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De-embedding a pair of dipoles to calibrate the mutual impedance plasma density diagnostic

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For two dipole antennas in a two-port system, an impedance calibration method is presented for situations where the dipole antennas are inaccessible during testing, but environmental changes require recalibration, such as during mutual-impedance measurements of a plasma inside a vacuum chamber. The antenna system of this study is composed of two dipoles connected by rigid coax cables to their respective calibration boxes. The boxes house surface mounted baluns, for balancing the dipoles, and surface mounted standards, for calibrating the cables. The cables connect the system through the chamber walls to the measurement hardware, a Vector Network Analyzer (VNA). The measured S-parameters are simulated in the python package *scikit-rf* to de-embed the dipoles. This new virtual approach incorporates the balun common mode and eliminates error from the conventional balun characterization where only the differential mode is considered. The presented calibration method has been validated with direct measurements of known cable loads and a COMSOL simulation for a range of 1 MHz–1 GHz, equating to an electron density range of roughly $10^{10} - 10^{16} \text{ m}^{-3}$. The calibration method de-embeds the two dipoles without using a “through” measurement, allowing for remote calibration and recalibration during testing. Since the eventual goal of this work is to apply the calibration method to an array of dipoles for plasma impedance tomography, the lack of a “through” measurement provides the following key capabilities that removes uncertainty and significantly reduces the setup time: 1) Calibration uncertainty in the cables due to thermal changes and movement are accounted for, which has previously not been possible during plasma testing. 2) The calibration procedure inside the chamber can be fully automated, greatly speeding up the calibration time for multiple antenna pairs. 3) Without the need for a “through” measurement, the number of calibration measurements within the chamber is reduced to the number of antennas (N) rather than the number of unique antenna pairs ($N(N-1)/2$). Extending this method to higher plasma densities could be possible with hardware that performs in the respective GHz range.

KEYWORDS

plasma tomography, mutual impedance probe, de-embedding method, plasma impedance probe, dipole calibration

1 Introduction

A plasma mutual impedance probe is a diagnostic that measures plasma parameters, such as electron density and electron temperature, by fitting a plasma impedance model to the real and imaginary components of the mutual impedance (Storey et al., 1998a). The mutual impedance probe is composed of two antennas that measure the frequency response of a signal transmitted through the plasma. Although, the calibration of two antennas has established methods in the electrical engineering discipline (C63.5, 2017), this work accounts for laboratory plasma testing conducted in vacuum chambers, where bending cables attached to translation stages and changing cable temperatures due to plasma sources, will change the calibration along a coaxial cable.

Storey et al. (1969), Storey et al. (1998a), Storey et al. (1998b) first adapted the mutual impedance method from its geophysics origin to measure characteristic resonance frequencies of a cold unmagnetized and magnetized plasma. Pottelette et al. (1975) adapted the theory for a dipole configuration and a warm isotropic plasma. Chasseriaux et al. (1972) and Debie et al. (1976) independently validated the measurement of electron density and electron temperature. Grard (1997) expanded the analysis to suprathermal electrons with multiple distributions. More recent laboratory work has been produced by Bucciantini et al. (2022), Bucciantini et al. (2023a), and Bucciantini et al. (2023b) has used the quadrupole configuration of Storey et al. (1969).

The original theory of plasma mutual impedance (Storey et al., 1969) was intended to eliminate the errors induced by the sheath capacitance, common for the self-impedance plasma density probe (Balmain, 1964). The theory was dependent upon: 1) the quasi-static assumption, 2) the sheath impedance being far less than the source impedance of the transmitting or receiving antenna, 3) the probes being smaller than the Debye length, and 4) the distance between the transmitters and receivers being several times the Debye length (Storey et al., 1969). These assumptions allowed the measured current and voltage, at a well-defined location, to be approximately the same on the surface of the probes as on the outer surface of the sheath surrounding the probes. The small probe size was intended to eliminate the non-uniformity of the potential across the sheath caused by the probe shape.

