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Higher Dimensional Bianchi Type-I Cosmological Models With Massive String in General Relativity

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Here we studied Bianchi type-I cosmological models with massive strings in general relativity in five dimensional space time. Out of the two different cases obtained here, one case leads to a five dimensional Bianchi type-I string cosmological model in general relativity while the other yields the vacuum Universe in general relativity in five dimensional space time. The physical and geometrical properties of the model Universe are studied and compared with the present day's observational findings. It is observed that our model is anisotropic, expanding, shearing, and decelerates at an early stage and then accelerates at a later time. The model expands along x, y, and z axes and the extra dimension contracts and becomes unobservable at $t \rightarrow \infty$. We also observed that the sum of the energy density (ρ) and the string tension density (λ) vanishes ($\rho + \lambda = 0$).

Keywords: five dimension, cloud string, bianchi type-I space-time, general relativity, anisotropic

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1 INTRODUCTION

Nowadays, there has been drastic interest in string cosmology because of its important role in the study of the origin of the Universe and its very early phases before the formation of particles. It is an interesting concept for cosmologists to study and discover its unknown phenomena that have yet to be observed to study and explore the hidden information of the Universe. So cosmologists have taken an enormous interest to understand the past evolution, present state, and future evolution of the Universe. The general relativistic study of string was started by Letelier (Letelier, 1983) and Stachel (Stachel, 1980), who developed a classical concept of the geometric strings. Due to the key role of strings in describing the evolution of the early stage of our Universe, these days, many distinguished authors are inquisitive about cosmic strings within the framework of general relativity (Kibble (Kibble, 1976; Kibble, 1980)) and it is believed that strings give rise to density perturbations leading to the creation of the massive scale structures (like galaxies) of the Universe (Zel'dovich (Zel'dovich et al., 1974; Zel'dovich, 1980)). These strings have stress-energy and they are classified as geometric strings and massive strings. The occurrence of strings in the Universe results in anisotropy in space-time, though the strings cannot be seen in the latest epoch. These strings are not harmful to the cosmological models, alternatively they can result in plenty of very interesting astrophysical outcomes. There was a spontaneous symmetry breaking of the Universe during the phase transition in the early stage of the Universe after the big-bang explosion and those cosmic strings which are very important topological defects arose in the early epoch as the cosmic temperature went down below a few critical temperature points which are consistent with grand unified theories (Everett (Everett, 1981), Vilenkin (Vilenkin, 1981a; Vilenkin, 1981b)).

The study of cosmological models in higher dimensional space-time provides us with an idea that our present Universe is much greater than the Universe at the early stage of evolution due to the accelerated expansion of the Universe. So, nowadays it is becoming very interesting to study string cosmology in higher-dimensional space-time in the framework of general relativity. The possibility of space-time possessing more than 4D (Higher Dimensions) has attracted many authors to study higher dimensional models to study cosmology. The higher dimensional model was introduced by Kaluza (Kaluza, 1921)and Klein (Klein, 1926) in an effort to unify gravity with electromagnetism. Higher-dimensional models can be regarded as a tool to illustrate the late time expedited expanding paradigm (Banik and Bhuyan (Banik and Bhuyan, 2017)). Investigation of higher-dimensional space-time can be regarded as a task of paramount importance as the Universe might have come across a higher dimensional era during the initial epoch (Singh et al.(Singh et al., 2004)). Marciano (Marciano, 1984) asserts that the detection of a time-varying fundamental constant can possibly show us the proof for extra dimensions. According to Alvax and Gavela (Alvarez and Gavela, 1983) and Guth (Guth, 1981), extra dimensions generate a huge amount of entropy which gives a possible solution to the fitness and horizon problems. Since we are living in a 4D space-time, the hidden extra dimension in 5D is highly likely to be associated with the invisible Dark Matter and Dark Energy (Chakraborty and Debnath (Chakraborty and Debnath, 2010)). Many researchers have investigated various Bianchi type models in the field of five -dimensional space-time to explore the hidden information of the Universe. Chatterjee (Chatterjee, 1993) constructed a cosmological model in higher dimensional inhomogeneous space-time with massive strings.

