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Abdelmalek Essaadi University, Morocco

*CORRESPONDENCE
Abdelfatah Kouidere
✉ kouidere89@gmail.com
Amine El Bhih
✉ elbhihamine@gmail.com

†These authors have contributed equally to this work

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Optimal control problem for mathematical modeling of Zika virus transmission using fractional order derivatives

Abdelfatah Kouidere[1*], Amine El Bhih[1,2*†], Issam Minifi[1†], Omar Balatif[3†] and Khalid Adnaoui[1†]

[1]Laboratory of Analysis Modeling and Simulation, Department of Mathematics and Computer Science, Faculty of Sciences Ben M'Sik, Hassan II University of Casablanca, Casablanca, Morocco, [2]Multidisciplinary Research and Innovation Laboratory (LPRI), Moroccan School of Engineering Sciences (EMSI), Casablanca, Morocco, [3]Laboratory of Mathematical Fundamental and Application (LMFA), Department of Mathematics, Faculty of Sciences El Jadida, Chouaib Doukkali University, El Jadida, Morocco

This study delves into the dynamics of Zika virus transmission by employing a mathematical model to explain virus spread with fractional order derivatives. The population is divided into two groups: the human group and the ticks group to accurately explain the transmission routes of the virus. The objective of this research is to protect susceptible individuals from infection and curb the spread of this endemic disease. To achieve this, we have included two control measures: the first is a sensibilization program, and the second is treatment. We investigate the use of optimal control strategies and fractional derivative techniques under the Caputo method to reduce the number of exposed and infected individuals. By employing the Pontryagin maximum principle to analyze and characterize the optimal controls, the proposed method is further validated through numerical simulations. The outcome of this study highlights the importance of containing the rate of dynamic dissemination in preventing the Zika epidemic.

KEYWORDS

Zika virus, optimal control, mathematical model, fractional-order model, epidemic

1 Introduction

Zika virus infection is a disease caused by a viral infection that spreads primarily through the bites of infected mosquitoes. While some person may not experience any noticeable symptoms, others may develop fever, rash, joint pain, or conjunctivitis (known as pink eye). The consequences of Zika virus infection can be particularly devastating for pregnant women, as it can lead to severe birth defects such as microcephaly, where a baby is born with an abnormally small head, as well as eye abnormalities. Apart from mosquito bites, the virus can also be transmitted through sexual contact, blood transfusion, and from a pregnant woman to her fetus before or during birth.[1] Although symptoms, if any, are generally mild, healthcare professionals often consider a person's symptoms and recent history to suspect Zika virus infection. Vis–vis to confirm the diagnosis, blood or urine tests are usually performed.[2]

1 https://www.who.int/news-room/fact-sheets/detail/zika-virus?gclid=Cj0KCQjwuLShBhC_ARIsAFod4fKV2P6U3A_KwDudnmZawXdqhB7fwGDuSd412P5HnQHhJKP9_dh_gP8aAsMPEALw_wcB

2 https://www.who.int/westernpacific/health-topics/zika-virus-disease

Zika virus infection can be effectively prevented by taking certain precautions. First, it is important to avoid mosquito bites completely. Although there is currently no specific treatment for Zika virus infection, there are measures that can help relieve symptoms.[3] Adequate rest, drinking plenty of fluids, and using acetaminophen can help reduce fever and relieve pain. Zika virus spreads in a similar way to other arthropod-borne viruses such as dengue, yellow fever, and chikungunya. It is transmitted primarily by certain types of mosquitoes called Aedes mosquitoes, which lay their eggs in areas with stagnant water. These mosquitoes are particularly active during the day, both indoors and in shaded areas outdoors [1].

The Zika virus was first identified in the Zika Forest of Uganda in 1947. It remained relatively unknown until 2007 when significant outbreaks began occurring in the South Pacific islands. The situation escalated in May 2015 when cases of local transmission were reported in South America, followed by Central America and the Caribbean. Eventually, the virus reached Mexico in late November 2015. Local transmission refers to the transmission of the virus through mosquito bites in areas where people live or work, rather than contracting it during their travels. During the initial week of infection, the Zika virus can be detected in the bloodstream. When an infected person is bitten by a mosquito, the mosquito ingests the virus by feeding on the infected blood. The virus then multiplies within the mosquito's body, allowing it to transmit the virus to another person when it bites again after a few days. Consequently, individuals who have traveled to regions with a high prevalence of Zika virus infection may carry the virus in their bloodstream upon returning home, potentially leading to local transmission of the Zika virus [2].

Zika virus can be spread through various means, including sexual contact, mother-to-child transmission, blood transfusion or organ transplantation, and accidental exposure in laboratories. Understanding these modes of transmission is essential for implementing effective preventive measures against Zika virus [3].

To prevent Zika virus infection, it is important to take precautions and use acetaminophen to treat fever and relieve pain. Zika virus is spread by Aedes mosquitoes, similar to dengue, yellow fever, and chikungunya. These mosquitoes are most active during the day, especially a few hours after sunrise and before sunset. They can also sting at night. Zika virus was first identified in the Zika Forest, Uganda, in 1947, but gained more attention after a large outbreak in the South Pacific islands in 2007. It subsequently spread to South America, Central America, and the Pacific Rim, Caribbean, and Mexico. Local transmission occurs when infected mosquitoes bite people in their living or work areas. While cases have been reported in certain parts of the United States, as of December 2019, no new cases of local transmission have been recorded. However, Zika virus infections have been reported among travelers returning from affected countries. The Centers for Disease Control and Prevention (CDC) permanent diagnosis for travelers is essential [4–7].