For space satellite measurements, these assumptions have held and have been well demonstrated on several space flights to measure the plasma density for earth and interplanetary missions: Viking I (Bahnsen et al., 1988), GEOS-1 (Decreau et al., 1978a; Decreau et al., 1978b), AUREOL-3 (ARCAD-3 Project) (Beghin et al., 1982), BepiColombo (Trotignon et al., 2006; Gilet et al., 2019), and ROSETTA (Trotignon et al., 2007; Johansson et al., 2021; Wattieaux et al., 2020). These plasma densities cover the range of $8 \times 10^7 - 3 \times 10^{11} \text{ m}^{-3}$ with their respective plasma frequencies 0.08–5 MHz. However, at higher plasma densities, these assumptions for mutual impedance probes begin to break down or become prohibitive to maintain. In addition, more precise calibration for lower hybrid measurements (Storey et al., 1998a) and improved space plasma metrology (Storey et al., 2013) are two needs asserted by Storey, the original developer of the mutual impedance method.

At low MHz plasma frequencies (1–5 MHz), the electrical length of the system components can be small, which allows

the system to operate without compensating for electrical length. However, at higher frequencies (>100 MHz), cable lengths and circuit component sizes begin to change the measured signal. The added phase shifts and losses, due to the components and cabling, requires calibrating out these electrical lengths. This leads to a different measurement technique for higher frequencies: using a Vector Network Analyzer (VNA). VNA calibration requires attaching and detaching known loads or standards to the ends of the VNA's cables, where the measurement will be performed. When the dipole system is in a vacuum chamber, changing loads is not possible unless the chamber is brought up to atmosphere to recalibrate. Even when the dipoles are calibrated prior to closing the chamber, the plasma conditions can heat the cabling, changing the permittivity (Carlisle, 2017; Ehrlich, 1953; Riddle et al., 2003; Krupka et al., 1998; Burger et al., 2018) so that the calibration is no longer accurate. A similar issue occurs with moving cables, where tight bends along the cable length change the cables impedance (Mini-Circuits, 2018) affecting the calibration. To compensate for these issues, this work presents a method for remotely calibrating the system, once the system is setup inside the vacuum chamber.

Plasma interferometry, which measures the phase delay as a function of the line integrated plasma density, uses a similar setup: two antennas transmitting through the plasma. (Dittmann et al., 2012; Greenberg and Hebner, 1993; Hebner et al., 1988; Kovtun et al., 2018; Krämer et al., 2006; Lukas et al., 1999; Nagora et al., 2020; Niemöller et al., 1997; Overzet, 1995; Wharton and Slager, 1960). However, plasma interferometry only requires the phase shift to calculate the plasma density. Since the probing frequency is anywhere from 8–300 GHz and the demonstrated plasma densities are in the range of $10^{15} - 10^{17} \text{ m}^{-3}$, the resulting phase shifts are fractions of a degree (Dittmann et al., 2012). This does not require the calibration of the horn antennas, but rather just an initial vacuum condition as a datum from which to measure the phase shift. Both Overzet (Overzet, 1995) and Hebner et al. (1988), used references, placed into the test region, to calibrate the output value. Since mutual impedance requires magnitude and phase effect of only the dipoles, a full calibration of the dipoles is needed.

Several antenna calibration methods are listed in the American National Standards (C63.5, 2017), including: 1) the standard site method (Newell et al., 1973; Smith, 1982) (i.e., the three antenna method), 2) the standard antenna method (IEEE Standard Methods for Measuring, 1991), 3) the standard field method, (IEEE Standard Methods for Measuring, 1991), and 4) the equivalent capacitance substitution method (Knight et al., 2004). The majority of these methods focus on resolving the antenna factor (Balanis, 2015) for characterizing the antennas. These methods require inserting a secondary radiating element into the system or knowing the propagating medium between the antennas. For calibrating dipoles in a vacuum chamber surrounded by plasma, these methods are not viable.

