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Interval observers design for systems with ostensible Metzler system matrices

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This paper attempts to resolve the problem concerning the interval observers design for linear systems with ostensible Metzler system matrices. Because system dynamics matrices are partially different from strictly Metzler structures, a solution is achieved by constructing a composed system matrix representation, which combines pre-compensated interval matrix structures fixed with a prescribed region of D-stability and the reconstructed strictly Metzler matrix structure, related to the original interval system matrix parameter definition. A novel design procedure is presented, which results in a strictly positive observer gain matrix and guarantees that the lower estimates of the positive state variables are non-negative when considering the given system structure and the non-negative system state initial values. The design is computationally simple since it is reduced to the feasibility of the set of linear matrix inequalities.

KEYWORDS

Metzler systems, parametric constraints, diagonal stabilization, linear matrix inequalities, applied interval analysis, interval observers

1 Introduction

Interval observers have appeared as an alternative technique for robust state estimation (Moisan et al., 2009). Whilst, when using the technique based on classical observers, only the initial condition is assumed to be unknown (Luenberger, 1971), interval observers structures are constructed assuming that the upper and lower bounds of the initial conditions are known (Raïssi and Efimov, 2018; Khan et al., 2020). The main limitation to the interval observers theory is that the trajectories of the system that start from an internally bounded initial condition will enclose the stable system trajectory only if the system is positive that its system matrix is Metzler and Hurwitz and that other matrix parameters are non-negative (Farina and Rinaldi, 2000). Thus, the positivity of interval estimation error dynamics is one of the most restrictive assumptions for interval observers design. When restricted to the Metzler structure of system matrices, as well as to non-negative input and output matrices, such systems are referred to as Metzler systems (Nikaido, 1968; Smith, 1995; Liu et al., 2011), with a stringent approach that reflects the diagonal stabilization principle. Although a certain class of systems can be transformed through a change of coordinates into positive cooperative systems (Mazenc and Bernard, 2011; Mazenc and Bernard, 2014), no general technique exists for such construction.

When maintaining platforms for positive systems with nonnegative states (Nikaido, 1968; Smith, 1995; Moisan et al., 2009), the theory of Metzler matrices (Berman et al., 1989) implies some additional parametric constraints to reflect the system positiveness (Shorten et al., 2009) and to construct the system representation (Son and Hinrichsen, 1996; Gao et al., 2005; Liu et al., 2017; Ito and Dinh, 2020). Since the linear time-invariant system theory cannot be directly used for linear positive systems, various combinations of linear programming and linear matrix inequalities (LMI) are generally used to represent Metzler systems (Ait Rami and Tadeo, 2006; Shu et al., 2008; Anderson and Murray, 2018; Guo et al., 2020). The benefits of a potential unification are presented (Krokavec and Filasová, 2018) when reflecting diagonal stabilization and associated Metzler system matrix parametric representations by a specific set of LMIs.

The system matrix parametric constraints give rise to substantially complex design methods when applied to positive systems with interval-defined model parameters (Ganesan, 2007). To demarcate the object of study in this field, Metzler matrix transforms are reflected for interval observers analysis (Mason, 2012; Chambon et al., 2015). Interval observers design for linear time-varying (LTV) systems, as well as for a class of non-linear time-varying systems with output specifications, exploits static coordinate transformation (Efimov et al., 2013; Raïssi and Efimov, 2018) when translating a stable LPV system to another stable and cooperative LPV system. The LMI-based conditions applicable in interval observers design for positive Metzler systems have also been studied (Krokavec and Filasová, 2020b). The utilization of interval observers for interconnected schemes is often applied in relation to distributed interval estimation and distributed feedback control (Wang et al., 2020; Wang X L et al., 2022; Zhang et al., 2022); this also reflects that their application for continuous linear large-scale systems is limited due to the system's complexity (Wang T et al., 2022). These problems are still open in distributed applications since it is difficult to ensure that the system state will be enclosed by the cooperative estimated upper and lower bounds of the observed system state (Huong, 2022; Li et al., 2022), as well as in interval estimation strategy for anti-disturbance control of drones (Yong et al., 2020).

This paper contributes to the properties of the interval state estimation for linear systems with ostensible Metzler system matrices. It outlines a new LMI-based approach to determine interval observers with positive observer gains using a combined representation of the ostensible Metzler system matrix (Krokavec and Filasová, 2022). Design conditions are formulated using LMIs, respecting the diagonal stabilization principle, Metzler system matrix parametric constraints, and given interval matrix bounds. Because linear systems with ostensible Metzler matrices are not positive, even if their matrix parameters are non-negative, the main limitation to the solution is that the interval estimation only works for system state variables whose trajectories for a non-negative initial state are non-negative.

The outline of this paper is as follows. Following an introduction in Section 1, the basic preliminaries are discussed in Section 2. Section 3 presents the LMI structures necessary for observer stability and the positive gain, and the design method of interval observers for a given class of positive systems is presented in Section 4. To illustrate the design task, its efficiency is demonstrated by numerical solutions in Section 5; in Section 6, conclusions are briefly presented.

Throughout the paper, $\boldsymbol{X} \prec 0$ conveys briefly that a real square matrix $\boldsymbol{X}$ is a symmetric and negative definite, notations $\boldsymbol{x}^T$ and $\boldsymbol{X}^T$ identify the transpose of a vector or a matrix, $\boldsymbol{I}_n$ indicates the nth order unit matrix, function $\rho(\cdot)$ reflects the eigenvalue spectrum of a real square matrix, diag $[\,\cdot\,]$ enters a block diagonal matrix, the symbol * is used as ellipsis in a symmetric matrix, $\mathbb{L}_\infty$ is the set of real function $z(t)$ with the property $\|z(t)\| < \infty$, the relations $\boldsymbol{x}_1 \leq \boldsymbol{x}_2$ and $\boldsymbol{X}_1 \leq \boldsymbol{X}_2$ operate on corresponding elements element-wise, $\mathbb{R}$ ($\mathbb{R}_+$) qualifies the set of (non-negative) real numbers, $\mathbb{R}^{n\times r}$ ($\mathbb{R}_+^{n\times r}$) refers to the set of $n \times r$ (non-negative) real matrices, and $\mathbb{M}_{-+}^{n\times n}$ denotes the set of strictly Metzler square matrices.

2 Basic preliminaries

To explain the technique used, the main question can be illustrated using the linear Metzler systems given as follows:

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{D}\boldsymbol{d}(t), \tag{1}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t), \tag{2}$$

where $\boldsymbol{q}(t) \in \mathbb{R}_+^n$, $\boldsymbol{u}(t) \in \mathbb{R}^r$, and $\boldsymbol{y}(t) \in \mathbb{R}_+^m$ are non-negative vectors of the system, input and output, disturbance $\boldsymbol{d}(t) \in \mathbb{L}_\infty$, $\boldsymbol{d}(t) \in \mathbb{R}_+^{r_d}$ is norm-bounded and non-negative, and $\boldsymbol{B} \in \mathbb{R}_+^{n\times r}$, $\boldsymbol{D} \in \mathbb{R}_+^{n\times r_d}$, and $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$ are non-negative matrices.

DEFINITION 1. *Berman et al. (1989). A square matrix $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is a strictly Metzler matrix if all its diagonal elements are negative and all its off-diagonal elements are positive.*

This study also uses the term "a purely Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-\oplus}^{n\times n}$" if all diagonal elements of $\boldsymbol{A}$ are negative and all its off-diagonal elements are non-negative, and "an ostensible Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-\ominus}^{n\times n}$" if all diagonal elements of $\boldsymbol{A}$ are negative and at least one off-diagonal element is negative while the number of non-negative off-diagonal elements is prevalent.

2.1 Positive continuous-time linear systems

The following assumptions make it possible to cover constraints on the parameters of system (1) and (2) when studying the system positivity and the conditions of its diagonal stabilizability.

LEMMA 1. *Tanaka and Langbort (2011). Disturbance-free system (1), (2) is internally positive if and only if $\boldsymbol{A}$ is (strictly, purely) Metzler and $\boldsymbol{B}$, $\boldsymbol{C}$ are entry-wise non-negative.*

In consequence, any solution of an autonomous and disturbance-free linear system with a Metzler matrix $\boldsymbol{A} \in \mathbb{M}_{-\oplus}^{n\times n}$ is element-wise non-negative for all $t \geq 0$, provided that $\boldsymbol{q}(0) \geq \boldsymbol{0}$. The output solution $\boldsymbol{y}(t)$ for such a defined solution is non-negative if $\boldsymbol{C} \in \mathbb{R}_+^{m\times n}$. Such dynamical systems are called non-negative only if initial conditions in $\mathbb{R}_+^n$ are considered.

REMARK 1. *A strictly Metzler matrix $\boldsymbol{A} = \{a_{ij}\}_{I,j=1}^n$ makes of n^2 constraints*

$$a_{ii} < 0 \;\forall\; i = 1,\ldots,n, \quad a_{ij,i\neq j} > 0 \;\forall\; i, \quad j = 1,\ldots,n. \tag{3}$$

These parametric constraints imply the strict application of the diagonal stabilization principle (Shorten et al., 2009; Mason, 2012) in analysis. If a strictly Metzler $\boldsymbol{A} \in \mathbb{M}_{-+}^{n\times n}$ is represented (in relation to the observer design) in the following rhombic form, where the diagonal items reflected by the column index define multiple circular shifts of elements of the columns of $\boldsymbol{A}$ as (Krokavec and Filasová, 2020b)

$$\boldsymbol{A}_\ominus = \begin{bmatrix} -a_{11} & & & & \\ a_{21} & -a_{22} & & & \\ a_{31} & a_{32} & -a_{33} & & \\ \vdots & \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & -a_{nn} \\ & a_{12} & a_{13} & \cdots & a_{1n} \\ & & a_{23} & \cdots & a_{2n} \\ & & & \ddots & \vdots \\ & & & & a_{n-1,n} \end{bmatrix}, \tag{4}$$

the diagonal stabilization principle can be appropriately respected using the derived diagonal matrix structures related to $\boldsymbol{A}_\ominus$ as

$$\boldsymbol{A}(l,l) = diag\left[\,-a_{11}\;\; -a_{22}\;\; \cdots\;\; -a_{nn}\,\right] \prec 0, \tag{5}$$
$$\boldsymbol{A}(l+h,l) = diag\left[\,a_{1+h,1}\;\; \cdots\;\; a_{n,n-h}\;\; a_{1,n-h+1}\;\; \cdots\;\; a_{h,n}\,\right] \succ 0, \tag{6}$$

for $h = 0, 1, \ldots, n-1$.

