Mirror Mode Junctions as Sources of Radiation

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Mirror modes in collisionless high-temperature plasmas represent macroscopic high-temperature quasi-superconductors with bouncing electrons in discrete-particle resonance with thermal ion-sound noise contributing to the ion-mode growth beyond quasilinear stability. In the semi-classical Ginzburg-Landau approximation the conditions for phase transition are reviewed. The quasi-superconducting state is of second kind causing a magnetically perforated plasma texture. Focussing on the interaction of mirror bubbles we apply semi-classical Josephson conditions and show that a mirror perforated plasma emits weak electromagnetic radiation which in the magnetosheath should be in the sub-millimeter, respectively, infrared range. This effect might be of astrophysical importance.

1 INTRODUCTION

The mirror mode (Chandrasekhar, 1961; Vedenov et al., 1961; Hasegawa, 1969; Davidson, 1972; Hasegawa, 1975; Gary, 1993; Southwood and Kivelson, 1993; Kivelson and Southwood, 1996; Pokhotelov et al., 2000; Pokhotelov et al., 2001; Constantinescu, 2002; Pokhotelov et al., 2002; Constantinescu et al., 2003; Pokhotelov et al., 2004; Sulem, 2011; Rincon et al., 2015; Noreen et al., 2017; Yoon, 2017) which, in high temperature plasma, evolves under anisotropic $A_i = P_{i\perp}/P_{i\parallel} - 1 > 0$ pressure conditions, can be interpreted as a phase transition from (unstable) normal to a (stationary) second-kind quasi-superconducting state (Treumann and Baumjohann, 2019; Treumann and Baumjohann, 2020). It causes the plasma to become magnetically perforated. This raises the question, investigated in this letter, in what way closely spaced mirror bubbles may interact, possibly producing identifiable effects other than localized diamagnetic field depletions.

This second-kind superconducting phase transition (Ginzburg and Landau, 1950) is known from low temperature solid state physics (Bardeen et al., 1957; Callaway, 1990), evolving Meissner diamagnetism based on electron pairing and condensation that pushes the magnetic field locally out. In mirror modes the possibility of similar condensations has recently been demonstrated (Treumann and Baumjohann, 2019). The transition is initiated by the mirror instability, which starts under the necessary condition of positive ion pressure anisotropy $A_i > 0$. In addition, the sufficient condition for instability requires the magnetic field strength B to drop below a threshold B_c

$$B < B_c \approx \sqrt{2\mu_0 N T_{i\perp} A_i} |\sin\theta_m| \tag{1}$$

(with temperature in energy units) which follows from the linear ion-mirror growth rate

$$\frac{\gamma}{\omega_{ci}} \approx \frac{k_\parallel \lambda_i}{1 + A_i} \sqrt{\frac{\beta_{p\parallel}}{\pi}} \left[A_i + \sqrt{\frac{T_{e\perp}}{T_{i\perp}}} A_e - \frac{k^2}{k_\perp^2 \beta_{p\perp}} \right] \tag{2}$$

The electron anisotropy $A_e \approx 0$ is assumed negligible initially, $k_\parallel \ll k_\perp, \lambda_i = c/\omega_i$ ion inertial length, plasma $\beta_p = 2\mu_0 N(T_i + T_e)/B_0^2$. Sometimes this is inverted into a condition on the (arbitrary) angle of wave propagation (Southwood and Kivelson, 1993) $\theta_m = \tan^{-1}(k_\perp/k_\parallel)$ but the relevant physics is contained in the field threshold. The two conditions together yield that the critical (ion) temperature is $T_{ci\perp} = T_{i\parallel}$ which means that

$$\begin{array}{l} T_{ci\perp} - T_{i\perp} \geq 0 \text{ stability, "normal" state} \\ T_{ci\perp} - T_{i\perp} < 0 \text{ instability, "quasi} - \text{superconducting"} \end{array} \quad (3)$$

which indeed reminds at the solid state superconducting phase transition.

The mirror instability readily saturates quasilinearly on the expense of the ion anisotropy (cf., e.g., Davidson, 1972; Treumann and Baumjohann, 1997) forming elongated $k_\parallel \ll k_\perp$ magnetic bottles. Landau diamagnetic theory (cf., e.g., Huang, 1973) suggests that any finite temperature diamagnetism is macroscopically very small, which is confirmed by simulations (Noreen et al., 2017) which show the saturation amplitude to remain minuscule. The observation of large-amplitude localized quasi-stationary magnetic depletions of $\lesssim 50\%$ (see Treumann and Baumjohann, 2018a, for examples, in high resolution) must be enforced by conditions which are not included in linear or quasilinear theory (Treumann et al., 2004). We do not go into discussing this problem here as it has been the subject of previous publications (cf., Treumann and Baumjohann, 2018b; Treumann and Baumjohann, 2019). We just note that a number of simulations (Rincon et al., 2015; Yao et al., 2019) and theory (Sulem, 2011) claim that nonlinear interactions between ions and waves provide large magnetic amplitudes (see the discussion in Treumann and Baumjohann, 2019). In simple words, nonlinear scattering off waves, as sometimes assumed, increases diffusivity and entropy which primarily thermodynamically inhibits structure formation. The argument of increased internal pressure fails because pressure is compensated by the large elastic volume of the environment. It dilutes the magnetic field only infinitesimally. Bubbles can deepen only on the expense of their neighbours along the same flux tube by sucking in plasma, which contradicts the observation of long mirror chains while supporting observation of isolated bubbles (Luehr and Kloecker, 1987; Treumann et al., 1990). In thermodynamic terms, entropic structure formation requires chaotic single particle interactions. These are provided by condensate formation.