The Zika virus has received significant attention because of its potential impact on public health. While many individuals infected with Zika virus do not experience any symptoms, others may develop a range of mild symptoms such as fever, conjunctivitis, joint and muscle pain, headache, and rash. In rare cases, the virus has been linked to the Guillain-Barr syndrome, a neurological disorder characterized by muscle weakness and sensory abnormalities. One of the most troubling aspects of Zika virus infection is its potential to cause serious complications in pregnant women and their unborn babies. In addition, children infected with the virus before birth may face various developmental challenges, including delayed speech and motor skills, intellectual disabilities, seizures, and movement difficulties [8, 9].

A number of contributions have been published in related topics. Tesla et al. [10] studied temperature drives Zika virus transmission: evidence from empirical and mathematical models. Agusto et al. [11] considered a mathematical model for Zika virus dynamics with sexual transmission route. Khan et al. [9] studied a dynamical model of asymptomatic carrier Zika virus with optimal control strategies. By replacing the ordinary normal equation with a fractional derivative equation, the fractional derivatives approach achieves an important goal of controlling the fact that the system dynamics are affected by memory. Ahmed et al. [12] studied equilibrium points, stability, and numerical solutions of fractional-order predator–prey and rabies models. Zeb et al. [13] studied the optimal campaign strategies in fractional-order smoking dynamics. Sardar et al. [14] proposed a mathematical model of dengue transmission with memory. Kouidere et al. [15] studied the analysis and optimal control of a mathematical modeling of the spread of African swine fever virus with a case study of South Korea and cost-effectiveness. Raza et al. studied and proposed the number model in Zika virus and the epedmic model related in it [16–20]. And papers related it [21–33].

Considering the above, they did not take into account the ability of the memory to contain the spreading of the Zika virus, as well as the transmission of the Zika virus model to the fractional derivative.

Among the models used for epidemic analysis of Zika virus, the majority was formulated using ODE's, while others were based on fractional calculus. Recall that the fractional calculus is applied in different directions of physics, mathematical biology, fluid mechanics, electrochemistry, signal processing, viscoelasticity, finance, and many others. Fractional derivatives were used in the literature to monitor the effect of memory on the system dynamics by replacing the normal derivative arrangement with a fractional derivative arrangement. In epidemic modeling, fractional derivatives and fractional integrals are important aspects, because the effect of memory plays an important role in the spread of the disease. The presence of memory effects on past events will affect the spread of the disease in the future so that the disease can be controlled in the future, and the distance of the memory effect indicates the date of the disease spread. Thus, the effects of memory on the spread of infectious diseases can be verified using fractional derivatives. The fractional calculus adds an extra dimension in the study of dynamics of epidemiological models, especially for COVID-19 pandemic. Therefore, the fractional version of many epidemic models has been investigated as in Ahmed et al. [12] and Khan and Atangana [34].

This article is organized using the following methodology: in the first section, we present some basic characteristics and definitions of the integral and fractional derivatives. Because of its applicability to the initial conditions of differential equations, we

3 https://www.who.int/news-room/feature-stories/detail/the-history-of-zika-virus

will apply Caputo's definition to fractional order differentiation. In Section 2, we describe the problem of optimal command on the basis of the suggested model, and we also characterize the terms of optimal command by utilizing the principle of Pontryagin's maximum. In Section 4, we performed some computational simulations. Finally, in Section 5, the article is terminated.

2 Methods

2.1 Mathematical model

A mathematical model $S_P E_P I_P R_P S_M I_M$ is considered, which represents the spread of the Zika virus through a population. As a result, we divided the N population into six compartments: To begin, the people susceptible S_P, the people exposed E_P, the people infected I_P, the people recovered R_P, the ticks susceptible S_M, and the ticks infected I_M.

In Figure 1, the considered model is graphically represented.

As a result, we present the fractional optimal command mathematical model of the Zika virus, which is governed by the system of differential equations shown below Equation 1:

$$\begin{cases} D^\alpha S_P(t) = \xi_P - \mu_P S_P(t) - \beta_1 \frac{S_P(t)E_P(t)}{N} - \beta_2 \frac{S_P(t)I_P(t)}{N} - \beta_3 \frac{S_P(t)I_T(t)}{N} \\ D^\alpha E_P(t) = \beta_1 \frac{S_P(t)E_P(t)}{N} + \beta_2 \frac{S_P(t)I_P(t)}{N} + \beta_3 \frac{S_P(t)I_T(t)}{N} - (\mu_P + \alpha)E_P(t) \\ D^\alpha I_P(t) = \alpha E_P(t) - (\mu_P + \sigma + \delta) I_P(t) \\ D^\alpha R_P(t) = \sigma I_P(t) - \mu_P R_P(t) \\ D^\alpha S_T(t) = \xi_T - \mu_M S_T(t) - \beta_4 \frac{S_T(t)I_T(t)}{N} \\ D^\alpha I_T(t) = \beta_3 \frac{S_T(t)I_T(t)}{N} - \mu_M I_T(t) \end{cases} \tag{1}$$

where $S_P(0) \geq 0$, $E_P(0) \geq 0$, $I_P(0) \geq 0$, $R_P(0) \geq 0$, $S_T(0) \geq 0$ and $I_T(0) \geq 0$ are the initial state.

