies have been carried out to try to close the gap between theoretical calculations and the experiments (Iglesias, 2015; Blancard et al., 2016; Nahar and Pradhan, 2016; Iglesias and Hansen, 2017; Pain et al., 2018; Pradhan and Nahar, 2018; Zhao et al., 2018), some of which finding opacity increases compatible with the experimental results (Bailey et al., 2015). The debate is, however, still very much open and will probably require further extensive theoretical computations and comparisons with experiments.

2.3. Equation of State

Another key elements of solar models is the equation of state. Throughout the years, refinements to the equation of state have also contributed to improve the agreements of solar models with helioseismic constraints.

Two different approaches are used to compute an equation of state for stellar models. The first and most common approach is the so-called “chemical picture,” where the thermodynamical quantities are computed from a free-energy minimization

approach. The chemical picture has been used in the computation of the CEFF (Christensen-Dalsgaard and Däppen, 1992), FreeEOS (Irwin, 2012), SAHA-S (Gryaznov et al., 2004; Gryaznov et al., 2006, 2013; Baturin et al., 2013), and MHD (Däppen et al., 1988; Hummer and Mihalas, 1988; Mihalas et al., 1988, 1990) equations of state. Moreover, in the regimes of astrophysical applications, effects as those of radiation pressure, relativistic corrections and electron degeneracy, amongst other, have to be included in the free energy and included consistently in the equation of state. Slight differences between various equations of state using the “chemical picture” might however result from different hypotheses made when taking into account these effects.

The other approach used in equation of state calculations is the so-called “physical picture,” which uses fundamental constituents and computes their interactions *ab initio*. Namely, this formalism considers separately atomic nuclei and electrons and describes their states using quantum wavefunctions. Again, additional corrections are included for astrophysical considerations. This approach has been used to compute the OPAL equation of state (Rogers et al., 1996; Rogers and Nayfonov, 2002).

The equation of state is a fundamental constituent of solar models, as it impacts indirectly multiple processes acting in solar and stellar interiors. For example, it influences the ionization levels of the chemical elements, which impacts the opacity at various temperatures. In some cases, differences between opacity tables do actually stem from the fact that a different equation of state has been associated with the computations. Consequently, one should in principle use opacity tables with the same equation of state employed in their computation. This is however unfortunately not always possible. The impact of the equation of state can also be directly seen in the sound-speed profile of solar models. This is particularly important when comparing models with various constituents with helioseismic inferences, as the equation of state will impact the results in a significant manner. The induced variations are such that it is often stated that inversions of density profile should not be done using kernels such as the (ρ, Y) kernels, as they lead to biases in the inferred profiles (see Basu et al., 2009, for a discussion and an illustration of this effect).

Moreover, differences in ionization level will impact diffusion velocities and hence the transport of chemicals during the evolution of the Sun. The equation of state also affects the adiabatic temperature gradient, which will influence the onset of convective transport and hence macroscopic mixing in solar and stellar models.

Various studies have been performed to improve the current equation of state in the solar models by carrying out inversions of the profile of the adiabatic exponent, $\Gamma_1 = \frac{\partial \ln P}{\partial \ln \rho} \Big|_S$ (e.g., Elliott, 1996; Basu and Christensen-Dalsgaard, 1997; Vorontsov et al., 2013).

2.4. Mixing of Chemical Elements

2.4.1. Microscopic Diffusion

It is well-known from first principle that a slow transport of the chemical elements is present in stellar radiative layers. This

transport process is called microscopic diffusion and is linked to the various effects of temperature, pressure and composition gradients as well as the effects of ionization and radiation pressure with the various chemical elements of the stellar plasma. These effects induce chemical composition gradients in the stellar radiative regions and thus drastically change the expected initial chemical composition of solar models and their structure. In terms of nomenclature, solar models including the effects of microscopic diffusion are called “standard solar models” whereas models not including this transport process are called “classical solar models.” It was one of the big successes in the early days of helioseismology to show that diffusion was acting in the Sun and thus had to be included in stellar model computations (e.g., Christensen-Dalsgaard et al., 1993; Basu and Antia, 1994; Basu et al., 1996).

While it has been proven that solar models including microscopic diffusion are by far superior to models neglecting it, there are still some uncertainties linked to details in the physical processes underlying the generic term “microscopic diffusion” described in textbooks such as Burgers (1969), Chapman and Cowling (1970), Ferziger and Kaper (1972), and Michaud et al. (2015). As such, various approaches for its implementation exist in the literature (Michaud et al., 1976; Noerdlinger, 1977; Paquette et al., 1986; Michaud and Proffitt, 1993; Thoul et al., 1994), with various hypotheses linked to the components of the stellar plasma and the physical processes considered. It is also worth noticing that many standard solar models do not consider the effects of partial ionization nor the effects of radiation pressure when computing the transport of chemicals by microscopic diffusion. Including radiation pressure can be done in various ways; a simple approximate formula has been derived by Alecian and LeBlanc (2002), to avoid the full computation of radiative accelerations for each element, which is very expensive numerically. Indeed, computing the effects of radiation in a fully consistent manner requires to compute the opacities for each chemical element on the fly at the given conditions of the layer of stellar material. This requires to interpolate in the individual opacity tables whenever these are made available and is computationally very expensive. In the solar case, Turcotte et al. (1998) have demonstrated that these effects are negligible for the solar case. However, other studies have shown that slight modifications should be expected (Schlattl, 2002; Gorshkov and Baturin, 2008, 2010) while the radiative accelerations for certain elements will of course be ultimately influenced by potential significant opacity modifications. Other effects, such as quantum corrections on diffusion coefficients will also slightly affect the transport of chemical elements in the Sun and thus alter the (dis)agreement with helioseismic constraints (Schlattl and Salaris, 2003). Recently, careful investigations of the numerical integrations of the resistance coefficients have also been undertaken by Zhang (2017). This study found slight but significant modifications to the properties of solar models, resulting from singularities in the case of an attractive screened Coulomb potential. In addition, while many of these effects might well be of small importance, when not completely negligible for the Sun, this hypothesis does not hold for other stars (see e.g., Richard et al., 2002a,b; VandenBerg et al., 2002; Théado et al.,

2005; Michaud and Richer., 2008; Theado and Vauclair, 2010; Deal et al., 2018).

2.4.2. Macroscopic Chemical Mixing at the Base of the Convective Zone

Besides microscopic diffusion, macroscopic motions of the solar plasma are also responsible for alterations of the chemical stratification inside the Sun. The most well-known process is turbulent convection, which occurs in the upper layers of the solar envelope. The modeling of convection is one of the most central problem in stellar astrophysics, as most of the current stellar evolution codes use the so-called mixing length theory (MLT) which is a very crude representation of the turbulent motions occurring in stellar conditions (Böhm-Vitense, 1958; Cox and Giuli, 1968). For the solar modeling problem, the shortcomings of the MLT are especially crucial for the positioning of the base of the convective zone and the transition from convective to radiative transport of energy. Indeed, the largest differences between the Sun and standard models are found right below the base of the convective zone.

The problem is linked to the criterion used to determine the extension of convective region, the so-called Schwarzschild criterion (Schwarzschild, 1906). This criterion is based on the cancelation of the convective flux, which translates into a local criterion for the temperature gradients inside the star. However, the cancelation of the flux does not necessarily imply a cancelation of the velocity of the convective elements, which is the parameter determining the extent of the mixed region. This extra-mixed region and its thermal stratification are still uncertain, although hydrodynamical simulations can provide some guidelines in the computation of this so-called “overshooting” or “penetrative convection” at the base of the solar convective zone (Xiong and Deng, 2001; Rempel, 2004; Li and Yang, 2007; Yang and Li, 2007; Viallet et al., 2015; Hotta, 2017). Helioseismology can also be used to provide some insights on the transition of the temperature gradient from adiabatic to radiative in this region (Monteiro et al., 1994; Christensen-Dalsgaard et al., 2011) but unfortunately, it is difficult to disentangle the effects of overshoot from the effects of opacities which can also alter the temperature gradient in these layers.

Besides the effects of overshooting, the base of the solar convective zone is also affected by the effects of rotation in a thin region called the tachocline (Spiegel and Zahn, 1992). In this region of around $0.04R_{\odot}$ wide (Corbard et al., 1999; Elliott and Gough, 1999), the rotational profile of the Sun changes from differentially rotating in latitude to solid body rotating. This transition implies shear-induced mixing of the chemical elements. However, comparisons of helioseismic inversions of the solar rotation profile to rotating models have shown that the effects of meridional circulation and shear-induced turbulence were insufficient to reproduce the inferred properties. Hence, additional processes linked to magnetism or internal gravity waves have to be invoked to reproduce the solar rotation profile (Gough and McIntyre, 1998; Charbonnel and Talon, 2005; Eggenberger et al., 2005). These effects impact the chemical evolution of the Sun, being for example thought to be responsible for the observed lithium depletion and influencing the evolution

of the solar convective zone. These effects, while localized, also slightly influence the calibration procedure and hence the initial chemical composition of the standard solar models (Proffitt and Michaud, 1991; Richard et al., 1996; Gabriel, 1997; Brun et al., 2002). It should be noted, however, that including these processes in a calibration procedure is extremely difficult and somewhat dangerous as they introduce additional parameters which are not constrained from first principles. Hence, further theoretical work is required to avoid the artificial fine-tuning of correlated parameters which could lead to spurious solutions. The recent detection of gravity modes by Fossat et al. (2017) could prove to be a game changer in that respect, by providing an average rotation of the solar core. This would provide a link between mean molecular weight and potential rotation gradients, providing very stringent constraints on the nature of the physical process responsible for the flat rotation profile of the upper radiative layers (Eggenberger et al., 2005). This detection, however, still needs to be confirmed independently as it has already triggered some controversy (Schunker et al., 2018).

The extra-mixing below the envelope is often treated in a parametric way, by introducing an additional turbulent diffusion coefficient depending on various parameters. In our study, we parametrize this diffusion coefficient as a function of ρ_{cz} , the density value at the base of the convective zone

$$D_{Turb} = D \left(\frac{\rho_{cz}}{\rho(r)} \right)^N, \quad (1)$$

with the free parameters D [cm^2s^{-1}], and N which were fixed to 7500 and 3 respectively in the work of Proffitt and Michaud (1991).

2.5. Early Evolution

In the previous sections, we discussed mainly effects that occurred largely on the main sequence and consisted in the “usual suspects” of the solar modeling problem. There are, however, other sources of uncertainties in the early solar evolution that could have an impact on the present-day solar structure as seen from helioseismic constraints.

These include accretion of material during the early stages of the formation of the solar system. This would lead to a contrast in the models, where the internal structure would behave as if the model had a high metallicity, whereas the upper layers would have the observed photospheric abundances. Accretion of low metallicity material was considered by Winnick et al. (2002), Guzik et al. (2006), or Castro et al. (2007). The proposed scenario was that 98% in mass of the Sun could have formed from metal-rich material, in agreement with the GS98 or GN93 abundances, while the last 2% of material would be metal-poor or metal-free and would have been accreted after the apparition of the radiative core of the Sun, to avoid a full mixing of the elements. This scenario provided some improvement in the position of the base of the convective envelope, the helium abundance in the convective zone and to some extent in the sound speed profile [at least in Guzik et al. (2006), whereas Castro et al. (2007) still find large discrepancies just below the convective zone].

Serenelli et al. (2011) have tested the accretion scenario using various metallicities, masses and times at which accretion took place. They found that accretion alone could not solve the solar problem, as metal-rich accretion led to a good agreement in the position of the base of the convective zone and sound speed profile, but reduced the agreement in helium abundance. Metal-poor accretion only provided a good agreement in helium abundance in the convective zone in their tests. They also noted that accretion of material could easily lead to a strong disagreement in lithium abundances, implying that at least additional mixing would be required to reproduce the proper lithium depletion.

Besides accretion, the so-called “faint young Sun paradox” has also motivated non-standard computations of the evolution of the Sun, including exponentially decaying mass loss on the main-sequence. The paradox resides in the fact the solar luminosity on the zero-age main sequence, according to a standard model evolution, would be around 70% of its current luminosity, which is insufficient to explain the presence of liquid water on Mars and the Earth at an early stage of the evolution of the solar system. Other solutions have been suggested to explain these discrepancies, such as greenhouse gases (see Forget et al., 2013; Airapetian et al., 2016; Wordsworth, 2016; Bristow et al., 2017; Turbet et al., 2017), a revision of the carbon cycle in the early Earth’s atmosphere (Charnay et al., 2017) or a slightly more massive young Sun (Sackmann and Boothroyd, 2003; Minton and Malhotra, 2007; Turck-Chièze et al., 2011; Weiss and Heners, 2013).

