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# Bulk Viscous Bianchi Type-V Cosmological Model in *f*(*R*, *T*) Theory of Gravity

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This paper deals with the bulk viscous Bianchi type-V cosmological model with an exponential scale factor in Lyra geometry based on $f(R, T)$ gravity, by considering a time dependent displacement field. To determine the nature and physical properties of the model, we considered Harko et al. (Harko et al., Phys. Rev. D, 2011, 84, 024020) [proposed the linear form $f(R, T) = f_1(R) + f_2(T)$], in which the barotropic equation of state for pressure, density, and bulk viscous pressure is proportional to energy density. The kinematical properties of the model are also discussed in the presence of bulk viscosity. Evolution of energy conditions is also studied and examined the behaviour of that in examined in order to explain the late-time cosmic acceleration.

**Keywords: bianchi type-V, lyra geometry, bulk viscous, dark energy, Λ CDM model, f(R, T) gravity**



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## 1 INTRODUCTION

As evidenced by various observational data from Garnavich (Garnavich et al., 1998a; Garnavich et al., 1998b), Riess (Riess et al., 1998) and Perlmutter (Perlmutter et al., 1997; Perlmutter et al., 1999) the expansion of the universe is accelerating. The cause of this observed acceleration is still unknown, and the "dark energy" problem is commonly used to describe it. Two methods have been proposed to overcome this problem, one is to develop several dark energy candidates and the other is to modify Einstein's theory of gravitation. It is very exclusively well known that modification of Einstein's theory plays an important role in explaining the late time acceleration and negative pressure. Among these modifications of Einstein's theories are Brans Dicke (BD) theory, $f(R)$ gravity: Carroll (Carroll et al., 2004), Nojiri (Nojiri and Odintsov, 2007), Bertolami (Bertolami et al., 2007), Capozziello (Capozziello et al., 2008), Sotiriou (Sotiriou and Faraoni, 2010), Capozziello (Capozziello and Vignolo, 2010), Iosifidis (Iosifidis et al., 2019), Gogoi (Gogoi and Dev Goswami, 2020), $f(T)$ gravity: Yang (Yang, 2011), Tamanini (Tamanini and Böhmer, 2012), Paliathanasis (Paliathanasis et al., 2016), Bamba (Bamba et al., 2017a), Ferraro (Ferraro and Guzmán, 2018), Bahamonde (Bahamonde et al., 2019), $f(G)$ gravity: De Felice (De Felice and Tsujikawa, 2009), Abbas (Abbas et al., 2015), Bamba (Bamba et al., 2017b), Sharif (Sharif and Saba, 2018), and $f(R, T)$ gravity: De Felice (De Felice et al., 2011), De Laurentis (De Laurentis and Lopez-Revelles, 2014), De Laurentis (De Laurentis et al., 2015), Odintsov (Odintsov et al., 2019), where $R$, $T$, and $G$ indicate the scalar curvature, the torsion scalar, and the Gauss Bonnet scalar respectively. But in a recent cosmological model, $f(R)$ gravity has become a more attractive theory to represent the behaviour of the expansion of the universe, known as $f(R, T)$ gravity, where the matter Lagrangian is given by an arbitrary function of the Ricci scalar $R$ and the trace of the energy momentum tensor $T$. Recently, Rao (Rao and Papa Rao, 2015), Kanakavalli (Kanakavalli et al., 2016), Sahoo (Sahoo et al., 2017), Nath (Nath and Sahu, 2019), and Sharma (Sharma et al., 2019), investigated the nature of the universe in $f(R, T)$ gravity in different cosmological models in various space times. Very recently, Arora (Arora et al., 2021) discussed the late-time viscous cosmology in $f(R, T)$ gravity considering a bulk viscous fluid with viscosity coefficient.