2 QUASI-SUPERCONDUCTING PHASE TRANSITION

One way out of the above mentioned basic physical dilemma between observation and theory may be related to the resonance of bouncing particles in the mirror bubble and the persistent thermal ion-acoustic background noise which is independent of the presence of mirror modes (Rodriguez and Gurnett, 1975; Treumann and Baumjohann, 2018a; Treumann and Baumjohann, 2019). These resonant bouncing particles (we

here restrict to electrons, but ions if bouncing could contribute in a similar way as well) form the required condensate for phase transition.

2.1 Condensate Formation
The resonance is a discrete particle effect. It applies to all electrons in the Debye sphere and differs from the small number of Landau-cyclotron-resonant electrons (like in the radiation belts, Kennel and Petschek, 1966) which generate the banded whistler lion roars (cf., e.g., Smith and Tsurutani, 1976; Tsurutani et al., 1982; Zhang et al., 1998; Baumjohann et al., 1999; Maksimovic et al., 2001; Ahmadi et al., 2018; Breuillard et al., 2018; Giagkiozis et al., 2018) in mirror bubbles.

Trapped electrons, in contrast, performing their bounce motion in the quasilinearly-stable mirror bubble, resonate with the permanently present thermal ion-acoustic background noise of frequency ω_s through $v_\parallel \approx \omega_s/k$ near their mirror points where their parallel velocity $v_\parallel \approx c_s$ becomes comparable to the ion-sound speed $c_s = \sqrt{T_e/m_i}$. This applies to a large number N_p of electrons. In resonance they become locked to the wave and drop out of their bounce motion while maintaining their large energy anisotropy. Many such locked electrons form the condensate. Their anisotropy further increases when they move with the ion-acoustic wave into the strong magnetic field beyond their mirror points. This mechanism also causes a weak attractive potential field at distance in the wake of each resonant electron which by trapping another electron acts as Pippard's correlation length in solid state physics (Fetter and Walecka, 1971)[1] The fractional number density of condensate electrons $\mathcal{N} = N_p/N_0 < 1$ may in this case not be small. Under the assumption that initially $A_e \approx 0$, the condensate anisotropy becomes

$$A_e = \frac{2T_e}{m_e c_s^2} - 1 = \frac{2m_i}{m_e} - 1 \approx \frac{2m_i}{m_e} \quad (4)$$

which enters the above ion-mode growth rate at quasilinear stability with quasilinearly compensated ion contribution (Treumann and Baumjohann, 2018b), causing the instability to grow beyond the quasilinear limit. This effect corresponds to the noted condensate formation by pairing in metals though is basically different as here it is a high temperature purely classical effect. Further evolution implies pressure balance and the cause of surface gradient currents which has the effect of generating the partial London-Meissner diamagnetic phase transition. This phase transition can be treated in analogy to Ginzburg-Landau theory (Treumann and Baumjohann, 2020).

It is, here, instructive to point out that the above noted discrete-particle ion-resonance with ion-sound waves, though possible, as can be easily shown in the same way as for electrons, yields that the condensate-ion anisotropy becomes merely $A_i \approx T_i/T_e - 1$. This ion-condensate anisotropy is positive only in high ion temperature plasma $T_i > T_e$ where

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[1]Below we will make considerable use of definitions taken from the theory of super-conductivity. For clarity we refer the reader to the basic and quite understandable presentation of the quantum theory of super conductivity in (Fetter and Walecka, 1971), chapters 2 and 3.

ions have become heated, for instance by the presence of a shock and behind it, as would be the case in the magnetosheath. However, in order to compete with the electron anisotropy $T_i \gg T_e$ is required which is probably unrealistic.

2.2 Ginzburg-Landau Theory

The first Ginzburg-Landau equation (Huang, 1973) in the semi-classical final stationary state is obtained by putting $\hbar \to 0$ (Treumann and Baumjohann, 2020). With $\mathbf{A}$ magnetic vector potential it reads

$$\frac{e^2}{2m_e}\mathbf{A}^2\psi + \alpha\psi + \beta|\psi|^2\psi = 0 \qquad (5)$$

where ψ is the final condensate wave function, α, β constants. Clearly $\beta > 0$. It has the symmetric normal state solution $\psi = 0$ for $\alpha > 0$, and in condensate formation $\alpha = -a < 0$ after (spontaneous gauge) symmetry breaking the solution

$$\beta|\psi|^2 = a - \frac{e^2}{2m_e}\mathbf{A}^2 > 0 \qquad (6)$$

which means that the density N_p of resonant electrons is finite. Here, $|\psi|^2 \sim N_p/N_0 \equiv \mathcal{N}$ as we will also use it below when discussing radiation. Hence phase transition occurs if only $a > (e^2/2m_e)\mathbf{A}^2$. In the magnetosheath we have $|\mathbf{A}| \sim B\Delta L \approx 10^{-3}$ Vs/m. Hence the requirement is that numerically $a \gtrsim 10^{-14}$ VAs or $\sim 10^5$ eV, which looks large. However, β is of the same order of magnitude thus balancing it.

One may note that for ion-condensate formation this value reduces by more than three orders of magnitude and might thus energetically favour ions even though their anisotropy cannot compete with that of electrons, a case which we do not investigate further here but might turn out important in application to the magnetosheath or cases where heavy cold ions come into play which easily resonate with the thermal acoustic noise for instance when produced by charge exchange as is the case in the interaction of stellar winds and the cold interstellar matter. Hence separate investigation of ion-condensate formation is of vital interest. We here remain in the electron picture for reasons which will become clear below.