REMARK 2. *Defining the matrix* $\boldsymbol{L} \in \mathbb{R}^{n\times n}$ *in the circulant permutation form (Horn and Johnson, 1995)*

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{0}^{\mathrm{T}} & 1 \\ \boldsymbol{I}_{n-1} & \boldsymbol{0} \end{bmatrix}, \quad \boldsymbol{L}^{-1} = \boldsymbol{L}^{\mathrm{T}} \tag{7}$$

and considering a diagonal matrix $\boldsymbol{Z} = diag\left[\,z_{11}\;\; z_{22}\;\; \cdots\;\; z_{nn}\,\right]$ *where* $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$ *then*

$$\boldsymbol{L}^{\mathrm{T}}\boldsymbol{Z}\boldsymbol{L} = diag\left[\,z_{22}\;\; \cdots\;\; z_{nn}\;\; z_{11}\,\right]. \tag{8}$$

The aforementioned results can be combined and reflected by the following lemma.

LEMMA 2. *Krokavec and Filasová (2020a) If a positive matrix* $\boldsymbol{J} \in \mathbb{R}_{+}^{n\times m}$ *forces the strictly Metzler matrix* $\boldsymbol{A}_e = \boldsymbol{A} - \boldsymbol{J}\boldsymbol{C} \in \mathbb{R}_{-+}^{n\times n}$*, where* $\boldsymbol{A} \in \mathbb{R}_{-+}^{n\times n}$ *is strictly Metzler and* $\boldsymbol{C} \in \mathbb{R}_{+}^{m\times n}$ *is non-negative, then* $\boldsymbol{A}_e$ *is parameterized as*

$$\boldsymbol{A}_e = \sum_{h=0}^{n-1}\left(\boldsymbol{A}(l+h,l) - \sum_{k=0}^{r}\boldsymbol{J}_{jh}\boldsymbol{C}_k\right)\boldsymbol{L}^{h\mathrm{T}}, \tag{9}$$

$$\boldsymbol{A}(l,l) - \sum_{k=0}^{r}\boldsymbol{J}_k\boldsymbol{C}_k \prec 0, \tag{10}$$

$$\left(\boldsymbol{A}(l+h,l) - \sum_{k=0}^{r}\boldsymbol{J}_{kh}\boldsymbol{C}_k\right)\boldsymbol{L}^{h\mathrm{T}} \succ 0, \tag{11}$$

where $h = 1, \ldots, n-1$ *and the diagonal matrices* $\boldsymbol{J}_k, \boldsymbol{J}_{kh}, \boldsymbol{C}_k \in \mathbb{R}_{+}^{n\times n}$ *are composed in the following ways:*

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{c}_1^{\mathrm{T}} \\ \vdots \\ \boldsymbol{c}_m^{\mathrm{T}} \end{bmatrix}, \quad \boldsymbol{C}_k = diag\left[\,\boldsymbol{c}_k^{\mathrm{T}}\,\right], \tag{12}$$

$$\boldsymbol{J} = \left[\,\boldsymbol{j}_1\;\; \cdots\;\; \boldsymbol{j}_m\,\right], \quad \boldsymbol{J}_k = diag\left[\,\boldsymbol{j}_k\,\right], \quad \boldsymbol{J}_{kh} = \boldsymbol{L}^{h\mathrm{T}}\boldsymbol{J}_k\boldsymbol{L}^{h}. \tag{13}$$

The proof of the aforementioned lemma is based on the fact that only diagonal matric representations are applicable for the diagonal stabilization of positive systems.

2.2 Ostensible Metzler matrices

Given a system with the dynamical model (1) and (2) and considering that $\boldsymbol{A} \in \mathbb{R}_{-\oplus}^{n\times n}$ is an ostensible Metzler matrix, then inclusion of the negative off-diagonal elements of $\boldsymbol{A}$ into the design task is built on the following basic facts from the theory of matrices.

DEFINITION 2. *Shores (2007) Matrix* $\boldsymbol{X} \in \mathbb{R}^{n\times n}$ *is similar to the matrix* $\boldsymbol{\Lambda} \in \mathbb{R}^{n\times n}$ *if there exists an invertible similarity transform matrix* $\boldsymbol{S} \in \mathbb{R}^{n\times n}$ *such that*

$$\boldsymbol{S}^{-1}\boldsymbol{X}\boldsymbol{S} = \boldsymbol{\Lambda}. \tag{14}$$

If $\boldsymbol{X}$ and $\boldsymbol{\Lambda}$ are similar, then they have the same eigenvalues, their algebraic multiplicities are the same, their characteristic polynomials are the same, and their determinants and traces are the same.

REMARK 3. *Let* $\{\boldsymbol{v}_k \in \mathbb{C}^n\}_{k=1}^n$ *be the set of eigenvectors for a matrix* $\boldsymbol{X} \in \mathbb{R}^{n\times n}$ *and* $\{\lambda_k \in \mathbb{C}\}_{k=1}^n$ *is the associated set of eigenvalues of* $\boldsymbol{X}$ *such that eigenvalues are all distinct, then (14) implies*

$$\boldsymbol{V} = \left[\,\boldsymbol{v}_1\;\; \boldsymbol{v}_2\;\; \cdots\;\; \boldsymbol{v}_n\,\right], \quad \boldsymbol{\Lambda} = diag\left[\,\lambda_1\;\; \lambda_2\;\; \cdots\;\; \lambda_n\,\right] \tag{15}$$

and $\{\boldsymbol{v}_k \in \mathbb{C}^n\}_{k=1}^n$ *are linearly independent.*

Theorem 1. *Shores (2007) If for* $\boldsymbol{X}, \boldsymbol{Y} \in \mathbb{R}^{n\times n}$ *it can be set* $\boldsymbol{Y} = c\boldsymbol{X} + d\boldsymbol{I}_n$ *with scalars* $c, d \in \mathbb{R}$, $c \neq 0$, *and* $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$*, then the eigenvalues of* $\boldsymbol{Y}$ *are*

$$\xi_k = c\lambda_k + d, \tag{16}$$

where λ_k *runs over* $\rho(\boldsymbol{X})$ *with* $k = 1, \ldots, n$*, and the eigenvectors of* $\boldsymbol{X}$ *and* $\boldsymbol{Y}$ *are identical.*

Supposing that $\boldsymbol{A}$ is ostensible Metzler, then the proposed idea means decoupling the system matrix $\boldsymbol{A}$ so that $\boldsymbol{A} = \boldsymbol{A}_p + \boldsymbol{A}_m$, where $\boldsymbol{A}_p$ is strictly Metzler and $\boldsymbol{A}_m$ is entry-wise negative and Hurwitz.

LEMMA 3. *Krokavec and Filasová (2022)* A strictly Metzler $\boldsymbol{A}_p \in \mathbb{M}_{-+}^{n\times n}$ and an entry-wise negative and Hurwitz $\boldsymbol{A}_m \in \mathbb{R}^{n\times n}$ to the composed form of the ostensible Metzler matrix $\boldsymbol{A} = \boldsymbol{A}_p + \boldsymbol{A}_m \in \mathbb{M}_{-\ominus}^{n\times n}$ exist if there exist positive scalars $\eta, \delta \in \mathbb{R}_+$ such that with

$$\lambda_o = \max_k\left(\lambda_k^+ \mid \lambda_k^+ = real\left(\lambda_k\right) > 0\right), \quad \lambda_k \in \rho\left(\boldsymbol{A}^{\circ}{}_m\right)), \tag{17}$$

$$p = \lambda_o + \delta, \quad \boldsymbol{A}_d + p\boldsymbol{I}_n \prec 0, \quad \boldsymbol{A}_d = diag\left[\,-a_{11}\;\; \cdots - a_{nn}\,\right],$$
$$\boldsymbol{A} = \left\{a_{ij}\right\}_{i,j=1}^{n}, \tag{18}$$

it yields

$$\begin{aligned}\boldsymbol{A}_p &= \boldsymbol{A}_d + \boldsymbol{A}^+ + \eta\boldsymbol{\Sigma} + p\boldsymbol{I}_n = \boldsymbol{A}^{\circ}{}_p + p\boldsymbol{I}_n,\\ \boldsymbol{A}_m &= \boldsymbol{A}^- - \eta\boldsymbol{\Sigma} - p\boldsymbol{I}_n = \boldsymbol{A}^{\circ}{}_m - p\boldsymbol{I}_n,\end{aligned} \tag{19}$$

where

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0 & 1 & \cdots & 1 & 1 \\ 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \tag{20}$$

and

$$\boldsymbol{A}^- = \begin{bmatrix} 0 & a_{12}^- & \cdots & a_{1,n-1}^- & a_{1n}^- \\ a_{21}^- & 0 & \cdots & a_{2,n-1}^- & a_{2n}^- \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-1,1}^- & a_{n-1,2}^- & \cdots & 0 & a_{n-1,n}^- \\ a_{n1}^- & a_{n2}^- & \cdots & a_{n-1,n-1}^- & 0 \end{bmatrix},$$
$$\boldsymbol{A}^+ = \begin{bmatrix} 0 & a_{12}^+ & \cdots & a_{1,n-1}^+ & a_{1n}^+ \\ a_{21}^+ & 0 & \cdots & a_{2,n-1}^+ & a_{2n}^+ \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-1,1}^+ & a_{n-1,2}^+ & \cdots & 0 & a_{n-1,n}^+ \\ a_{n1}^+ & a_{n2}^+ & \cdots & a_{n-1,n-1}^+ & 0 \end{bmatrix}, \tag{21}$$

$$a_{ij}^+ = \begin{cases} a_{ij} & \text{if } a_{ij} > 0, \\ 0 & \text{if } a_{ij} < 0, \end{cases} \quad a_{ij}^- = \begin{cases} a_{ij} & \text{if } a_{ij} < 0, \\ 0 & \text{if } a_{ij} > 0, \end{cases} \tag{22}$$

are defined for $i, j \in \langle 1, n\rangle$, $i \neq j$*, whilst* $\rho(\boldsymbol{A}^{\circ}{}_m)$ *is the set of eigenvalues of the matrix* $\boldsymbol{A}^{\circ}{}_m$*.*

REMARK 4. *Structure (21) implies, since the sum of the eigenvalues of a matrix equals its trace,*

$$tr\left(\boldsymbol{A}^{\circ}{}_m\right) = \sum_{k=1}^{n}\lambda_k^{\circ m} = 0 \tag{23}$$

and so the set $\{\lambda_k^{\circ m} \in \mathbb{C}\}_{k=1}^n$ *consists of stable and unstable eigenvalues. For repeated eigenvalues, one must add them according to their multiplicity, but this does not change the existence of a real eigenvalue with a maximal positive value to be compensated by the construction used.*