Bulk viscosity is particularly essential in current cosmology, because it plays a key role in the inflationary expansion. Misner (Misner, 1968) has mentioned that when neutrinos disconnect from the viscosity of the cosmic fluid, an effective process of entropy production may emerge during cosmic evolution. Barrow (Barrow, 1986) and Padmanabhan (Padmanabhan and Chitre, 1987) have pointed out that in the FRW space time, the presence of bulk viscosity causes inflationary solutions. Subsequently, Johri (Johri and Sudharsan, 1989)have studied the inflationary solutions in the presence of bulk viscosity in the Brans Dicke theory. The presence of bulk viscosity institutes a number of intriguing characteristics to the universe's dynamics. It was once thought that neutrino viscosity may smooth out primordial anisotropies, resulting in the isotropic universe we see today. The big-bang singularity can also be avoided if bulk viscosity is present. A phenomenological process of particle production in a strong gravitational field can also be explained by bulk viscosity. A bulk viscous fluid can be used to represent the back-reaction consequences of string formation. The bulk viscosity cosmological models have been discussed, and the model's nature has been discussed by Fabris (Fabris et al., 2006), Singh (Singh and Baghel, 2009), Yadav (Yadav and Yadav, 2011), Singh (Bali et al., 2012), Kiran (Kiran and Reddy, 2013), Mahanta (Mahanta, 2014), Rao (Bhaskara Rao et al., 2015), Tiwari (Tiwari and Tiwari, 2017), and Sahoo (Sahoo and Reddy, 2018). Very recently Goswami (Goswami et al., 2021) and Kotambkar (Kotambkar et al., 2021) have investigated the modeling of accelerating and the dynamical behaviours of Chaplygin gas, cosmological and gravitational "constants" with cosmic viscous fluid in different contexts.

The expansion of the universe, which is considered to be due to large scale recession of galaxies, was uknown during that period, and for that reason Einstein had to include the cosmological constant into the field equations when discussing cosmological solutions. Not only has the theory proved successful in describing gravitational phenomena, but it has also served as the foundation for cosmological models of the universe. Classical physics does not, however, describe gravity as the only force. Electromagnetic forces are also important, and they are not explained as geometric phenomena by general relativity. Many attempts to unify electromagnetic and gravitation have been made. Weyl proposed a modified Riemannian geometry theory to unify electromagnetic and gravitation in 1918. However, due to the non-integrability of length transfer, this hypothesis was later dismissed. By inserting a gauge function into the structureless manifold and removing the non-integrability of length transfer, G. Lyra (Lyra, 1951) introduced a new sort of alternative theory. Sen (Sen, 1957); Sen (Sen and Dunn, 1971) suggested a scalar-tensor theory of gravity based on this theory, which was an analogue of the Einstein field equations. Later, Halford (Halford, 1972)concluded that the constant displacement vector used in Lyra's geometry acts as a cosmological constant in the typical general relativistic interpretation. Within observational constraints, the scalar-tensor theory based on Lyra's geometry predicts the same result as the Einstein theory. Subsequently, Soleng (Soleng, 1987) looked into cosmological models based on Lyra geometry and discovered that the displacement field contains either a creation field equal to a specific vacuum field that can be regarded as a cosmological term when combined with a gauge vector. A lot of authors have looked into cosmological theories based on Lyra geometry as Pradhan (Pradhan and Vishwakarma, 2004), Singh (Singh, 2008), Singh (Singh and Kale, 2009), Ram (Ram et al., 2010), Adhav (Adhav, 2011), Kumari (Kumari et al., 2013), Singh (Singh and Rani, 2015), Maurya (Maurya and Zia, 2019), Ram (Ram et al., 2020), Hegazy (Hegazy, 2020), where they have investigated the different nature of the model in different cosmological models so far. Recently Brahma (Brahma and Dewri, 2021)has investigated the $f(R, T)$ gravity for Bianchi type–V metric in Lyra geometry and to get the deterministic solution of the model one special form of Harko et al. (2011) is used with linearly varying deceleration parameter to investigate the myterious nature of the dark energy. Here, in this study we are interested in Bianchi type-v cosmological model, because it describes homogeneous and anisotropic universes with various scale factors along each spatial direction, which is a natural generalisation of the FRW model of the universe. The above exclusive analysis and investigation encourage us to study the Bianchi type V cosmological model with $f(R, T)$ gravity in the presence of viscous fluid based on Lyra geometry.