Since the goal is to know the S-parameters, conventional S-parameter calibration methods (Rumiantsev and Ridler, 2008; Anritsu (2012)) were considered including Short, Open, Load, Through (SOLT) (Kruppa and Sodomsy, 1971); Through, Reflected, Line (TRL) (Engen and Hoer, 1979); and Line, and Reflected, Match (LRM) (Barr and Pervere, 1989). These rely on measuring known standards to calculate the error terms for 8- or 12-term error networks that model the transmitted, reflected,

and absorbed signal. These errors are then removed from the experimental measurement as a calibration. The downside to these approaches is that a Through or Line standard is required to be attached between the two port terminals. When the chamber is pumped down to vacuum for plasma testing, this is not feasible.

An alternative to calibrating the system as a whole is the “de-embedding” method (Caspers, 2010; Keysight, 2024). The attachments leading up to the antennas are characterized and removed from the final measurement. The process works by mathematically modeling the cabling and circuit components between the VNA and dipoles as S-parameters, which characterize the reflection, transmission, and absorption characteristics of a circuit. Then removing these components from the VNA measurement with matrix operations.

Ferrero and Pisani (1992) developed a calibration method where they characterized the two extensions leading to the two-port terminals with a Short, Open, Load (SOL) method (Caspers, 2010) and then removed these components from the full measurement, similar to de-embedding. However, they included a factor (α) that resolves a sign ambiguity for the S_{21} and S_{12} terms of the attachments, requiring them to make an unknown reciprocal through measurement. Ferrero and Piani termed it the Reciprocal, Short, Open, Load (RSOL) method, but it is better known as the “unknown thru” method. They demonstrated that the method was in close agreement with the LRM method. Hirose and Komiyama (2007), Hirose and Komiyama (2010) demonstrated the RSOL method for two dipoles, showing that the signal-to-noise ratio (SNR) between SOLT and RSOL were in close agreement. This is similar to the de-embedding method used in this work, however, the sign ambiguity of the cable is resolved without the through measurement, as discussed in Section 3.1.3. This alternative method is used since the reciprocal requirement of the through measurement is not true for plasma when a significant magnetic field is present, creating directionally dependent plasma wave oscillation from the anisotropic permittivity (Pozar, 2011; Swanson, 2003).

Salter and Alexander (1991) demonstrate how to characterize the cable and the balun attachments of a pair of dipoles as S-parameters. The matrix operations require that the balun, a 3-port system used to balance the dipole signal, be reduced to a 2-port system, since a 1-port to 2-port matrix cannot be inverted (Frei et al., 2008). The balun is reduced to its differential mode, ignoring the common mode, to satisfy this requirement (Issakov et al., 2017). The cabling leading up to the balun is characterized by performing a SOL calibration (Steer, 2013). However, the SOL process is conventionally conducted by hand, requiring access to the antenna system during calibration.

In this work, we present a calibration method similar to the RSOL method, but with the addition of measuring the S-parameters of the cabling remotely, incorporating the full S-parameters of the balun, and resolving the sign ambiguity of the cable by examining its phase transition. The S-parameters of the cabling is measured remotely using embedded standards on the PCB that also mount the balun. A radio frequency (RF) switch allows the standards to be individually measured through the cabling. The inclusion of this remote access allows for remote recalibration of the system during testing, with the one caveat that the S-parameters of the circuit board (balun and measured standards) do not change during testing from environmental conditions. Both the differential and common mode