From the observational data, we found that our Universe is homogeneous and isotropic on a large scale, however, no physical evidence denies the chance of an anisotropic Universe. In fact, theoretical arguments are presently promoting the existence of an anisotropic phase of the Universe that approaches the isotropic phase as suggested by Charles (Charles, 1968), Hinshaw et al. (Hinshaw et al., 2003), Page et al. (Page et al., 2007). Anisotropy plays a vital role in the early phase of the evolution of the Universe and so studying homogeneous and anisotropic cosmological models is considered important. Generally, the Bianchi-type models are spatially homogeneous and are in general anisotropic. The simplicity of the field equations made Bianchi type space-time useful in constructing models which are spatially homogeneous and isotropic. For the Bianchi type I cosmological models the corresponding anisotropy parameters are time-dependent. As time increases, for a suitable choice of the scalars, the Universe which was initially anisotropic starts to become isotropic and finally attains isotropy after some large cosmic time, which agrees with the present-day observational data such as cosmic microwave background (CMB) and type Ia supernovae.

We can find fascinating studies considering the anisotropic Universe in Bianchi type-I space-time by Mohanty et al. (Mohanty et al., 2002), Sahoo et al. (Sahoo et al., 2017). A Bianchi type-I cosmological model in higher dimensional space-time with string was investigated by Krori et al. (Krori et al., 1994) where they found that the strings and matter coexist throughout the evolution of the Universe. Rahaman et al.(Rahaman et al., 2003) obtained the exact solutions of the field equations of a five-dimensional space-time within the framework of Lyra manifold with massive string as a source of gravitational field. Mohanty and Samanta (Mohanty and Samanta, 2010a) constructed an LRS Bianchi type-I inflationary string cosmological model in five-dimensional space-time with massive scalar field in general relativity and obtained that the sum of energy density and tension density is zero. Also, Mohanty and Samanta (Mohanty and Samanta, 2010b) constructed string cosmological models with massive scalar field in five dimensional space-time considering Lyra manifold and obtained that the models avoid the initial singularity. Bianchi type-III cosmological models in five-dimensional space-time in general relativity with massive string as an origin of gravitational field were constructed by Samanta et al. (Samanta et al., 2011). Bianchi type-III string cosmological models in general relativity in presence of magnetic field were investigated by Kandalker et al. (Kandalkar et al., 2012), where they solved the field equations by using the Reddy string condition ($\rho + \lambda = 0$). Samanta and Debata (Samanta and Debata, 2011) constructed a five dimension Bianchi type-I string cosmological model in the framework of Lyra manifold. Singh and Mollah (Sing and Mollah, 2016) studied an LRS Bianchi type-I cosmological model with perfect fluidity in the framework of Lyra geometry in five dimensional space-time by using constant deceleration parameter. Kaiser (Kaiser and Stebbins, 1984), Banerjee (Banerjee et al., 1990), Wang (Wang, 2005), Bali et al. (Bali et al., 2007), Power and Deshmukh (Pawar and Deshmukh, 2010), Sahoo and Mishra (Sahoo and Mishra, 2015), Singh (Singh, 2013), Goswami (Goswami et al., 2016), Reddy (Reddy and Naidu, 2007), Khadekar (Khadekar et al., 2005; Khadekar et al., 2007; Khadekar and Tade, 2007), Yadav (Yadav et al., 2011), Ladke (Ladke, 2014), Singh and Baro (Singh and Baro, 2020), Baro and Singh (Baro and Singh, 2020) are some of the authors who studied different string cosmological models within the general relativity in a different contexts in various space-times. In addition to the above mentioned authors, recently Choudhury (Choudhury, 2017), Tripathi (Tripathi et al., 2017), Dubey et al. (Dubey et al., 2018), Tiwari et al. (Tiwari et al., 2019), Ram et al. (Ram and Verma, 2019), Mollah et al. (Mollah et al., 2019), and Baro et al.(Baro et al., 2021) investigated different string cosmological models in various space times.

The above discussion motivated us to investigate here the five-dimensional string cosmological models with particles attached to them in Bianchi type-I space-time in general relativity to investigate the different possibilities of the Bianchi type model Universe which transitions from anisotropic in early evolution to isotropic at a later time point where the survival field equations are solved by making some simplifying assumptions. Also, the physical and geometrical properties of some parameters of our model Universe are discussed in detail.

2 THE METRIC AND FIELD EQUATIONS

We consider the Bianchi type-I metric in five dimensional space-time in the form of

$$ds^2 = -dt^2 + a^2dx^2 + b^2dy^2 + c^2dz^2 + D^2dm^2 \tag{1}$$

where a, b, c and D are the metric functions of cosmic time 't' only. Here the extra (fifth) coordinate 'm' is taken to be space-like.