With

- ξ_P : recruitment rate of people susceptible.
- ξ_T : ratio of susceptible Zika being recruited.
- μ_P : death ratio in the population from natural causes.
- μ_T : death ratio in Zika due to natural causes.
- β_1: infection ratio among people through contact with exposed people.
- β_2: contamination ratio among people through contact with infected individuals.
- β_3: infection ratio among people through contact with infected Zika.
- β_4: infection rate among Zika through contact with infected individuals.
- α: the ratio of exposed becoming infected with the virus.
- σ: the ratio of infected become recovered from the virus.
- δ: death ratio caused by complications.

3 Optimal command problem with fractional derivation

3.1 Application of optimal command

Our strategy for managing the Zika virus focuses on reducing its spread among the population. The aims is to minimize both the number of individuals exposed to the virus and the number of those who become infected. To accomplish this, we developed a program that employs two distinct controls, denoted as $w_1(t)$ and $w_2(t)$, for a time period of *tinlbrack* $[0, T]$. The control $w_1(t)$ represents an awareness campaign launched at a specific time t, whereas the control $w_2(t)$ represents treatment options available at the same time t.

$$\begin{cases} D^\alpha S_P(t) = \xi_P - \mu_P S_P(t) - \beta_1(1 - w_1(t)) \frac{S_P(t)E_P(t)}{N} - \\ \beta_2(1 - w_1(t)) \frac{S_P(t)I_P(t)}{N} - \beta_3(1 - w_1(t)) \frac{S_P(t)I_T(t)}{N} \\ D^\alpha E_P(t) = \beta_1(1 - w_1(t)) \frac{S_P(t)E_P(t)}{N} + \\ \beta_2(1 - w_1(t)) \frac{S_P(t)I_P(t)}{N} + \beta_3(1 - w_1(t)) \frac{S_P(t)I_T(t)}{N} - (\mu_P + \alpha)E_P(t) \\ D^\alpha I_P(t) = \alpha E_P(t) - \sigma(1 - w_2(t))I_P - (\mu_P + \delta)I_P(t) \\ D^\alpha R_P(t) = \sigma(1 - w_2(t))I_P(t) - \mu_P R_P(t) \\ D^\alpha S_T(t) = \xi_T - \mu_T S_T(t) - \beta_3 \frac{S_T(t)I_T(t)}{N} \\ D^\alpha I_T(t) = \beta_3 \frac{S_T(t)I_T(t)}{N} - \mu_T I_T(t) \end{cases} \tag{2}$$

with $S_P(0) \geq 0$, $E_P(0) \geq 0$, $I_P(0) \geq 0$, $R_P(0) \geq 0$, $S_T(0) \geq 0$ and $I_T(0) \geq 0$ are the initial state (Equation 3).

$$J(w_1, w_2) = \int_0^T \left[E_P(t) + I_P(t) + \frac{A}{2} w_1^2(t) + \frac{B}{2} w_2^2(t) \right] dt \tag{3}$$

In this equation, we assign relative weights to the cost of implementing the awareness program and treatment controls using parameters A and B, where A and B are positive numbers. These weights assist us in determining the relative significance of the controls $w_1(t)$ and $w_2(t)$ at a given time t. To capture the non-linear interactions that arise from high levels of implementation, we use a quadratic cost function for the controls. This cost function includes the financial resources required for the treatment and implementation of an awareness campaign. The final time period in the implementation of the controls is represented by T.

To put it differently, we aim to find the optimal controls w_1^* and w_2^* that would minimize the cost function and meet the objectives of controlling the spread of the Zika virus.

To put it another way, we want to find the optimal controls w_1^* and w_2^* that minimize the cost function (Equation 4), while also controlling the spread of the Zika virus.

$$J(w_1^*, w_2^*) = \min_{(w_1, w_2) \in U} J(w_1, w_2) \tag{4}$$

Where U is the set of admissible commands defined by $U = \{(w_1, w_2)/0 \leq w_{1min} \leq w_{1i}(t) \leq w_{1max} \leq 1, 0 \leq w_{2min} \leq w_{2i}(t) \leq w_{2max} \leq 1, /\ t \in [0, T]\}$.

The primary objective in FOCPs is to determine the optimal command pair (w_1, w_2) by minimizing the objective function:

$$J(w_1, w_2) = \int_0^T \left[E_P(t) + I_P(t) + \frac{A}{2} w_1^2(t) + \frac{B}{2} w_2^2(t) \right] dt \tag{5}$$