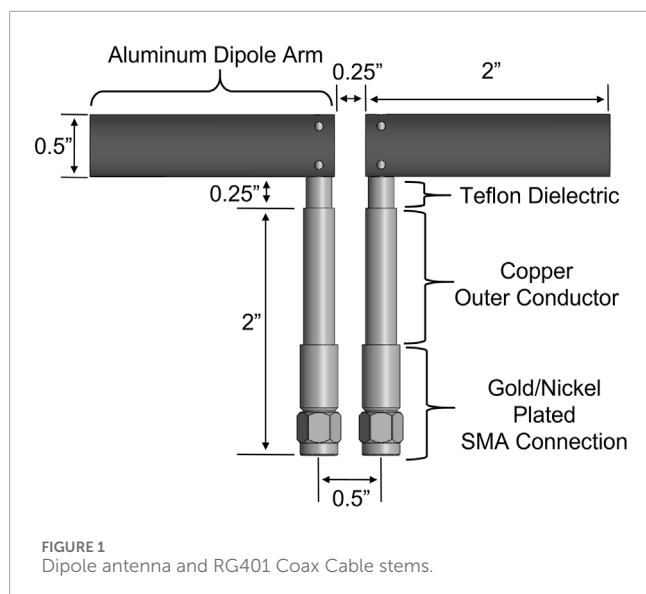
of the balun are incorporated by simulating the entire network, given all the different components’ S-parameters. McLean (2008) simulated a balun using a Thevenin equivalent model to include the radiating, non-radiating, and asymmetric components of a real balun when incorporating the VNA measured balun with a simulated dipole. However, we will be using the Python package *scikit-rf* (Arsenovic et al., 2022) to model the entire system as S-parameters. This allows the system to be virtually calibrated using the SOLT method (Steer, 2013) by comparing virtual standards, attached to the simulated stems, with the simulated response of the entire system. This is similar to the balun TRL two-port balun calibration demonstrated by Iwasaki and Tomizawa (2003) except that we perform the calibration in simulation space.

Part of this work’s motivation is in using mutual-impedance measurements for electrical impedance tomography (EIT) (Adler and Holder, 2021) of plasma which requires multiple measurements between an array of antennas. Previous calibration methods for non-plasma tomography has used test objects, placed within the array of antennas, to determine a conversion coefficient between the S-parameters measured by the VNA and S-parameters at each dipole (Cathers et al., 2023; Gilmore et al., 2010; Nemez et al., 2017; Ostadrahimi et al., 2011). This is not possible when calibrating a new plasma condition since the test object would be surrounded by an unknown density of plasma, and inserting test objects within the vacuum chamber would require extra cumbersome automation. Haynes et al. (2012) did calibrate their system using SOLT, but this required 99 different through measurements using a connecting cable. This highlights how tomography calibrations become prohibitive with added antennas for additional resolution. Haynes et al. could reduce their calibration time by applying the “unknown through” (Ferrero and Pisani, 1992) method. This would use the antenna-to-antenna measurement as the through, which could be automated with their switching system, eliminating their need for attaching and detaching a cable. However, as discussed previously, this approach is not possible with a non-reciprocal plasma.

This work shows the calibration steps for a plasma mutual impedance probe by de-embedding. It highlights best practices to incorporate the common mode of the balun, perform remote calibration in a vacuum chamber, reduce the number of calibration measurements for an antenna array, and mitigate sources of error due to cable line length, movement, and thermal changes in the plasma environment. The major advantages of this approach is that it makes S-parameter calibration of a mutual impedance probe, used to measure magnetized plasma within a vacuum chamber, possible at plasma densities above $10^{11} m^{-3}$. It also significantly reduces the calibration setup time for a plasma tomography array by eliminating the tedious physical through measurement. Since the through measurement is not needed, the number of calibration measurements reduces from the number of unique antenna pairs ($N(N-1)/2$) to the number of antennas (N).

2 Setup

The setup is a 2-port antenna system measured by a VNA (Agilent E5061B with a range of 5 Hz to 3 GHz) with data averaged over 100 samples to minimize error and obtain a consistent