REMARK 5. *An upper-bound parameter $\eta \in \mathbb{R}_+$ must be chosen such that, in the final result, it must be set $\lambda_o > \eta$. Since (19) is constructed as a strictly Metzler matrix, all the elements on its main diagonal must be negative. This implies the boundary condition in defining a stable D-stability region with $\delta \in \mathbb{R}_+$ such that*

$$\max_i \left((a_{ii} + p) : a_{ii} + p \in \boldsymbol{A}_d + p\boldsymbol{I}_n\right) < 0, \quad p = \lambda_o + \delta. \tag{24}$$

2.3 Intervally defined ostensible Metzler matrices

In this case, is assumed that $\boldsymbol{q}(0)$ and the ostensible Metzler system parameter $\boldsymbol{A}$ are unknown but bounded by constant bounding vectors and constant bounding matrices of appropriate dimensions in such a way that (these inequalities being understood element-wise (Jaulin et al., 2001))

$$0 \le \underline{\boldsymbol{q}}(0) \le \boldsymbol{q}(0) \le \bar{\boldsymbol{q}}(0), \quad \underline{\boldsymbol{A}} \le \boldsymbol{A} \le \bar{\boldsymbol{A}}. \tag{25}$$

Since the main goal is the design of an interval observer of the state, it is considered that $\boldsymbol{B} \in \mathbb{R}_+^{n \times r}$, $\boldsymbol{D} \in \mathbb{R}_+^{n \times r_d}$, and $\boldsymbol{C} \in \mathbb{R}_+^{m \times n}$ are known non-negative matrices, the system input $\boldsymbol{u}(t)$ is norm-bounded, and the following assumption is adopted.

ASSUMPTION 1. *The function bounds $\underline{\boldsymbol{d}}, \bar{\boldsymbol{d}} \in \mathbb{R}_+^{r_d}$ and $\underline{\boldsymbol{d}}, \bar{\boldsymbol{d}} \in \mathbb{L}_\infty$ are given such that*

$$0 \le \underline{\boldsymbol{d}} \le \boldsymbol{d}(t) \le \bar{\boldsymbol{d}}. \tag{26}$$

This assumption states that the disturbance is known up to some interval error $\boldsymbol{e}_d = \bar{\boldsymbol{d}} - \underline{\boldsymbol{d}}$.

COROLLARY 1. *Strictly Metzler $\underline{\boldsymbol{A}}_p, \bar{\boldsymbol{A}}_p \in \mathbb{M}_{-+}^{n \times n}$ and an entry-wise negative and Hurwitz $\underline{\boldsymbol{A}}_m, \bar{\boldsymbol{A}}_m \in \mathbb{R}^{n \times n}$ to the composed forms of the ostensible Metzler matrices $\underline{\boldsymbol{A}} = \underline{\boldsymbol{A}}_p + \underline{\boldsymbol{A}}_m$, $\bar{\boldsymbol{A}} = \bar{\boldsymbol{A}}_p + \bar{\boldsymbol{A}}_m \in \mathbb{M}_{-\ominus}^{n \times n}$ exist if there exist positive scalars $\underline{\eta}, \bar{\eta}, \underline{\delta}, \bar{\delta} \in \mathbb{R}_+$ such that, with*

$$\underline{\lambda}_o = \max_k \left(\underline{\lambda}_k^+ \mid \underline{\lambda}_k^+ = real(\underline{\lambda}_k) > 0\right), \quad \underline{\lambda}_k \in \rho(\underline{\boldsymbol{A}}^\circ{}_m)), \tag{27}$$

$$\bar{\lambda}_o = \max_k \left(\bar{\lambda}_k^+ \mid \bar{\lambda}_k^+ = real(\bar{\lambda}_k) > 0\right), \quad \bar{\lambda}_k \in \rho(\bar{\boldsymbol{A}}^\circ{}_m)), \tag{28}$$

$$\begin{aligned} &\underline{p} = \underline{\lambda}_o + \underline{\delta}, \quad \underline{\boldsymbol{A}}_d + \underline{p}\boldsymbol{I}_n \prec 0, \quad \underline{\boldsymbol{A}}_d = diag\left[-\underline{a}_{11} \ \cdots - \underline{a}_{nn}\right], \\ &\underline{\boldsymbol{A}} = \left\{\underline{a}_{ij}\right\}_{i,j=1}^{n}, \end{aligned} \tag{29}$$

$$\begin{aligned} &\bar{p} = \bar{\lambda}_o + \bar{\delta}, \quad \bar{\boldsymbol{A}}_d + \bar{p}\boldsymbol{I}_n \prec 0, \quad \bar{\boldsymbol{A}}_d = diag\left[-\bar{a}_{11} \ \cdots - \bar{a}_{nn}\right], \\ &\bar{\boldsymbol{A}} = \left\{\bar{a}_{ij}\right\}_{i,j=1}^{n}, \end{aligned} \tag{30}$$

it yields

$$\begin{aligned} \underline{\boldsymbol{A}}_p &= \underline{\boldsymbol{A}}_d + \underline{\boldsymbol{A}}^+ + \underline{\eta}\boldsymbol{\Sigma} + \underline{p}\boldsymbol{I}_n = \underline{\boldsymbol{A}}^\circ{}_p + \underline{p}\boldsymbol{I}_n, \\ \underline{\boldsymbol{A}}_m &= \underline{\boldsymbol{A}}^- - \underline{\eta}\boldsymbol{\Sigma} - \underline{p}\boldsymbol{I}_n = \underline{\boldsymbol{A}}^\circ{}_m - \underline{p}\boldsymbol{I}_n, \end{aligned} \tag{31}$$

$$\begin{aligned} \bar{\boldsymbol{A}}_p &= \bar{\boldsymbol{A}}_d + \bar{\boldsymbol{A}}^+ + \bar{\eta}\boldsymbol{\Sigma} + \bar{p}\boldsymbol{I}_n = \bar{\boldsymbol{A}}^\circ{}_p + \bar{p}\boldsymbol{I}_n, \\ \bar{\boldsymbol{A}}_m &= \bar{\boldsymbol{A}}^- - \bar{\eta}\boldsymbol{\Sigma} - \bar{p}\boldsymbol{I}_n = \bar{\boldsymbol{A}}^\circ{}_m - \bar{p}\boldsymbol{I}_n, \end{aligned} \tag{32}$$

where $\boldsymbol{\Sigma}$ is from (20) *and $\underline{\boldsymbol{A}}^+$, $\bar{\boldsymbol{A}}^+$, $\underline{\boldsymbol{A}}^-$,* and *$\bar{\boldsymbol{A}}^-$ are constructed by the rules defined in (21) and (22).*

3 General interval observers structure

Under these introduced assumptions, the interval observers equations for systems with intervally given ostensible Metzler matrices can be defined as follows:

$$\begin{aligned} \dot{\underline{\boldsymbol{q}}}_e(t) &= \underline{\boldsymbol{A}}\,\underline{\boldsymbol{q}}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{J}\left(\boldsymbol{y}(t) - \underline{\boldsymbol{y}}_e(t)\right) \\ &= \underline{\boldsymbol{A}}_e\bar{\boldsymbol{q}}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{J}\boldsymbol{y}(t), \end{aligned} \tag{33}$$

$$\begin{aligned} \dot{\bar{\boldsymbol{q}}}_e(t) &= \bar{\boldsymbol{A}}\bar{\boldsymbol{q}}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{J}\left(\boldsymbol{y}(t) - \bar{\boldsymbol{y}}_e(t)\right) \\ &= \bar{\boldsymbol{A}}_e\bar{\boldsymbol{q}}_e(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{J}\boldsymbol{y}(t), \end{aligned} \tag{34}$$

where $\underline{\boldsymbol{q}}_e(t) \in \mathbb{R}^n$ and $\bar{\boldsymbol{q}}_e(t) \in \mathbb{R}^n$ are, respectively, the lower and upper interval estimates for the state $\boldsymbol{q}(t)$ and

$$\bar{\boldsymbol{A}}_e = \bar{\boldsymbol{A}} - \boldsymbol{J}\boldsymbol{C}, \quad \underline{\boldsymbol{A}} = \underline{\boldsymbol{A}} - \boldsymbol{J}\boldsymbol{C}, \tag{35}$$

$$\begin{aligned} \underline{\boldsymbol{y}}(t) &= \boldsymbol{C}\underline{\boldsymbol{q}}(t), \quad \bar{\boldsymbol{y}}(t) = \boldsymbol{C}\bar{\boldsymbol{q}}(t), \\ \underline{\boldsymbol{y}}_e(t) &= \boldsymbol{C}\underline{\boldsymbol{q}}_e(t), \quad \bar{\boldsymbol{y}}_e(t) = \boldsymbol{C}\bar{\boldsymbol{q}}_e(t). \end{aligned} \tag{36}$$

Using the observation errors

$$\underline{\boldsymbol{e}}(t) = \boldsymbol{q}(t) - \underline{\boldsymbol{q}}_e(t), \quad \bar{\boldsymbol{e}}(t) = \boldsymbol{q}(t) - \bar{\boldsymbol{q}}_e(t), \tag{37}$$

it follows from (1), (33), and (34) that

$$\dot{\underline{\boldsymbol{e}}}(t) = \underline{\boldsymbol{A}}_e\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t), \qquad \dot{\bar{\boldsymbol{e}}}(t) = \bar{\boldsymbol{A}}_e\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t). \tag{38}$$

To construct a Hurwitz stable $\underline{\boldsymbol{A}}_e, \bar{\boldsymbol{A}}_e \in \mathbb{R}^{n \times n}$, guaranteeing also strictly Metzler and Hurwitz matrices $\underline{\boldsymbol{A}}_{pe}, \bar{\boldsymbol{A}}_{pe} \in \mathbb{M}_{-+}^{n \times n}$ when implementing for ostensible Metzler $\underline{\boldsymbol{A}}, \bar{\boldsymbol{A}} \in \mathbb{M}_{-\ominus}^{n \times n}$, $\underline{\boldsymbol{A}} = \underline{\boldsymbol{A}}_p + \underline{\boldsymbol{A}}_m \in \mathbb{M}_{-\ominus}^{n \times n}$, and $\bar{\boldsymbol{A}} = \bar{\boldsymbol{A}}_p + \bar{\boldsymbol{A}}_m \in \mathbb{M}_{-\ominus}^{n \times n}$, then (38) can be rewritten as

$$\begin{aligned} \dot{\underline{\boldsymbol{e}}}(t) &= \underline{\boldsymbol{A}}_{pe}\underline{\boldsymbol{e}}(t) + \underline{\boldsymbol{A}}_m\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t), \\ \dot{\bar{\boldsymbol{e}}}(t) &= \bar{\boldsymbol{A}}_{pe}\underline{\boldsymbol{e}}(t) + \bar{\boldsymbol{A}}_m\underline{\boldsymbol{e}}(t) + \boldsymbol{D}\boldsymbol{d}(t), \end{aligned} \tag{39}$$

where

$$\begin{aligned} \underline{\boldsymbol{A}}_{pe} &= \underline{\boldsymbol{A}}_p - \boldsymbol{J}\boldsymbol{C}, \quad \bar{\boldsymbol{A}}_{pe} = \bar{\boldsymbol{A}}_p - \boldsymbol{J}\boldsymbol{C}, \\ \underline{\boldsymbol{A}}_e &= \underline{\boldsymbol{A}}_{pe} + \underline{\boldsymbol{A}}_m, \quad \bar{\boldsymbol{A}}_e = \bar{\boldsymbol{A}}_{pe} + \bar{\boldsymbol{A}}_m. \end{aligned} \tag{40}$$

To apply the parametrization principle in designing this class of observer, the following corollary is objective.