## 2 OVERVIEW AND THE FIELD EQUATIONS OF *f*(*R*, *T*) GRAVITY

Here we consider Bianchi type -V space time in the following form

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}\left(B^2dy^2 + C^2dz^2\right) \tag{1}$$

where $A$, $B$, $C$ are functions of cosmic time $t$ and $m$ is a constant.

The action of $f(R, T)$ gravity, where we have obtained the various field equations and modification of $f(R, T)$ gravity with observational constraints in Lyra geometry by Harko et al.(Harko et al., 2011) [Taking $G = 1$] as

$$S = \frac{1}{16\pi}\int f\left(\tilde{R}, T\right)\sqrt{-g}\ d^4x + \int L_m\sqrt{-g}\ d^4x \tag{2}$$

where

$$\tilde{R} = R + 3\nabla_i\phi^i + \frac{3}{2}\phi^i\phi_i \tag{3}$$

In which $\tilde{R}$, $T$, and $L_m$ respectively denote the function of Ricci scalar $R$, the trace of the stress tensor, and the Lagrangian density of matter, where the stress- energy tensor of the matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}} \tag{4}$$

such that its trace is given by $T = g^{ij}T_{ij}$

Consider that the matter Lagrangian $L_m$ depends only on the metric tensor components $g^{ij}$ and does not depend on its derivatives, thus it reduces to

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \tag{5}$$

Now by varying the action S in **Eq. 2** with respect to metric tensor $g_{ij}$, the gravitational field equations of $f(\tilde{R}, T)$ gravity are obtained as

$$f_{\tilde{R}}(\tilde{R}, T)\tilde{R}_{ij} - \frac{1}{2}f(\tilde{R}, T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_{\tilde{R}}(\tilde{R}, T) = -\frac{8\pi}{c^2}T_{ij} - f_T(\tilde{R}, T)T_{ij} - f_T(\tilde{R}, T)\Theta_{ij} \tag{6}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{7}$$

Here $f_{\tilde{R}}(\tilde{R}, T) = \frac{\partial f(\tilde{R}, T)}{\partial \tilde{R}}$, $f_T(\tilde{R}, T) = \frac{\partial f(\tilde{R}, T)}{\partial T}$, and $\nabla_i$ denote the covariant derivative.

If the matter is considered as a perfect fluid, then the stress energy tensor of the matter Lagrangian is given by

$$T_{ij} = (\rho + \bar{p})u_i u_j + \bar{p}g_{ij} \tag{8}$$

Here $\rho$ denotes the energy density and $u^i = (0, 0, 0, 1)$ is the four velocity vector in the co-moving co-ordinate system satisfying the condition $u_i u^i = -1$ and $u^i\nabla_j u_i = 0$. Since there is no unique choice for matter Lagrangian, we assume a perfect fluid matter as $L_m = -\bar{p}$ and the trace of the total energy momentum tensor [Debnath (Debnath, 2019)] is given by $T = \rho - 3\bar{p}$, so that **Eq. 7** reduces as follows:

$$\theta_{ij} = -2T_{ij} - \bar{p}g_{ij} \tag{9}$$

As we know that the physical nature of the matter field depends on the metric tensor $\Theta_{ij}$ for the field equations of $f(\tilde{R}, T)$ gravity. So among the three cases of Harko et al.(Harko et al., 2011), we can obtain so many theoretical results by the choice of different explicit forms of $f(\tilde{R}, T)$ as

$$f(\tilde{R}, T) = \begin{cases} \tilde{R} + 2f(T) \\ f_1(\tilde{R}) + f_2(T) \\ f_1(\tilde{R}) + f_2(\tilde{R})f_3(T) \end{cases} \tag{10}$$

But, in this paper the second case is considered to describe the behaviour of the model in $f(\tilde{R}, T)$ gravity as

$$f(\tilde{R}, T) = f_1(\tilde{R}) + f_2(T) \tag{11}$$

where $f(\tilde{R}, T)$ is an arbitrary function of the trace of the stress tensor.