Physically, one makes the hypothesis that large mass loss on the pre-main sequence could still be present at the very beginning of the main-sequence. Indeed Wood et al. (2005) have observed large winds on young solar-like stars. Increasing the mass loss on the early main sequence implies that the mass and hence the solar luminosity at the zero-age main sequence would be slightly higher and could then provide the physical conditions required for the presence of liquid water. Typically, this effect is erased as the models including mass loss recover the standard evolution of luminosity at about 2 Gy. Of course, such a non-standard evolution leaves traces on observational constraints. Early works by Guzik et al. (1987) and Graedel et al. (1991) studied its impact on the lithium depletion problem and recently, Guzik and Mussack (2010) and Wood et al. (2018) investigated its impact on seismic properties and neutrino fluxes. It appears that such massive models improve the agreement of low metallicity models in the upper radiative layers but not in the core. The disagreement in the central regions has to be mitigated by modifying other physical ingredients such as the screening factors of nuclear reaction. Following Wood et al. (2018), this can be done using the dynamical screening factor of Mussack and Däppen (2011). As for the rotational profile of the Sun, the potential detection of solar gravity modes would provide stringent constraints on the solar core, which could eventually require to question key ingredients linked to the nuclear reactions. Spalding et al. (2018) suggest another way to test this hypothesis by analyzing terrestrial or martian sediments to look for traces of specific Milankovitch cycle imprints scaling with the solar mass. Detecting such frequencies at different epochs could provide a direct hint at

the history of the Sun and thus insights on the “young massive Sun hypothesis.”

Following the neutrino measurements by Davis et al. (1968) and their disagreement with the solar models of the time, Dilke and Gough (1972), adapting the formalism of (Defouw, 1970), suggested a mechanism that could alter the core properties of the Sun in its early evolution and provide a solar explanation for the some climate cycles on Earth. The mechanism received some criticism by Ulrich and Rood (1973), Ulrich (1974) and Ulrich (1975) and was further investigated by Unno (1975) and also discussed by various other authors (Christensen-Dalsgaard et al., 1974; Ledoux, 1974; Boury et al., 1975; Scuflaire et al., 1975; Shibahashi et al., 1975; Gabriel et al., 1976; Noels et al., 1976). The original idea was called the “solar spoon” and was linked to the potential intermittent mixing of the solar core as a result of gravity modes, which would be excited by a form of ϵ mechanism due to ^3He burning. In practice, the first appearance of overstability is favored by some form of mixing such as the aftermath of the intermittent convective core at early stages of solar evolution, some amount of rotational mixing or other unknown processes such as magnetic convection (Schatten, 1973). Once a favorable condition for overstability is provided at some point during the solar evolution, the oscillations can be excited by the intermittent burning of ^3He , which starts once the first oscillations have grown large enough. However, since the gravity modes are stabilized by radiative damping, a trapping condition has to be ensured so that they can grow large enough in the deep layers to trigger the intermittent burning and self-sustain the cycle.

Provided an adequate trapping of the modes, mixing of ^3He will occur as a result of the oscillation and the Brunt-Visli profile of the solar model will be altered. After a sufficient nuclear time linked to the ^3He , the depletion of nuclear fuel will induce the disappearance of the overstability. However, the overstability will propagate toward lower temperatures and thus subsist in regions where its timescale will be greater, provided that the ^3He profile is adequate for its development. The whole process will thus be quasi-periodic, as the different timescales involved will change over the course of the solar evolution.

In a recent paper Gough (2015) discussed the process and considered it ruled out. However, a steep ^3He is suggested by non-linear inversions of the solar core (Marchenkov et al., 2000), which could drive the overstable oscillations described by Dilke and Gough (1972) and Unno (1975). Roxburgh (1976, 1984) suggested that the instability would break down into mild turbulence and locally modify the sound-speed gradient. Various computations have been undertaken to investigate the stability of g modes to this form of ϵ mechanism. Christensen-Dalsgaard et al. (1974), Boury et al. (1975), and Shibahashi et al. (1975) found, using the quasi-adiabatic approximation, that some low order g modes could be unstable. Christensen-Dalsgaard and Gough (1975) investigated the issue using fully non-adiabatic computations and found the modes to be likely stable, as a result of significant damping in the upper layers of the convective envelope. Saio (1980), using a linear non-adiabatic analysis taking into account time-dependent convection, confirmed the instability of some g modes in early stages of the solar evolution.

Moreover, Saio (1980) confirmed the potential instability of the $g_2 \ell = 1$ mode in the present Sun and suggested that some higher degree modes could also be non-linearly coupled with the $g_2 \ell = 1$ mode. The issue was later investigated by Kosovichev and Severnyi (1985), which confirmed that mixing and a low-metallicity of the solar models would enhance the instability of the gravity modes with respect to the ϵ mechanism. The main difficulty in reaching a definitive answer on the issue is linked to the treatment of the behavior of the convective envelope in the stability analysis. Moreover, a fully non-linear analysis of the development of the instability is required to prove that it would lead to a significant transport of chemical elements which would self-sustain the process. Ulrich and Rood (1973) and Ulrich (1974) have stated that an additional agent was required to provide the necessary chemical mixing, as the non-radial oscillation would be insufficient to do so. Non-linear calculations of resonant coupling of gravity modes by Dziembowski (1983), using the approach of Dziembowski (1982), confirmed this criticism of the original formalism of Dilke and Gough (1972). Finally, the absence of undisputed detection and identification of gravity modes does not allow to close the debate. In the quest for solar g-modes, other excitation mechanisms have been suggested and investigated (see Appourchaux et al., 2010, for a review on solar gravity modes), predicting various detectability levels for these highly-sought pulsations.

In recent years, the ϵ mechanism has been reinvestigated in metal-poor low-mass main-sequence stars (Sonoi and Shibahashi, 2012a,b). In this case, the reduced size of the outer convective zone simplifies the treatment of the stability analysis, as it is thought to play a minor role in the total energy budget.

Other more subtle effects, like the low-temperature opacities (Guzik et al., 2006), the equation of state or the properties of the chemical mixing at the base of the convective zone in the early phases of the solar evolution (Baturin et al., 2015) could affect the observed properties of the current Sun and the conclusion we may draw from them. At first, these effects may seem negligible but they would actually impact the initial conditions of a solar calibration, hence leading to overall changes in the structure that cannot be fully neglected. A very stringent constraint on such effects is the lithium depletion observed in the solar photosphere, which is strongly affected by micro- and macrophysical effects in the solar models.

3. COMBINED STRUCTURAL INVERSIONS AND STRUCTURAL DIAGNOSTICS

In this section, we will present inversion results of solar models built with various physical ingredients. All models have been computed with the Lige stellar evolution code (CLES, Scuflaire et al., 2008b). Their oscillations have been computed using the Lige adiabatic oscillation code (LOSC, Scuflaire et al., 2008a) and the inversions have been carried out using the SOLA method (Pijpers and Thompson, 1994) implemented in the InversionKit software (Reese et al., 2012).

We followed the guidelines of Rabello-Soares et al. (1999) to adjust the trade-off parameters of the inversion techniques and

used the data of Basu et al. (2009) supplemented by an extension of BiSON observations of Davies et al. (2014) (as used in Buldgen et al., 2018). We computed inversions of the squared adiabatic sound speed $c^2 = \frac{\Gamma_1 P}{\rho}$, an entropy proxy, denoted $S_{5/3} = \frac{P}{\rho^{5/3}}$ presented in Buldgen et al. (2017e); and the Ledoux discriminant, defined as $A = \frac{d \ln \rho}{d \ln r} - \frac{1}{\Gamma_1} \frac{d \ln P}{d \ln r}$ as in Buldgen et al. (2017c).

We start in sections 3.1, 3.2, and 3.3 by presenting inversion results for solar models built using different physical ingredients. To test the dependency of standard solar models on chemical composition and opacities, we used models built using the AGSS09 and GS98 abundances tables, the OPAS, OPAL, and OPLIB opacity tables and models including the revision of the neon abundance found in Landi and Testa (2015) and Young (2018), hereafter AGSS09Ne. We also present results for various implementations of the mixing of chemical elements, namely the use of the Paquette et al. (1986) collision integrals in the diffusion coefficients, the effects of considering the partial ionization of the heavy elements in the computation of microscopic diffusion. Besides microscopic effects, we also consider macroscopic mixing, in the form of an adiabatic overshoot and in the form of turbulent diffusion. All models presented here have been built using the FreeEOS equation of state and the Adelberger et al. (2011) nuclear reaction rates, except for the model taking into account partial ionization in the computation of microscopic diffusion which used the SAHA-S equation of state (Gryaznov et al., 2004; Gryaznov et al., 2013; Baturin et al., 2013).

In **Figure 1**, we illustrate the effect of the changes in the properties of the chemical mixing on the metallicity profile of the solar models. Most of the trends can be easily understood. For example, taking into account partial ionization of the metals when computing microscopic diffusion as in the AGSS09-OPAL-PartIon model (blue) will lead to a slightly more efficient diffusion of these elements, as they encounter less repulsion near the base of the convective zone and thus will more easily fall down toward central layers. Using the screened Coulomb potentials in the diffusion coefficients, as in Paquette et al. (1986), in the AGSS09-OPAL-Paquette model, leads to a less efficient diffusion during the evolution, as the ions will experience more repulsion than in the case of the cut-off hypothesis over a Debye sphere used in the original Thoul et al. (1994) formalism. The models including turbulent diffusion, denoted “AGSS09-OPAL-DT” and “AGSS09-OPAL-Proffitt” in **Table 1**, show a much more different behavior. In the “AGSS09-OPAL-DT” model, we have fixed the D and N parameter of Equation (1) to respectively 50 and 2 and to 7500 and 3 the “AGSS09-OPAL-Proffitt” model. The peak stemming from the variations of diffusion velocity near the base of the convective envelope is erased by the turbulent mixing, which induces a very different metallicity profile. The disappearance of this metal-peak is actually seen in the Ledoux discriminant inversion through its impact on the temperature gradient around 0.65 solar radii (see section 3.3). It is also worth noticing that including the prescription of Proffitt and Michaud (1991) for turbulent diffusion has a sufficiently large impact on the calibration to alter the initial chemical composition of model “AGSS09-OPAL-Paquette,” as can also be seen from **Table 1**, whereas the coefficients used

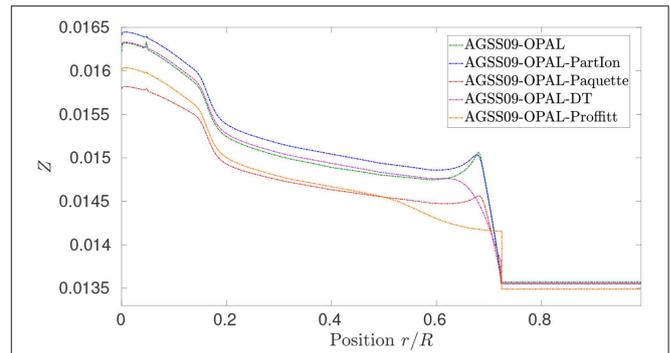


FIGURE 1 | Metallicity profile of the standard solar models of **Table 1** including various prescriptions for the transport of chemicals.