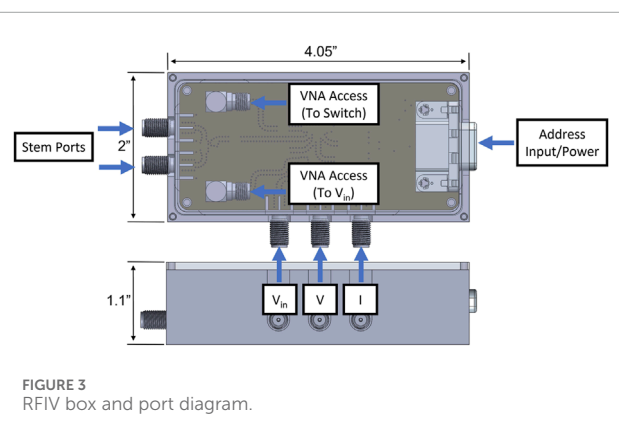
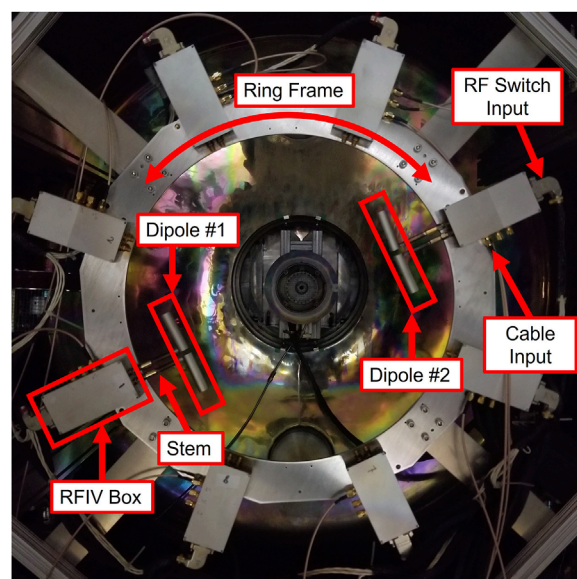


measurement. The antennas are dipoles, each fixed to their own PCB by semi-rigid coax cables (“stems”). The signals from the two stems of a dipole are combined into a single signal by a balun. The signal passes through an RF switch to an output port of the PCB and is connected to the VNA by an arbitrary length RG316 cable. The RF switch is also connected to 6 passive standards, that are used to measure the S-parameters between the VNA and the RF switch. Each component is described in detail below.

Each “element” of the dipole is constructed from a hollow aluminum cylinder, 0.5 inches in diameter and 2 inches long, as shown in Figure 1. The inner ends of the two cylinders are capped by aluminum. Each element is connected to the SMA ports by a stem constructed from an RG401 cable. Each dipole element is fixed to its stem by two fit screws that tighten onto the exposed center conductor of the stem. To isolate the dipole elements from the grounded outer conductor of the stem, 0.25 inches is removed from the outer conductor at the interface of the stem and element.

Each PCB is housed within a Radio Frequency Current-Voltage (RFIV) box. The RFIV boxes are attached to an aluminum ring frame, as shown in Figure 2. The ring is 3/16 inches thick, has an inner diameter of 16 inches, and an outer diameter of 20 inches. This keeps the dipoles aligned, in parallel, and at a fixed distance of 11.2 inches. The additional RFIV boxes placed around the ring, shown in Figure 2, form a circular antenna array, when the dipoles are attached. The array is designed to take multiple mutual-impedance measurements for plasma impedance tomography. The additional dipoles have been removed to avoid unintended capacitive paths between the two measured dipoles.

On the RFIV box, shown in Figure 3, there are five external SMA ports, two internal SMA ports, and a rear address/power port. The two external SMA ports, on the end of the RFIV box, attach to the dipole stems. The three SMA ports, on the side of the box, are: voltage in (V_{in}), voltage out (V), and current out (I). The V_{in} -port is the VNA input/output port used in this work. The V -port and I -port are connected to the same circuit path as the V_{in} port, but through transformers, as shown in Figure 4. Through



the transformers, the V -port and I -port directly measure the time-dependent voltage and current. These latter two ports are not used here since this work focuses on the time-averaged measurements from the VNA. Instead the ports are terminated by 50 Ω terminators. The rear address/power port supplies power and control signals to the 8-terminal RF Switch (Model# HMC253AQS24E, Manufacturer: Analog Devices).