Now, using **Eq. 11** in **Eq. 6**, we obtain

$$f_1'(\tilde{R}, T)\tilde{R}_{ij} - \frac{1}{2}f_1(\tilde{R})g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_1'(\tilde{R}) = -\frac{8\pi}{c^2}T_{ij} + f_2'(T)T_{ij} + \left[f_2'(T)\bar{p} + \frac{1}{2}f_2(T)\right]g_{ij} \tag{12}$$

The field equations of $f(\tilde{R}, T)$ gravity, for a perfect fluid matter source, by assuming $f_1 = \mu\tilde{R}$ and $f_2 = \mu T$, where $\mu$ is taken as arbitrary constant and with this condition the above **Eq. 12** reduces to

$$\tilde{R}_{ij} - \frac{1}{2}\tilde{R}g_{ij} = -\left(\frac{8\pi - \mu c^2}{c^2}\right)T_{ij} + \left[\bar{p} + \frac{1}{2}T\right]g_{ij} \tag{13}$$

Applying **Eq. 3** in **Eq. 13**, we obtain the field equations in Lyra geometry [Maurya (Maurya, 2020)] as given by

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_i\phi^j = -hT_{ij} + \left[\bar{p} + \frac{1}{2}T\right]g_{ij} \tag{14}$$

Here the displacement vector field is $\phi^i = (0, 0, 0, \beta(t))$ and $h = (\frac{8\pi - \mu c^2}{\mu c^2})$ is taken as unity.

For the metric (1), the Einstein field **Equation 14** reduces to the form as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\bar{p} + \left(\frac{\rho - \bar{p}}{2}\right) \tag{15}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{C}\dot{A}}{CA} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\bar{p} + \left(\frac{\rho - \bar{p}}{2}\right) \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} + \frac{3}{4}\beta^2 = -\bar{p} + \left(\frac{\rho - \bar{p}}{2}\right) \tag{17}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} - \frac{3}{4}\beta^2 = \rho + \left(\frac{\rho - \bar{p}}{2}\right) \tag{18}$$

$$\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0 \tag{19}$$

## 3 SOLUTIONS OF THE FIELD EQUATIONS

The spatial volume $(V)$ and the scale factor $a(t)$ are given by

$$V = a^3 = (ABC) \tag{20}$$

The generalized Hubble's parameter $(H)$ and the scalar expansion $(\theta)$ are defined as

$$H = \frac{\dot{a}}{a} = (H_1 + H_2 + H_3); \quad \theta = 3H \tag{21}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of the $X$, $Y$, and $Z$ axes respectively.

Integrating **Eq. 19**, we get

$$A^2 = k_1 BC \tag{22}$$

where $k_1$ is an integrating constant and without loss of generality, the constant of integration $k_1$ can be chosen as unity as

$$A^2 = BC \tag{23}$$

In the field **Equations 15–19**, we found that there are five equations involving seven unknowns. As the field equations are highly non-linear differential equations, we need some other condition to complete the field equations such as.

- we consider the shear scalar $(\sigma)$ is proportional to the expansion scalar $(\theta)$ [Collins et al. (Collins et al., 1980)]

$$B = C^n \tag{24}$$

where n is a non zero constant.