Since we require that $\mathcal{N} < 1$ for electron condensate formation the condition on the coefficients in GL-theory becomes simply that

$$1 < a/\beta < 1 + (\beta/10^5 \text{ eV})^{-1} \qquad (7)$$

The absolute values of these coefficients are unimportant. Approximate relations between these coefficients and the mirror conditions have also been obtained (Treumann and Baumjohann, 2020) but will not be repeated here. Hence there is some range where phase transition becomes probable which, for the purpose of this Letter, should suffice. The physical meaning is that the discrete resonant-electron condensate causes macroscopic diamagnetism which substantially diminishes the magnetic field locally.

2.3 Ginzburg-Landau Parameter

In mirror chains the magnetic field penetrates the quasi-superconducting region just up to a length $\lambda_m \approx \kappa_G\lambda_L \gtrsim \lambda_L$ with London skin depth $\lambda_L \approx \mathcal{N}^{-1/2}\lambda_i$. Here $\lambda_i = (m_i/m_e)^{1/2}\lambda_e$ is the ion skin depth, $\lambda_e = c/\omega_e$ and ω_e the electron plasma frequency, and κ_G the Ginzburg ratio, defined below.

In real mirror bubbles, however, depletion of the magnetic field is partial only. It is not complete, a point which is fundamental to the above mechanism of phase transition and the generation of chains of mirror bubbles. Maintenance of a magnetic field fraction is crucial because it maintains and thus enables the required trapped-electron bounce motion. The discrete-particle resonance is only temporary and resolves after a while but the large number and distribution of bouncing electrons over the whole bubble volume guarantees for the permanent presence of a locked electron population forming a condensate that is distributed over the volume of the mirror bubble. The property of a second-kind quasi-superconductor is provided by the Ginzburg-Landau ratio of skin depth to the above given correlation length $\xi \approx 1.5\lambda_D$ with λ_D Debye length

$$\kappa_G = \frac{\lambda_L}{\xi} > 1 \qquad (8)$$

The plasma perforates into a large number of bubbles (mirror chains) with local diamagnetism caused by the condensate in each bubble. It does not embrace the whole plasma volume. (Clearly, complete Meissner effects in space, for instance the magnetosheath, are unrealistic as they would deplete the entire plasma volume of magnetic fields on the large scale, which is not observed and thus does not take place). The correlation length ξ is the scale where the electrons feel their mutual attracting potentials close to all the continuously distributed mirror points of the trapped bouncing electrons. Thus it is a natural correlation length of the electrons in the mirror mode plasma. Clearly, the correlation length $\xi \approx 1.5\lambda_D < \lambda_m$. In the magnetosheath one then has about $10^2 < \kappa_G < 10^4$ which suggests strong magnetic perforation as is clearly observed and, in addition large skin depth, reflecting that the bubbles are only partially depleted of the magnetic field.

In this view mirror mode chains can be considered classical representations of a second-kind superconducting Ginzburg-Landau phase transition from normal to perforated plasma state in high temperature plasma. Their observation in the turbulent magnetosheath behind the bow shock, which is a strong shock, is due to the capacity of the shock to generate conditions in the transition region between the shock and magnetopause which satisfy both the necessary and sufficient conditions for the evolution of the mirror mode.

Once mirror chains have evolved and the plasma has become perforated by the quasi-superconducting phase transition described above, the question arises whether the closely spaced mirror bubbles may interact. In the following we focus on this interaction between mirror bubbles and its possible observational signature.

3 JOSEPHSON EFFECT IN MIRROR MODES

The problem of interaction of two superconductors (in our case two quasi-superconducting partially magnetic field-depleted mirror bubbles separated by a non-superconducting magnetized sheet) is the celebrated Josephson problem (Josephson, 1962; Josephson, 1964). It makes use of the Landau-Ginzburg mesoscopic theory of superconductivity (Ginzburg and Landau, 1950) which is applicable in this case. The order parameter is the expectation value of the wave function ψ given by $\langle \psi \psi^* \rangle = \mathcal{N}$, which in our case is the above introduced fractional density $\mathcal{N}$ of the bouncing electrons in resonance with the ion-acoustic thermal background fluctuations that form the condensate.

The interaction includes of course the boundaries of the two bubbles and hence takes into account the current while, in the mutual interaction, the interior is of little interest. It just responses by the exponential partial Meissner screening of the magnetic field $B = B_0 \exp(-x/\lambda_m)$ with λ_m the actual penetration depth which for $\mathcal{N} < 1$ is larger than the inertial length $\lambda_i = c/\omega_i \approx 43\lambda_e$ and the London length in a proton plasma. Since mirror bubbles are ion modes even though driven unstable by electrons, one here must use the ion inertial length, with ω_i the ion plasma frequency. Observations suggest that the mirror penetration length is roughly $\lambda_m \approx (10-20)\lambda_L$ which on its own suggests that $\mathcal{N} \lesssim 10^{-2}$.