Subject to the constraint

$D^\alpha S_P(t) = f_1(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$

$D^\alpha E_P(t) = f_2(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$

$D^\alpha I_P(t) = f_3(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$

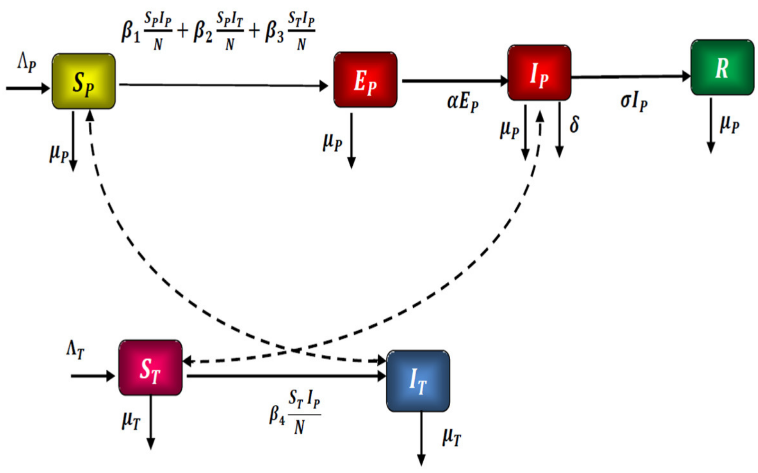

FIGURE 1
Figure population compartments model.

$D^\alpha R_P(t) = f_4(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$
$D^\alpha S_T(t) = f_4(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$
$D^\alpha I_T(t) = f_5(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2)$

The following expression defines a modified objective function (Equation 6):

$$\hat{J}(w_1, w_2) = \int_0^T \left[H^*(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t)\right] dt - \int_0^T \left[\sum_{i=1}^{6} \xi_i(t) f_i(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t)\right] dt \tag{6}$$

where $H(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t)$

$$H(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t) = E_P(t) + I_P(t) + \frac{A}{2} w_1^2(t) + \frac{B}{2} w_2^2(t) + \sum_{i=1}^{6} \xi_i(t) f_i(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t) \tag{7}$$

From Equations (5, 7), we can derive the following

$$\begin{array}{ll} D^\alpha S_P(t) = -\frac{\partial H^*(t)}{\partial \xi_1}(t) & D^\alpha E_P(t) = -\frac{\partial H^*(t)}{\partial \xi_2}(t) \\ D^\alpha I_P(t) = -\frac{\partial H^*(t)}{\partial \xi_3}(t) & D^\alpha R_P(t) = -\frac{\partial H^*(t)}{\partial \xi_4}(t) \\ D^\alpha S_T(t) = -\frac{\partial H^*(t)}{\partial \xi_5}(t) & D^\alpha I_T(t) = -\frac{\partial H^*(t)}{\partial \xi_6}(t) \end{array} \tag{8}$$

With the transversality conditions at time T :

$$\xi_1(T) = 0,\ \xi_2(T) = 0,\ \xi_3(T) = 0,\ \xi_4(T) = 0,\ \xi_5(T) = 0 \text{ and } \xi_6(T) = 0 \tag{9}$$

Equations (8, 9) outline the required conditions for the previously defined FOCP in terms of a Hamiltonian.

These conditions generate a series of fractional differential equations defined by the state variables $S_P, E_P, I_P, R_P, S_T, I_T$ and commands w_1, w_2, and solving for the Lagrange multiplier ξ_i can be accomplished through analytical, numerical, or a blended approach.

3.2 Characterization of optimal command

The Pontryagain's principle [35] to solve the optimal commands problem is given below. This method converted this into a problem of minimizing a Hamiltonian $H(t)$ at time t (Equation 10), which is defined by

$$H(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t) = E_P(t) + I_P(t) + \frac{A}{2} w_1^2(t) + \frac{B}{2} w_2^2(t) + \sum_{i=1}^{6} \xi_i(t) f_i(S_P, E_P, I_P, R_P, S_T, I_T, w_1, w_2, t) \tag{10}$$

That f_i is the right side of the differential equation of the i^{th} state variable of the system at time t.

Theorem 1. The optimals commands (w_1^*, w_2^*) and the solutions S_P^*, E_P^*, I_P^*, R_P^*, S_T^* and I_T^* of the corresponding co-state system (Equation 2) are given, the adjoint variables $\xi_1(t)$, $\lambda_2(t)$, $\xi_3(t)$, $\xi_4(t)$, $\xi_5(t)$, and $\xi_6(t)$ satisfying:

$$D^\alpha \xi_1(t) = -\xi_1(t)\left[-\mu_P - \beta_1(1 - w_1(t))\frac{E_P(t)}{N} - \beta_2 \frac{I_P(t)}{N} - \beta_3 \frac{I_T(t)}{N}\right] - \xi_2(t)\left(\beta_1(1 - w_1(t))\frac{E_P(t)}{N} - \beta_2 \frac{I_P(t)}{N} - \beta_3 \frac{I_T(t)}{N}\right)$$

$$D^\alpha \xi_2(t) = -\xi_1(t)\left[-\beta_1(1 - w_1(t))\frac{S_P(t)}{N}\right] - \xi_2(t)\left[\beta_1(1 - w_1(t))\frac{S_P(t)}{N} - (\mu_P + \alpha)\right] - \xi_3(t)\alpha + \xi_4(t)\beta_3 \frac{S_T(t)}{N} - \xi_5(t)\beta_3 \frac{S_T(t)}{N}$$

$$D^\alpha \xi_3(t) = -\xi_1(t)\left[-\beta_2(1 - w_1(t))\frac{S_P(t)}{N}\right] - \xi_2(t)\left[\beta_2(1 - w_1(t))\frac{S_P(t)}{N} - (\mu_P + \alpha\right]) - \xi_3(t)\left[(\mu_P - (1 - w_2(t))\sigma) - \xi_4(t)((1 - w_2(t))\sigma)\right]$$