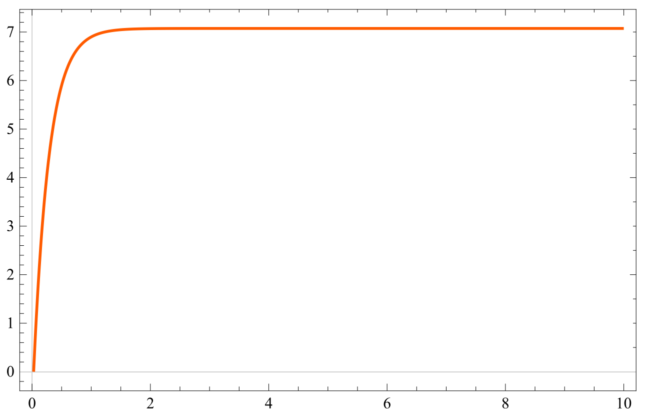

**FIGURE 1 |** Variation of density ($\rho$) *vs*. time ($t$).

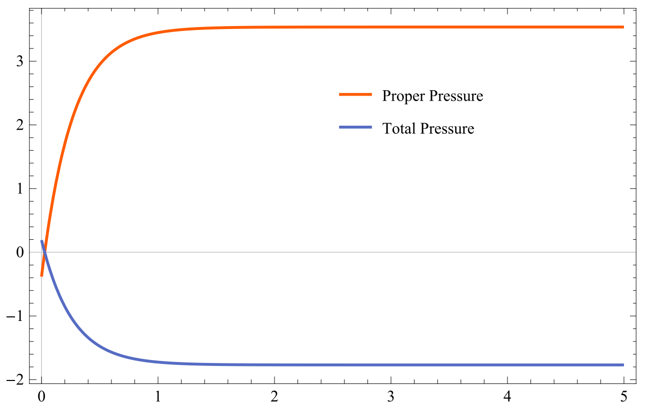


**FIGURE 2 |** Variation of total pressure ($\bar{p}$) and proper pressure ($p$) *vs*. time ($t$).

- Let us consider the combined effect of the proper pressure and the bulk viscous pressure, for a barotropic fluid can be expressed as follows:

$$\bar{p} = p - 3\xi H = \varepsilon\rho\ ;\quad p = \varepsilon_0 \rho \tag{25}$$

Such that $\varepsilon = \varepsilon_0 - \eta (0 \le \varepsilon_0 \le 1)$ and $\epsilon$, $\varepsilon_0$, and $\eta$ are constant. The symbols $\xi$ and $p$ are respectively known as the coefficient of bulk viscosity and proper pressure of the model.

Let us consider a time dependent displacement field scale factor [ Pradhan et al. (Pradhan et al., 2006)] as given by

$$a(t) = \alpha e^{\alpha_1 t} \tag{26}$$

where $\alpha$ and $\alpha_1$ are constants.

From **Eqs 23**, **24**, **26**, we get the metric potentials of the model which are

$$A = \alpha e^{\alpha_1 t},\ B = \left(\alpha e^{\alpha_1 t}\right)^{\frac{2n}{n+1}},\ C = \left(\alpha e^{\alpha_1 t}\right)^{\frac{2}{n+1}} \tag{27}$$