The wave function of superconduction which in the above spirit we apply to the case of mirror modes obeys the above used first Ginzburg-Landau equation (as was proposed in Treumann and Baumjohann, 2018b; Treumann and Baumjohann, 2019). The current, being purely electronic, is given by the well known quantum mechanical expression (Ginzburg and Landau, 1950; Bardeen et al., 1957; Huang, 1973), the second Ginzburg-Landau equation

$$\mathbf{j}(\mathbf{x}, t) = -\frac{ie\hbar N_0}{2m_e} \left(\psi^* \nabla \psi - \psi \nabla \psi^* \right) - |\psi|^2 \frac{N_0}{\kappa_G} \frac{\mathbf{A}}{\mu_0 \lambda_e^2} \quad (9)$$

$$= \frac{e\hbar N_0}{m_e} |\psi|^2 \left(\nabla \phi - \frac{e}{\hbar \kappa_G} \mathbf{A} \right) \quad (10)$$

with $\mathbf{A}$ the magnetic vector potential, ϕ the phase of the complex wave function, and boundary condition that the normal current must be continuous

$$\mathbf{n} \cdot \left[\left(\hbar \nabla - \frac{ie\mathbf{A}}{\kappa_G} \right) \psi \right]_{2-1} = 0 \quad (11)$$

where the brackets mean the difference between the quantities to both sides of the boundary, as indicated by the subscript $2 - 1$, and $\mathbf{n}$ is the normal to the boundary. Clearly, in a purely classical treatment only the last term in the current expression survives when putting $\hbar = 0$. In our semiclassical approach we retain the quantum part of the current, which is the Josephson approximation. Classically the quantum part is neglected, and one has

$$\mathbf{j}(\mathbf{x}) = -|\psi|^2 \frac{N_0}{\kappa_G} \frac{\mathbf{A}}{\mu_0 \lambda_e^2} \quad (12)$$

and from Mawell's equations trivially

$$\Box \mathbf{A} = -|\psi|^2 \frac{N_0}{\kappa_G} \frac{\mathbf{A}}{\mu_0 \lambda_e^2} \quad (13)$$

whose stationary solution in one dimension only is clearly $B = B_0 \exp(-x/\lambda_m)$, the explicit partial Meissner skin effect caused by the condensate $|\psi|^2$. (Note again that the current does not explicitly depend on mass, which implies that a hypothetical ion condensate would as well contribute to the phase transition.) Thus the important physics is contained in the generation of the condensate as described in the previous section. If assuming $B/B_0 \approx 0.5$ as is typical in the magnetosheath, one has that $\lambda_m \approx 1.5x$, where x is the measured penetration length. This, in the magnetosheath, is about $x \approx 100$ km. Hence, with $\lambda_i \sim 10$ km one confirms the order of magnitude of λ_m as given above.

Now, when in contrast to the above semi-classical use of the first Ginzburg-Landau equation considering the interaction of the mirror bubbles, the quantum property of the phase has to be retained because it is just the phase which contains the microscopic information. Moreover, space plasmas are ideal conductors and no resistors. Hence the normal current will naturally be different from zero and will reflect the microscopic effect of the interaction. For this reason the quantum part of the current must be retained. We will see that this is important in the case under consideration.

There is, however, a difference in the region between the two bubbles. It is void of any condensate and thus that narrow domain is void of the Meissner effect. The magnetic field and density it contains are spatially constant. Hence the difference between the two regions is just in the quantum mechanical term in the boundary condition and thus cannot be neglected while the conditions in the two adjacent bubbles may be different. Moreover, the tangential currents (which we do not consider here as they contribute to the partial Meissner effect but are not involved into the normal current which must by itself be continuous) flowing in the adjacent bubble boundaries are in opposite directions. This implies that the two bubbles do not merge. They do not attract each other because of the repulsive Lorentz forces such that they remain separated. Nevertheless one may assume that the separation is narrow with non-compensating currents.

Since all regions are conducting a normal current will necessarily flow. In real superconductors separated by insulators electron tunnelling takes care of normal currents. Here, in the classical case, these currents are real. Nevertheless because of the retained quantum mechanical part of the current, Josephson conditions of current continuity apply to both its sides are given as

$$\partial_n \psi_1 - \frac{ie}{\hbar \kappa_G} A_n \psi_1 = b\psi_2 \quad (14)$$

$$\partial_n \psi_2 - \frac{ie}{\hbar \kappa_G} A_n \psi_2 = -b\psi_1 \quad (15)$$

with b = const some real constant whose value is only of secondary importance here. These boundary conditions follow directly from the general condition of continuity of the normal current (11) writing it in the normal coordinates not as a difference but as the finite mismatch between the two wave functions. This is easily seen when subtracting them. As the mismatch is not known a priori because it depends on the properties of the transition region, it can be accounted for simply by the constant b. One may note that for a perfect insulator with no classical current flow $b = 0$. However, the medium between two mirror bubbles clearly permits current flow. So $b \neq 0$ is an appropriate assumption[2]. This means that seen from each bubble's side the effect of the other on the transition is constant and opposite. It is thus assumed that the transition layer between the bubbles is thin enough to consider a constant current when crossing it, thereby for simplicity neglecting any spatial fluctuation or divergence of the current. This may hold as long as the transition distance is short compared with the bubble diameter, a condition satisfied in the magnetosheath, for instance. Moreover, the layer is neither an ideal conductor nor an ideal insulator such that current flow across it is permitted. In the case when it is an ideal conductor it should be narrower than the skin depth outside the mirror bubbles, but even if this does not apply current flow is permitted anyway.

