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Editorial: Approximation methods and analytical modeling using partial differential equations

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Editorial on the Research Topic
Approximation methods and analytical modeling using partial differential equations

Adequate mathematical modeling is the key to success for many real-world projects in engineering, medicine, and other applied areas. Once a well-suited model is established, it can be thoroughly examined using a broad spectrum of analytical techniques. For example, compartmental models are frequently employed in epidemiology to simulate the spread of infectious diseases, and they are also instrumental in population genetics. Although one can often prove the existence of an optimal solution under certain conditions, this does not guarantee that the solution is easy to implement in practice. In many cases, obtaining a viable approximation presents a challenging research problem in itself.

This Research Topic focuses on modeling, analysis, and approximation problems whose solutions leverage the theory of partial differential equations. It aims to showcase new analytical tools for modeling challenges in applied sciences and practical fields. Researchers explore the qualitative behavior of weak solutions, including removability conditions for singularities, the influence of initial and boundary data on local asymptotic properties, and the existence of solutions. Many articles concentrate on anisotropic models, examining the prerequisites for anisotropy strength and comparing analytical estimates of solution growth near singularities with numerical simulations. The qualitative analysis and theoretical findings are validated through observed numerical behavior. Overall, this Research Topic introduces new theoretical tools and expands the scope of established applications.

We would like to emphasize the following main topics:

1 Modeling nonlinear processes in anisotropic and inhomogeneous media, as well as boundary value problems for linear and quasilinear hyperbolic systems, and elliptic and parabolic equations with a diffusion-absorption structure

The manuscript of Barkov and Shepelsky deals with a nonlinear PDE known as the modified complex short pulse equation in its focusing version. This model is closely related to the Short Pulse (SP) equation, which is known to be a useful model of propagation of ultra-short optical pulses as thus is studied extensively in the literature, and on the other hand, it is a complex version of one of two integrable cases (the first one being the Short Pulse equation itself) of nonlinear PDE with cubic nonlinearities. The existence of the Lax pair representation suggests, in principal, that it is possible to develop the formalism of the Inverse Scattering Transform (IST) method for studying various problems for this nonlinear PDE, including solving the Cauchy problem and constructing particular explicit solutions of soliton type.

Vasylyeva examines initial-boundary value problems for semilinear integro-differential equations with multi-term fractional Caputo derivatives, particularly in the context of oxygen diffusion in capillaries. The study establishes classical and strong solvability using the continuation method, supported by a priori estimates in fractional Hölder and Sobolev spaces. The research underscores the relevance of fractional differential equations in modeling biological and engineering processes, including polymer relaxation, chaotic neuron activity, and financial time series analysis. Each of these studies contributes to advancing mathematical models in epidemiology, thermal and mechanical wave propagation, fluid dynamics, and diffusion processes, showcasing the versatility of fractional calculus and PDE-based approaches in scientific research.

Bokalo et al. consider the problem without initial conditions for some strictly nonlinear functional-differential variational inequalities in the form of subdifferential inclusions with functionals. The main results concern the existence and uniqueness of a solution for this problem in the absence of restrictions on solution's behavior and the growth of input data when the time variable is directed to minus infinity.

Protsakh studies some inverse problem of finding the time-dependent source term in a third-order semi-linear hyperbolic equation with a strong damping term. This equation is considered under Dirichlet boundary and integral over-determination conditions. The existence and uniqueness of the solution are established using Galerkin's method. She also proposes the Fourier truncation method for stabilizing the ill-posed problem.

The manuscript of Langemann and Savchenko is concerned with the numerical validation of theoretical results for the removability of singularities in anisotropic parabolic partial differential equations of porous-medium type. Numerical solution was built and compered with the theoretical apriori estimates.

Andreieva and Buryachenko's study focuses on proving the analog of the maximum principle for fourth-order hyperbolic equations, emphasizing its importance for ensuring the qualitative properties of solutions, such as uniqueness and existence. This is a significant contribution to the field, particularly given the lack of existing results for such higher-order hyperbolic equations.