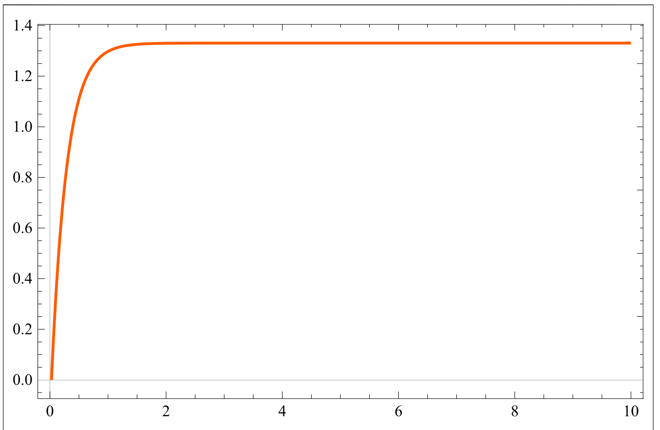

**FIGURE 3 |** Variation of coefficient bulk viscosity ($\xi$) *vs*. time ($t$).

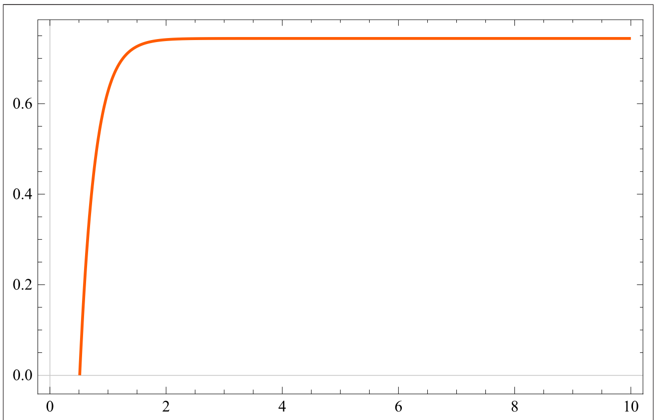

**FIGURE 4 |** Variation of displacement vector ($\beta^2$) *vs*. time ($t$).

Then **Eq. 1** reduces to

$$ds^2 = -dt^2 + \left(\alpha e^{\alpha_1 t}\right)^2 dx^2 + e^{-2mx}\left[\left(\alpha e^{\alpha_1 t}\right)^{\frac{4n}{n+1}} dy^2 + \left(\alpha e^{\alpha_1 t}\right)^{\frac{4}{n+1}} dz^2\right] \tag{28}$$

## 4 PHYSICAL PROPERTIES OF THE MODEL IN *f*(*R*, *T*) GRAVITY

The Physical Parameters of the model are obtained as follows:

Spatial Volume:

$$V = a^3(t) = \left(\alpha e^{\alpha_1 t}\right)^3 \tag{29}$$

Hubble's Parameter:

$$H = \alpha\alpha_1 \tag{30}$$

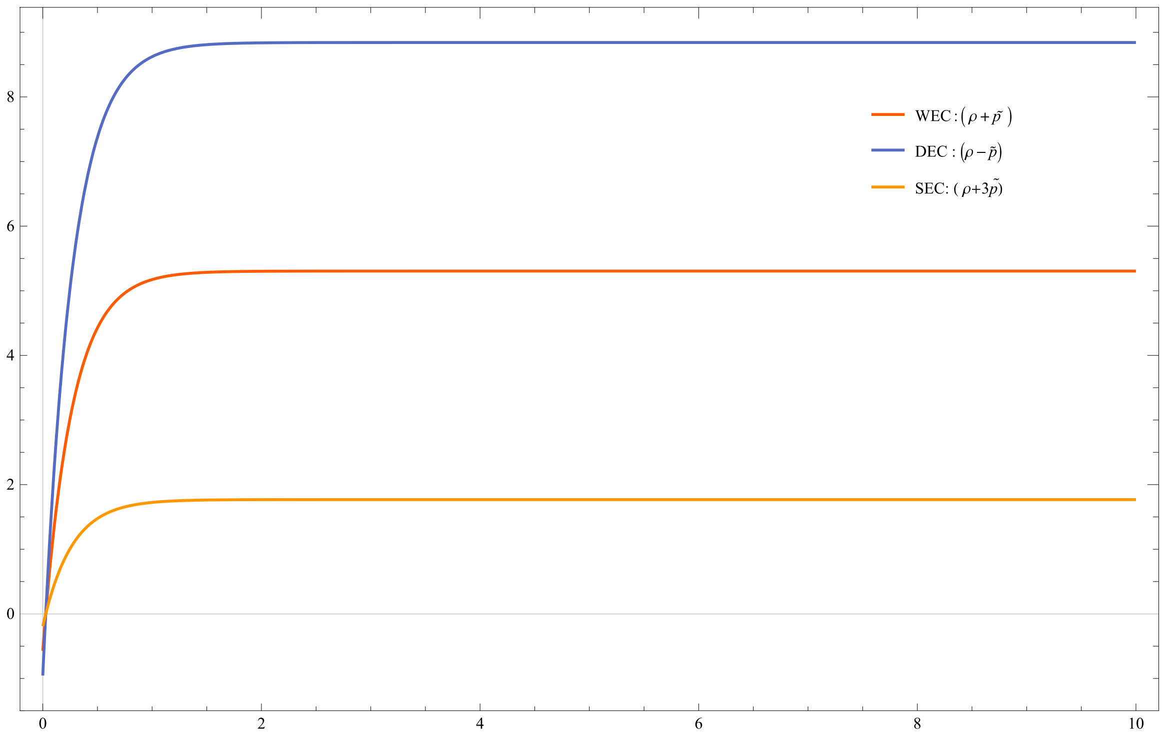


**FIGURE 5 |** Variation of strong energy condition *vs*. time (*t*).

The Expansion Scalar:

$$\theta = 3\alpha\alpha_1 \tag{31}$$

The Shear Scalar:

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2 (\alpha\alpha_1)^2 \tag{32}$$

Anisotropy parameter:

$$A_m = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{33}$$

Deceleration parameter:

$$q = -1 \tag{34}$$

Adding **Eqs 15–17** and applying in **Eq. 18**, we have the energy density given by

$$\rho = \frac{1}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{35}$$

Also the total pressure, proper pressure, the co-efficient of bulk viscosity and the displacement vector are given by

$$\bar{p} = \frac{\epsilon}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{36}$$

$$p = \frac{\epsilon_0}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{37}$$

$$\xi = \frac{\epsilon_0-\epsilon}{3(2\epsilon-1)(\alpha\alpha_1)}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{38}$$

$$\frac{3}{4}\beta^2 = -3(\alpha\alpha_1)^2 - \frac{m^2}{(\alpha e^{\alpha_1 t})^2} + \frac{1-3\epsilon}{2(2\epsilon-1)}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] + \left(\frac{n-1}{n+1}\right)^2(\alpha\alpha_1)^2 \tag{39}$$

The Trace $(T = \rho - 3\bar{p})$, function of Ricci-Scalar $(\tilde{R})$ and the $f(\tilde{R}, T)$ gravity are given by

$$T = \frac{1-3\epsilon}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{40}$$

$$\tilde{R} = \left(\frac{\epsilon}{1-2\epsilon}\right)\frac{4m^2}{(\alpha e^{\alpha_1 t})^2} - 6\left(\frac{3\epsilon-2}{2\epsilon-1}\right) + \frac{18}{\sqrt{3}}(\alpha\alpha_1)Z_1 + 2\sqrt{3}m^2\left(\frac{8\epsilon-3}{2\epsilon-1}\right)\left(\frac{\alpha\alpha_1 e^{\alpha_1 t}}{(\alpha\alpha_1 e^{\alpha_1 t})^3}\right)Z_2 \tag{41}$$

$$\frac{1}{\mu}f(\tilde{R}, T) = \left(\frac{\epsilon}{1-2\epsilon}\right)\frac{4m^2}{(\alpha e^{\alpha_1 t})^2} - 6\left(\frac{3\epsilon-2}{2\epsilon-1}\right) + \frac{18}{\sqrt{3}}(\alpha\alpha_1)Z_1 + 2\sqrt{3}m^2\left(\frac{8\epsilon-3}{2\epsilon-1}\right)\left(\frac{\alpha\alpha_1 e^{\alpha_1 t}}{(\alpha\alpha_1 e^{\alpha_1 t})^3}\right)Z_2 + \frac{1-3\epsilon}{2\epsilon-1}\left[-6(\alpha\alpha_1)^2 + \frac{4m^2}{(\alpha e^{\alpha_1 t})^2}\right] \tag{42}$$

where $Z_1 = [\frac{3\epsilon}{2\epsilon-1}(\alpha\alpha_1)^2 + \frac{1-8\epsilon}{2\epsilon-1}\frac{m^2}{(\alpha\alpha_1 e^{\alpha_1 t})^2}]^{\frac{-1}{2}}$ and $Z_2 = [\frac{3\epsilon}{2\epsilon-1}(\alpha\alpha_1)^2 + \frac{1-8\epsilon}{2\epsilon-1}\frac{m^2}{(\alpha\alpha_1 e^{\alpha_1 t})^2}]^{\frac{1}{2}}$ are functions of cosmic time $t$.