As shown in the circuit board schematic of Figure 4, this switch changes the V_{in} path between: the 6 different standards, the path to the dipole through the balun, and a reference TANK circuit used to check the cable calibration. The two internal SMA ports, shown in Figure 3, are the “VNA Access” ports. They connect the V_{in} path with the switch input via an RG316 cable “jumper”. When the jumper is removed, the standards can be directly measured through the VNA Access port. These six measured standards form a set of “known” values, for calibrating the cable and circuit path

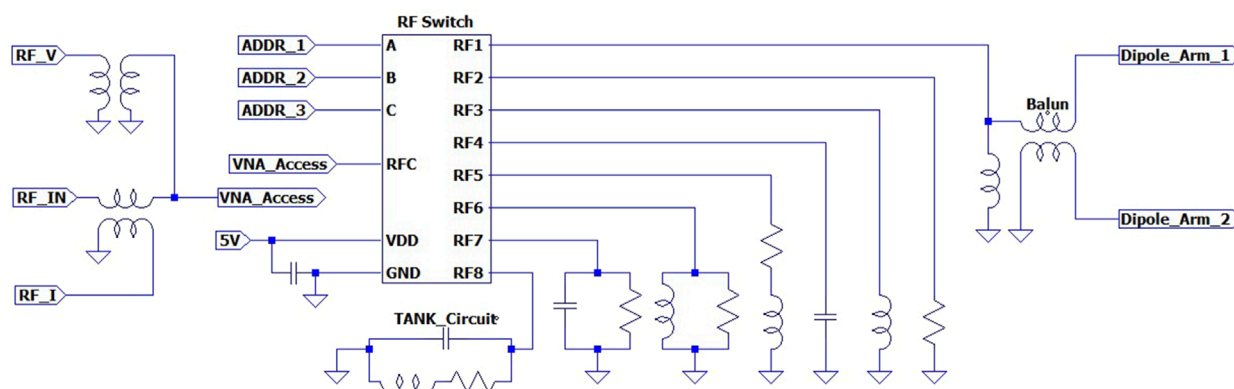


FIGURE 4
RFIV circuit board diagram. The input signal at RF_IN is routed to the dipole (RF1), one of the six calibration standards (RF2–RF7), or the TANK Circuit (RF8) by the RF Switch.

between the VNA and the switch. The balun (Model# MABA-010247-2R1250, Manufacturer: MACOM) is a transformer that balances the signal between the two stems of the dipole. This makes both elements of the dipole carry the same signal but with reversed polarity (180° out of phase). It allows each dipole to have only one SMA output, so that there is one dipole per VNA port. However, it also makes the balun a 3-port system, with specific calibration considerations, discussed in [Section 3.1.4](#).

3 Method

De-embedding ([Keysight, 2024](#)) is used to obtain the S-parameters, [Equation 1](#), of the dipoles when they are surrounded by other components. S-parameters are characteristic measurements of a circuit's reflection, transmission, or absorption due to an incident voltage wave, as a function of frequency ([Caspers, 2010](#)). To aid the de-embedding process, the system is divided by “calibration planes” (cal. plane) into the different components, as shown in [Figure 5](#). A calibration plane is a reference point along a transmission line from which the waves are measured. After measuring the S-parameters between each cal. plane, the de-embedding process removes each component layer, until only the S-parameters of the dipoles and separating media remain.

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad (1)$$

3.1 Measuring component S-parameters

The de-embedding process requires knowing the S-parameters of the components surrounding the dipoles prior to calibration or to be able to measure them during the calibration. The semi-rigid cable of the stems are modeled as transmission lines, with propagation constants from the published data sheets for RG401 ([Pasternack Inc, 2013](#)). By contrast, the far longer cable and circuit S-parameters, extending from the calibrated VNA to the RFIV board, can change