2 The nonlinear transmission problem for composite beams, hyperbolic models in flow dynamics and viscoelasticity

Fastovska et al. investigate the nonlinear transmission problem associated with a composite Bresse beam consisting of a damped part. They prove the well-posedness in energy space of the PDEs describing the dynamics of the beam; establish existence of a regular global attractor under specific conditions on nonlinear parameters and damping coefficients of the damped part, and, finally, study some singular limits of the proposed problem which tend to solutions to a transmission problem for the Timoshenko beam and to solutions to a transmission problem for the Kirchhoff beam with rotational inertia. All theoretical results are validated by numerical simulation.

By means of the Cauchy-Stieltjes transform of a copolynomial, Gefter and Piven present and study a multiplication of copolynomials. They examine a Cauchy problem for the nonlinear partial differential equation in the ring of copolynomials and find a solution by using the series in powers of the δ-function. Such theoretical results are very essential and can be applied to a Cauchy problem for the Euler-Hopf equation, for the Hamilton-Jacobi type equation and for the Harry Dym equation.

Al-Lehaibi introduces a new mathematical model for analyzing thermal conduction in viscothermoelastic ceramic micro-circular rings using Kirchhoff's love plate theory. The model incorporates fractional derivatives (Caputo and Caputo-Fabrizio) to study vibration distribution under thermal loading. Results show that fractional derivatives and resonator thickness significantly affect mechanical waves, while ramp heat parameters play a crucial role in energy damping. Numerical and graphical analyses illustrate the impact of fractional-order derivatives on thermal and mechanical wave behavior.

3 Recent advances in numerical methods for fractional partial differential equations and for models with complicated geometry

The manuscript of Rassokhina and Krizhanovski concerns very popular systems used in planar microwave technology - open stubs. This work is interesting because of a lot physical applications and at the same time the nontrivial mathematical modeling background. Authors present a methodology for analyzing symmetric open stubs in a microstrip transmission line using the method of transverse resonance. This method is suitable for a variety of geometries and materials, allowing for the investigation of a wide range of stub configurations. Moreover, the method of transverse resonance is easier to implement compared to more complex numerical methods.

In the research of Noor et al. new approaches for solving fractional nonlinear Korteweg-de Vries (KdV) and coupled Burger's equations using the Aboodh residual power series and Aboodh transform iteration methods were explored. The fractional derivatives, defined in the Caputo sense, provide accurate and efficient solutions. These methods allow for explicit numerical approximations of fractional partial differential equations (FPDEs), which are widely used in physics and engineering. The study emphasizes the importance of fractional calculus in various scientific applications, including electromagnetics, fluid mechanics, and wave propagation.

4 Modern methods in approximation theory and their applications

Prestin and Semenova investigate the approximation error of trigonometric interpolation for multivariate functions of bounded variation in the sense of Hardy-Krause. The authors consider interpolation operators based on both the tensor product and sparse grids on the multivariate torus. A key aspect of their study is the focus on functions that are generally non-continuous, extending known results for smooth functions. They derive error estimates in the L_p norm and compare the accuracy of these approaches in relation to the number of grid nodes. Notably, while existing interpolation error estimates apply to smooth function spaces, e.g., Sobolev spaces H_p^r with $r > 1/p$, the authors establish convergence rates for the broader class of functions of bounded variation, achieving results analogous to the case $r = 1/p$.

Rovenska explores the approximation of classes of periodic functions using rectangular linear means of Fourier series. The study derives asymptotic equalities for upper bounds of deviations of Fejér means in the uniform norm for multivariable function classes defined by sequences tending to zero at a geometric rate. In the one-dimensional case, such classes include Poisson integrals, which admit analytic continuation in a fixed strip of the complex plane. These findings generalize known one-dimensional results and contribute to the theory of function approximation via Fourier summation methods, offering potential applications to similar upper bound problems in other settings.