From **Eqs 28**, **31**, we obtain the statefinder parameters, which is defined as $r = \frac{\dddot{a}}{aH^3}$ and $s = \frac{r-1}{3(q-\frac{1}{2})}$, exactly gives the value 1 and 0 respectively.

## Energy Conditions

The energy conditions are constructed from the Raychaudhuri equation, which are very important tools to describe the behavior of the compatibility of timelike, lightlike, or spacelike curves and singularities. Very recently Alvarenga (Alvarenga et al., 2013), Moraes (Moraes and Sahoo, 2017), Zubair (Zubair et al., 2018), and Ahmed (Ahmed and Abbas, 2020) have tested the energy conditions in f (R, T) theory of gravity and to check the energy conditions for the present model, they defined the weak energy conditions (WECs), dominant energy conditions (DECs), and the strong energy conditions (SEC) as given by

1) $\rho \geq 0$, $\rho + p \geq 0$ 2) $\rho - p \geq 0$ (iii)$\rho + 3p \geq 0$

To observe the absolute observational data, we have plotted the graphs for energy conditions in terms of $\rho$ and $\bar{p}$, and we extensively observe from the graph that all the three energy conditions are satisfied in the present model.

## 5 CONCLUSION

In this study, a completely spatially homogeneous and anisotropic Bianchi type-V cosmological model has been discussed in the presence of a bulk viscous fluid based on Lyra geometry, with an exponential form of scale factor. We employed a barotropic equation of state for pressure and energy density to determine the nature and deterministic solution of the highly non linear differential equation. Furthermore, we assumed that bulk viscous pressure is proportional to energy density [Naidu et al. (Naidu et al., 2013)]. As per recent observational data in combination with Baryonic Acoustic Oscillations (BAO), Cosmic Microwave Background (CMB), from Type Ia Supernova (SN Ia), the model found in this paper is in conformative. The model 28) found here is shearing, expanding, and anisotropic which is similar to [Zia (Zia et al., 2018), Tiwari (Tiwari et al., 2020), and Desikan (Desikan, 2020)]. At $t = 0$, we found that the model has no singularity. Subsequently, we can see from **Eqs 30**, **31** that the Hubble's parameter ($H$) and the expansion scalar remain constant throughout the expansion, implying that model 28) represents a uniform expansion. It is evident from **Eq. 29** that the volume ($V$) of the universe rises with cosmic time ($t$), and that as $V$ approaches infinity, for $t \to \infty$. We observed from **Eq. 38** that the bulk viscosity coefficient increases with time and approaches infinity as t approaches infinity. The model's energy density, total pressure, coefficient of bulk viscosity, and displacement vector all rise positively, but they all yield a constant value for $t$ tending to infinity (**Figures 1–4**). The model predicts an accelerating phase of the universe for $q = -1$, which is given by **Eq. 34**. In the current model of the universe, there is a dark energy due to negative pressure in the presence of bulk viscous fluid based on Lyra geometry with $f(R, T)$ gravity, as shown in **Figure 2**. All the three energy conditions are satisfied as in **Figure 5**. The Trace and the Ricci scalar are always positive throughout the cosmic time $t$, and for $t \to \infty$, it offers a constant value, as shown in **Eqs 40**, **41**. Furthermore, $r$ and $s$ tend to 1 and 0 respectively, indicating that the current universe model approaches the $\Lambda CDM$ model. There have been many works done by researchers in the area of Lyra geometry, but Lyra geometry with $f(R, T)$ gravity is a very new concept and there is scope for the continuation of work.

## DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding authors.

## AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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