COROLLARY 2. *State observation error dynamics (40) entail the parameterizations of the strictly Metzler matrices $\underline{\boldsymbol{A}}_p$* and *$\bar{\boldsymbol{A}}_p$ as follows:*

$$\begin{aligned} \underline{\boldsymbol{A}}_p(l,l) &= diag\left[\underline{a}_{p11} \ \cdots \ \underline{a}_{pnn}\right], \\ \bar{\boldsymbol{A}}_p(l,l) &= diag\left[\bar{a}_{p11} \ \cdots \ \bar{a}_{pnn}\right], \end{aligned} \tag{41}$$

$$\underline{\boldsymbol{A}}_p(l+h,l) = diag\left[\underline{a}_{p,1+h,1} \ \cdots \ \underline{a}_{p,n,n-h} \ \ \underline{a}_{p,1,n-h+1} \ \cdots \ \underline{a}_{ihn}\right], \tag{42}$$

$$\bar{\boldsymbol{A}}_p(l+h,l) = diag\left[\bar{a}_{p,1+h,1} \ \cdots \ \bar{a}_{p,n,n-h} \ \ \bar{a}_{p,1,n-h+1} \ \cdots \ \bar{a}_{phn}\right], \tag{43}$$

$$\underline{\boldsymbol{A}}_{pe} = \sum_{h=0}^{n-1}\left(\underline{\boldsymbol{A}}_p(l+h,l) - \sum_{j=0}^{r}\boldsymbol{J}_{jh}\boldsymbol{C}_j\right)\boldsymbol{L}^{h\mathrm{T}}, \tag{44}$$

$$\bar{\boldsymbol{A}}_{pe} = \sum_{h=0}^{n-1}\left(\bar{\boldsymbol{A}}_p(l+h,l) - \sum_{j=0}^{r}\boldsymbol{J}_{jh}\boldsymbol{C}_j\right)\boldsymbol{L}^{h\mathrm{T}}, \tag{45}$$

while the parameterizations (12) and (13) stay unchanged.

Provided that (25) is satisfied, then for all $t \in \mathbb{R}_+$, the estimates $\underline{q}_e(t)$ and $\bar{q}_e(t)$ are bounded with the limit properties, illustrated by the following remark.

REMARK 6. *Performing an inner adjustment for (38) as*

$$\dot{q}(t) - \dot{\underline{q}}_e(t) = \underline{A}_e\left(q(t) - \underline{q}_e(t)\right) + Dd(t), \tag{46}$$

$$\dot{\underline{q}}_e(t) = \dot{q}(t) - \underline{A}_e q(t) + \underline{A}_e \underline{q}_e(t) - Dd(t), \tag{47}$$

respectively, and substituting (1) in (47) yields

$$\begin{aligned}\dot{\underline{q}}_e(t) &= (A - (\underline{A} - JC))q(t) + \underline{A}_e\underline{q}_e(t) + Bu(t)\\ &= (A - \underline{A})\,q(t) + JC\,q(t) + \underline{A}_e\underline{q}_e(t) + Bu(t)\\ &= (A - \underline{A})\,q(t) + JC\,q(t) + \left(\underline{A}_{ep} + \underline{A}_m\right)\underline{q}_e(t) + Bu(t),\end{aligned} \tag{48}$$

and, if $\underline{A}_e = \underline{A}_{ep} + \underline{A}_m$ is Hurwitz, $C \in \mathbb{R}_+^{m\times n}$ is nonnegative, $J \in \mathbb{R}_+^{n\times m}$ is positive, and $\underline{A} \le A$, then the lower system state estimate produced by the interval observer constructed on the system model with the ostensible Metzler matrix converges to a non-negative trajectory if $JC\,q(t) > 0$. Consequently, provided that $\underline{q}(0) \le q(0) \le \bar{q}(0)$, then for all $t \in \mathbb{R}_+$, the estimates $\underline{q}(t)$ and $\bar{q}(t)$ given by (33) and (34) produce the interval bounds only to those system state variables $q_i(t)$, $i = 1, \ldots, n$ which are non-negative.

4 Interval observers design

The design goals are Hurwitz stable matrices $\underline{A}_e \in \mathbb{R}^{n\times n}$ and $\bar{A}_e \in \mathbb{R}^{n\times n}$ and strictly Metzler and Hurwitz matrices $\underline{A}_{pe} \in \mathbb{M}_{-+}^{n\times n}$ and $\bar{A}_{pe} \in \mathbb{M}_{-+}^{n\times n}$ when implementing for ostensible Metzler $\underline{A} \in \mathbb{M}_{-\ominus}^{n\times n}$ and $\bar{A} \in \mathbb{M}_{-\ominus}^{n\times n}$. A solution method, resulting in positive matrix gain $J \in \mathbb{R}_+^{n\times m}$, is given in Theorem 2.

Theorem 2. *The matrices $\underline{A}_{ep}, \bar{A}_{ep}, \in \mathbb{R}_{-+}^{n\times n}$ are strictly Metzler and Hurwitz and the matrices $\bar{A}_e$ and $\underline{A}_e \in \mathbb{R}_{-\oplus}^{n\times n}$ are Hurwitz if, for the given ostensible Metzler matrices $\underline{A}, \bar{A} \in \mathbb{R}_{-\oplus}^{n\times n}$ and non-negative $C \in \mathbb{R}_+^{m\times n}$, there exist positive definite diagonal matrices $P, R_k \in \mathbb{R}_+^{n\times n}$ and positive scalars $\underline{\mu}, \bar{\mu} \in \mathbb{R}_+$ that for $h = 1, \ldots, n-1$, $l^{\mathrm{T}} = [\,1 \;\cdots\; 1\,]$ and the parameters from Corollary 2 satisfy the LMIs*

$$P \succ 0, \quad R_k \succ 0, \tag{49}$$

$$\begin{bmatrix} \underline{\Omega} & * & * \\ D^{\mathrm{T}}P & -\underline{\mu}I_{r_d} & * \\ C & 0 & -\underline{\mu}I_m \end{bmatrix} \prec 0, \quad \begin{bmatrix} \bar{\Omega} & * & * \\ D^{\mathrm{T}}P & -\bar{\mu}I_{r_d} & * \\ C & 0 & -\bar{\mu}I_m \end{bmatrix} \prec 0, \tag{50}$$

$$P\bar{A}_p(l,l) - \sum_{k=1}^{m} R_k C_k \prec 0, \quad P\underline{A}_p(l,l) - \sum_{k=1}^{m} R_k C_k \prec 0, \tag{51}$$

$$PL^h\underline{A}_p(l+h,l)L^{h\mathrm{T}} - \sum_{k=1}^{m} R_k L^h C_k L^{h\mathrm{T}} \succ 0,$$

$$PL^h\bar{A}_p(l+h,l)L^{h\mathrm{T}} - \sum_{k=1}^{m} R_k L^h C_k L^{h\mathrm{T}} \succ 0, \tag{52}$$

$$\underline{\Omega} = P\underline{A}_p + \underline{A}_p^{\mathrm{T}}P + P\underline{A}_m + \underline{A}_m^{\mathrm{T}}P - \sum_{k=1}^{r}\left(R_k l l^{\mathrm{T}} C_k + C_k l l^{\mathrm{T}} R_k\right), \tag{53}$$

$$\bar{\Omega} = P\bar{A}_p + \bar{A}_p^{\mathrm{T}}P + P\bar{A}_m + \bar{A}_m^{\mathrm{T}}P - \sum_{k=1}^{r}\left(R_k l l^{\mathrm{T}} C_k + C_k l l^{\mathrm{T}} R_k\right). \tag{54}$$

Confirming the feasible task, the interval observer gain is given as

$$J_k = P^{-1}R_k, \quad j_k = J_k l, \quad J = \left[\,j_1 \;\cdots\; j_m\,\right]. \tag{55}$$

PROOF. To respect the diagonal stabilization principle, $v(\underline{e}(t))$ is served as a Lyapunov function for (37) using a symmetric positive definite matrix $P \in \mathbb{R}_+^{n\times m}$ and a positive scalar $\underline{\mu} \in \mathbb{R}_+$ such that

$$v(\underline{e}(t)) = \underline{e}^{\mathrm{T}}(t)P\underline{e}(t) + \underline{\mu}^{-1}\int_0^t\left(\underline{e}_y^{\mathrm{T}}(\tau)\underline{e}_y(\tau) - \underline{\mu}^2 d^{\mathrm{T}}(\tau)d(\tau)\right)\mathrm{d}\tau > 0, \tag{56}$$

whose time-derivative for the observer error trajectory must satisfy

$$\begin{aligned}\dot{v}(\underline{e}(t)) &= \dot{\underline{e}}^{\mathrm{T}}(t)P\underline{e}(t) + \underline{e}^{\mathrm{T}}(t)P\dot{\underline{e}}(t) + \underline{\mu}^{-1}\underline{e}_y^{\mathrm{T}}(t)\underline{e}_y(t)\\ &\quad - \underline{\mu}\, d^{\mathrm{T}}(t)d(t) < 0.\end{aligned} \tag{57}$$