Bilet and Dovgoshey analyze conditions under which a given set of metric-preserving functions can be represented as the set of all such functions associated with a certain class of metric spaces. They demonstrate that this representation holds when the given set forms a monoid with respect to the operation of function composition. In particular, they establish the existence of a class of metric spaces for which the set of all amenable sub-additive increasing functions coincides with the set of metric-preserving functions preserving that class. These results enhance the theory of metric transformations and provide new insights into the structural properties of function classes preserving various types of metric spaces.

Petrov et al. obtain generalizations of well-known fixed point theorems, including those of Banach, Kannan, Chatterjea, and Ćirić-Reich-Rus, as well as the fixed point theorem for mappings contracting the perimeters of triangles. They consider these mappings in semimetric spaces with triangle functions introduced by Bessenyei and Páles. This approach allows them to extend fixed point results to various types of semimetric spaces, demonstrating their validity in metric, ultrametric, and b-metric settings. The significance of these generalizations extends across multiple disciplines, including optimization, mathematical modeling, and computer science, where they may serve to establish stability conditions, demonstrate the existence of optimal solutions, and improve algorithm design.

Langemann and Zavarzina study plastic and non-plastic subspaces of the real line $\mathbb{R}$ with the standard Euclidean metric. They investigate non-expansive bijections, prove properties of such maps, and demonstrate their relevance through examples. The authors show that plasticity of a subspace contains two complementary questions: a purely geometric one and a topological one. Both aspects contribute to plasticity and become more critical in higher dimensions or abstract metric spaces.

Kovalyov and Levina investigate the Darboux transformation of symmetric Jacobi matrices and Toda lattices. They examine the conditions under which a symmetric Jacobi matrix can be factorized into lower and upper triangular matrices. In this case, the Darboux transformation of the symmetric Jacobi matrix produces another symmetric Jacobi matrix, which is associated with a different Toda lattice. The authors study both the Darboux transformation with and without parameters, providing insights into the relationships between Jacobi matrices, orthogonal polynomials, moment sequences, m-functions, and Toda lattices.

5 Partial differential equations based models as approximations of Markov chain dynamics. Modeling complex systems with stochastic partial differential equations

The study of Taranets et al. focuses on a time-dependent Susceptible-Infectious-Susceptible (SIS) partial differential equation (PDE) model derived from a Markov chain approach. The authors analyze the qualitative behavior of weak solutions, exploring their local asymptotic properties, existence of Dirac delta function solutions, and long-term dynamics. Numerical computations confirm their findings. The paper highlights the continued importance of epidemiological modeling despite advancements in medical treatments and the emergence of new infectious diseases like COVID-19.

A Geometric Brownian Motion (GBM) represents a classical model for stock market since 1965 by the very fruitful proposal of P. Samuelson, a famous economist. Since that time the GBM as a financial model became many extensions, especially, due to a volatility coefficient. But there is much less attention to the drift coefficient as another possibility for model transformations. Golomoziy et al. investigated the model in which the drift coefficient is modeled with the help of a Markov chain. They developed a natural asymptotic technique showing the weak convergence of a discrete scheme to the corresponding continuous time GBM. So, this work is devoted to an interesting market model, which is important historically and have a non-trivial mathematical background.

Arif et al. propose an innovative stochastic finite difference approach for modeling unsteady non-Newtonian mixed convective fluid flow with variable thermal conductivity and mass diffusivity.

Through these diverse contributions, our Research Topic provides high-quality fundamental, applied, and industry-focused research that stresses analytical aspects, novel problems, and their solutions. It provides a high-visibility, open-access publishing outlet for researchers in mathematical analysis, differential equations, numerical analysis, and other mathematical disciplines, while also fostering collaboration between these fields and related applied areas.

Author contributions

KB: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. MC: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. YK: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

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Conflict of interest

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