$$D^\alpha \xi_4(t) = \xi_4(t)\mu_P$$

$$D^{\alpha}\xi_5(t)=\xi_5(t)(\mu_M I_T(t)+\beta_4\frac{E_P(t)}{N})-\xi_6(t)(\beta_4\frac{I_T(t)}{N})$$

$$D^{\alpha}\xi_6(t)=\xi_1(t)(\beta_3\frac{S_P(t)}{N})-\xi_2(t)(\beta_3\frac{S_P(t)}{N})+\xi_5(t)(S_T(t)+\beta_4\frac{E_P(t)}{N})-\xi_6(t)(\mu_M+\beta_4\frac{S_T(t)}{N})$$

We find the transversality conditions at time T: $\xi_1(T)=0$, $\lambda_2(T)=-1$, $\xi_3(T)=-1$, $\xi_4(T)=0$; $\xi_5(T)=0$ and $\xi_6(T)=0$.

Furthermore, for $t\in[0,T]$, the optimal controls w_1^* and w_2^* are given by

$$w_1^*=\min\left(w_{1\max},\max\left(w_{1\min},\frac{(\lambda_1(t)-\xi_2(t))}{A}\times\beta_1\frac{S_P^*(t)E_P^*(t)}{N}+\beta_2\frac{S_P^*(t)I_P^*(t)}{N}+\beta_3\frac{S_P^*(t)I_T^*(t)}{N}\right)\right)\quad(11)$$

$$w_2^*=\min\left(w_{2\max},\max\left(w_{2\min},\frac{\lambda_3(t)-\xi_4(t))}{B}\times I_P^*(t)\right)\right)\quad(12)$$

Proof. Here, we applied the Pontryagain's principle [35] to solve the optimal command problem given below.

So, we defined the Hamiltonian H by Equation 13:

$$\begin{aligned}H^*(t) = {} & E_P(T)+I_P(T)+\frac{A}{2}w_1^2(t)+\frac{B}{2}w_2^2(t)\\
&+\xi_1(t)\left[\xi_P-\mu_PS_P(t)-\beta_1(1-w_1(t))\frac{S_P(t)E_P(t)}{N}-\beta_2(1-w_1(t))\frac{S_P(t)I_P(t)}{N}-\beta_3(1-w_1(t))\frac{S_P(t)I_T(t)}{N}\right]\\
&+\xi_2(t)\left[\beta_1(1-w_1(t))\frac{S_P(t)E_P(t)}{N}+\beta_2(1-w_1(t))\frac{S_P(t)I_P(t)}{N}+\beta_3(1-w_1(t))\frac{S_P(t)I_T(t)}{N}-(\mu_P+\alpha)E_P(t)\right]\\
&+\xi_3(t)\left[\alpha E_P(t)-\sigma(1-w_2(t))I_P-(\mu_P+\delta)I_P(t)\right]\\
&+\xi_4(t)\left[\sigma(1-w_2(t))I_P(t)-\mu_PR_P(t)\right]\\
&+\xi_5(t)\left[\xi_T-\mu_TS_T(t)-\beta_3\frac{S_T(t)I_T(t)}{N}\right]\\
&+\xi_6(t)\left[\beta_3\frac{S_T(t)I_T(t)}{N}-\mu_TI_T(t)\right]\end{aligned}\quad(13)$$

For $t\in[0,T]$, the adjoint equations and transversality conditions can be obtained by using the Pontryagin's maximum principle [35–38] such that

$$\begin{aligned}D^{\alpha}\xi_1(t) = {} & -\frac{\partial H(t)}{\partial S_p(t)}=-\xi_1(t)\left[-\mu_P-\beta_1(1-w_1(t))\frac{E_P(t)}{N}-\beta_2\frac{I_P(t)}{N}-\beta_3\frac{I_T(t)}{N}\right]\\
&-\xi_2(t)\left(\beta_1(1-w_1(t))\frac{E_P(t)}{N}-\beta_2\frac{I_P(t)}{N}-\beta_3\frac{I_T(t)}{N}\right)\end{aligned}$$

$$\begin{aligned}D^{\alpha}\xi_2(t) = {} & -\frac{\partial H(t)}{\partial E_p(t)}=-\xi_1(t)\left[-\beta_1(1-w_1(t))\frac{S_P(t)}{N}\right]-\xi_2(t)\left[\beta_1(1-w_1(t))\frac{S_P(t)}{N}-(\mu_P+\alpha)\right]\\
&-\xi_3(t)\alpha+\xi_4(t)\beta_3\frac{S_T(t)}{N}-\xi_5(t)\beta_3\frac{S_T(t)}{N}\end{aligned}$$

$$\begin{aligned}D^{\alpha}\xi_3(t) = {} & -\frac{\partial H(t)}{\partial I_p(t)}=-\xi_1(t)\left[-\beta_2(1-w_1(t))\frac{S_P(t)}{N}\right]-\xi_2(t)\left[\beta_2(1-w_1(t))\frac{S_P(t)}{N}-(\mu_P+\alpha\right])\\
&-\xi_3(t)\left[(\mu_P-(1-w_2(t))\sigma)-\xi_4(t)((1-w_2(t))\sigma)\right]\end{aligned}$$

