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Corrigendum: Expectation-maximization alternating least squares for tensor network logistic regression

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A Corrigendum on
Expectation-maximization alternating least squares for tensor network logistic regression

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In the original published article, there were typographical errors in mathematical formulas (Equations 58, 59, 73, and 74). The equations were derived and implemented correctly in the computer program; however, mistakes occurred during the writing of the paper. Corrections have been made to Equations 58, 59 in Section 4.2.2 *EM-ALS algorithm* and Equations 73, 74 in Section 4.3.2 *EM-ALS for learning multi-class TN classifiers*.

Equations 58, 59, 73, and 74 previously stated:

$$\hat{\boldsymbol{\beta}}_m = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m)^{-1} \mathbf{Z}_m^\top (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t), \tag{58}$$

$$\hat{\boldsymbol{\beta}}_m^{(\epsilon)} = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m + \epsilon \mathbf{I})^{-1} \mathbf{Z}_m^\top (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t), \tag{59}$$

$$\hat{\boldsymbol{\beta}}_m = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m)^{-1} \mathbf{Z}_m^\top (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t), \tag{73}$$

$$\hat{\boldsymbol{\beta}}_m^{(\epsilon)} = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m + \epsilon \mathbf{I})^{-1} \mathbf{Z}_m^\top (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}). \tag{74}$$

The corrected Equations appear below:

$$\hat{\boldsymbol{\beta}}_m = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m)^{-1} \mathbf{Z}_m^\top \boldsymbol{\kappa}, \quad (58)$$

$$\hat{\boldsymbol{\beta}}_m^{(\epsilon)} = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m + \epsilon \mathbf{I})^{-1} \mathbf{Z}_m^\top \boldsymbol{\kappa}, \quad (59)$$

$$\hat{\boldsymbol{\beta}}_m = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m)^{-1} \mathbf{Z}_m^\top \boldsymbol{\kappa}, \quad (73)$$

$$\hat{\boldsymbol{\beta}}_m^{(\epsilon)} = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m + \epsilon \mathbf{I})^{-1} \mathbf{Z}_m^\top \boldsymbol{\kappa}. \quad (74)$$

1 Derivation of corrections

Here we only show the derivation of Equation (58). The remaining three corrections can be derived in a similar manner.

First, Equation 58 is the weighted least squares solution for

$$g(\boldsymbol{\mathcal{A}}_m, \nu | \theta^{(t)}) = \| \boldsymbol{\Omega}_t^{\frac{1}{2}} (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t) - \boldsymbol{\Omega}_t^{\frac{1}{2}} (\mathbf{L}^{(m)} \odot \boldsymbol{\Phi}^{(m)} \odot \mathbf{R}^{(m)})^\top \mathrm{vec}(\boldsymbol{\mathcal{A}}_m) + \nu \boldsymbol{\Omega}_t^{\frac{1}{2}} \mathbf{1} \|_2^2, \quad (57)$$

where we put $\boldsymbol{\Omega}_t = \mathrm{diag}(\hat{\omega}_1^{(t)}, \hat{\omega}_2^{(t)}, ..., \hat{\omega}_N^{(t)}) \in \mathbb{R}^{N \times N}$, $\boldsymbol{\omega}_t = [\hat{\omega}_1^{(t)}, \hat{\omega}_2^{(t)}, ..., \hat{\omega}_N^{(t)}]^\top \in \mathbb{R}^{N \times N}$, $\boldsymbol{\kappa} = [\kappa_1, \kappa_2, ..., \kappa_N]^\top \in \mathbb{R}^N$, and $\oslash$ stands for entry-wise division. Let us put $\boldsymbol{\beta}_m = [\mathrm{vec}(\boldsymbol{\mathcal{A}}_m)^\top, \nu]^\top$ and $\mathbf{Z}_m = [(\mathbf{L}^{(m)} \odot \boldsymbol{\Phi}^{(m)} \odot \mathbf{R}^{(m)})^\top, \mathbf{1}]$, the cost function is rewritten as

$$\begin{aligned}
&\| \boldsymbol{\Omega}_t^{\frac{1}{2}} (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t) - \boldsymbol{\Omega}_t^{\frac{1}{2}} (\mathbf{L}^{(m)} \odot \boldsymbol{\Phi}^{(m)} \odot \mathbf{R}^{(m)})^\top \mathrm{vec}(\boldsymbol{\mathcal{A}}_m) + \nu \boldsymbol{\Omega}_t^{\frac{1}{2}} \mathbf{1} \|_2^2 \\
&= \| \boldsymbol{\Omega}_t^{\frac{1}{2}} (\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t) - \boldsymbol{\Omega}_t^{\frac{1}{2}} \mathbf{Z}_m \boldsymbol{\beta}_m \|_2^2 \\
&= \| \boldsymbol{\Omega}_t^{\frac{1}{2}} (\mathbf{Z}_m \boldsymbol{\beta}_m - \boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t) \|_2^2 \\
&= \| \boldsymbol{\Omega}_t^{\frac{1}{2}} (\mathbf{Z}_m \boldsymbol{\beta}_m - \boldsymbol{\Omega}_t^{-1} \boldsymbol{\kappa}) \|_2^2 \\
&= (\mathbf{Z}_m \boldsymbol{\beta}_m - \boldsymbol{\Omega}_t^{-1} \boldsymbol{\kappa})^\top \boldsymbol{\Omega}_t (\mathbf{Z}_m \boldsymbol{\beta}_m - \boldsymbol{\Omega}_t^{-1} \boldsymbol{\kappa}) \\
&= \boldsymbol{\beta}_m^\top \mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m \boldsymbol{\beta}_m - 2 \boldsymbol{\beta}_m^\top \mathbf{Z}_m^\top \boldsymbol{\kappa} + \boldsymbol{\kappa}^\top \boldsymbol{\Omega}_t^{-1} \boldsymbol{\kappa}.
\end{aligned}$$

Note that $\boldsymbol{\kappa} \oslash \boldsymbol{\omega}_t = \boldsymbol{\Omega}_t^{-1} \boldsymbol{\kappa}$. Optimality condition is given by

$$\left. \frac{\partial g}{\partial \boldsymbol{\beta}_m} \right|_{\boldsymbol{\beta}_m = \hat{\boldsymbol{\beta}}_m} = 2 \mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m \hat{\boldsymbol{\beta}}_m - 2 \mathbf{Z}_m^\top \boldsymbol{\kappa} = \mathbf{0}. \quad (1)$$

Finally, the minimizer is given in closed form as

$$\hat{\boldsymbol{\beta}}_m = (\mathbf{Z}_m^\top \boldsymbol{\Omega}_t \mathbf{Z}_m)^{-1} \mathbf{Z}_m^\top \boldsymbol{\kappa}. \quad (58)$$

The original article has been updated.

Publisher's note