Applying in inequality (57) the observer error dynamics (37) gives the following:

$$\begin{aligned}\dot{v}(\underline{e}(t)) &= \underline{e}^{\mathrm{T}}(t)\left(\underline{A}_e^{\mathrm{T}}P + P\underline{A}_e\right)\underline{e}(t) + \underline{e}^{\mathrm{T}}(t)PDd(t) + d^{\mathrm{T}}(t)D^{\mathrm{T}}P\,\underline{e}(t))\\ &\quad + \underline{\mu}^{-1}\underline{e}^{\mathrm{T}}(t)C^{\mathrm{T}}C\,\underline{e}(t) - \underline{\mu}\,d^{\mathrm{T}}(t)d(t) \qquad < 0.\end{aligned} \tag{58}$$

Thus, constructing a common notation $\underline{e}_d(t)$ that is readily representable for the used variables as

$$\underline{e}_d^{\mathrm{T}}(t) = \left[\,\underline{e}^{\mathrm{T}}(t)\;\; d^{\mathrm{T}}(t)\,\right], \tag{59}$$

then there is reasonable grounds to conclude that

$$\dot{v}(\underline{e}_d(t)) = \underline{e}_d^{\mathrm{T}}(t)\,\underline{\Omega}^{\circ}\,\underline{e}_d(t) < 0, \tag{60}$$

where, for the covered systematization,

$$\underline{\Omega}^{\circ} = \begin{bmatrix} \underline{A}_e^{\mathrm{T}}P + P\underline{A}_e + \underline{\mu}^{-1}C^{\mathrm{T}}C & PD \\ D^{\mathrm{T}}P & -\underline{\mu}I_{r_d} \end{bmatrix} \prec 0. \tag{61}$$

Therefore, the new form of LMI after applying the property of the Schur complement is

$$\begin{bmatrix} P\underline{A}_e + \underline{A}_{ei}^{\mathrm{T}}P & * & * \\ D^{\mathrm{T}}P & -\underline{\mu}I_{r_d} & * \\ C & 0 & -\underline{\mu}I_m, \end{bmatrix} \prec 0 \tag{62}$$

and, using (13) and 40, it can be set as

$$P\underline{A}_{pe} = P\left(\underline{A}_p - JC\right) = P\underline{A}_p - \sum_{k=1}^{m} Pj_k c_k^{\mathrm{T}} = P\underline{A}_p - \sum_{k=1}^{m} PJ_k l l^{\mathrm{T}} C_k, \tag{63}$$

where the column vector l is used to uncover the diagonal matrix structures. Thus, (62) implies (50) and (53) when substituting

$$PJ_k = R_k, \quad \underline{A}_e = \underline{A}_{pe} + \underline{A}_m. \tag{64}$$

Separating $h = 0$ from (44) diagonal part and multiplying its left side by P yields

$$P\underline{A}_p(l,l) - \sum_{j=1}^{m} PJ_jC_j \prec 0 \tag{65}$$

and using notation (64) then (65) implies (51). Analogously, it can be obtained when taking from (44) a component for $h \neq 0$ and multiplying its left side by PL^h (since $L^hL^{h\mathrm{T}} = I_n$) that

$$PL^h\underline{A}_p(l,l+h)L^{h\mathrm{T}} - \sum_{k=1}^{m} PL^hL^{h\mathrm{T}}J_kL^hC_kL^{h\mathrm{T}} \succ 0 \tag{66}$$

and, using notation (64), then (66) implies (52).

Analogously, all this can be carried out for the upper bound parameters. This concludes the proof.

5 Illustrative examples

In this section, two examples are presented to demonstrate the effectiveness of the interval observers design.

Example 1. To illustrate the proposed design principles, the stable interval ostensible strictly Metzler systems (1) and (2) are constructed on the matrices

$$\underline{\boldsymbol{A}} = \begin{bmatrix} -0.172 & 1.94 & 1.45 \\ -0.142 & -1.96 & -0.38 \\ 0.100 & 0.17 & -2.91 \end{bmatrix}, \quad \bar{\boldsymbol{A}} = \begin{bmatrix} -0.158 & 2.06 & 1.55 \\ -0.142 & -1.64 & -0.32 \\ 0.200 & 0.17 & -2.55 \end{bmatrix},$$

$$\boldsymbol{D} = \begin{bmatrix} 0.12 \\ 1.09 \\ 0.21 \end{bmatrix}, \quad \boldsymbol{C}^{\mathrm{T}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

To apply Theorem 2 conditions, the derived design parameters are selected as

$$\underline{\boldsymbol{A}}_d = \mathrm{diag}\begin{bmatrix} -0.172 & -1.96 & -2.91 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_d = \mathrm{diag}\begin{bmatrix} -0.158 & -1.64 & -2.55 \end{bmatrix},$$

and the related matrix structures are constructed from the system matrix bounds as follows:

$$\underline{\boldsymbol{A}}^- = \begin{bmatrix} 0 & 0 & 0 \\ -0.142 & 0 & -0.38 \\ 0 & 0 & 0 \end{bmatrix}, \quad \underline{\boldsymbol{A}}^+ = \begin{bmatrix} 0 & 1.94 & 1.45 \\ 0 & 0 & 0 \\ 0.10 & 0.17 & 0 \end{bmatrix},$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix},$$

$$\bar{\boldsymbol{A}}^- = \begin{bmatrix} 0 & 0 & 0 \\ -0.142 & 0 & -0.32 \\ 0 & 0 & 0 \end{bmatrix}, \quad \bar{\boldsymbol{A}}^+ = \begin{bmatrix} 0 & 2.06 & 1.55 \\ 0 & 0 & 0 \\ 0.20 & 0.17 & 0 \end{bmatrix}.$$

Thus, using $\boldsymbol{\Sigma}$ and $\eta = 0.005$ yields for $\boldsymbol{A}^{\circ}_m$ that

$$\underline{\boldsymbol{A}}^{\circ}_m = \underline{\boldsymbol{A}}^- - \eta\boldsymbol{\Sigma} = \begin{bmatrix} 0 & -0.005 & -0.005 \\ -0.147 & 0 & -0.325 \\ -0.005 & -0.005 & 0 \end{bmatrix},$$

$$\rho(\underline{\boldsymbol{A}}^{\circ}_m) = \begin{Bmatrix} -0.0541 \\ 0.0050 \\ 0.0491 \end{Bmatrix}, \quad \underline{\lambda}_0 = 0.0491,$$

$$\bar{\boldsymbol{A}}^{\circ}_m = \bar{\boldsymbol{A}}^- - \eta\boldsymbol{\Sigma} = \begin{bmatrix} 0 & -0.005 & -0.005 \\ -0.147 & 0 & -0.325 \\ -0.005 & -0.005 & 0 \end{bmatrix},$$

$$\rho(\bar{\boldsymbol{A}}^{\circ}_m) = \begin{Bmatrix} -0.0511 \\ 0.0050 \\ 0.0461 \end{Bmatrix}, \quad \bar{\lambda}_0 = 0.0461.$$

Setting $\underline{\delta} = \bar{\delta} = 0.003$ means $\underline{p} = \underline{\lambda}_0 + \underline{\delta} = 0.0521$ and $\bar{p} = \bar{\lambda}_0 + \bar{\delta} = 0.0491$, and so $\underline{\boldsymbol{A}}_m = \underline{\boldsymbol{A}}^{\circ}_m - \underline{p}\boldsymbol{I}_n$, $\bar{\boldsymbol{A}}_m = \bar{\boldsymbol{A}}^{\circ}_m - \bar{p}\boldsymbol{I}_n$ take the values

$$\underline{\boldsymbol{A}}_m = \begin{bmatrix} -0.0521 & -0.0050 & -0.0050 \\ -0.1470 & -0.0521 & -0.3850 \\ -0.0050 & -0.0050 & -0.0521 \end{bmatrix}, \quad \rho(\underline{\boldsymbol{A}}_m) = \begin{Bmatrix} -0.1062 \\ -0.0471 \\ -0.0030 \end{Bmatrix},$$

$$\bar{\boldsymbol{A}}_m = \begin{bmatrix} -0.0761 & -0.0050 & -0.0050 \\ -0.1470 & -0.0761 & -0.3250 \\ -0.0050 & -0.0050 & -0.0761 \end{bmatrix}, \quad \rho(\bar{\boldsymbol{A}}_m) = \begin{Bmatrix} -0.1272 \\ -0.0711 \\ -0.0300 \end{Bmatrix}.$$

Furthermore, $\underline{\boldsymbol{A}}_p = \underline{\boldsymbol{A}}_d + \underline{\boldsymbol{A}}^+ + \eta\boldsymbol{\Sigma} + \underline{p}\boldsymbol{I}_n$ and $\bar{\boldsymbol{A}}_p = \bar{\boldsymbol{A}}_d + \bar{\boldsymbol{A}}^+ + \eta\boldsymbol{\Sigma} + \bar{p}\boldsymbol{I}_n$ are computed as

$$\underline{\boldsymbol{A}}_p = \begin{bmatrix} -0.1199 & 1.9450 & 1.4550 \\ 0.0050 & -1.9079 & 0.0050 \\ 0.1050 & 0.1750 & -2.8579 \end{bmatrix}, \quad \rho(\underline{\boldsymbol{A}}_p) = \begin{Bmatrix} -0.0596 \\ -1.9133 \\ -2.9128 \end{Bmatrix},$$

$$\bar{\boldsymbol{A}}_p = \begin{bmatrix} -0.0683 & 2.0650 & 1.5550 \\ 0.0050 & -1.5503 & 0.0050 \\ 0.2050 & 0.1750 & -2.4603 \end{bmatrix}, \quad \rho(\bar{\boldsymbol{A}}_p) = \begin{Bmatrix} 0.0652 \\ -1.5572 \\ -2.5869 \end{Bmatrix},$$

which are strictly Metzler, and their rhombic representations imply the diagonal matrices for the observer synthesis

$$\underline{\boldsymbol{A}}_p(l,l) = \mathrm{diag}\begin{bmatrix} -0.1199 & -1.9079 & -2.8579 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_p(l,l) = \mathrm{diag}\begin{bmatrix} -0.0683 & -1.5503 & -2.4603 \end{bmatrix},$$
$$\underline{\boldsymbol{A}}_p(l+1,l) = \mathrm{diag}\begin{bmatrix} 0.0050 & 0.1750 & 1.4550 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_p(l+1,l) = \mathrm{diag}\begin{bmatrix} 0.0050 & 0.1750 & 1.5550 \end{bmatrix},$$
$$\underline{\boldsymbol{A}}_p(l+2,l) = \mathrm{diag}\begin{bmatrix} 0.1050 & 1.9450 & 0.0050 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_p(l+2,l) = \mathrm{diag}\begin{bmatrix} 0.2050 & 2.0650 & 0.0050 \end{bmatrix},$$

whilst straightforward calculations give

$$\boldsymbol{C}_1 = \mathrm{diag}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{C}_2 = \mathrm{diag}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{l}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix},$$