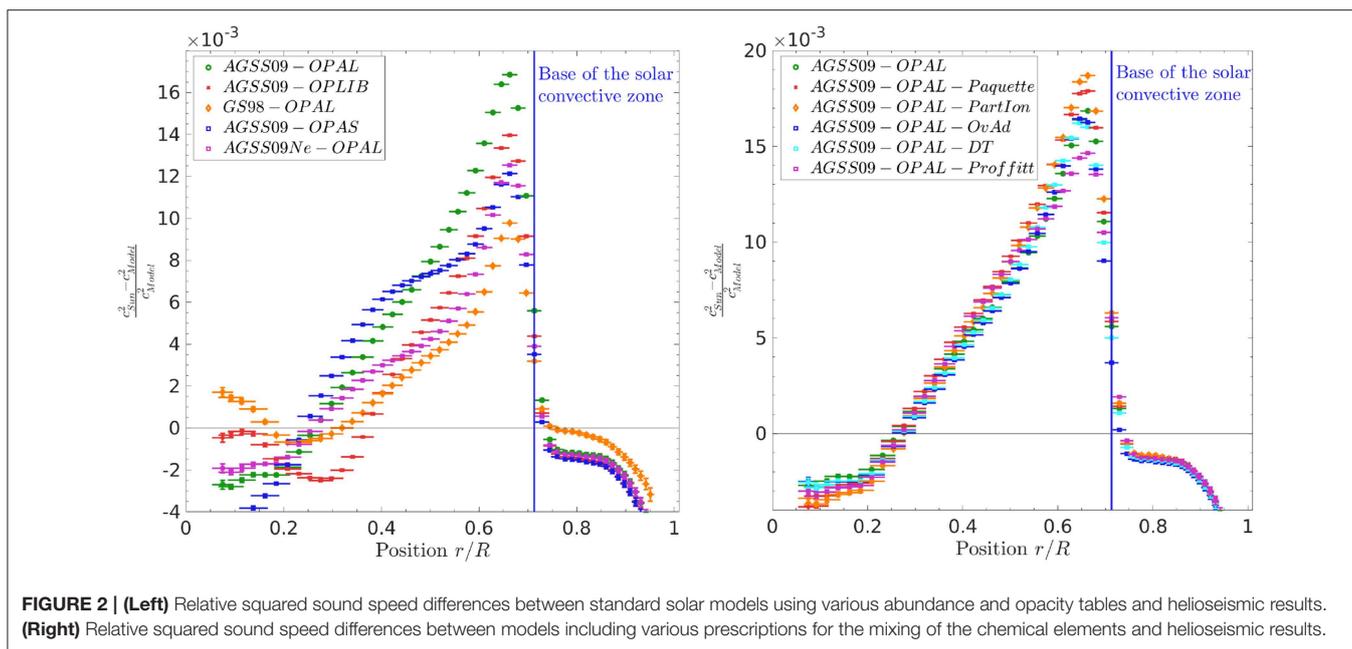
TABLE 1 | Physical ingredients of the standard solar models used in this study.

Name	EOS	Opacity	Abundances	Diffusion	Convection
AGSS09-OPAL	FreeEOS	OPAL	AGSS09	Thoul	MLT
AGSS09-OPLIB	FreeEOS	OPLIB	AGSS09	Thoul	MLT
AGSS09-OPAS	FreeEOS	OPAS	AGSS09	Thoul	MLT
AGSS09-OPAL-Paquette	FreeEOS	OPAL	AGSS09	Paquette	MLT
GS98-OPAL	FreeEOS	OPAL	GS98	Thoul	MLT
AGSS09Ne-OPAL	FreeEOS	OPAL	AGSS09Ne	Thoul	MLT
AGSS09-OPAL-PartIon	SAHA-S	OPAL	AGSS09	Thoul + PartIon	MLT
AGSS09-OPAL-OvAd	FreeEOS	OPAL	AGSS09	Thoul + OvAd (0.1H _p)	MLT
AGSS09-OPAL-DT	FreeEOS	OPAL	AGSS09	Thoul + DT	MLT
AGSS09-OPAL-Proffitt	FreeEOS	OPAL	AGSS09	Thoul + Proffitt	MLT

in Buldgen et al. (2017c) have a negligible impact on the initial conditions.

3.1. Sound Speed Inversions

We start with classical sound speed inversions, presented in **Figure 2**. In the left panel of **Figure 2**, we present results for standard solar models built with various abundances and opacities. We can see the illustration of the well-known solar modeling problem when comparing the standard AGSS09 models, in green, with the GS98 standard model, in orange. However, it appears that considering the 40% increase of the Ne/O ratio derived independently by Landi and Testa (2015) and Young (2018) provides a significant improvement of the agreement between AGSS09 models and helioseismic inversions. This is not a surprise, since a neon increase, although much larger, was already suggested by Antia and Basu (2005), Zaatari et al. (2007), and Basu and Antia (2008) as a potential solution



to the solar modeling problem. Similarly, using the more recent OPAS or OPLIB opacity tables also leads to a non-negligible improvement of the agreement of low-metallicity models and helioseismic results. However, this significant improvement is restricted to the radiative layers. Indeed, large discrepancies in sound speed in the convective envelope are still present for all the AGSS09 models. This is likely due to the large discrepancies in helium in the convective envelope, since, as we will see in section 3.4, none of the models presented in **Figure 2** shows a good agreement with the helioseismic helium abundance.

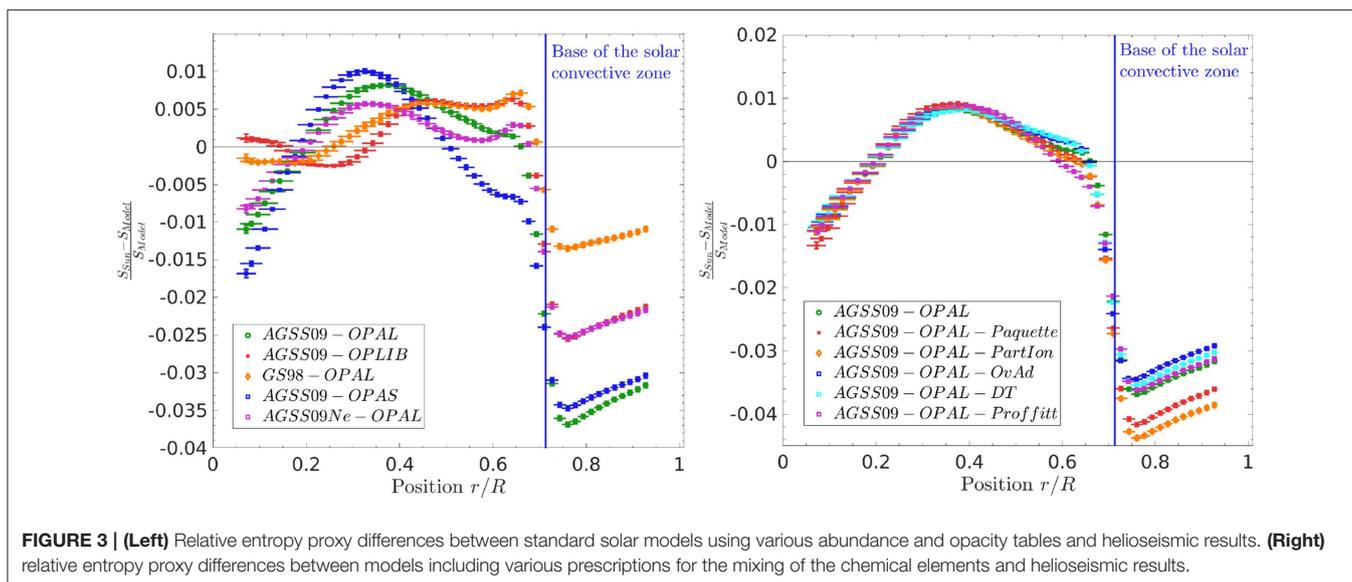
In the right panel of **Figure 2**, we illustrate squared adiabatic sound speed inversions for models including various prescriptions for the mixing of the chemical elements. Using the Paquette et al. (1986) collision integrals or considering partial ionization in the computation of microscopic diffusion leads to an increase of the disagreements in the sound speed profile just below the convective envelope (orange and red symbols in the right panel of **Figure 2**). Adding a form of macroscopic mixing improves the agreement of AGSS09 models and helioseismic inversions, as can be seen from the models including either turbulent diffusion or a form of overshooting. The best agreement is found for the polynomial formulation of turbulent diffusion used in Proffitt and Michaud (1991) to reproduce the solar lithium abundances (purple symbols in the right panel of **Figure 2**). However, the improvement is very localized and the mixing has little to no impact on the deeper radiative layers. This demonstrates, as is now well-known, that the solar modeling problem cannot stem only from an inaccuracy of the mixing of the chemical elements, but that other ingredients such as the radiative opacities, may be partially responsible for the discrepancies.

3.2. Entropy Proxy Inversions

In addition to squared adiabatic sound speed, other structural quantities can be inverted, such as for example the density, using the (ρ, Γ_1) structural pair (see e.g., Antia and Basu, 1994) or the squared isothermal sound speed (see e.g., Dziembowski et al., 1990; Gough and Thompson, 1991). Recently, we presented in Buldgen et al. (2017b) approaches to change the structural variables of the variational equations which could in turn be used in helio- and asteroseismology. In Buldgen et al. (2017e), we presented inversion results of an entropy proxy, denoted $S_{5/3} = \frac{p}{\rho^{5/3}}$ which provides interesting insights on the solar structure. In **Figure 3**, we show the inversion results of this structural quantity for the models discussed in section 3.1.

From **Figure 3**, a slightly different picture of the problem is drawn. In the left panel, the model built with the OPAS opacity tables which performed quite well in the sound speed inversion does not provide a good agreement in the entropy proxy, whereas the OPLIB opacities provide a very significant improvement over the OPAL opacities, similar to the effect of the neon revision. Nevertheless, it is clear that the performance of the AGSS09 models is still very far from the agreement obtained using the GS98 abundances. The performance of the models built with the OPLIB opacities and the revision of the neon abundance is due to the steeper temperature gradient of these models below the convective envelope due to either the behavior of the opacity profile (see Colgan et al., 2016; Guzik et al., 2016, for a discussion) or simply the increase of neon which leads to an increased opacity. Indeed, from Blancard et al. (2012), it appears that neon is the third most important contributor to the opacity at the base of the solar convective zone.

In the right panel of **Figure 3**, we can see again that none of the modifications of the transport of chemical elements have led to a large improvement of the performance of low-metallicity



solar models. Slight modifications to the $S_{5/3}$ profile are seen, with the model including adiabatic overshooting performing slightly better than the models including turbulent diffusion. Again, we also see that the models including the Paquette et al. (1986) collision integrals or partial ionization when computing microscopic diffusion lead to an increase of the disagreements with helioseismic results. Overall, this inversion confirms that the solution to the solar modeling problem is not to be found from the mixing of the chemical elements alone, but also that some distinction can be made over the type of mixing if one refines the diagnostic by combining it to a quantity more sensitive to local variations. This will be further discussed in section 3.3, when presenting the results of the Ledoux discriminant inversions.

3.3. Ledoux Discriminant Inversions

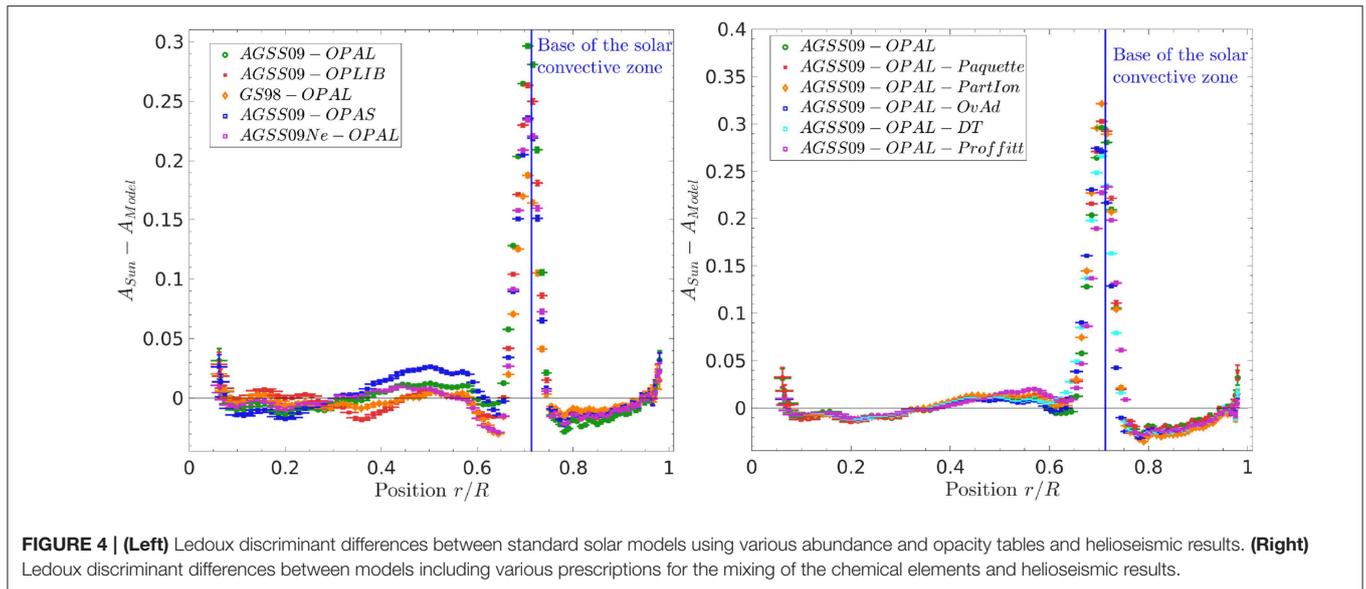
In section 3.2, we discussed the results of inversions of an entropy proxy and showed the importance of combining the information from various inversion techniques to lift potential degeneracies that could hinder our understanding of the solar modeling problem. This thinking can be pushed even further by carrying out inversions of the Ledoux discriminant. These inversions were already presented in Gough and Kosovichev (1993b), Elliott (1996), Kosovichev (1999), and Takata and Montgomery (2002) but have not been exploited to analyse the discrepancies found for models built with the recent abundance tables of Asplund et al. (2009). This analysis was carried out in Buldgen et al. (2017e, 2018), where in this last paper, an extended set of models is analyzed.