The Einstein's field equation in general relativity is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij} \tag{2}$$

The energy-momentum tensor for a cloud string is taken as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j \tag{3}$$

where ρ, λ, ρ_p are the energy density of cloud of strings, tension density, particle density of the string respectively and they satisfy the equation $\rho = \rho_p + \lambda$. The co-ordinates are co-moving, x_i is a unit space-like vector towards the direction of strings and u_i is the five velocity vector which satisfies the conditions given below.

$$u^i u_i = -x^i x_i = -1 \tag{4}$$

$$and \quad u^i x_i = 0 \tag{5}$$

$$u^i = (0,0,0,0,1) \quad and \quad x^i = \left(a^{-1},0,0,0,0\right) \tag{6}$$

For the metric 1 by using **Equations 3–6** in the field **Equation 2** yields

$$\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\lambda \tag{7}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{c}}{c} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{c}\dot{D}}{cD} = 0 \tag{8}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{D}}{D} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{D}}{bD} = 0 \tag{9}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ab} + \frac{\dot{b}\dot{c}}{bc} = 0 \tag{10}$$

$$\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{a}\dot{D}}{aD} + \frac{\dot{b}\dot{c}}{bc} + \frac{\dot{b}\dot{D}}{bD} + \frac{\dot{c}\dot{D}}{cD} = 8\pi\rho \tag{11}$$

where an over dot and double over dot denote the first derivative and the second derivative w.r.t. cosmic time '*t*' respectively.

3 SOLUTION OF THE FIELD EQUATIONS

In this section we find physically meaningful solutions of the set of field **Equations 7–11** by taking some simplifying assumptions.

3.1 Case-I(Isotropic Model)

Let us consider the Isotropic Model as

$$a = b = c = t^{l_1} \quad and \quad D = t^{l_2} \tag{12}$$

where l_1 and l_2 are arbitrary constants.

By using **Eq. 12** in **Equations 7–11**, we get

$$\frac{1}{t^2}\left(3l_1^2 + 2l_1 l_2 - 2l_1 - l_2 + l_2^2\right) = 8\pi\lambda \tag{13}$$

$$\frac{1}{t^2}\left(3l_1^2 + 2l_1 l_2 - 2l_1 - l_2 + l_2^2\right) = 0 \tag{14}$$

$$\frac{1}{t^2}\left(2l_1^2 - l_1\right) = 0 \tag{15}$$

$$\frac{1}{t^2}\left(3l_1^2 + 3l_1 l_2\right) = 8\pi\rho \tag{16}$$

Now from **Equation 15** we get.

$l_1 = 0 \ \ or \ \ l_1 = \frac{1}{2}$

For $l_1 = 0$, from **(14)** we obtained

For $l_1 = \frac{1}{2}$, from **(14)** we obtained.

$l_2 = \frac{1}{2} \quad or \quad l_2 = -\frac{1}{2}$

Eq. 12 shows that, with the increases of time t the Universe expands indefinitely if $l_1 > 0$ and the extra dimension "m" contract to a Planckian length as $t \to \infty$ if $l_2 < 0$. The string cosmological model will be physically realistic only if we take $l_1 = \frac{1}{2} > 0$ and $l_2 = -\frac{1}{2} < 0$.

In this case the geometry of the model is described by the metric

$$ds^2 = -dt^2 + t\left(dx^2 + dy^2 + dz^2\right) + t^{-1} dm^2 \tag{17}$$

Using $l_1 = \frac{1}{2}$ and $l_2 = -\frac{1}{2}$ in **Equation 13**, the string tension density is obtained as

$$\lambda = 0 \tag{18}$$

Using $l_1 = \frac{1}{2}$ and $l_2 = -\frac{1}{2}$ in **Equation 16**, the energy density is obtained as

$$\rho = 0 \tag{19}$$

And using **18** and **19**, the particle density is obtained as

$$\rho_p = 0 \tag{20}$$

This shows that the five-dimensional isotropic Bianchi type-I model in general relativity with strings do not survive and so it results in the five-dimensional vacuum Universe in the context of the general theory of relativity.

3.2 Case-II(Anisotropic Model)

The models with the anisotropic background are the most suitable models to describe the early stages of the Universe. Bianchi type-I models are among the simplest models with the anisotropic backgrounds.

In this case, let us consider

$$a = t^{k_1}, b = t^{k_2}, c = t^{k_3} \quad and \quad D = t^{k_4} \tag{21}$$

Where k_1, k_2, k_3 *and* k_4 are arbitrary constants.