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{0}^{\mathrm{T}} & 1 \\ \boldsymbol{I}_3 & \boldsymbol{0} \end{bmatrix}.$$

Using LMIs defined by Theorem 2, the feasible matrix variables result in the non-negative gain matrix when applying the SeDuMi package (Peaucelle et al., 2002)

$$\boldsymbol{P} = \mathrm{diag}\begin{bmatrix} 2.4234 & 3.1289 & 2.7125 \end{bmatrix} \succ 0,$$
$$\boldsymbol{R}_1 = \mathrm{diag}\begin{bmatrix} 3.0845 & 0.0048 & 0.0988 \end{bmatrix} \succ 0,$$
$$\boldsymbol{R}_2 = \mathrm{diag}\begin{bmatrix} 2.3627 & 1.1504 & 0.1488 \end{bmatrix} \succ 0,$$
$$\underline{\mu} = 4.0252, \quad \bar{\mu} = 4.1147, \quad \boldsymbol{J} = \begin{bmatrix} 1.2728 & 0.9750 \\ 0.0015 & 0.3677 \\ 0.0364 & 0.0548 \end{bmatrix},$$

This infuses the strictly Metzler and Hurwitz matrices $\underline{\boldsymbol{A}}_{pe} = \underline{\boldsymbol{A}}_p - \boldsymbol{JC}$ and $\bar{\boldsymbol{A}}_{pe} = \bar{\boldsymbol{A}}_p - \boldsymbol{JC}$ as

$$\underline{\boldsymbol{A}}_{pe} = \begin{bmatrix} -1.3927 & 0.9700 & 1.4550 \\ 0.0035 & -2.2756 & 0.0050 \\ 0.0686 & 0.1202 & -2.8579 \end{bmatrix}, \quad \rho(\underline{\boldsymbol{A}}_{pe}) = \begin{Bmatrix} -1.3235 \\ -2.2794 \\ -2.9233 \end{Bmatrix},$$

$$\bar{\boldsymbol{A}}_{pe} = \begin{bmatrix} -1.3411 & 1.0900 & 1.5550 \\ 0.0035 & -1.9180 & 0.0050 \\ 0.1686 & 0.1202 & -2.4603 \end{bmatrix}, \quad \rho(\bar{\boldsymbol{A}}_{pe}) = \begin{Bmatrix} -1.1366 \\ -1.9236 \\ -2.6591 \end{Bmatrix},$$

where $\underline{\boldsymbol{A}}_{pe} \le \bar{\boldsymbol{A}}_{pe}$. In addition, it can be seen that, due to the structure of matrix $\boldsymbol{C}$, the elements on the third columns of matrices $\underline{\boldsymbol{A}}_{pe}$ and $\bar{\boldsymbol{A}}_{pe}$ have not changed compared to $\underline{\boldsymbol{A}}_p$ and $\bar{\boldsymbol{A}}_p$.

Applying the same gain matrix to the ostensible Metzler matrices yields

$$\underline{\boldsymbol{A}}_e = \underline{\boldsymbol{A}} - \boldsymbol{JC} = \begin{bmatrix} -1.4448 & 0.9650 & 1.4500 \\ -0.1435 & -2.3277 & -0.3800 \\ 0.0636 & 0.1152 & -2.9100 \end{bmatrix},$$

$$\bar{\boldsymbol{A}}_e = \bar{\boldsymbol{A}} - \boldsymbol{JC} = \begin{bmatrix} -1.4308 & 1.0850 & 1.5500 \\ -0.1435 & -2.0077 & -0.3200 \\ 0.1636 & 0.1152 & -2.5500 \end{bmatrix}.$$

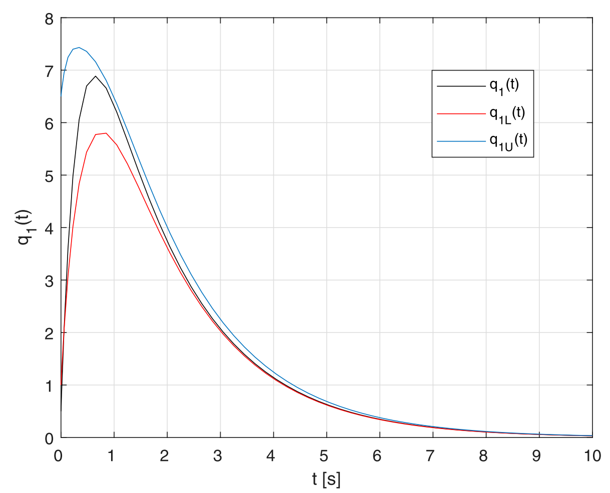

FIGURE 1
Estimation of the first state variable in the autonomous mode.

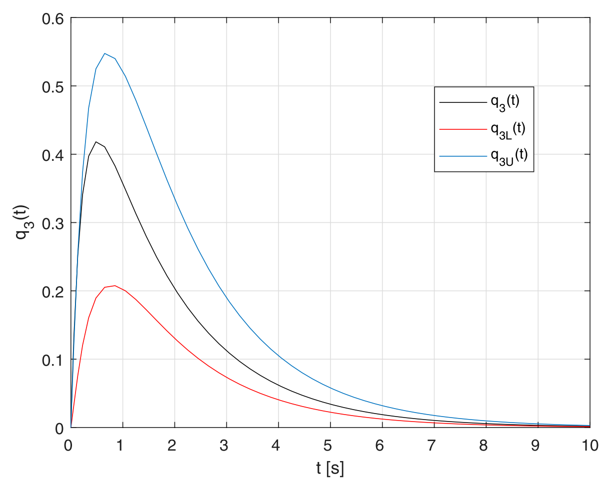

FIGURE 2
Estimation of the third state variable in the autonomous mode.

It should be noted that the positions of the negative off-diagonal elements in $\underline{A}$ and $\underline{A}_e$, as well as in $\bar{A}$ and $\bar{A}_e$, have been preserved. In addition, in the considered case, $\underline{A}_e \le \bar{A}_e$.

By simulating the response of the autonomous system with considered interval ostensible Metzler parameters to better illustrate the ostensible Metzler phenomena, the dynamics of the system were

$$\boldsymbol{A} = \begin{bmatrix} -0.165 & 2.00 & 1.50 \\ -0.142 & -1.80 & -0.35 \\ 0.150 & 0.17 & -2.73 \end{bmatrix}, \qquad \underline{\boldsymbol{A}} \le \boldsymbol{A} \le \bar{\boldsymbol{A}},$$

the initial system state was set as $\boldsymbol{q}(0) = [\ 0.5\ \ 7.5\ \ 0\ ]^T$, $\underline{\boldsymbol{q}}_e(0) = \bar{\boldsymbol{q}}_e(0) = \boldsymbol{0}$, and $\sigma_d^2 = 0.04$. The simulation is executed in the MATLAB framework using Simulink.

Figure 1 depicts the time responses of the first system state variable and its upper and lower estimations; Figure 2 shows the time responses for the third system state variable. Although the given system is not positive, it can be seen from Figures 1 and 2 that the behaviors of these state variables are correctly intervally estimated by the proposed interval observer if the components $\underline{\boldsymbol{A}}_m\boldsymbol{q}(t)$ and $\bar{\boldsymbol{A}}_m\boldsymbol{q}(t)$, indicated in (48), are compensated by suitably choosing the observer initial states, satisfying conditions $\underline{\boldsymbol{q}}_e(0) \le \boldsymbol{q}(0) \le \bar{\boldsymbol{q}}_e(0)$. Since the state variable $q_2(t)$ is undefined in sign, its interval estimation is also undefined in sign. This case is trivial and is not presented.

Moreover, considering the effect of the fixed uncompensated part with prescribed D-stability region related to $\underline{\boldsymbol{A}}_m$, $\bar{\boldsymbol{A}}_m$, the proposed approach leads to a structure that has the properties of a stable system. By using the tuning parameter δ, the D-stability region can be analytically continued.

Example 2. To demonstrate the application validity of the suggested interval observer, the second example is presented on the linearized dynamic model of a U.S. Navy F-404 engine which powers the F/A-18 aircraft (Kwon et al., 1999). The corresponding dynamic model is a stable interval ostensible purely Metzler system (1), (2), written as

$$\underline{\boldsymbol{A}} = \begin{bmatrix} -1.4600 & 0 & 2.4280 \\ -0.8357 & -2.4 & -0.3788 \\ 0.3107 & 0 & -2.1300 \end{bmatrix},$$

$$\bar{\boldsymbol{A}} = \begin{bmatrix} -1.4600 & 0 & 2.4280 \\ -0.3357 & -1.4 & -0.3788 \\ 0.3107 & 0 & -2.1300 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0.4182 & 5.2030 \\ 0.3901 & -0.1245 \\ 0.5186 & 0.0236 \end{bmatrix}.$$

Since the interval matrices of the system are purely Metzler, due to the structure of their second column, it is advantageous if the measurement system corresponds the following matrix $\boldsymbol{C}$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{D} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Applying analogously as the aforementioned conditions of Theorem 2, the resulting matrix representations are

$$\underline{\boldsymbol{A}}_m = \begin{bmatrix} -0.0788 & -0.0050 & -0.0050 \\ -0.8407 & -0.0788 & -0.3838 \\ -0.0050 & -0.0050 & -0.0788 \end{bmatrix}, \quad \rho(\underline{\boldsymbol{A}}_m) = \begin{Bmatrix} -0.1596 \\ -0.0738 \\ -0.0030 \end{Bmatrix},$$

$$\bar{\boldsymbol{A}}_m = \begin{bmatrix} -0.0877 & -0.0050 & -0.0050 \\ -0.3407 & -0.0877 & -0.3838 \\ -0.0050 & -0.0050 & -0.0877 \end{bmatrix}, \quad \rho(\bar{\boldsymbol{A}}_m) = \begin{Bmatrix} -0.1504 \\ -0.0827 \\ -0.0300 \end{Bmatrix},$$

$$\underline{\boldsymbol{A}}_p = \begin{bmatrix} -1.3812 & 0.0050 & 2.4330 \\ 0.0050 & -2.3212 & 0.0050 \\ 0.3157 & 0.0050 & -2.1512 \end{bmatrix}, \quad \rho(\underline{\boldsymbol{A}}_p) = \begin{Bmatrix} -0.8089 \\ -2.3213 \\ -2.7234 \end{Bmatrix},$$

$$\bar{\boldsymbol{A}}_p = \begin{bmatrix} -1.3723 & 0.0050 & 2.4330 \\ 0.0050 & -1.3123 & 0.0050 \\ 0.3157 & 0.0050 & -2.1423 \end{bmatrix}, \quad \rho(\bar{\boldsymbol{A}}_p) = \begin{Bmatrix} -0.7999 \\ -1.3124 \\ -2.7145 \end{Bmatrix},$$

where $\underline{\delta} = \bar{\delta} = 0.003$, $\underline{\lambda}_0 = 0.0758$, $\bar{\lambda}_0 = 0.0577$, $\underline{p} = \underline{\lambda}_0 + \underline{\delta} = 0.0788$, and $\bar{p} = \bar{\lambda}_0 + \bar{\delta} = 0.0607$.