In **Figure 4**, we present inversion results for the models of sections 3.1 and 3.2. The first striking feature of these inversions is the large disagreements at the base of the convective zone which is found for any opacity tables, chemical abundances and mixing considered. These discrepancies illustrate clearly the fact that the standard solar models are unable to reproduce the transition in both temperature and chemical composition gradient at the base

of the convective envelope. The main difficulty is to separate each of their contributions to the A inversion.

Overall, the results are again quite mixed. In the left panel of **Figure 4**, we find that the increase in neon provides the largest improvement, bringing the models to an agreement nearly as good as that found in the GS98 models. The OPAS and OPLIB opacity tables also significantly improve the behavior of the AGSS09 models. Nevertheless, the results are far from convincing. Moreover, even the GS98 models show large deviations below the convective zone, as deep as 0.6 solar radii, thus in a region supposedly fully radiative. This emphasizes that while the potentially missing macroscopic mixing process is certainly very localized, it can still have an impact in deeper radiative layers. Indeed, it will influence the initial chemical abundances required to reproduce the solar surface metallicity, luminosity and temperature (or radius) at a solar age and thus the whole structure to a level that is detectable with helioseismic data.

This is confirmed by the right panel of **Figure 4**, where we can see the impact of extra-mixing below the convective zone. Again, the parametrization of Proffitt and Michaud (1991) provides the largest improvement for the low-metallicity models, while the second parametrization used in Buldgen et al. (2017c) provides a similar agreement to that of using a convective overshoot and the use of Paquette et al. (1986) collision integrals and considering partial ionization in the computation of microscopic diffusion leads to larger deviations. This does not mean, however, that turbulent mixing is not occurring at the base of the convective zone and that such approaches should not be explored. However, it clearly shows that mixing alone is not sufficient to solve the solar modeling problem and other ingredients have to be revised. Hence, it is of crucial importance to compare physical ingredients, formalisms and numerical techniques to fully assess their importance for the current issue, in a similar fashion to what has been done in Boothroyd and Sackmann (2003), Lebreton et al. (2007), and Montalbán et al. (2007).



On a sidenote, we would like to emphasize the degeneracy at play in the analysis of helioseismic inversions. Even when combining the results of sound speed, entropy proxy, and Ledoux discriminant, we cannot fully distinguish between thermal and compositional effects. Moreover, the inverted results are not independent. They could in principle all be deduced from the solar density profile. In that sense, they all provide the same information about the solar structure. For example, if one uses the Ledoux discriminant inversion to correct the A profile of a standard solar model and integrate the other variables, assuming Γ_1 known, the agreement in both sound speed and entropy proxy is very significantly improved. However, combining the inversions is useful when trying to link an improvement with respect to the helioseismic inversions to a change in the physical ingredients of the models, as the degeneracy at play between compositional and thermal effects will not act in the same way for all structural variables.

Despite these differences in their behaviors, one cannot fully separate thermal and compositional effects without further assumptions linked to the equation of state of stellar material and the chemical composition or the temperature stratification. This degeneracy is a consequence of the physical dependencies of the quantities for which inversions can be carried out, and of the fact that the changes considered in the models will always impact both temperature and chemical gradients. For example, changing the opacity tables will alter the position of the base of the convective zone in the models and thus alter the diffusion history, thus the chemical gradients. Similarly, adding an additional mixing at the base of the convective zone will alter the chemical history of the model and thus the temperature gradients, near the base of the convective zone but also deeper, as seen from the A inversion of the model including turbulent diffusion following Proffitt and Michaud (1991). In that sense, one should emphasize the value of independent constraints such as neutrino measurements, which provide crucial additional informations about the deep layers of the Sun particularly the temperature.

3.4. Convective Envelope Properties and Frequency Separation Ratios

Besides looking at seismic inversions, it is also interesting to analyse other complementary constraints. Indeed, solar models do not have only to show a good agreement in terms of inferred quantities, but should also reproduce the correct position of the base of the convective zone (Christensen-Dalsgaard et al., 1991; Kosovichev and Fedorova, 1991; Basu and Antia, 1997), determined to be around 0.713 solar radii and the helium abundance in the convective envelope, determined by (Vorontsov et al., 1991, 2013) and found to be above 0.245¹. In addition, a classical seismic diagnostic of the solar models are the so-called frequency separation ratios

$$r_{0,2} = \frac{\nu_{n,0} - \nu_{n-1,2}}{\nu_{n,1} - \nu_{n-1,1}}, \quad (2)$$

$$r_{1,3} = \frac{\nu_{n,1} - \nu_{n-1,3}}{\nu_{n+1,0} - \nu_{n,0}}, \quad (3)$$

following the definitions of Roxburgh and Vorontsov (2003b). They showed that these ratios are very sensitive to the deep layers of stellar structure. In this section, we present in **Table 2** the above properties of the solar models presented in the preceding sections and illustrate in **Figure 5** the frequency separation ratios of some of our models and those obtained from BiSON data.

From **Table 2**, we can see that models with the AGSS09 abundances all have a surface helium abundance well below the seismically determined intervals and a too shallow convective envelope. A more worrying result is found for the models built with the latest OPLIB and OPAS opacity tables, as their helium abundance is even lower than those built using the OPAL tables. This is a consequence of the reduction of the opacity in an extended part of the solar radiative zone, which implies a higher

¹We consider here a conservative approach given the differences in the precision of the helium determination found by various studies.

TABLE 2 | Parameters of the standard solar models used in this study.

Name	$(r/R)_{BCZ}$	$(m/M)_{CZ}$	Y_{CZ}	Z_{CZ}	Y_0	Z_0
AGSS09-OPAL	0.7224	0.9785	0.2363	0.01361	0.2664	0.01511
AGSS09-OPLIB	0.7205	0.9777	0.2300	0.01372	0.2588	0.01520
AGSS09-OPAS	0.7196	0.9779	0.2322	0.01368	0.2614	0.01516
AGSS09-OPAL-Paquette	0.7235	0.9788	0.2373	0.01359	0.2648	0.01480
GS98-OPAL	0.7157	0.9764	0.2465	0.01706	0.2765	0.01887
AGSS09Ne-OPAL	0.7207	0.9780	0.2373	0.01393	0.2655	0.01547
AGSS09-OPAL-Partlon	0.7240	0.9790	0.2378	0.01355	0.2690	0.01524
AGSS09-OPAL-OvAd	0.7207	0.9780	0.2372	0.01356	0.2666	0.01514
AGSS09-OPAL-DT	0.7230	0.9786	0.2375	0.01355	0.2666	0.01514
AGSS09-OPAL-Proffitt	0.7244	0.9790	0.2411	0.01349	0.2650	0.01486

TABLE 3 | Physical ingredients of the solar models with modified opacities and additional mixing used in this study.

Name	EOS	Opacity	Abundances	Diffusion	Convection
AGSS09Ne-Poly	SAHA-S	OPAL+Poly	AGSS09Ne	Thoul	MLT
AGSS09Ne-Poly-DT	SAHA-S	OPAL+Poly	AGSS09Ne	Thoul+ D_{Turb}	MLT
AGSS09Ne-Poly-Prof	SAHA-S	OPAL+Poly	AGSS09Ne	Thoul+ D_{Turb} –Prof	MLT
AGSS09Ne-Poly-Rad	SAHA-S	OPAL+Poly	AGSS09Ne	Thoul+Ov–Rad (0.3 H_p)	MLT
AGSS09Ne-Poly-Ad	SAHA-S	OPAL+Poly	AGSS09Ne	Thoul+Ov–Ad (0.3 H_p)	MLT

initial hydrogen abundance of the model to allow them to reproduce the solar luminosity at the solar age.

The only AGSS09 model to show a significant improvement in the helium abundance is the one using the Proffitt and Michaud (1991) parametrization of turbulent diffusion. However, this is made at the expense of a larger disagreement of the position of the base of the convective zone. The model including the revised neon abundance, for example, does not significantly improve the helium abundance problem while it reduced the discrepancies observed in structural inversions. All other modifications lead to somewhat similar conclusions, with neither the macroscopic mixing nor the modifications to microscopic diffusion implying a decisive improvement of the models. However, the similarities between the parameters of the models including partial ionization in the computation of microscopic diffusion and those including macroscopic mixing

illustrate the importance of combining the structural inversions, since they could differentiate between both effects.

In **Figure 5**, we compare the frequency separation ratios of theoretical models including revised abundance and opacity tables. In previous papers, the good agreement between the frequency separation ratios of the GS98 and the solar data was considered as a strong argument against the revised abundances. We see in **Figure 5** that a similar agreement can be obtained by using the OPLIB opacities in AGSS09 models and that the neon revision also provided a significant improvement of the agreement. This results from the fact that the frequency separation ratios are sensitive to the sound-speed derivative. Hence, they are sensitive to both the temperature and chemical composition gradients and not only to the chemical composition of the solar radiative layers.

Consequently, the frequency separation ratios cannot be used as a direct constraint on the solar chemical composition. However, they certainly provide some additional information to dissect the current solar modeling problem. For example, the fact that the model built with the OPAS opacity tables, while it provided a quite good improvement in the squared sound speed inversion, demonstrates that there is a clear issue. Similarly, since the AGSS09 OPLIB model reproduces quite well the ratios implies that the gradient of the ratio of temperature over mean molecular weight must be quite close to the solar one, but clearly fails at reproducing the mean molecular weight itself, since the helium abundance in the convective envelope is far too low.

3.5. Modified Solar Models

In addition to the models presented in the previous sections, we also carried out inversions for models built using a modified profile of the mean Rosseland opacity and taking into account the recent revision of the neon abundance. The modification is implemented as a combination of a polynomial and a Gaussian peaked around $\log T = 6.35$. The general behavior of the considered alteration of the opacity profile is motivated by the current discussions in the opacity community regarding uncertainties in conditions similar to those of the base of the solar convective envelope. These models also include an additional macroscopic mixing of the chemical elements at the base of the convective zone in the form of either turbulent diffusion or overshoot.

The opacity modification is implemented as a multiplicative factor to the mean Rosseland opacity

$$\kappa' = (1 + f_{\kappa}(T)) \kappa, \quad (4)$$

with κ the original value of the mean Rosseland opacity, κ' the modified value and $f_{\kappa}(T)$ the parametric function considered. An illustration of $f_{\kappa}(T)$ is provided in **Figure 6**, the modification is cut at lower temperatures than those of the position of the base of the convective zone, as these regions will not affect the solar modeling problem. However, in stars other than the Sun, modifications can also be expected in other regimes and their amplitude might be higher than what is found in the solar case. As can be seen, most of the alteration is localized below the base of the solar convective zone, and the order of magnitude is similar

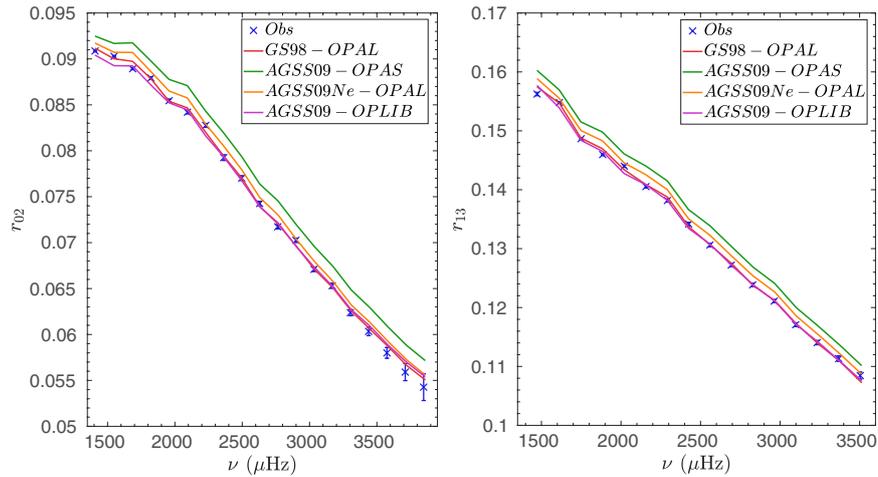


FIGURE 5 | Comparison between the observed frequency separation ratios $r_{0,2}$ and $r_{1,3}$ from the BiSON data and those of solar models built with various abundance and opacity tables.