$$D^{\alpha}\xi_4(t)=-\frac{\partial H(t)}{\partial R_p(t)}=\xi_4(t)\mu_P$$

$$D^{\alpha}\xi_5(t)=-\frac{\partial H(t)}{\partial S_T(t)}=\xi_5(t)(\mu_M I_T(t)+\beta_4\frac{E_P(t)}{N})-\xi_6(t)(\beta_4\frac{I_T(t)}{N})$$

$$D^{\alpha}\xi_6(t)=-\frac{\partial H(t)}{\partial I_T(t)}=\xi_1(t)(\beta_3\frac{S_P(t)}{N})-\xi_2(t)(\beta_3\frac{S_P(t)}{N})+\xi_5(t)(S_T(t)+\beta_4\frac{E_P(t)}{N})-\xi_6(t)(\mu_M+\beta_4\frac{S_T(t)}{N})$$

With the transversality conditions at time T: $\xi_1(T)=0$, $\lambda_2(T)=-1$, $\xi_3(T)=-1$, $\xi_4(T)=0$; $\xi_5(T)=0$ and $\xi_6(T)=0$.

For, $t\in[0,T]$ the optimal commands w_1 and w_2 can be solved from the optimality condition

$$-\frac{\partial H(t)}{\partial w_1(t)}=0\Rightarrow -Aw_1(t)-(\xi_2(t)-\xi_1(t))\beta_1\frac{S_P^*(t)E_P^*(t)}{N})+\beta_2\frac{S_P^*(t)I_P^*(t)}{N})+\beta_3\frac{S_P^*(t)I_T^*(t)}{N}=0$$

$$-\frac{\partial H(t)}{\partial w_2(t)}=0\Rightarrow -Bw_2(t)-(\xi_4(t)-\xi_3(t))I_P(t)=0$$

We have

$$w_1(t)=\frac{(\xi_1(t)-\xi_2(t))}{A}\times\beta_1\frac{S_P(t)E_P(t)}{N})+\beta_2\frac{S_P(t)I_P(t)}{N})+\beta_3\frac{S_P(t)I_T(t)}{N}$$

$$w_2(t)=\frac{\lambda_3(t)-\xi_4(t))}{B}\times I_P(t)$$

By the bounds in U of the commands, it is easy to obtain w_1^* and w_2^*, which are given by Equations (11, 12) in the form of system (Equation 8).

4 Discussion

This section delves into the outcomes obtained through the numerical resolution of the optimality system. In our problem of control, the state variables have initial conditions and the adjoint conditions have terminal conditions. In other words, the optimality system is a boundary problem with two points and distinct boundary conditions for the time steps $i=t_0$ and

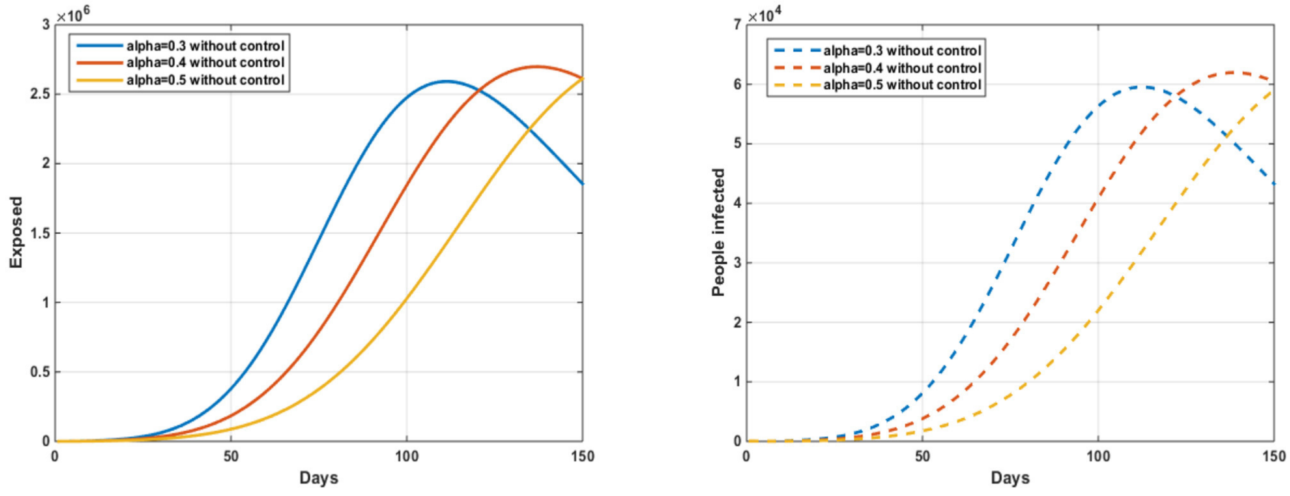

FIGURE 2
The evolution of the number of exposed and infected people without control.

TABLE 1 The values of the parameters used in the numerical simulation.