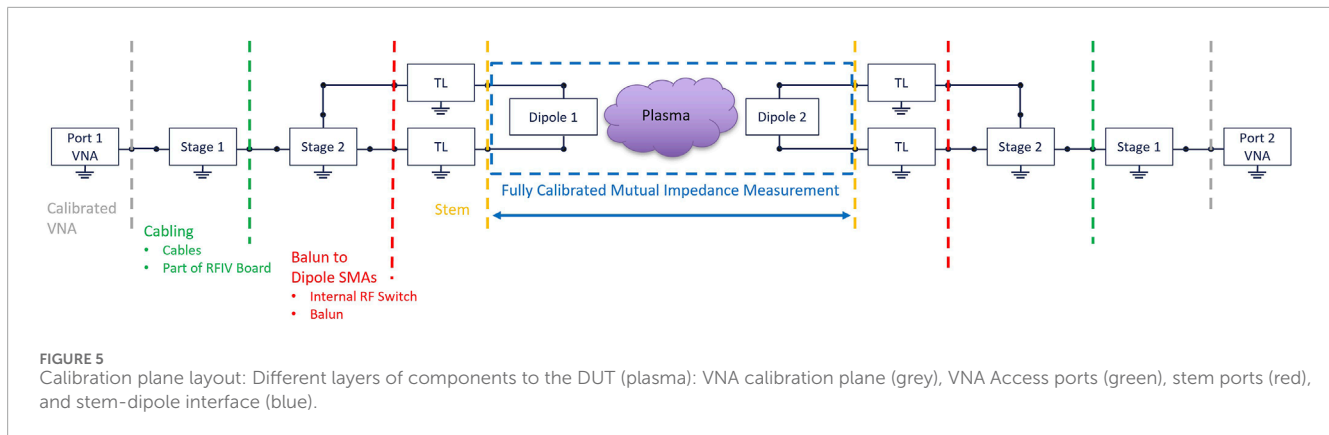
due to position or temperature. These cables must be measured *in-situ*, using the standards built into the RFIV board. Since the balun and cable standards are embedded on the RFIV PCB, these components are less susceptible to temperature variation and movement, and are measured prior to setting up the dipoles. As long as the components on the RFIV board do not change due to environmental conditions (e.g., temperature changes beyond the tolerance of the PCB or electrical discharge), their S-parameters may be assumed constant.

3.1.1 Initial VNA calibration

To acquire all S-parameters, the VNA is first calibrated using a 2-port SOLT calibration ([Keysight, 2024](#)) so that there is a known calibration plane from which to measure. The initial VNA calibration includes a through measurement, which the de-embedding builds upon. This eliminates the need for a through measurement between the dipoles. The VNA is calibrated with a BNC Calibration Kit (Maury Microwave Corp. Model No. 8550G). The calibration kit has BNC connections, but SMA connections are used on the RFIV box. So BNC-SMA adapters are attached to SMA cables leading from the VNA. The BNC calibration standards are used to establish the VNA calibration plane at the end of these cables (Additional cables leading from this calibration plane, through the vacuum chamber, and to the RFIV box will be addressed in [Section 3.1.3](#).) The BNC-SMA adapters are removed from the SMA cables and a phase shift is applied to the measured data to move the calibration plane from the BNC connection to the SMA connection. This procedure is described in more detail in Section VI A of the [Supplementary Appendix](#), as it is only an issue when there is a mis-match between the calibration kit and the cable connectors.

3.1.2 Stem S-Parameters

The S-parameters of the dipole stems (S_{TL}) are modeled as ideal transmission lines, using [Equation 2](#) and plotted in [Figure 6](#) ([Caspers, 2010](#)). The propagation constant ($\gamma = \alpha + i\beta$) is made up of the attenuation coefficient (α) and phase constant (β) ([Pozar, 2011](#)). These values are calculated from the published ([Author anonymous, 2013](#)) frequency dependent attenuation per unit length (A_L) and the velocity factor (VF). Since the attenuation per unit length is reported in dB/100 m, it is converted to the attenuation constant



by Equation 3. The data sheet reported the attenuation per unit length at four sparse frequencies, so a power model (aw^b) was fit to this data before solving for the attenuation constant. The velocity factor is converted to the attenuation coefficient by Equation 4. As an alternative, a method for directly measuring the attenuation coefficient and phase constant, from a length of cable, is included in Section VI B of the Supplementary Appendix. In this work, the stem's 2 inch length extends from the calibration plane of the SMA connection to the cable cross-section where the outer conductor was removed, as shown in Figure 1.