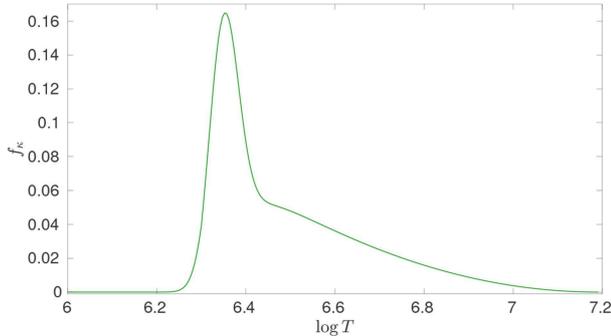


FIGURE 6 | Modification to the opacity profile used in the solar models denoted as “Poly.” $f_k(T)$ is the increase in relative opacity applied during the evolution.

to the value given by Pradhan and Nahar (2018) and Zhao et al. (2018)² whereas at higher temperatures, the modification quickly drops to values of the same order of magnitude as the various standard opacity tables. From a physical point of view, the sharp decrease in opacity uncertainties at higher temperatures due to the higher ionization state of the various chemical elements and the reduced contribution of photon absorption to the total opacity budget. In our study, the opacity modification is applied throughout the evolution and each of these “corrected” models is recalibrated individually.

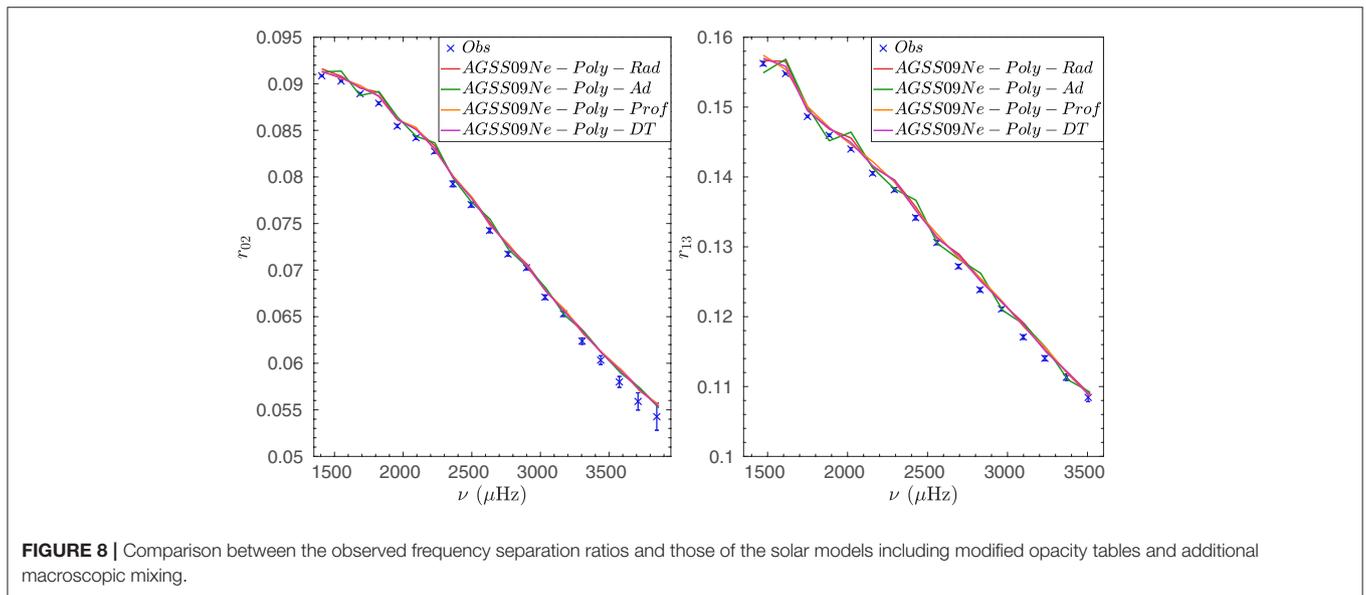
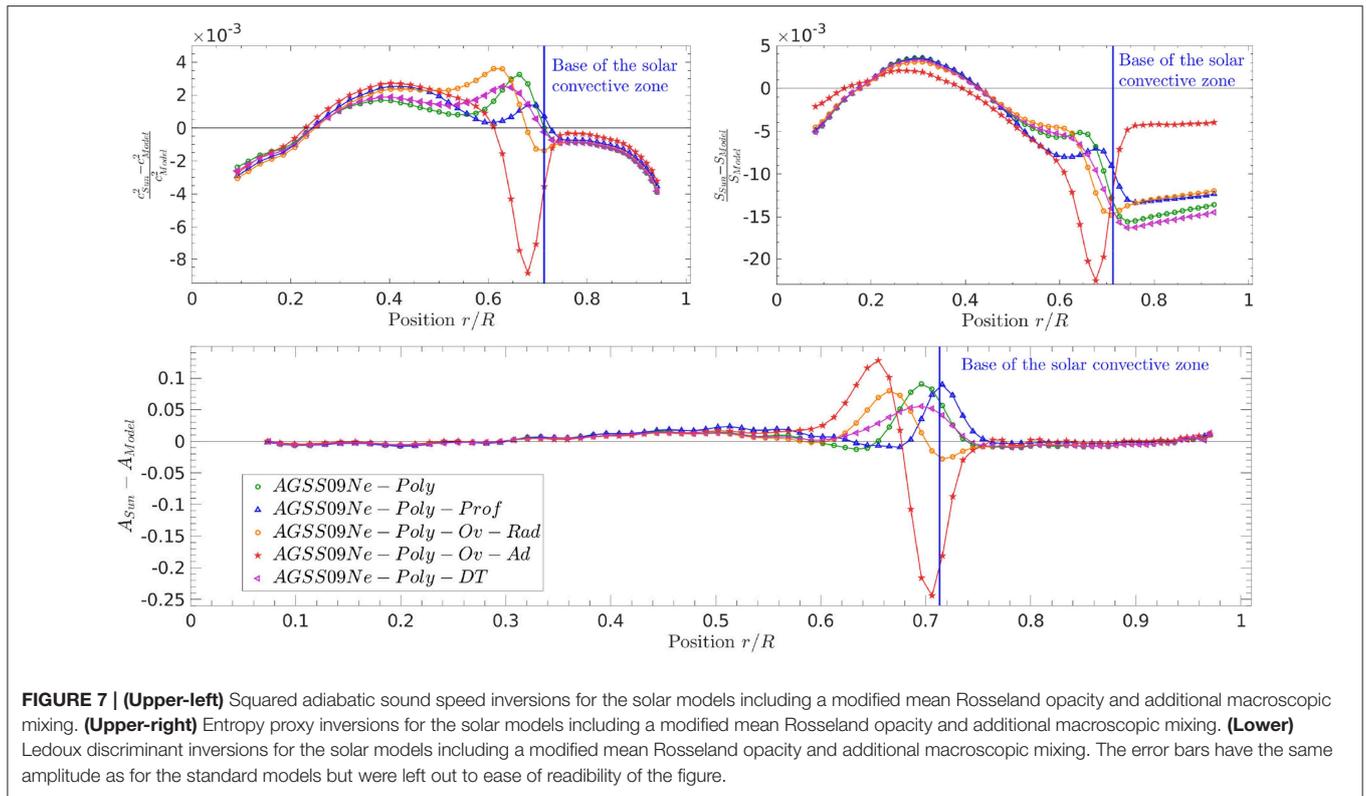
We considered models built with the AGSS09 abundance tables, including the corrected neon abundance, the OPAL opacity tables and the SAHA-S equation of state. The motivation behind the use of the SAHA-S EOS was to include the most recent version of an EOS relying on the chemical picture. We summarize

²A. Pradhan, private communication.

the physical ingredients of these modified solar models in **Table 3**. All these models have been built with the idea of seeing how well the agreement with all the seismic diagnostics could be improved and what we could learn about the degeneracies of the solar modeling problem. A more extended study can be found in Buldgen et al. (2018) where we have investigated various modifications to the opacity profile using various standard opacity tables as a starting point. Similar studies using modified models can also be found in Montalbán et al. (2006), Christensen-Dalsgaard et al. (2009), Christensen-Dalsgaard and Houdek (2010), Ayukov and Baturin (2011), Ayukov and Baturin (2017), and Christensen-Dalsgaard et al. (2018).

We illustrate in **Figure 7** the results of the c^2 , $S_{5/3}$ and A inversions for these various modified models. In **Figure 8**, we compare the frequency separation ratios of these models to those of BiSON data and in **Table 4**, we give the values of various parameters of these solar models of direct interest for helioseismology. We have used the following naming convention for the additional mixing at the base of the convective envelope: “AGSS09Ne-Poly-DT” denotes a model where we used Equation (1) with the values of 50 and 2 for the D and N coefficients respectively, whereas “AGSS09Ne-Poly-Prof” denotes the use of the values 7500 and 3 for these coefficients. “AGSS09Ne-Poly-Rad” denotes the uses of a step overshoot function of $0.3H_p$ using the radiative temperature gradient in the overshooting region and an instantaneous mixing of the chemical elements whereas “AGSS09Ne-Poly-Ad” denotes the uses of the same step overshoot function but fixing the temperature gradient to the adiabatic gradient.

As can be seen from **Table 4**, the parameters of these models are in much better agreement with helioseismology. For nearly all models, the position of the base of the convective zone is in near perfect agreement with helioseismic constraints. The only exception being the model including adiabatic overshoot, which leads to an extension of the base of the convective zone far beyond



what is expected from helioseismology and generates an glitch in the sound speed profile due to the too steep position of the transition in temperature gradients. This is perfectly illustrated in the c^2 and A profiles (in red in **Figure 7**) which show large deviations in the transition region.

All models also present a significant increase in the helium abundance in the convective zone. This is a direct consequence of the extended region over which the opacity is increased,

which leads to a reduced initial hydrogen abundance and thus a higher initial helium abundance. However, the values still remain slightly lower than the helioseismic value³, implying that, while the base of the convective zone is placed at the right position, something is still amiss in the solar models. This is confirmed

³We consider here a conservative interval between 0.245 and 0.26 in agreement with recent studies (Vorontsov et al., 2013).

TABLE 4 | Parameters of the solar models with modified opacities and additional mixing used in this study.

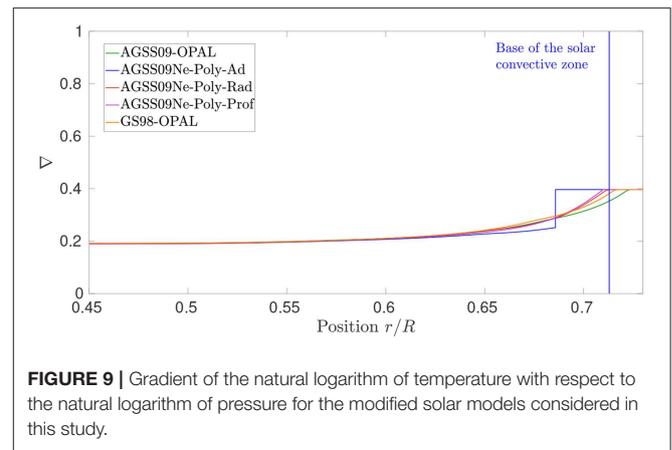
Name	$(r/R)_{BCZ}$	$(m/M)_{CZ}$	Y_{CZ}	Z_{CZ}	Y_0	Z_0
AGSS09Ne-Poly	0.7122	0.9757	0.2416	0.01385	0.2692	0.01494
AGSS09Ne-Poly-DT	0.7106	0.9762	0.2425	0.01383	0.2685	0.01466
AGSS09Ne-Poly-Prof	0.7121	0.9756	0.2460	0.01376	0.2696	0.01500
AGSS09Ne-Poly-Rad	0.7118	0.9757	0.2437	0.01381	0.2692	0.01495
AGSS09Ne-Poly-Ad	0.6871	0.9757	0.2438	0.01381	0.2700	0.01506

by a closer analysis of the inversion results and the frequency separation ratios.

From the upper-left panel of **Figure 7**, we can see that the sound speed profile is in very good agreement with helioseismic results. However, considering the amount of ingredients that have been fine-tuned, the presence of very significant deviations below 0.6 solar radii indicate that something still needs to be corrected in these models. Similarly, the height of the entropy plateau is still off by more than 1% for all models, except the model including adiabatic overshoot which strongly disagrees with the base of the convective zone. Such disagreements give weight to the hypothesis that, beyond corrections to the radiative opacities, the modeling of the transition in temperature gradient at the base of the convective region will have a strong impact on the inversion results. This is a well-known fact, which has been analyzed and discussed by Christensen-Dalsgaard et al. (2011, 2018). However, the current results show in a new way the potential of helioseismic data to provide very stringent constraints on the solar structure.