Inserting these boundary conditions into the current **Eq. 9** and cancelling some terms yields for the perpendicular current crossing the thin layer that

$$j_n = -\frac{ieb\hbar N_0}{2m_e}\left(\psi_1^*\psi_2 - \psi_1\psi_2^*\right) = j_J \sin\left(\phi_2 - \phi_1\right) \qquad (16)$$

where $j_J = (eb\hbar/m_e)N_0|\psi|^2$ is the Josephson current, and we for simplicity assumed condensate symmetry $|\psi_1| = |\psi_2|$ up to the phases. (One may note that this current is a pure electron current; any ion-condensate contribution can be neglected because of the inverse proportionality to the mass. Clearly this is an effect of the large electron mobility.) This is most easily seen when replacing the functions $\psi_{1,2}$ by the sum of their real and imaginary parts $\psi = \psi_r + i\psi_i$. The difference in the round brackets is the phase difference between the two wave functions $\psi = |\psi|e^{i\phi}$ in the two bubbles which distinguishes them.

This transverse current is a retained quantum effect even in the macroscopic case. One might argue that the two phases might be the same and thus cancel the current. There is, however, no reason for this to happen even when the condensates are identical. Continuity of the current does not require equal phases if only it can be achieved otherwise. Its importance comes into account when remembering that the gauge potentials are defined just up to additional functions which leave the fields unaffected. The

vector potential $\mathbf{A} \rightarrow \mathbf{A} + \nabla U$ is defined up to the gradient of a potential U, and consequently the electric field $\mathbf{E} = -\nabla V - \partial_t \mathbf{A}$ up to its time derivative $\partial_t U$. This implies from **Eq. 9** that the phase changes as $\phi \rightarrow \phi + eU/\hbar\kappa_G$ and the electric potential as $V \rightarrow V - \partial_t U$ or after comparison and elimination of U

$$\partial_t \phi = eV/\hbar\kappa_G \qquad (17)$$

showing that the phase is affected by the gauge potential thereby exhibiting a real change in the phase.

Before continuing, it is most interesting to reflect about what has happened. In principle the electrodynamic equations are gauge invariant which means that the vector and scalar potentials can be changed by adding particular gauge functions while leaving the fields unchanged. This it true also here. However, by applying an external potential V to the two mirror bubbles one fixes one particular gauge. This still does not change anything on the fields, it however breaks the gauge symmetry locally. By providing the mirror modes with a particular electric potential field $V(x)$ they shift to one special particular gauge, and no other gauge can be chosen anymore. In the following we will see which consequences this produces.

The gauge is in fact a (Weyl) gauge like in field theory. Time integration, with applied constant external potential V, yields the well known form of the Josephson phase

$$\phi_2 - \phi_1 = \left(\phi_2 - \phi_1\right)_0 - \omega_J t, \qquad \omega_J = \frac{e}{\hbar\kappa_G}\left(V_2 - V_1\right) \qquad (18)$$

which enters into the exponent of the wave function ψ. In the presence of a potential difference $V_2 - V_1$ the current (16) in the junction consisting of the two mirror bubbles with their common boundary of finite thickness will thus oscillate at the Josephson circular frequency ω_J. Correspondingly the normal current becomes

$$j_n = j_J \sin\left(\Delta\phi_0 - \omega_J t\right), \Delta\phi_0 = \left(\phi_2 - \phi_1\right)_0 \qquad (19)$$

This current is a *real oscillating classical normal current* flowing in the boundary region of the two adjacent mirror bubbles. It is a current that varies with time, oscillating back and forth between the bubbles (For instance, for $\Delta\phi_0 = 0$ one has

$$j_n = -j_J \sin\omega_J t \qquad (20)$$

showing that the normal current in the boundary oscillates spatially back and forth between the two bubbles, which is a real classical spatially localized effect.) Physically it is not difficult to understand the origin of this oscillation. The two oppositely directed tangential currents at the boundaries of the mirror bubbles could indeed be closed by a normal current across the highly conducting gap which separates them. In the presence of an electric potential this happens on the small scale when the transverse current temporarily for a very short time breaks through and connects the two tangential currents. This break through happens however on the microscopic scale and transports very few magnetic flux elements only. Therefore its high frequency. Interestingly, there should as well be a spatial dependence of this process along the bubbles which we have not

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[2]Classically indeed the normal derivatives would vanish, as one would naively assume. Retaining them just retains the quantum effect even under classical condition. This is the essence of what is done below though it cannot be anticipated from the equations by looking at them. In fact its realization was the great discovery of Josephson which was honoured by the Nobelprize and subsequently was most successfully been applied in SQUIDS.

considered here. It causes a tangential variation of the oscillation frequency.

Denoting the potential difference as $V_2 - V_1 = \Delta V$ and introducing the magnetic flux quantum $\Phi_0 = \pi\hbar/e$, the observable Josephson frequency becomes

$$\nu_J = \frac{\Delta V}{\Phi_0 \kappa_G}, \quad \Phi_0 = 2 \times 10^{-15} \text{ Vs.} \tag{21}$$

This frequency corresponds to an energy $\kappa_G \hbar \omega_J \approx 1 \text{ } ev$ multiplied by the applied potential difference ΔV. For $\Delta V \approx 1 \text{ } V$ the oscillation frequency is $\kappa_G \nu_J \approx 5 \times 10^{14} \text{ Hz}$, which is in the near-optical infrared. The applied potential is measured in units of the elementary magnetic flux. It thus reflects the oscillations or transport of elementary flux tubes at high frequency, perfectly suited to measure very small potential differences as used in Josephson SQUIDs.

4 RADIATION

This result discovered by Josephson (Josephson, 1962) is remarkable as, according to the above discussion, it also occurs under semi-classical conditions if only an electric potential $V(t)$ is applied to the two adjacent mirror bubbles. This is the case in a streaming plasma with plasma flow across the magnetically depleted region or where an externally applied cross potential exists.