Analogously constructing the diagonal matrices for the interval observer synthesis from the rhombic representations of the interval system matrices and for the used matrix $\boldsymbol{C}$, the feasible matrix variables resulting from the conditions defined by Theorem 2 are

$$\boldsymbol{P} = \mathrm{diag}\left[\, 2.2680 \;\; 3.3453 \;\; 2.4098 \,\right] \succ 0, \quad \boldsymbol{R}_1 = \mathrm{diag}\left[\, 1.4454 \;\; 0.0062 \;\; 0.2474 \,\right] \succ 0, \quad \boldsymbol{R}_2 = \mathrm{diag}\left[\, 2.8826 \;\; 0.0060 \;\; 1.1582 \,\right] \succ 0, \quad \boldsymbol{J} = \begin{bmatrix} 0.6373 & 1.2710 \\ 0.0018 & 0.0018 \\ 0.1027 & 0.4806 \end{bmatrix},$$
$$\underline{\mu} = 4.2335, \;\; \bar{\mu} = 4.4557.$$

This infuses the strictly Metzler and Hurwitz matrices $\underline{\boldsymbol{A}}_{pe} = \underline{\boldsymbol{A}}_p - \boldsymbol{JC}$ and $\bar{\boldsymbol{A}}_{pe} = \bar{\boldsymbol{A}}_p - \boldsymbol{JC}$ as

$$\underline{\boldsymbol{A}}_{pe} = \begin{bmatrix} -2.0185 & 0.0050 & 1.1620 \\ 0.0032 & -2.3212 & 0.0032 \\ 0.2130 & 0.0050 & -2.6318 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_{pe} = \begin{bmatrix} -2.0096 & 0.0050 & 1.1620 \\ 0.0032 & -1.3123 & 0.0032 \\ 0.2130 & 0.0050 & -2.6229 \end{bmatrix},$$
$$\rho\left(\underline{\boldsymbol{A}}_{pe}\right) = \{-1.7406 \;\; -2.3213 \;\; -2.9096\},$$
$$\rho\left(\bar{\boldsymbol{A}}_{pe}\right) = \{-1.3122 \;\; -1.7319 \;\; -2.9007\},$$

where $\underline{\boldsymbol{A}}_{pe} \le \bar{\boldsymbol{A}}_{pe}$. In addition, it can be seen that, due to the structure of matrix $\boldsymbol{C}$, the elements on the second columns of matrices $\underline{\boldsymbol{A}}_{pe}$ and $\bar{\boldsymbol{A}}_{pe}$ have not changed compared to $\underline{\boldsymbol{A}}_p$ and $\bar{\boldsymbol{A}}_p$.

Using these ostensible purely Metzler matrices results in stable, purely Metzler structures

$$\underline{\boldsymbol{A}}_e = \underline{\boldsymbol{A}} - \boldsymbol{JC} = \begin{bmatrix} -2.0973 & 0 & 1.1570 \\ -0.8375 & -2.4 & -0.3806 \\ 0.2080 & 0 & -2.6106 \end{bmatrix},$$
$$\bar{\boldsymbol{A}}_e = \bar{\boldsymbol{A}} - \boldsymbol{JC} = \begin{bmatrix} -2.0973 & 0 & 1.1570 \\ -0.3375 & -1.4 & -0.3806 \\ 0.2080 & 0 & -2.6106 \end{bmatrix}.$$

By simulating the response of the observer in the forced mode, it is set as $\underline{\boldsymbol{A}} \le \boldsymbol{A} \le \bar{\boldsymbol{A}}$

$$\boldsymbol{A} = \begin{bmatrix} -1.4600 & 0 & 2.4280 \\ -0.5857 & -1.9 & -0.3788 \\ 0.3107 & 0 & -2.1300 \end{bmatrix}, \quad \boldsymbol{u}(t) = \boldsymbol{W}\boldsymbol{w}(t),$$
$$\boldsymbol{W} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \; \boldsymbol{w}(t) = \begin{bmatrix} 0.352 \\ 0.076 \end{bmatrix}, \; \sigma_d^2 = 0.01^2,$$
$$\boldsymbol{q}(0) = \left[\, 0.250 \;\; 3.750 \;\; 0.025 \,\right]^{\mathrm{T}}, \; \underline{\boldsymbol{q}}_e(0) = \left[\, 0.15 \;\; 0 \;\; 0 \,\right]^{\mathrm{T}},$$
$$\bar{\boldsymbol{q}}_e(0) = \left[\, 0.4 \;\; 0 \;\; 0.1 \,\right]^{\mathrm{T}}, \; \underline{\boldsymbol{q}}_e(0) \le \bar{\boldsymbol{q}}_e(0).$$

Figure 3 depicts the time responses of the first system state variable and its upper and lower estimations; Figure 4 shows the time responses for the third system state variable. Although the given system is not positive, it can be seen from Figure 3 and Figure 4 that the behaviors of these state variables are correctly intervally estimated by the proposed interval observer.

Note for both examples, since $\underline{\mu}, \bar{\mu}$ are scalar variables defined directly by a feasible solution of LMIs, they can be indicated as the values at the disturbance attenuation levels. The scalar variable p provides an additional degree of freedom in solving the problem of the dynamics of an interval observer, which should generally be faster than the dynamics of the system. Because the synthesis method is a two-step procedure, it is possible to sequentially define the locations of the stable regions first for the uncontrolled stable dynamics of $\underline{\boldsymbol{A}}_m$ and $\bar{\boldsymbol{A}}_m$ by defining the D-region of stability using the parameter $p > 0$ ($\nu > 0$ is just some small positive value by means of which Σ is regularized) and then, indirectly via LMIs, finding a solution that guarantees the required rate of estimation error convergence.

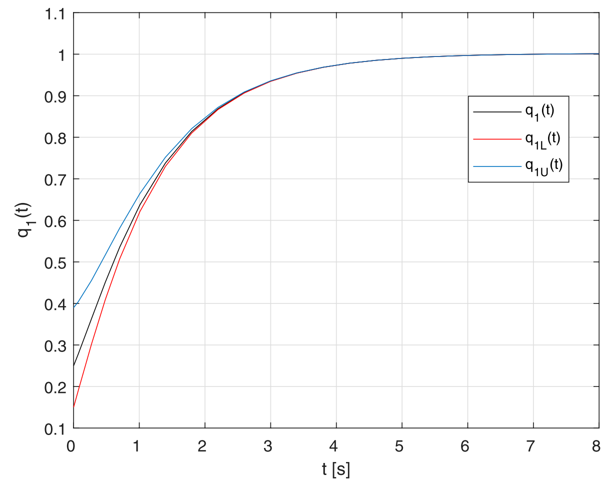

FIGURE 3
Estimation of the first state variable in the forced mode.

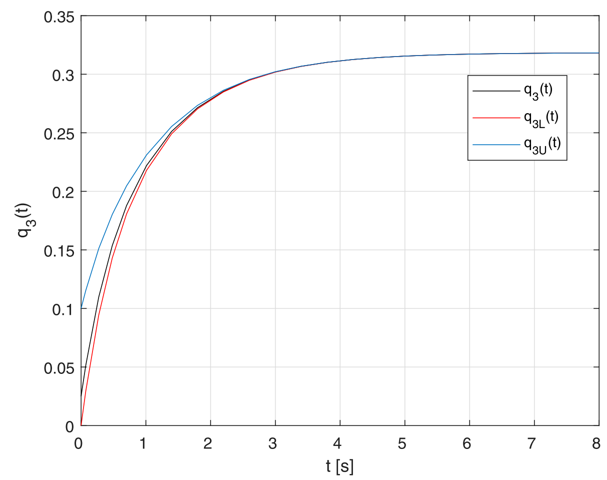

FIGURE 4
Estimation of the third state variable in the forced mode.

Both tasks are parametrically dependent, while mutual interaction in the resulting dynamics of the observer is defined by the parameter $p > 0$, and its interactive setting is, as a rule, sufficient.

6 Concluding remarks

This paper presents new results concerning the interval state estimation of intervally defined ostensible Metzler systems. It proposes how this problem can be formulated using a positive parametric representation and how a constructive procedure based on LMIs can be used respecting the diagonally stabilization principle. It is therefore proven that the gain matrix of the interval observer can be constructed for strict positivity when the stability of the interval observer is defined for a strictly Metzler approximation of the ostensible Metzler system matrix in combination with its stable complement, having a prescribed region of D-stability. The intention was to define the synthesis conditions based only on the quadratic Lyapunov function and to suppress the influence of disturbance in the state estimation by setting the upper bounds of the H_∞ norm of its transfer function matrix. The proposed synthesis conditions are not singular, ensuring fast enough convergence of estimation errors, and do not require prior knowledge of the disturbance boundary. With a constant output matrix and the fact that only the upper and lower bounds of the system dynamics matrix are required, such interval observers have relatively high robustness to changes in system parameters. No comparable results in the field of interval estimators for systems with Metzler dynamics seem to have been published so far.

The use of the class of application models was strictly limited by the occurrence of the description of dynamics in the form of Metzler matrices, a class which also includes models of turbo engines applied in the field of networked aircraft fault tolerant control and diagnosis (Jin and Chen, 2014; Li et al., 2020). The goal of the idea was to derive a method for application in the context of interval observer-based methodology for aircraft engine diagnosis and fault-tolerant control (Lamouchi et al., 2022). It is still left as an open question.