The Ledoux discriminant inversions also illustrate the full potential of these diagnostics. In the lower panel of **Figure 7**, we can see that the inversions clearly show the different behaviors of various types of chemical mixing unlike the results illustrated in **Figure 4**. The reason why these different behaviors were not visible in **Figure 4** is due to the fact that the mixed region was actually compared to a region that is fully mixed in the Sun. It seems that any type of mixing could provide a significant improvement in the Ledoux discriminant, especially the adiabatic overshooting. In the modified models, since the fully mixed region in the models more closely resembles that of the Sun, the Ledoux discriminant is far more useful in disentangling the various types of macroscopic mixing occurring below the transition in temperature gradient.

In **Figure 8**, we compare the values of the frequency separation ratios from our to observations. From a comparison between **Figures 5, 8**, we can see that the opacity modification induces a slight improvement in comparison with the standard AGSS09Ne model, especially at lower frequencies. However, it appears that the model built with the GS98 abundances still performs better. This also advocates for change in opacity over a wider range of temperature, which could be linked to a revision

**FIGURE 9** | Gradient of the natural logarithm of temperature with respect to the natural logarithm of pressure for the modified solar models considered in this study.

of the equation of state used in the opacity computations in such regimes. In practice, opacity computations are expected to be more robust at higher temperatures, as less transitions come into play. The equation of state used by different groups to compute the tables may explain some of the differences⁴. However, the large differences observed in the Bailey et al. (2015) experiment results may also be linked to other issues in current opacity computations (Krief et al., 2016; Nahar and Pradhan, 2016; Pain and Gilleron, 2019). (Krief et al., 2016; Nahar and Pradhan, 2016; Pain and Gilleron, 2019).

Indeed, changes of even a few percent at higher temperatures could significantly affect the frequency separation ratios, as well as the agreement with the helioseismic helium abundance in the convective envelope. Modifications of such amplitude are within the uncertainties of the opacity tables (Guzik et al., 2005, 2006) and thus do not imply significant revisions of the physics in opacity computations, unlike the modifications required at the base of the solar convective zone. Amongst the modified models, the addition of macroscopic mixing of the chemical elements does not have a significant impact on the ratios, with the exception of the adiabatic overshoot, which adds an oscillatory signal due to the large mismatch in the position of the base of the convective zone in this model.

In **Figure 9**, we illustrate the gradient of the natural logarithm of temperature with respect to the natural logarithm of pressure ($\nabla = \frac{d \ln T}{d \ln P}$) for the various modified solar models considered in our study. As can be seen, the combination of both opacity modifications and chemical mixing allows to place the base of the convective zone in very good agreement with helioseismology, with the exception of course of the adiabatic overshoot. One can also note the slight differences in temperature gradient near the base of the convective zone for the various mixing. As the mean molecular weight gradients will be also very different, it can also be easily understood why the Ledoux discriminant inversions offer a great opportunity to probe chemical mixing just below the base of the convective zone. In the **Appendix** section, we discuss

⁴This statement is however, difficult to assess, as the equation of state used for opacity computations is often not available, at least for the recent OPAS and OPLIB tables.

how the Ledoux discriminant can be separated in its chemical and thermal components and how additional insights could be gained from these inversions. It also appears that the temperature gradient quickly follows a very similar behavior at 0.5 solar radii for all models, as expected from the small amplitude of the considered opacity modifications at higher temperatures. These small shifts are however of constant sign over the whole radiative layers and thus still impact the initial hydrogen abundance of the calibrated model and its present-day helium abundance in the convective zone.

4. IMPACT OF SOLAR MODEL MODIFICATIONS ON THE 16CYG BINARY SYSTEM

4.1. From Global Helioseismology to Asteroseismology

With the advent of the CoRoT (Baglin et al., 2009) and *Kepler* missions (Borucki et al., 2010), asteroseismology of solar-like oscillators has become the golden path to characterize other stars than the Sun. Today, asteroseismic modeling is considered a standard tool to derive precise values of stellar fundamental parameters of stars, namely mass, radius and age which are of particular interest for fields such as exoplanetology and Galactic archaeology. While the high precision of these determinations is undisputable, as they result from the high precision of the seismic data, their accuracy will of course depend on the actual accuracy of the underlying stellar evolution models.

Consequently, efforts have recently been made to quantify the impact of physical ingredients on the determination of these fundamental parameters. In parallel, the wealth of seismic and non-seismic data led to the development of sophisticated modeling tools (Bazot et al., 2012; Gruberbauer et al., 2012; Rendle et al., 2019) and new analyses techniques (see for example Verma et al., 2014; Roxburgh, 2016; Farnir et al., 2019). Most notably, the advent of space-based photometric data allowed the extension of seismic inversion techniques to other targets than the Sun. The use of these methods had been discussed with artificial data in a few pioneering works (see for example Gough and Kosovichev, 1993a,b; Roxburgh et al., 1998; Roxburgh and Vorontsov, 2002).

From a seismic point of view, the first obvious targets to attempt seismic inversions are low-mass main-sequence solar-like oscillating stars observed by *Kepler* during the whole duration of the nominal mission. These stars have been assembled in a single catalog, called the *Kepler* LEGACY sample (Lund et al., 2017; Silva Aguirre et al., 2017). Amongst these stars, the most constrained targets are the components of the 16Cyg binary systems. In addition to high-quality seismic data, interferometric, photometric, and spectroscopic constraints are also available, providing an unprecedented dataset for such solar twins. Various studies have been dedicated to their modeling using forward and inverse approaches.

Given its extensive datasets, the 16Cyg binary systems offers an excellent opportunity to test the ingredients of stellar models to a degree of sophistication similar to helioseismic

TABLE 5 | Parameters of the 16Cyg models with modified opacities and additional mixing used in this study.

	16Cyg A	16Cyg B
$M (M_{\odot})$	1.06	1.02
$R (R_{\odot})$	1.22	1.12
X_0	0.700	0.700
Z_0	0.022	0.022
α_{MLT}	1.82	1.82

investigations. From a physical point of view, one can consider the targets of the *Kepler* LEGACY sample as additional experimental points to understand the solar modeling problem. In this section, we will carry out the academic exercise of considering the impact of the solar modeling problem on the seismic constraints of the 16Cyg binary system.

In our exercise, we computed different sets of models for the 16Cyg binary systems using the same initial conditions, summarized in **Table 5**, but various physical ingredients. These values have been taken from a preliminary modeling of the 16Cyg binary system presented in Farnir et al. (2019)⁵. The first set of models is composed of standard models of both stars built using the AGSS09 abundances, the FreeEOS equation of state, the OPAL opacities, following the diffusion formalism of Thoul et al. (1994), the classical mixing-length theory of convection and using an Eddington gray atmosphere. First, we test opacity modifications, considering that a re-investigation of the 16Cyg binary system would be required should updated opacity tables be made available⁶. Hence, we computed a second set of models of 16CygA&B that includes the polynomial opacity modification that we considered for model “AGSS09Ne-Poly” of section 3.5., represented in **Figure 6**. Finally, the third set of models considers both this opacity modification and the parametric macroscopic mixing of Proffitt and Michaud (1991) as in model “AGSS09Ne-Poly-Prof” of section 3.5. The properties of the models of the various sets are summarized in **Table 6**. The models have been calibrated by evolving them until they reach the radius values determined using interferometry (White et al., 2013). Hence, each has a different age, as a result of the differences in their physical ingredients. All ages are however consistent between the components of the binary system.

The goal of this exercise is fairly simple: to illustrate the impact a potential solution to the solar modeling problem could have on the determination of stellar fundamental parameters such as mass, radius, and age for stars other than the Sun. Hence, it also serves the purpose of reminding the model-dependence of asteroseismic investigations, but also how the high-quality asteroseismic data can help us better understand the physical processes acting inside stars by providing other

⁵Namely, the results are only presented for the A component in Table 2, second column of Farnir et al. (2019), but the modeling was carried out for both components.

⁶Buldgen et al. (2016b) also investigated the use of the OPAS opacities in their modeling, but this should be done more thoroughly.

TABLE 6 | Parameters of the 16Cyg models with modified opacities and additional mixing used in this study.

Name	$(r/R)_{BCZ}$	$(m/M)_{CZ}$	Y_{CZ}	Z_{CZ}	Age
CygA-Std	0.7070	0.9793	0.2300	0.01863	7.18 Gy
CygB-Std	0.6979	0.9739	0.2332	0.01887	7.48 Gy
CygA-Poly	0.7000	0.9771	0.2312	0.01876	7.30 Gy
CygB-Poly	0.6.850	0.9691	0.2348	0.01898	7.61Gy
CygA-Poly-Prof	0.7000	0.9765	0.2454	0.01950	7.48 Gy
CygB-Poly-Prof	0.6860	0.9699	0.2438	0.01958	7.76 Gy

experimental conditions to those at hand in helioseismology. A good illustration of the limitations of stellar models, and thus a central point for which a revision of their ingredients could have a significant impact is found for example in the current discussions related to the transport of angular momentum on both the main-sequence and the red giant branch (Eggenberger et al., 2017, 2019; Benomar et al., 2018; Ouazzani et al., 2018). In those cases, the sensitivity of the proposed mechanisms to the chemical composition gradients, and thus their validity, could be influenced by revisions of some of the physical ingredients at play when studying the solar modeling problem.

4.2. Impact of the Solar Problem on Classical Seismic Indices

Forward seismic modeling techniques can vary quite extensively depending on the quality of the data. The crudest approach uses the so-called scaling laws to infer stellar properties, whereas the most sophisticated techniques use various combinations of the individual frequencies. Amongst them, the use of the $r_{0,2}$ and $r_{1,3}$ ratios defined by Roxburgh and Vorontsov (2003b) and defined in Equations (2, 3) allow to infer the internal structure without too much dependency on the upper layers. Indeed, the direct use of the individual frequencies is not optimal for solar-like oscillators, as they are strongly influenced by the “surface effect” problem and lead to unrealistic precisions on stellar parameters⁷.

In **Figure 10**, we compare the frequency separation ratios of the various models with respect to the observations. As can be seen, none of the models fit very well the seismic data at hand, although the agreement is not catastrophic either. This is not surprising, as they have not been fitted to the individual frequency ratios. However, what is more striking is that the variations between the theoretical models that are induced by the opacity modifications and the additional turbulent mixing is significant with respect to the observational uncertainties for most of the data. This is not really surprising but emphasizes the model-dependence of seismic modeling results. As expected, the variations observed here are also reflected in the fundamental parameters. However, this behavior should be inspected for a

⁷Indeed, early studies already discussed the fact that individual frequencies did not constitute independent constraints on stellar structure and should not be used directly as inputs of stellar forward modeling.

given fit of the seismic constraints to be certain that the accuracy of the inferences is so significantly affected.

Unsurprisingly, the most affected parameter is the age, for which variations between the standard models and the models including opacity modifications and additional mixing can reach around 4%, whereas including only the opacity modifications only induces a variation of almost 2%. These variations are of course quite small. However, they are as large as the uncertainties on the fundamental parameters derived from seismic modeling studies using the whole *Kepler* dataset (see for example Metcalfe et al., 2015; Silva Aguirre et al., 2017) and are of course only indicative of the impact of the changes of given ingredients for a given set of initial conditions for the evolutionary models. Taking into account the uncertainties on the other fundamental parameters for a given set of constraints could lead to a larger spread in age.

In Buldgen et al. (2016a), we demonstrated that a similar spread in age could also be seen by altering the efficiency of microscopic diffusion. Hence, we can state that modifying the formalism of microscopic diffusion, using the Paquette et al. (1986) approach and considering partial ionization when computing microscopic diffusion would also cause a change in age of the order of one per cent. Consequently, we can confirm that for the current best *Kepler* targets (Borucki et al., 2010), as well as for future TESS and PLATO targets (Rauer et al., 2014; Ricker et al., 2015), the main contributors to the fundamental parameters will not be the propagation of the observational uncertainties onto the inferred parameters, but the physical ingredients of the underlying grids of evolutionary models. In such a context, very high precision results, for example as those of Metcalfe et al. (2015) or (Buldgen et al., 2016a,b) for the 16Cyg binary system, should only be taken as valid for a given set of physical ingredients.

To illustrate some of the differences between the various models considered here, we show in **Figure 11** the temperature gradients inside the models. As can be seen, the modification of the mean Rosseland opacity does not induce any significant variations in the deep layers. The main variation is unsurprisingly located at the base of the convective zone. A small modification is also seen in luminosity but well below the observational error bars. Interestingly, the inclusion of turbulent diffusion has altered the deep layers of the model. The variations are actually due to a change of the hydrogen abundance in the central layers. Indeed, X_C is 0.038 in the model including turbulent diffusion and 0.029 in the standard model. This could be due to the inhibition of microscopic diffusion that is induced by the turbulent mixing.