Examples are the magnetosheath (Lucek et al., 2005) or other regions like, for instance, mirror mode chains in the solar wind (Winterhalter et al., 1994; Zhang et al., 2008). Other examples are collisionless shocks (Balogh and Treumann, 2013) which have comparably narrow transition scales, develop current sheet overshoots between magnetic depletions resembling a similar kind of junctions. Ion-inertial scale plasma turbulence or flow-driven reconnection are further examples.

The Josephson frequency of oscillation is comparably high. Its large value is due to retaining the quantum effect which implies normalization of the potential difference ΔV to the small flux quantum Φ_0. Thus the Josephson current oscillates at high frequency. For any classical macroscopic process it averages out as neither the flow nor the density can follow the fluctuation. It is just the current whose real phase oscillates between the two adjacent mirror bubbles. The quantum effect on the macroscopic plasma behaviour of the fields thus disappears. This is, of course, what is expected if considering the flow or evolution of the magnetic field.

However, there is one effect that, in addition to the fluctuating electric potential, is retained even in classical physics. This is radiation which can, in principle, be observed even though its cause is to be found in quantum physics. In this sense the Josephson effect and the frequency resemble the generation of electromagnetic radiation by atomic processes, which are pure quantum effects with macroscopically measurable consequences: emission of radiation. In close similarity the Josephson radiation can, in principle, be observed from remote by monitoring its intensity.

Oscillating currents represent sources of electromagnetic radiation, as prescribed by the wave equation

$$\square \mathbf{A}_{rad}(\mathbf{x}, t) = -\mu_0 \mathbf{j}(\mathbf{x}, t) \tag{22}$$

where $\mathbf{A}_{rad}$ is the radiated vector potential, and $\mathbf{j} = j_n \mathbf{n}$, the sinusoidal real current **Eq. 19**, where it should once more be noted that this is a real classical current. Any natural system which acts, even semi-classically, like a Josephson junction should therefore emit electromagnetic radiation at frequency around $\nu_J = \omega_J/2\pi$. The spectral width of the radiation depends on the spectral width $\Delta \omega$ of the applied time dependent external potential $V(t)$, whose Fourier transform is

$$V(\omega) = \int dt \, V(t) e^{i\omega t} \tag{23}$$

whose width can be quite large, compared with the theoretical sharpness of the Josephson frequency, in particular when the flow is highly turbulent. It leads to a time dependent Josephson phase

$$\phi_J(t) = \Delta \phi_0 - \frac{e}{\hbar \kappa_G} \int_0^t dt' \, V(t') \tag{24}$$

and, consequently, to a radiation spectrum of some typical width $\Delta \nu$ in frequency. An electric oscillation spectrum $V(\omega)$ yields for the fluctuating part of the phase

$$\phi_J(t) - \Delta \phi_0 = -\frac{e}{2\pi i \hbar \kappa_G} \int \frac{d\omega}{\omega} V(\omega) \left(e^{-i\omega t} - 1 \right) \tag{25}$$

which, for the realistic case of comparably low frequency oscillations $\omega t \ll 1$ yields that the Josephson frequency becomes

$$\nu_J \approx \frac{1}{4\pi \kappa_G \Phi_0} \int d\omega \, V(\omega) \approx \frac{V(\omega_{max})}{4\pi \kappa_G \Phi_0} \Delta \omega \tag{26}$$

The emission spectrum is only as broad as $\Delta \omega$. Since the electric potential arises from external motions in plasma its spectrum is limited from above by the plasma frequency $\Delta \omega \lesssim \omega_e \ll \omega_J$. As the plasma frequency is low, any radiation of Josephson frequency ν_J will therefore remain to be of very narrow bandwidth $\Delta \nu \ll \nu_J$ and thus, when observed, will form a narrow emission line. Any spectral broadening would then have other reasons having to be retraced to the angle or the tininess of the applied potential. One does of course not expect that this kind of radiation would be intense. Its intensity per unit volume and frequency is well known (Jackson, 1975) to be proportional to the average spectral square of the radiated vector potential,

$$\frac{d^2 I}{d\Omega d\omega} \propto |\mathbf{A}_{rad}(\omega)|^2 \propto \mu_0^2 |j_J(\omega)|^2 \propto \left(\frac{eb\hbar}{m} \right)^2 \mu_0^2 N_0^2 \mathcal{N}^2 \tag{27}$$

where we have just noted the proportionalities without solving the above wave equation as this would go beyond the purpose of the present letter.

Because of the weakness of the maximum Josephson current j_J, any susceptibly high enough radiation intensity requires a large number of emitters closely distributed in the volume, i.e., a large number of interacting mirror bubbles and hence large volumes, a condition that is probably not realistic in near-Earth space but

may sometimes be realized under astrophysical conditions. Hence there should be little doubt about the presence of such a radiation effect, its detectability might however be questioned.