Parameter	Description	Value in d^{-1}
ξ_P	The rate at which susceptible people are recruited	0.02
Λ_M	The rate at which susceptible Zika are recruited	0.2
μ_P	The mortality rate due to natural causes among people	0.1
μ_T	The mortality rate due to natural causes among Zika	0.8
β_1	The ratio of infection transmission among individuals through contact with exposed individuals	0.4
β_2	The ratio of infection transmission among individuals through contact with infected individuals	0.2
β_3	The ratio of infection transmission among individuals through contact with the infected Zika	0.1
β_4	The ratio of Zika become infected with the virus by contact with the infected Zika	0.1
α	The ratio of exposed becoming infected with the virus	0.2
σ	The ratio of people infected become recovered from the virus	0.2
δ	The ratio of death resulting from complications	0.3

$i = t_f$. The optimality problem is resolved iteratively, in which the state system is progressed forward and the adjoint system is progressed backward. We initially estimate the controls during the first iteration and subsequently update them using characterization before the next iteration commences. We proceed until we reach the convergence of the successive iterations. An optimality program is compiled and written in MATLAB according to the data below.

As the control and state functions have different magnitudes, the weight constant is set to the following values: $A = 100$ and $B = 100$.

4.1 Scenario without control

In this scenario, we use any control.

Based on Figure 2 and the parameter values in Table 1, we noted an increase in the number of exposed and infected people with the Zika virus, which leads to an increase in the number of deaths due to the lack of any strategy to contain the disease and prevent its spread.

4.2 Scenario 1: awareness program: education

In this scenario, we use only the optimal control $w_1(t)$.

The number of people exposed to and infected with the Zika virus has decreased, as shown in Figure 2. Due to the implementation of the strategy, sensitization campaigns will be conducted for people who are likely to be infected. Moreover, due to the risks of mixing with people infected with the virus, this biological strategy aims to prevent the spread of the disease. The number of people exposed without control ranges from 150 ($alpha = 0.3$) to 3.03105 ($alpha = 0.6$) and from 150 ($alpha = 0.4$) to 3.42106 ($alpha = 0.4$) with control. The number of people infected without treatment ranges from 50$*$ ($alpha = 0.6$) to

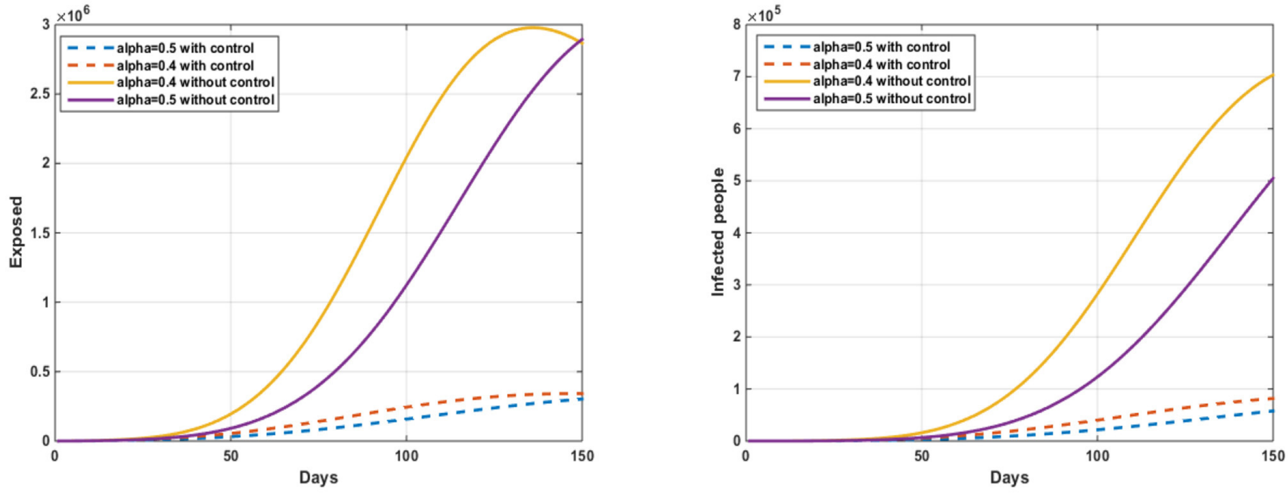

FIGURE 3
The evolution of the number of exposed and infected people without and with control $w_1(t)$.