The extreme impact of the Proffitt and Michaud (1991) parametric approach to turbulent diffusion can also be seen in **Figure 12**, where we illustrate the metallicity profile of the models of the 16Cyg binary system. One can see the influence of turbulent diffusion on the surface abundance of metals that is significantly higher than in the standard model and the model including modified opacity. It is also interesting to note the slight differences between these two models. We emphasize here that there is no modification to the mixing of chemicals. However, there is an indirect impact of the opacity modification on chemicals through the modification of the position of the

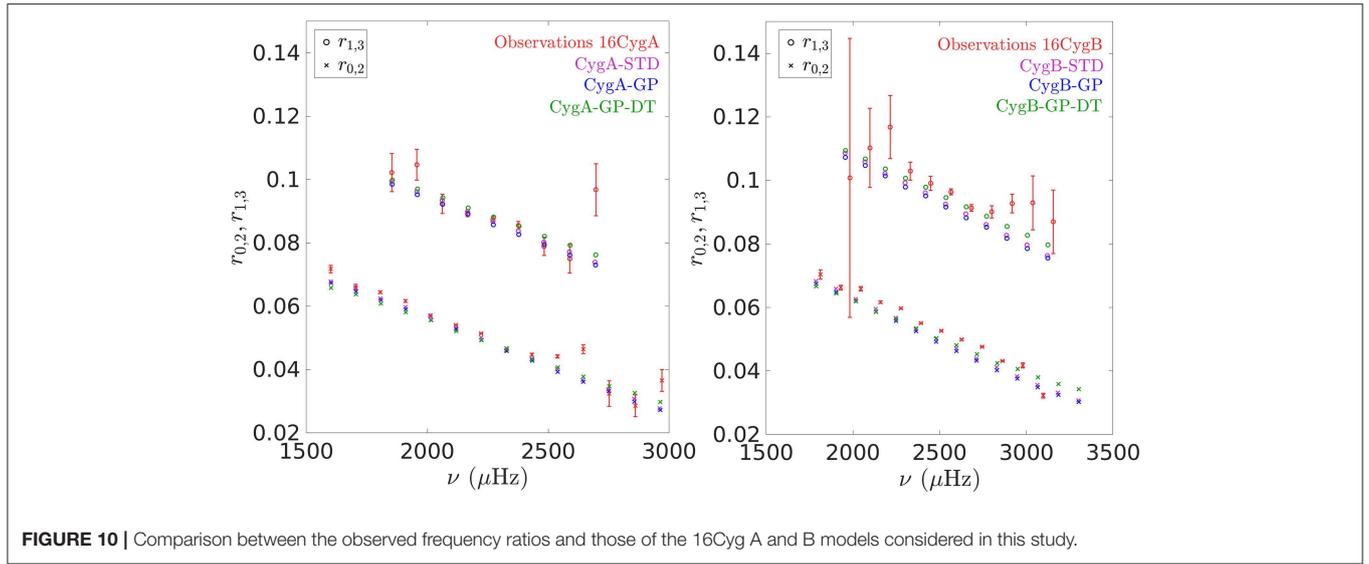


FIGURE 10 | Comparison between the observed frequency ratios and those of the 16Cyg A and B models considered in this study.

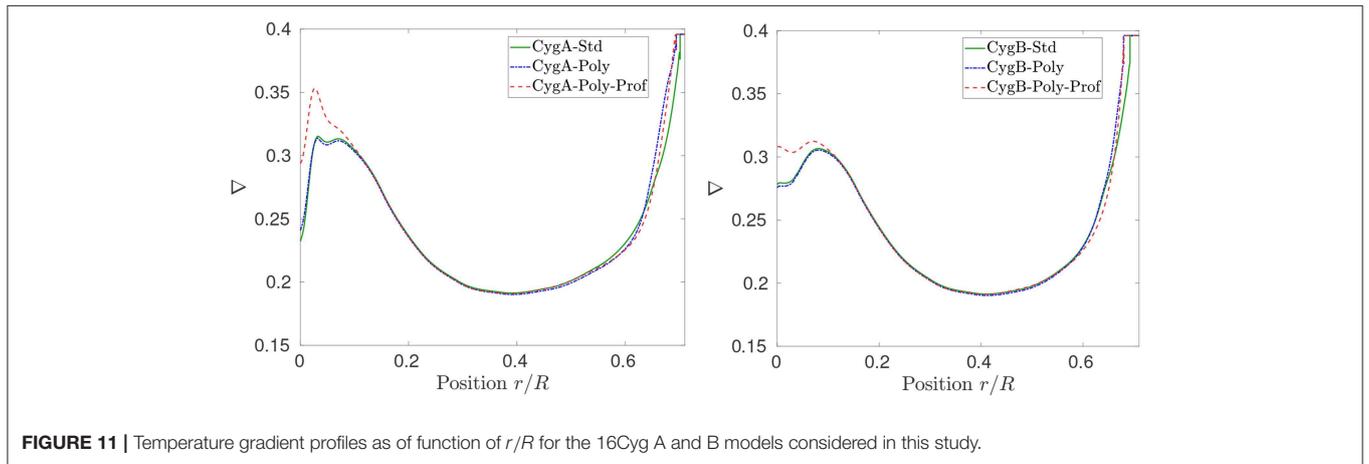


FIGURE 11 | Temperature gradient profiles as of function of r/R for the 16Cyg A and B models considered in this study.

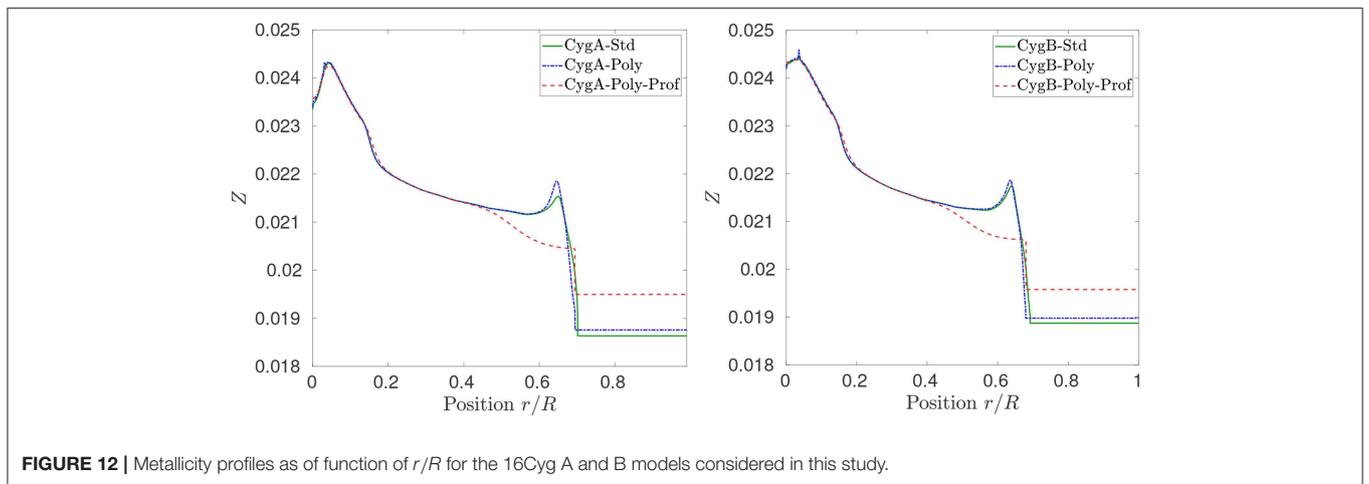


FIGURE 12 | Metallicity profiles as of function of r/R for the 16Cyg A and B models considered in this study.

base of the convective envelope. Here, the higher opacity leads to a larger convective envelope, extending at higher temperatures. This implies that microscopic diffusion will be slightly less

efficient and thus, that a higher metallicity will be found in the envelope but also just below the envelope, where metals tend to accumulate over the duration of the evolution. This is

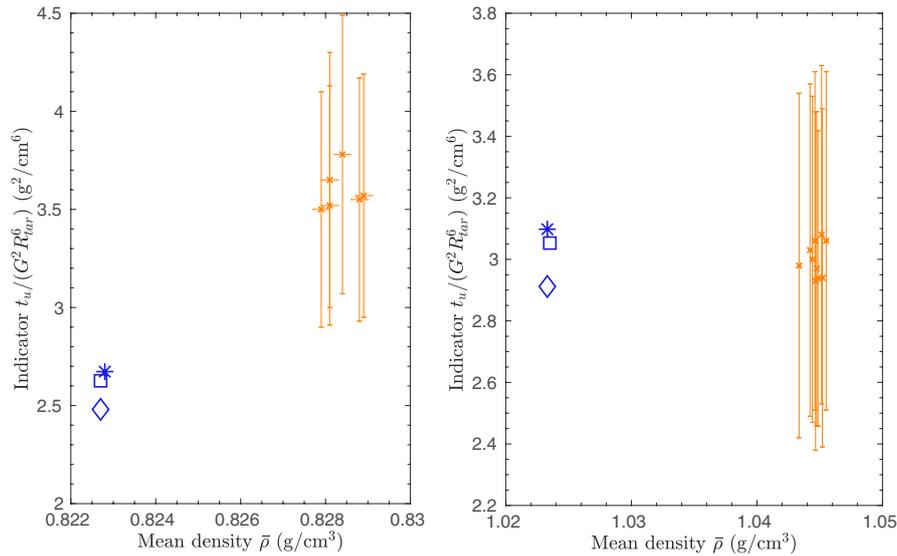


FIGURE 13 | Comparison of t_u and $\bar{\rho}$ inversions for 16CygA and B for the models considered in this study. The blue symbols refer to the reference models: * for the standard models, \square for the models with modified opacity and \diamond for the models including both the opacity modification and turbulent diffusion.

particularly well seen in the left panel of **Figure 12** in the case of 16CygA.

Overall, the modifications we see in the models of both components remain quite small. They could, however, be modified by the seismic optimization procedure which will alter the initial conditions of the evolutionary sequence. Therefore, some variations seen in the models assuming the same initial parameters but different approaches for the mixing of the chemical elements might be erased at the expense of a change of fundamental parameters such as mass, radius and age, as was noted in Buldgen et al. (2016b). On this matter, the case of 16Cyg is particularly interesting and promising, as both stars form a binary system. This adds another level of constraint on their initial composition and their age, further reducing the amplitude of the changes one can make to the models.

4.3. Impact of the Solar Problem on Indicator Inversions

In addition to classical seismic forward modeling, Reese et al. (2012), Buldgen et al. (2015b), Buldgen et al. (2015a), and Buldgen et al. (2018) developed inversions of so-called structural indicators, defined as integrated quantities, which can offer additional constraints beyond the use of classical seismic indices. In this section, we briefly discuss the potential variations in these seismic indicators that can be expected from the modifications of the physical ingredients of the 16Cyg binary system models. It should however be noted that these results are preliminary and that the true diagnostic potential of the inversions might be further improved. For example, the use of non-linear inversions, following the formalism of Roxburgh and Vorontsov (2003a) may provide an excellent complement, less sensitive to surface effects, to the classical formalism used in global helioseismology.

Here, we limit ourselves to a brief discussion on the diagnostic potential of structural indicators, namely the t_u indicator from Buldgen et al. (2015a), defined as

$$t_u = \int_0^R f(r) \left(\frac{du}{dr} \right)^2 dr, \quad (5)$$

with $u = P/\rho$, R the stellar radius and $f(r)$ a suitably chosen parametric weight function (see Buldgen et al., 2015a, for details).