5 EXAMPLES

5.1 Streaming Mirror Mode Plasmas

The case of a streaming plasma is of particular interest. Let the plasma, like that in the magnetosheath, flow at a convection speed $\mathbf{v}$ such that in a frame stationary with respect to the plasma the measured electric field is $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, where $\mathbf{B}$ is the local magnetic field. Then we have dimensionally

$$\Delta V \approx \frac{v\Phi}{\Delta L} \sin\theta \qquad (28)$$

where ΔL is the typical length scale, $\Phi = N_\Phi \Phi_0$ the magnetic flux in the magnetic field, N_Φ the usually large total number of flux elements, and θ the angle between the velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$. The Josephson frequency then becomes

$$\nu_J = \frac{v\Phi}{\Delta L \Phi_0 \kappa_G} \sin\theta \approx \frac{v N_\Phi}{\Delta L \kappa_G} \sin\theta \ \mathrm{Hz} \qquad (29)$$

As for an example, in the magnetosheath we have $v \gtrsim 10^4 \ \mathrm{ms}^{-1}$, $B \gtrsim 10 \ \mathrm{nT}$, and $\Delta L \sim 10^4 \ \mathrm{m}$, which gives $\kappa_G \nu_J \gtrsim 5 \times 10^{14} \sin\theta$ Hz just the above estimated frequency. This may become substantially reduced when the angle of plasma flow is close to parallel.

A texture of mirror bubbles closely spaced to each other in the magnetosheath should thus glow in the infrared, a frequency which can, without any problem leave the region of its excitation. Mirror mode chains in the solar wind on the other hand are roughly perpendicular to the flow and of generally larger extension. Hence their frequency will be higher closer to the optical range in the very near infrared where they occasionally could be observed. However they seem to occur rather rarely which is in contrast to the region behind shocks like the bow shock. Here they seem to be present almost at any time.

Similarly one expects that the heliosheath region behind the heliospheric termination shock evolves into magnetic turbulence where the mirror mode will constitute its lowest frequency contribution. The flow speed of the solar wind will become reduced to values similar to the magnetosheath, while the magnetic field drops to $B \approx 0.14 \ \mathrm{nT}$, as expected roughly by two orders of magnitude (Burlaga et al., 2016; Fichtner et al., 2020). The unknown length scale will partially compensate for this drop. One may thus expect that the heliosheath in the region where the mirror mode will become excited behind the termination shock will also glow in the infrared, possibly however at slightly longer wavelengths. In general, any infrared glow around stellar winds might indicate the position of their external boundaries by this kind of Josephson effect which evolves solely in mirror mode turbulences. For very weak magnetic fields or otherwise reduced magnetic fields or slower speeds the frequency of this glow may drop into the microwave domain.

5.2 The Case of Reconnection

In this context it is of particular interest to refer to reconnection as these results are independent of the direction of the magnetic field. Reconnection and mirror modes may be closely related (Volwerk et al., 2003; Phan et al., 2005) as one can easily imagine that mirror modes when encountering an antiparallel magnetic field could ignite reconnection. Moreover, they may also evolve in the reconnection process as recent MMS observations (Hau et al., 2020) of reconnection at the magnetopause and Grad-Shafranov reconstructions suggest.

Consider the case of reconnection when two plasmas of oppositely directed magnetic fields approach each other. Let them be separated by a non-magnetic plasma which by definition is ideally conducting. Then the magnetic fields penetrate it only up to their skin depth λ_e. When the two plasmas approach each other the nonmagnetic sheet between them will become compressed and ultimately some plasma will be squeezed out into jets escaping from the compressed region in all directions but ultimately preferably parallel/antiparallel to the two external magnetic fields (Savin et al., 2008). When the two plasmas are roughly $\sim 2\lambda_e$ apart, the separating sheet is a field-free superconductor in whose center two antiparallel exponentially weak fields get into contact and merge. However, the current normal to the boundaries of the sheet will be a Josephson current and oscillate at frequency ν_J in the potential ΔV of the approaching plasmas. Hence the reconnection region should radiate at high frequency. If many such regions contact in a large volume the volume may glow in the emitted frequency. Since the plasma is highly diluted, there is no problem for the radiation to escape and become visible from remote if only being composed of many such radiators such that the volume emissivity becomes susceptibly large. A single reconnection region will of course emit very weak radiation only.

This is probably the case in completely evolved low frequency plasma turbulence. Recently (Treumann and Baumjohann, 2015) we suggested that the main energy dissipation in fully developed plasma turbulence may be provided at the shortest (electron) scales $\lambda \sim \lambda_e$ by reconnection in the turbulently generated small-scale current vortices into which the streaming turbulent plasmas will necessarily decay on these scales. In general this reconnection in each single small-scale (microscopic) current sheet is weak. However since there are very many such current vortices distributed over the large-scale turbulent volume the integrated dissipation will become substantial such that it under stationary conditions will balance the mechanical energy input at large scales by the large scale flow of the plasma. Since such plasmas are filled by a multitude of small-scale reconnection regions each of them representing a magnetic field-free small region, adjacent reconnection sites represent Josephson junctions and thus should radiate at the local Josephson frequency which when measured provides direct information about the reconnection potential ΔV_{rec}, a quantity which is highly desired to know.

Since the reconnection potential is a most interesting quantity, it would be worth the effort to measure it. The Josephson effect could provide such a possibility by putting a SQUID onto a

spacecraft or otherwise trying to measure radiation in the infrared from reconnection sites.