Now from **Equations 7**, **11**, by the use of **21**, we find

$$\lambda = \frac{1}{8\pi t^2}\left[k_2^2 + k_3^2 + k_4^2 + k_2 k_3 + k_3 k_4 + k_4 k_2 - (k_2 + k_3 + k_4)\right] \tag{22}$$

$$\rho = \frac{1}{8\pi t^2}\left[k_1(k_2 + k_3 + k_4) + k_2 k_3 + k_3 k_4 + k_4 k_2\right] \tag{23}$$

and particle density is

$$\rho_p = \frac{1}{8\pi t^2}\left[(k_1 + 1)(k_2 + k_3 + k_4) - \left(k_2^2 + k_3^2 + k_4^2\right)\right] \tag{24}$$

We observed that the anisotropic three space will expand as $t \to \infty$ when k_1, k_2 and k_3 are all positive and the extra dimension will contract as $t \to \infty$ if $k_4 < 0$.

The Geometry of the model is described by the metric

$$ds^2 = -dt^2 + t^{2k_1} dx^2 + t^{2k_2} dy^2 + t^{2k_3} dz^2 + t^{2k_4} dm^2 \tag{25}$$

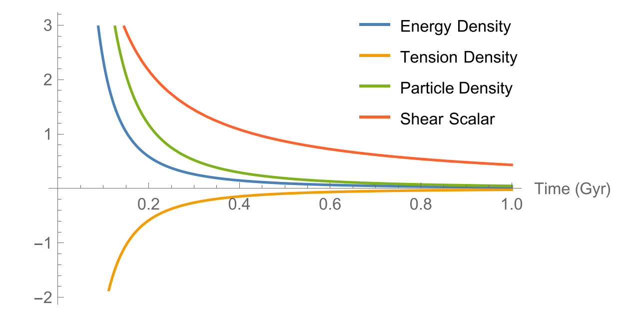

FIGURE 1 | Variation of ρ, λ, ρ_p, σ vs time t.

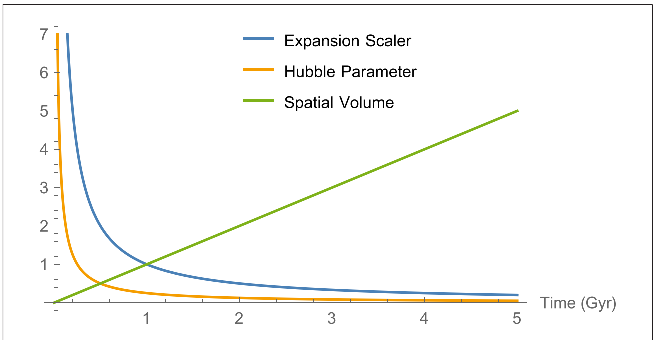

FIGURE 2 | Variation of θ, H, V vs time t.

The Scalar Expansion θ for model 25 is given by

$$\theta = \frac{l}{t} \tag{26}$$

Where, $l = k_1 + k_2 + k_3 + k_4$

The Hubble Parameter is given by

$$H = \frac{l}{4t} \tag{27}$$

The Spatial Volume of the Universe is obtained as

$$V = t^l \tag{28}$$

The Shear Scalar is obtained as

$$\sigma^2 = \frac{1}{2t^2}\left[k_1^2 + k_2^2 + k_3^2 + k_4^2 - \frac{1}{4}\right] \tag{29}$$

Deceleration Parameter q is given by

$$q = \frac{4}{l} - 1 \tag{30}$$

In this case, it is observed that the value of the deceleration parameter q is a positive constant when $l < 4$ which implies that our model Universe 25 decelerates in the standard way and the value of the deceleration parameter q is a negative constant when $l > 4$ which implies that our model Universe accelerates in the standard way. However, in the early stage of the evolution of the Universe the Bianchi type models represent the cosmos and though the Universe decelerates in the standard way in the early Universe, it will accelerate in finite time because of cosmic recollapse where the Universe in turns inflates “decelerates and then accelerates” (Kandalkar and Samdurkar (Kandalkar and Samdurkar, 2015)).

4 PHYSICAL INTERPRETATIONS OF THE SOLUTIONS

Case I: From the case I, it is observed that $\rho = \lambda = \rho_p = 0$, which results in the five-dimensional vacuum Universe in general relativity. So, the isotropic Bianchi type-I five dimensional cosmic strings Universe do not survive in general Relativity.

Case II: In case II, we have constructed the anisotropic Bianchi type-I string cosmological model in general relativity in five-dimensional space-time given by **Equation 25**. Taking $k_1 = \frac{2}{5}$, $k_2 = \frac{3}{5}$, $k_3 = \frac{1}{4}$ and $k_4 = \frac{-1}{4}$, the variation of the parameters of model 25 are shown by **Figures 1, 2**. The physical and geometrical behavior of the model can be discussed as.