The impact of the changes in physics on the t_u indicator can be seen in **Figure 13**, where we recreate the figures from Buldgen et al. (2016a,b) presenting the inversion results. As can be seen, the variations are quite small compared to the uncertainties of the inversions. However, the variations of the fundamental parameters will strongly affect the values of the structural indicators. As the t_u indicator scales with M^2 , its value is also strongly dependent on the mass and radii inferred from the forward modeling procedure. Including a small mixing at the base of the envelope of 16CygB, Buldgen et al. (2016b) found a variation of the t_u indicator between 12 and 20% between some models. However, it is very unlikely that this variation only results from changes in the structure of the models, but rather stems also from inaccuracies in the stellar fundamental parameters. This is illustrated in **Figure 13**, since the models are built using the same mass and evolved until they have the same radius. From these tests, we can see that the maximum variations in t_u at a given mass and radius are of approximately 6%; this is well below the uncertainties of the t_u indicator that can reach values around 16%. On a sidenote, it also appears that the mean density value is not well reproduced for 16CygB, with a difference of around 2% between the reference models and the inverted value. The t_u value seems however to be in very good

agreement with the inversions, but it remains to be seen whether one can obtain a good agreement for all inverted quantities. In addition to the t_u and $\bar{\rho}$ inversions, additional indicators presented in Buldgen et al. (2018) can be used to constrain the internal structure of the star. Moreover, taking into account the lithium and beryllium abundances, whenever measured, are also key additional observations to accurately depict the evolution of solar-like stars (Deal et al., 2015; Thévenin et al., 2017). In the case of the 16Cyg binary system, the lithium abundance for both stars has been determined (King et al., 1997; Tucci Maia et al., 2014), finding the lithium abundance to be more depleted in the B component by approximately a factor 5. Deal et al. (2015) suggested that those differences could have been provoked by an accretion of planetary matter on 16CygB which would have triggered thermohaline mixing. Following these results, the differences observed in both the indicator inversions and the lithium abundance motivated the study of Buldgen et al. (2016b) who assessed the impact of extra-mixing on the inversion results. However, since various physical ingredients could affect the inversion result, Buldgen et al. (2016b) concluded that the problem might be degenerate and required a careful re-study. Moreover, it is still unclear whether the process leading to the far more significant lithium depletion in the B component would still leave a mark on its present-day structure.

Results for some *Kepler* LEGACY stars have already been presented in Buldgen et al. (2017a), showing the diagnostic potential of these additional indicators. However, Appourchaux et al. (2015) demonstrated the potential of the method of Roxburgh and Vorontsov (2003a) by providing an inversion of the whole hydrostatic structure of a *Kepler* target. There is no doubt that the use of such an approach on the targets of the *Kepler* LEGACY sample in the Gaia era will provide invaluable information for stellar modelers, allowing to test with unprecedented thoroughness our depiction of stellar structure in a much more model-independent way than what is achievable with linear asteroseismic inversions.

5. PROSPECTS AND DISCUSSION

In the previous sections, we presented the current state of the solar modeling problem, with a strong emphasis on helioseismic diagnostics and their capabilities. While it is obvious that global helioseismology is an essential tool to probe the internal structure of the Sun, it does not imply that other fields cannot also reshape the picture of the current solar issue. As mentioned earlier, constraints provided by helioseismic inversions are somewhat degenerate. Indeed, they do not give direct constraints on the temperature gradients inside the Sun as they also probe variables related to a combination of temperature and mean molecular weight.

For example, measurements of neutrinos fluxes also provide stringent complementary constraints on the temperature of the most central regions of the solar core, probing a zone inaccessible to global helioseismology. Recent simultaneous measurements of all neutrinos of the pp-chain (Borexino Collaboration et al., 2018) provide a very complete picture of the solar core. In the future,

measurements of the CNO neutrinos could provide more direct constraints on the chemical composition of the solar core (see Gough, 2019, for a recent discussion), in particular, its oxygen abundance, offering strong constraints on the chemical mixing during the evolution of the Sun. However, it is also clear that the neutrino fluxes measurements could also be significantly affected by a revision of the electronic screening formulas used in stellar models (Mussack and Däppen, 2011), as mentioned by Vinyoles et al. (2017).

Similarly, the solar lithium and beryllium abundances also play a key role in understanding the evolution of the Sun (Richard et al., 1996; Piau and Turck-Chièze, 2001). They are closely linked to the intensity and the extent of the mixing at the base of the convective zone, thus constraining the physical processes that can be at play in this narrow region. As such, a key point for the future of solar modeling is understanding the nature and impact of so-called “non-standard” processes often treated using *ad-hoc* prescriptions. The stakes of the solar modeling problem are not so much to validate a value of the solar metallicity, but to trigger the development of new generations of stellar models.

In this perspective, the advent of space-based photometry missions and the rapid development of asteroseismology offers an unprecedented opportunity for stellar modelers. Today, we can use seismology to precisely probe the interior of thousands of stars, providing stellar modelers with additional experimental measurements to refine their understanding of the theory of stellar structure and evolution, from the microscopic scales of nuclear reactions and radiative transfer to the large scale of turbulent hydrodynamical motions.

In a provocative way, one could state that stellar physics is far from being reduced to an optimization problem and that the main concerns of asteroseismic modelers should not be on providing extremely precise stellar fundamental parameters. Indeed, those will always be model-dependent. Thus, their precision will always be overestimated, as the systematic differences that can result from inaccurate physical ingredients are difficult to estimate. On the contrary, stellar and solar seismologists should focus on the quality and relevance of their inferences and the connection between their data and the actual physical constraints that they contain. With this mindset, asteroseismology will truly fulfill its role of complementary, “experimental” domain of theoretical stellar physics.

Of course, the progress of stellar physics will require a strong effort on the modeling side. Improvements of the physical ingredients of stellar models are the keys to the solution of the solar modeling problem. From a macrophysical point of view for example, the development of hydrodynamical simulations also offers great potential for our understanding of turbulence in stellar conditions (see for example Jørgensen et al., 2018, for an application in the solar case). Linking these simulations to a formalism that can be used in stellar evolution codes is one of the key challenges of the coming years, especially for our depiction of the evolution of convective cores. From a microphysical point of view, further improvements of radiative opacities, microscopic diffusion, or the equation of state will also lead to revolutions in the field and will certainly play a key role for the solar problem.

A first step in this direction is to compare various evolution codes to separate the numerical contributions to the uncertainties to those that clearly result from physical inaccuracies. This approach, although time-consuming and not very rewarding, is also crucial to motivate further developments and improvements of stellar evolution codes from a numerical point of view. In that respect, it is of course pointless to claim the superiority of one code over others, as much as it is useless to use them as blackboxes. It is clear that the numerical development of some codes has been focused on implementing thoroughly specific aspects (e.g., rotation, magnetic instabilities and internal gravity waves for the GENEC code Eggenberger et al., 2008, microscopic diffusion including a complete treatment of radiative acceleration for the Montral-Montpellier code Turcotte et al., 1998; Richer et al., 2000; Richard et al., 2001 and the Toulouse-Geneva evolution code Théado et al., 2012, or the consistent evolution of convective boundary and quality of the models for seismology of the Liège code Scuflaire et al., 2008b).

6. CONCLUSION

This paper has focused on providing a brief review of the solar modeling problem, mainly from a helioseismic perspective. We have discussed in section 2 the various contributors to the current issue. Unsurprisingly, the opacity remains the usual suspect and probably the most significant contributor to the disagreements between standard solar models and helioseismic constraints. Beside the opacities, the mixing of the chemical element and the equation of state are the other usual suspects who could have a significant impact on the solar structure. In section 2.5, we also briefly presented some additional processes that could impact the present-day solar structure and thus the current discrepancies. While they are not commonly presented in the literature, they should perhaps not be totally dismissed.

In section 3, we presented inversion results, frequency separation ratios and convective envelope properties of a sample of solar models built with various physical ingredients. The constraining nature of combining this entire set of information into one consistent study is very clear, as it allows to isolate the effect of the various contributors to the solar problem. We showed the impact of an extended modification of the mean Rosseland opacity, for which the largest amplitude of the correction lay in the conditions of the iron opacity peak at $\log T = 6.35$. We also show that the increased neon abundance found by Landi and Testa (2015) and Young (2018) significantly reduces the discrepancies of the low-metallicity solar models. In addition, we show that the combined inversions could provide stringent constraints on the type of mixing at the base of the solar convective zone. Further extensions of this study using a non-linear inversion technique and/or the phase shift of the mode frequencies to properly reproduce the transition in temperature gradients will provide key constraints for the physical implementation of overshooting at the base of stellar envelopes.

In section 4, we have briefly discussed the impact of the solar modeling problem on the structure of the best *Kepler* targets, the components of the 16Cyg binary system. To do so, we have computed models with a given set of physical ingredients using the standard solar model framework, including the opacity modification we used for our modified solar models or including both the opacity modification and turbulent diffusion using the parametric approach of Proffitt and Michaud (1991). We demonstrated that the impact of such modifications would be significant at the level of precision required from asteroseismic investigations. In that respect, improving the current seismic inference techniques is crucial to better exploit the constraints on these uncertain processes and, by providing more stringent analyses, to ensure the success of future space missions such as PLATO and to bring theoretical stellar physics to a new level of accuracy. Improving indicator inversions but also generalizing the use of non-linear inversions can be foreseen as the most promising way to fully exploit the data. However, new approaches to treat the seismic information in forward modeling methods also provide important insights in the limitations of seismic information and define the necessary reference models for seismic inversions (Farnir et al., 2019).

Ultimately, the extension of such advanced modeling strategies further away from the solar conditions will allow to truly probe the limitations of the current state of theoretical stellar physics. In conclusion, the future of asteroseismology is deeply rooted in its history and the early developments of helioseismology. From these solid grounds, asteroseismologists can further develop this young and successful research field. This requires to solve the solar modeling problem, to promote synergies between stellar physicists, seismic modelers and experts in hydrodynamical simulations without perhaps falling into the trap of a race to precision of stellar parameters that are intrinsically model-dependent.

DATA AVAILABILITY

Publicly available datasets were analyzed in this study. This data can be found here: <https://academic.oup.com/mnras/article/446/3/2959/2893058#supplementary-data>.

AUTHOR CONTRIBUTIONS

GB, SS, and AN contributed equally to the paper.

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Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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APPENDIX

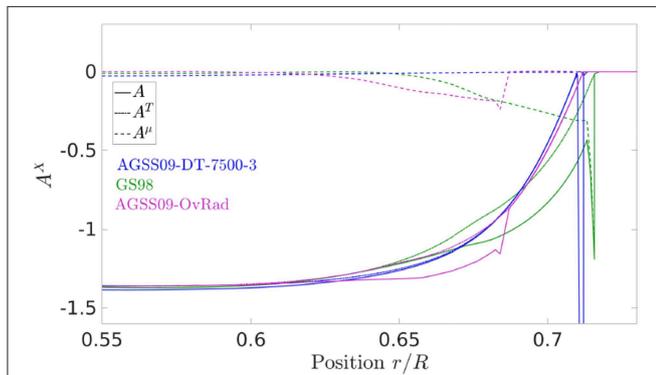


FIGURE A1 | Decomposition of the Ledoux discriminant in its thermal and chemical composition gradients contributions for a model built with the GS98 abundances, and other models including modified opacity tables and macroscopic chemical mixing.

To have a closer look at the impact of extra mixing at the base of the convective zone, we illustrate in **Figure A1** the various contributions to the Ledoux discriminant, A . We use the following definition

$$A = -\frac{r\delta}{H_p} \left(\nabla_{Ad} - \nabla + \frac{\phi}{\delta} \nabla_{\mu} \right), \quad (\text{A1})$$

with H_p the pressure scale height, μ the mean molecular weight, P the pressure and $\delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$, $\phi = \left(\frac{\partial \ln \rho}{\partial \ln \mu}\right)_{P,T}$, $\nabla = \frac{d \ln T}{d \ln P}$, $\nabla_{Ad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_S$, with S the entropy, $\nabla_{\mu} = \frac{d \ln \mu}{d \ln P}$. We define the

thermal and chemical contributions to the Ledoux discriminant, A^T and A^{μ} as

$$A^{\mu} = -\frac{r\phi}{H_p} \nabla_{\mu}, \quad (\text{A2})$$

$$A^T = -\frac{r\delta}{H_p} (\nabla_{Ad} - \nabla). \quad (\text{A3})$$

From **Figure A1**, we see that the thermal contribution largely dominates the behavior of the Ledoux discriminant, with the exception of the last percent of the radiative regions. There is a clear difference between a fully mixed overshoot and that of turbulent diffusion. This opens up the possibility to combine seismic diagnostics to distinguish the form of the macroscopic mixing at the base of the solar convective zone and explains the observed differences in **Figure 7**. Using non-linear inversions, as in Corbard et al. (1999), may also help to further constrain the profile of the Ledoux discriminant between 0.67 and 0.71 solar radii. There also appears to be a clear difference in the temperature contribution, A^T , between the various models, around 0.65 solar radii. This difference is due to the too steep temperature gradient at this position, seen for the GS98 model and the model built using radiative overshoot. A parametric modeling of the transition in both temperature and chemical gradients should shed new light the existing degeneracies and provide seismic constraints to works aiming at including hydrodynamical prescriptions in stellar evolutionary codes. To that end, a combination of the analysis of the phase-shift of frequencies, as carried out by Christensen-Dalsgaard et al. (2011) or the use of a non-linear RLS method following the approach of Corbard et al. (1999) would allow to probe the sharp transition in Ledoux discriminant at the base of the convective zone.