5.3 Thermal Background Effects

In the magnetosheath like in any other high temperature plasma mirror modes are embedded into a relatively intense thermal background of ion-sound fluctuations (Rodriguez and Gurnett, 1975; Lund et al., 1996). The mean thermal level of these fluctuations (Treumann and Baumjohann, 1997), assuming an isotropic Maxwellian background (Krall and Trivelpiece, 1973; Baumjohann and Treumann, 2012), is

$$W_s \equiv \left\langle \frac{1}{2}\epsilon_0 |\delta E|^2 \right\rangle \approx \frac{T_e}{\lambda_D^3} \qquad (30)$$

where λ_D is the Debye length. At temperature $T_e \approx 30\,\text{eV}$, density $N_0 \sim 10^7\,\text{m}^{-3}$ this gives an average electric fluctuation amplitude level of $\langle \delta E \rangle \approx 3 \times 10^{-5}\,\text{Vm}^{-1}$. Since $\langle \delta E \rangle \approx \langle \Delta V \rangle \Delta L^{-1}$ this yields a Josephson frequency

$$\kappa_G \nu_J \gtrsim 10^{10} \Delta L \ \text{Hz} \qquad (31)$$

If $\Delta L \approx 10\lambda_D \sim 10^2\,\text{m}$ is a typical wavelength of the ion sound noise, then the Josephson frequency is in the range $\sim 10^3$ GHz, and the radiation produced is of wavelength $\lambda \sim 10^{-4}\,\text{m}$, in the far infrared or millimeter radio wavelength range. For larger ΔL it again shifts closer to the infrared. Considerations of this kind may be of interest in astrophysical objects, in particular in regions of high streaming velocities across collisionless shock waves. Observations in the infrared and short wavelength radio wave spectrum could provide information about its origins.

On the other hand radiation in some frequency domain may provide information about the potential difference ΔV of natural systems where either mirror modes evolve or the semi-classical superconducting Meissner effect is generated. For instance radiation at $\hbar \omega_J = 100\,\text{eV}$ in the X-ray domain caused by the Josephson effect indicates the presence of potential differences of the order of $10^2\,V$. These may not necessarily belong to very strong electric fields, as their strength depends on the scale of the potential differences.

5.4 Remarks on Shocks

In collisionless shocks this may indeed be of particular interest. They separate regions of vastly different magnetic field strengths while on the scales of the shock remain collisionless. Moreover, shock fronts exhibit various regions of different properties with spatially highly variable magnetic fields evolving into overshoots and, relative to the shock also "holes" in both quasi-perpendicular and quasi-parallel shocks (Balogh and Treumann, 2013). Some of these regions may well be considered of similar properties as natural Josephson junctions. Since there the cross shock flow naturally applies a substantial electric potential difference $\Delta V \neq 0$, shock transitions of such properties, in particular when relativistic (Bykov and Treumann, 2011) should become visible as sources of soft X-rays becoming emitters, as is frequently observed in astrophysics. Such X-ray radiation is conventionally attributed to shock acceleration of electrons when interacting with the shock front. However, part of the radiation may also be caused by the Josephson junction effect inside the shock as well as in any texture of mirror modes downstream of the shock. The distinction between the two regions is given by a difference in the emitted spectrum. The much higher velocity difference prevalent to the shock transition than downstream places the former into the X-ray domain, while downstream radiation would be substantially softer reaching into the optical to infrared spectral ranges. Of course, in any case the intensity of the radiation will be low, depending on the number of mirror bubbles, the velocity of the cross flow, and magnetic field. On would expect the highest intensity from strong relativistic shocks or otherwise from a large volume of turbulence.

In the magnetosheath, the region where near Earth one observes mirror modes, the conditions are that the plasma is dilute of the order of $N_0 \sim 10^6\,\text{m}^{-3}$. The responsible applied electric field is that of the streaming plasma which in mirror modes is quite slow, of the order of $v \approx 10^4\,ms^{-1}$ in a magnetic field of some $B \approx 10\,nT$. This yields an electric field of the order of $E \approx Bv\sin\theta \sim 10^{-4}\sin\theta\,\text{vm}^{-1}$. This gives an oscillation frequency $\nu_J \sim 10^{10}\Delta L$ Hz, where ΔL is the width of the sheet crossed by the magnetosheath flow in meters. It probably compensates for the reduction of the frequency, but if the flow is mostly parallel to the magnetic field, then the oscillation frequency may be reduced substantially. A nearly parallel flow may nevertheless bring it down into the microwave domain of GHz. In any case, the mirror mode should become a high frequency radiator. Thus, if a comparably large volume is filled with an ensemble of mirror modes it may manifest itself as a source of incoherent radiation at high frequency if the plasma experiences a cross flow.

6 SUMMARY

Following earlier work on condensate formation in magnetic mirror modes we have provided the conditions for a quasi-superconducting phase transition in high temperature plasma, following the linear mirror instability. In this process bouncing charged particles in discrete particle resonance with the thermal ion acoustic background noise lock to the ion sound wave and temporarily escape from bounce motion while generating a large anisotropy. These particles form a condensate in the mirror bubble. Since bouncing particles are abundant, the condensate is quite dense and permanently present, longer than the life time of the discrete resonance of each single particle. It continuously reforms. Through production of a weak attracting electric potential in their wakes the condensate particles give rise to a correlation length ξ. The phase transition is governed by the semi-classical GL theory and results in a second-kind quasi-superconducting state exhibiting a partial Meissner effect. Since the Ginzburg ratio $\kappa_G > 1$ is large, the phase transition perforates the plasma causing a magnetic texture which consists of chains of mirror bubbles.

We then investigated the interaction of two closely spaced bubbles finding that it can be described as a Josephson junction which produces a classical signature in weak high frequency electromagnetic radiation at frequency depending on the equivalent electric field and direction of the plasma flow. Its frequency is sufficiently far above the plasma frequency cut-off such that it would be observable from remote. Though weak and if observable it maps the mirror mode region into frequency space. Similar effects are expected in reconnection and shocks and could be of interest in application to astrophysical objects. On the other hand, putting SQUIDS onto spacecraft in order to measure potential differences produced in mirror bubbles, reconnection, and shocks with extremely high accuracy *in situ* might be advantageous.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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