This approach requires further theoretical investigation, especially if the considered continuous-time systems have ostensible Metzler system matrices that have a dominant number of negative and zero elements outside the main diagonal. Further research is thus envisaged on both theoretical and applied aspects in anti-disturbance tracking control for unmanned aerial vehicles and drones considering ostensible Metzler and Hurwitz model parameter setting (Yong, 2022; Song et al., 2023).

Data availability statement

The raw data supporting the conclusion of this article will be made available by the authors, without undue reservation.

Author contributions

AF elaborated the principles of the observer parameter synthesis and implemented their numerical validation. DK addressed the design and constraint principle assembling into a set of LMIs in design for ostensible Metzler continuous-time linear MIMO systems.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

References

Ait Rami, M., and Tadeo, F. (2006). "Linear programming approach to impose positiveness in closed-loop and estimated states," in *Proceedings of the 17th international symposium on mathematical theory of networks and systems MTNS 2006*, 2470–2477.

Anderson, J., and Murray, R. (2018). "Structured feedback optimization for Metzler dynamics," in *Proceedings of the 57th IEEE conference on decision and control CDC 2018*, 4417–4424.

Berman, A., Neumann, M., and Stern, R. (1989). *Nonnegative matrices in dynamic systems*. New York: John Wiley and Sons.

Chambon, E., Apkarian, P., and Burlion, L. (2015). "Metzler matrix transform determination using a nonsmooth optimization technique with an application to interval observers," in *Proceedings of the SIAM conference on control and its applications CT 15*, 205–211.

Efimov, D., Raïssi, T., Chebotarev, S., and Zolghadri, A. (2013). Interval state observer for nonlinear time varying systems. *Automatica* 49 (1), 200–205. doi:10.1016/j.automatica.2012.07.004

Farina, L., and Rinaldi, S. (2000). *Positive linear systems. Theory and applications*. New York: John Wiley and Sons.

Ganesan, K. (2007). On some properties of interval matrices. *Int. J. Math. Comput. Sci.* 1 (1), 35–42.

Gao, H., Lam, J., Wang, C., and Xu, S. (2005). Control for stability and positivity. Equivalent conditions and computation. *IEEE Trans. Circuits Syst. II. Express Briefs* 52 (9), 540–544. doi:10.1109/tcsii.2005.850525

Guo, Z., Henry, D., Guo, J., Wang, Z., Cieslak, J., and Chang, J. (2020). Metzler matrix-based switching control scheme for linear systems with prescribed performance guarantees. *IFAC-PapersOnLine* 53 (2), 6428–6433. doi:10.1016/j.ifacol.2020.12.1784

Horn, R. A., and Johnson, C. R. (1995). *Matrix analysis*. New York: Cambridge University Press.

Huong, D. (2022). Secure interval estimations for time-varying delay interconnected systems using novel distributed functional observers. *Int. J. Adapt. Control Signal Process.* 36, 1373–1393. doi:10.1002/acs.3400

Ito, H., and Dinh, T. D. (2020). Asymptotic and tracking guarantees in interval observer design for systems with unmeasured polytopic nonlinearities. *IFAC-PapersOnLine* 53 (2), 5010–5015. doi:10.1016/j.ifacol.2020.12.1099

Jaulin, L., Kieffer, M., Didrit, O., and Walter, E. (2001). "Applied interval analysis with examples in parameter and state estimation," in *Robust control and robotics* (London: Springer-Verlag).

Jin, Y., and Chen, W. (2014). "Robust fault detection and estimation for turbofan engines subject to adaptive controllers via observer and ToMFIR techniques," in *Proceedings of the 9th IEEE conference on industrial electronics and applications*, 672–677.

Khan, A., Xie, W., Zhang, L., and Liu, L. W. (2020). Design and applications of interval observers for uncertain dynamical systems. *IET Circuits, Devices Syst.* 14, 721–740. doi:10.1049/iet-cds.2020.0004

Krokavec, D., and Filasová, A. (2020a). "Control design for linear strictly Metzlerian descriptor systems," in *Proceedings of the 18th European control conference ECC 2020*, 2092–2097.

Krokavec, D., and Filasová, A. (2020b). "Interval observer design for uncertain linear continuous-time Metzlerian systems," in *Proceedings of the 28th mediterranean conference on control and automation MED 2020*, 15–18.

Krokavec, D., and Filasová, A. (2018). "LMI based principles in strictly Metzlerian systems control design," in *Mathematical problems in engineering 2018*, 1–14. doi:10.1155/2018/9590253

Krokavec, D., and Filasová, A. (2022). "State control of linear systems with potentially Metzler dynamics," in *CONTROLO 2022, lecture notes in electrical engineering*. Editors L. B. Palma, R. Neves-Silva, and L. Gomes (Cham: Springer Nature), Vol. 930, 689–701.

Kwon, W. H., Kim, P. S., and Park, P. G. (1999). A receding horizon Kalman FIR filter for linear continuous-time systems. *IEEE Trans. Automatic Control* 44 (11), 2115–2120. doi:10.1109/9.802927

Lamouchi, R., Raissi, T., Amairi, M., and Aoun, M. (2022). Interval observer-based methodology for passive fault tolerant control of linear parameter-varying systems. *Trans. Inst. Meas. Control* 44 (5), 986–999. doi:10.1177/01423312211040370

Li, D., Chang, J., and Chen, W. (2022). Event-triggered controller design for LTI systems. A distributed interval observer-based approach. *ISA Trans.* 131, 146–159. doi:10.1016/j.isatra.2022.04.049

Li, T., Tang, X., Ge, J., and Fei, S. (2020). Event-based fault-tolerant control for networked control systems applied to aircraft engine system. *Inf. Sci.* 512, 1063–1077. doi:10.1016/j.ins.2019.10.039

Liu, L. J., Zhao, X., Sun, X. M., and Wang, W. (2017). "New approaches to positive observer design of linear positive systems," in *Proceedings of the 2017 Chinese automation congress CAC 2017*, 7195–7198.

Liu, X., Xia, Y., and Xi, H. (2011). "Estimating adverse selection and moral hazard effects with hospital invoices data in a government-controlled healthcare system," in *Proceedings of the international conference on electronics, communications and control ICECC 21*, 883–901. doi:10.1002/hec.1756

Luenberger, D. G. (1971). An introduction to observers. *IEEE Trans. Automatic Control* 16 (6), 596–602. doi:10.1109/tac.1971.1099826

Mason, O. (2012). Diagonal Riccati stability and positive time-delay systems. *Syst. Control Lett.* 61 (1), 6–10. doi:10.1016/j.sysconle.2011.09.022

Mazenc, F., and Bernard, O. (2011). Interval observers for linear time-invariant systems with disturbances. *Automatica* 47 (1), 140–147. doi:10.1016/j.automatica.2010.10.019

Mazenc, F., and Bernard, O. (2014). ISS interval observers for nonlinear systems transformed into triangular systems. *Int. J. Robust Nonlinear Control* 24, 1241–1261. doi:10.1002/rnc.2937

Moisan, M., Bernard, O., and Gouzé, J. L. (2009). Near optimal interval observers bundle for uncertain bioreactors. *Automatica* 45 (1), 291–295. doi:10.1016/j.automatica.2008.07.006

Nikaido, H. (1968). *Convex structures and economic theory*. New York: Academic Press.

Peaucelle, D., Henrion, D., Labit, Y., and Taitz, K. (2002). User's Guide for SeDuMi interface 1.04 *(toulouse*. LAAS-CNRS.

Raïssi, T., and Efimov, D. (2018). Some recent results on the design and implementation of interval observers for uncertain systems. *Automatisierungstechnik* 66 (3), 213–224. doi:10.1515/auto-2017-0081

Shores, T. S. (2007). *Applied linear algebra and matrix analysis*. New York: Springer.

Shorten, R., Mason, O., and King, C. (2009). An alternative proof of the Barker, Berman, Plemmons (BBP) result on diagonal stability and extensions. *Linear Algebra its Appl.* 430 (1), 34–40. doi:10.1016/j.laa.2008.06.037

Shu, Z., Lam, J., Gao, H., Du, B., and Wu, L. (2008). Positive observers and dynamic output-feedback controllers for interval positive linear systems. *IEEE Trans. Circuits Syst. I. Regul. Pap.* 55 (10), 3209–3222. doi:10.1109/tcsi.2008.924116

Smith, H. L. (1995). *Monotone dynamical systems. An introduction to the theory of competitive and cooperative systems*. Providence: American Mathematical Society.

Son, N. K., and Hinrichsen, D. (1996). Robust stability of positive continuous time systems. *Numer. Funct. Analysis Optim.* 17 (5-6), 649–659. doi:10.1080/01630569608816716

Song, Y., Yong, K., and Wang, X. (2023). Disturbance interval observer-based robust constrained control for unmanned aerial vehicle path following. *Drones* 7, 90. doi:10.3390/drones7020090

Tanaka, T., and Langbort, C. (2011). The bounded real lemma for internally positive systems and H-infinity structured static state feedback. *IEEE Trans. Automatic Control* 56 (9), 2218–2223. doi:10.1109/tac.2011.2157394

Wang T, T., Li, Y., and Xiang, W. (2022). Design of interval observer for continuous linear large-scale systems with disturbance attenuation. *J. Frankl. Inst.* 359 (8), 3910–3929. doi:10.1016/j.jfranklin.2022.03.014

Wang, X., Jiang, G. P., Yang, W., Su, H., and Wang, X. (2020). Neighborhood interval observer based coordination control for multi-agent systems with disturbances. *IFAC-PapersOnLine* 53 (2), 10994–10999. doi:10.1016/j.ifacol.2020.12.023

Wang, X. L., Luo, H., Zhao, J., and Yin, Q. (2022). Consensus of time-varying interval uncertain multiagent systems via reduced-order neighborhood interval observer. *Complexity* 2022, 1–14. doi:10.1155/2022/5800097

Yong, K., Chen, M., Shi, Y., and Wu, Q. (2020). Hybrid estimation strategy-based anti-disturbance control for nonlinear systems. *IEEE Trans. Automatic Control* 66 (10), 4910–4917. doi:10.1109/tac.2020.3043998

Yong, K. (2022). Disturbance interval observer-based carrier landing control of unmanned aerial vehicles using prescribed performance. *Sci. Sin. Inf.* 52 (9), 1711–1726.

Zhang, H., Huang, J., and He, S. (2022). Fractional-order interval observer for multiagent nonlinear systems. *Fractal Fract.* 6 (7), 355. doi:10.3390/fractalfract6070355