1) We observed that our model expanded along x, y, and z axes as $t \to \infty$ when, k_1, k_2 and k_3 are all positive and the extra dimension contracts and becomes unobservable at $t \to \infty$, when $k_4 < 0$.
2) It is observed that at the initial epoch, i.e., $t = 0$, the energy density $\rho \to \infty$ and $\rho \to 0$ as $t \to \infty$(shown by **Figure 1**) and it satisfies the reality condition when, $k_1 + k_2k_3 + k_3k_4 + k_4k_2 > k_1^2$
3) It is also observed that at the initial epoch, i.e., $t = 0$, the string tension density $\lambda \to -\infty$ and $\lambda \to 0$ as $t \to \infty$ (**Figure 1**). From **Eqs. 22, 23**, we obtained an equation of state $\rho + \lambda = 0$, which occurs naturally in our case.
4) Also from **Figure 1**, it is observed that the particle density (ρ_p) is infinite when $t = 0$ and as time increases it decreases and finally it becomes 0 as $t \to \infty$. It satisfies the reality condition when, $(k_1^2 + k_2^2 + k_3^2 + k_4^2) < 1$
5) The spatial volume V in this model is 0 at initial epoch $t = 0$, and it increases w. r.t time which shows that our model Universe is expanding with the evolution of time, which is clearly shown in **Figure 2**.
6) The expansion scalar $\theta \to \infty$ at initial epoch $t = 0$, and as the time progresses gradually it decreases and finally it becomes 0 when $t \to \infty$(as shown in **Figure 2**). Hence the model shows that the Universe is expanding with the increase of time but the rate of expansion is slower as time increases and the expansion stops at $t \to \infty$.
7) It is observed that the value of deceleration parameter q is positive when $l < 4$ which implies that our model Universe decelerates for an instant. It is also observed that the value of deceleration parameter q is negative when $l > 4$, which implies that our model Universe accelerates in the standard way which is in accordance with the present-day observational scenario of accelerating Universe. Here, for our proposed model it may be noted that Bianchi type models represent the cosmos in its initial stage of evolution and there may be some possibilities to have an anisotropic Universe for some finite duration but the initial anisotropy of the Bianchi type -I Universe quickly dies away and the Universe turns to an isotropic one in the late

Universe. However, though the Universe decelerates in the standard way for an instant, it will accelerate in finite time because of cosmic recollapse where the Universe in turns inflates "decelerates and then accelerates" (Kandalkar and Samdurkar (Kandalkar and Samdurkar, 2015)). The decelerating behavior of the expansion in the early stage and the accelerating behavior of the expansion of the present Universe has been indicated by many cosmological observations such as cosmic microwave background (CMB), clusters of galaxies, and type Ia supernovae, etc. which have suggested that the reason for this transition from deceleration to acceleration may be due to the presence of an anti-self attraction of matter. This shows that the Universe attains isotropy at late times and transits to the accelerating Universe which is consistent with the present day observational data such as cosmic microwave background (CMB) and type Ia supernovae. We may note that according to CMB and Planck results, our Universe is homogeneous and isotropic on a large scale, however, no physical evidence denies the chances of an anisotropic Universe. In fact, theoretical arguments are presently promoting the existence of an anisotropic phase of the Universe that approaches the isotropic phase as suggested by Charles (Charles, 1968), Hinshaw et al. (Hinshaw et al., 2003), Page et al. (Page et al., 2007).

5 CONCLUSION

Here we investigated an anisotropic five dimensional Bianchi type-I string cosmological model in the context of the general theory of relativity. The model represents an expanding Universe that starts at the time $t = 0$ with a volume $V = 0$ and expands with acceleration after an epoch of deceleration. Our model Universe satisfies the energy conditions $\rho \geq 0$ and $\rho_p \geq 0$. The model Universe can represent a stage of evolution from deceleration to acceleration. The deceleration parameter "q" is decelerating at the initial stage of the evolution of the Universe and then accelerates after some finite time because of the cosmic recollapse, indicating inflation in the model after an epoch of deceleration which is in accordance with the present-day observational scenario of the accelerated expansion of our Universe as claimed by type Ia supernovae [Riess et al. (Riess et al., 1998) and Perlmutter et al. (Perlmutter et al., 1999)]. It is observed that our model Universe is anisotropic, expanding, shearing, and the sum of the energy density (ρ) and the string tension density (λ) vanishes ($\rho + \lambda = 0$) for this model.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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