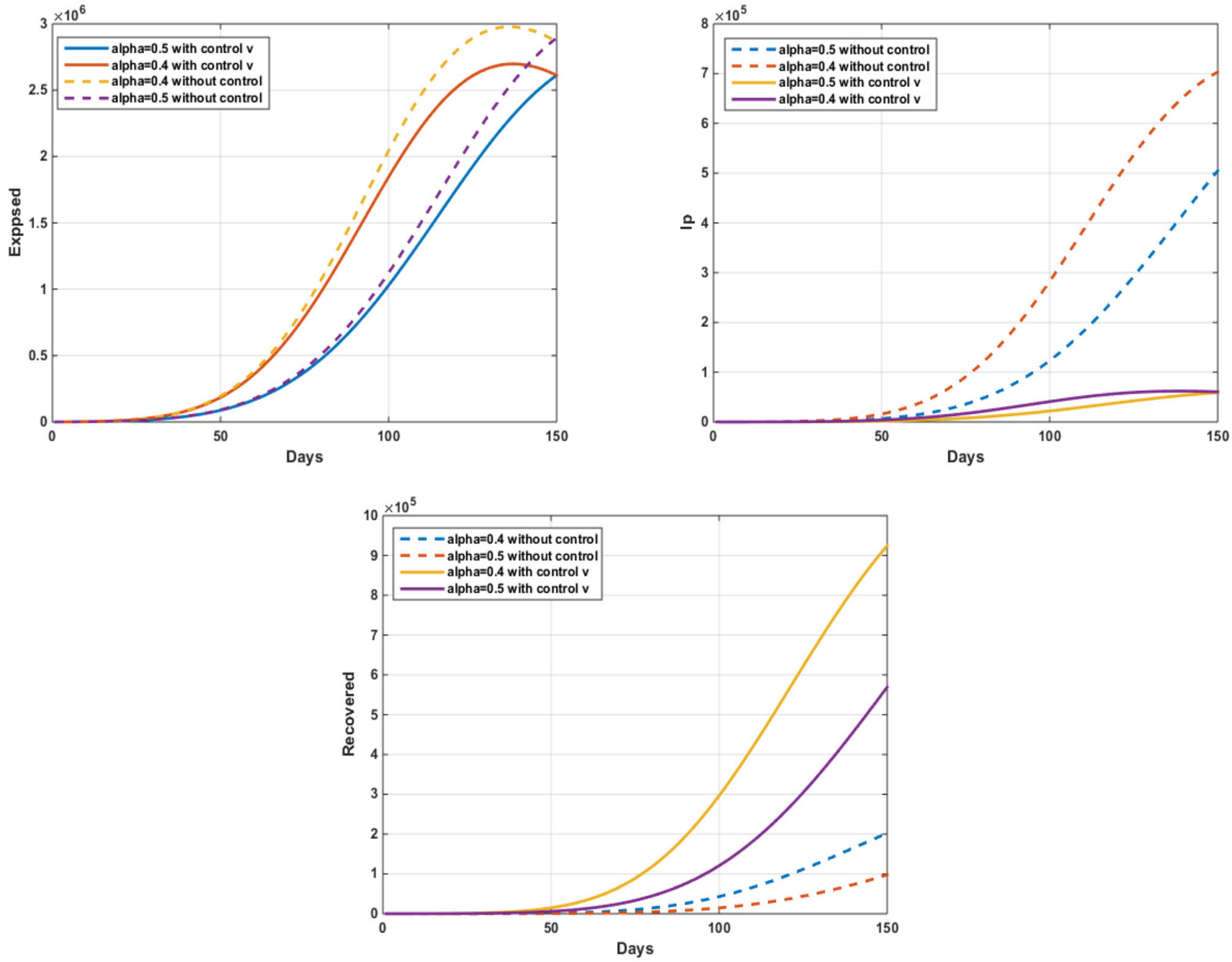

FIGURE 4
The evolution of the number of infected pigs and ticks susceptible to infection with and without controls $w_2(t)$.

$9.06, 104$ ($alpha = 0.3$) with treatment, and from 50 ($alpha = 0.4$) to $8.14, 104$ ($alpha = 0.4$) without treatment.

On the other hand, the fractional derivatives have a significant influence on the description of memory effects in dynamic systems. Memory's effects diminish as *alpha* approaches 1. Furthermore, in ordinary differential equations, the derivative order fractional *alpha* acts as a time delay. Figure 3 shows that the memory effect of the system increases when the order of the derivative *alpha* is equal to 1, and as a result, the number of infected people decreases over time.

4.3 Scenario 2: education and treatment

In this scenario, we use only the optimal control $w_2(t)$.

In Figure 4, we notice an important decrease in the number of people infected with a virus, that after 150 days, the number of exposed without control is 150 ($\alpha = 0.3$) to $1.85.10^6$ ($\alpha = 0.6$) and 150 ($\alpha = 0.4$) to $2.61.10^6$ ($\alpha = 0.4$) with control. The number of infected without control is 50 ($\alpha = 0.6$) to $4.32.10^4$ ($\alpha = 0.3$) and 50 ($\alpha = 0.4$) to $6, 04.10^4$ ($\alpha = 0.4$) with control. After applying this strategy of treatment as a vaccine and other treatments and quarantines, which will have important results in limiting the spread of the virus, the results were positive.

5 Conclusion

Our study has successfully employed fractal derivatives to establish a theoretical and computational analysis for a mathematical model of Zika transmission through a population. The model is a fractional mathematical model called $S_PE_PI_PR_PS_TI_T$, and it depicts the spread of the Zika virus throughout the population, which contained two controls. Our study has effectively integrated two control measures, an awareness program and a treatment, into the mathematical model of Zika transmission.

We have also conducted a thorough investigation of the optimal control strategy, which aims to minimize the spread of the virus by reducing the number of exposed and infected individuals. Utilizing the principles of control theory, we were able to derive the characterization of optimal control. The numerical simulations of our results have confirmed the effectiveness of the proposed containment strategies. Our research has conclusively shown that controlling the rate of dynamic dissemination is critical in containing the Zika epidemic.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

AK: Data curation, Methodology, Validation, Writing – original draft, Writing – review & editing. AE: Writing – original draft, Writing – review & editing. IM: Writing – original draft, Writing – review & editing. OB: Writing – original draft, Writing – review & editing. KA: Writing – original draft, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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