$$S_{TL} = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \quad (2)$$

$$\alpha = \frac{1}{L} \ln \left(\frac{A_{L/100}}{10} \frac{L}{20} \right) \quad (3)$$

$$\beta = \frac{\omega}{VF c_0} \quad (4)$$

3.1.3 Cable and board S-Parameters

The S-parameters of the cable and PCB path, (S_{Cbl}), between the VNA calibration plane and the VNA Access, are calculated during testing by equating the S-parameters of the transmission line path to the error terms for a one-port calibration, Equation 5 (Steer, 2013). Although we have six standards on the PCB, only three are needed to form a solvable set of linear equations for S_{11} , S_{22} , and $S_{12}S_{21}$. The “known” reflection coefficients (Γ) are the VNA measurements of the six standards on the PCB, measured at the VNA Access port, and shown as solid black in Figure 7. These are measured prior to the calibration process inside the vacuum chamber. The “measured” reflection coefficients (Γ') are the VNA measurements of the six standards on the PCB, measured at the end of the cabling and circuit path, shown as dotted purple in Figure 7. The inverse of Equation 5 is plotted, using the measured data and the solved values (S_{11} , S_{22} , and $S_{12}S_{21}$), as dashed yellow in Figure 7 to demonstrate the accuracy of the solutions.

$$\Gamma' = S_{11} + \frac{\Gamma S_{12}S_{21}}{(1 - S_{22}\Gamma)} \quad (5)$$

Since the cable and board are passive, the system is reciprocal: $S_{12} = S_{21}$ (Steer, 2013). However, since $S_{12}S_{21}$ is replaced by S_{21}^2

in solving Equation 5, there is a sign ambiguity. By examining the phase of S_{21} , which should be continuous if unwrapped, the sign is chosen so that the phase changes sign at 180° rather than at 90° . The 90° sign change occurs when $\sqrt{(S_{21})^2}$ is limited to a positive value of S_{21} . Figure 8 presents the resulting S-parameters of the cable and circuit path leading from the VNA to the VNA Access port.

3.1.4 Balun S-Parameters

The balun is characterized by a 3-port S-parameter matrix, Equation 6. To measure the 3-port balun with a 2-port VNA, a measurement is made between the two ports while the third port is terminated with a known standard (Poole and Darwazeh, 2015; Satoda and Bodway, 1968). The 2-port S-parameters of the 3-port system are related by Equation 7a, where i , j , and k are the indices of the three different ports. This includes the 2-port S-parameters ($[S]_{ij}'$), between the two measured ports, as well as the reflection coefficient (Γ_k) at the third port. This is repeated for each port pair to produce Equation 7b - Equation 7d. If a 50Ω matched terminator is used, the reflection coefficient ($\Gamma = 0$) eliminates the higher order terms in Equation 7a and directly equates the measurements to the true values ($S' = S$). The S-parameters of the balun from this method are plotted in Figure 9 as dashed lines. However, the balun may not have a known terminal impedance or be poorly matched to the load. In this case, Equation 7b - Equation 7d only forms a system of 27-unknowns and 12-equations using one unique standard (Each S-parameter matrix (Equation 1) has four terms, so that each Equation 7a introduces four unique linearly independent equations.)

$$S_{balun} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad (6)$$

$$[S]_{ij}' = [S]_{ij} + \frac{\Gamma_k [S]_{ik} [S]_{kj}}{1 - [S]_{kk} \Gamma_k} \quad (7a)$$

$$\begin{bmatrix} S_{11}' & S_{12}' \\ S_{21}' & S_{22}' \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} + \frac{\Gamma_3}{1 - S_{33}\Gamma_2} \begin{bmatrix} S_{13}S_{31} & S_{13}S_{32} \\ S_{23}S_{31} & S_{23}S_{32} \end{bmatrix} \quad (7b)$$

$$\begin{bmatrix} S_{11}' & S_{13}' \\ S_{31}' & S_{33}' \end{bmatrix} = \begin{bmatrix} S_{11} & S_{13} \\ S_{31} & S_{33} \end{bmatrix} + \frac{\Gamma_2}{1 - S_{22}\Gamma_2} \begin{bmatrix} S_{12}S_{21} & S_{12}S_{23} \\ S_{32}S_{21} & S_{32}S_{23} \end{bmatrix} \quad (7c)$$

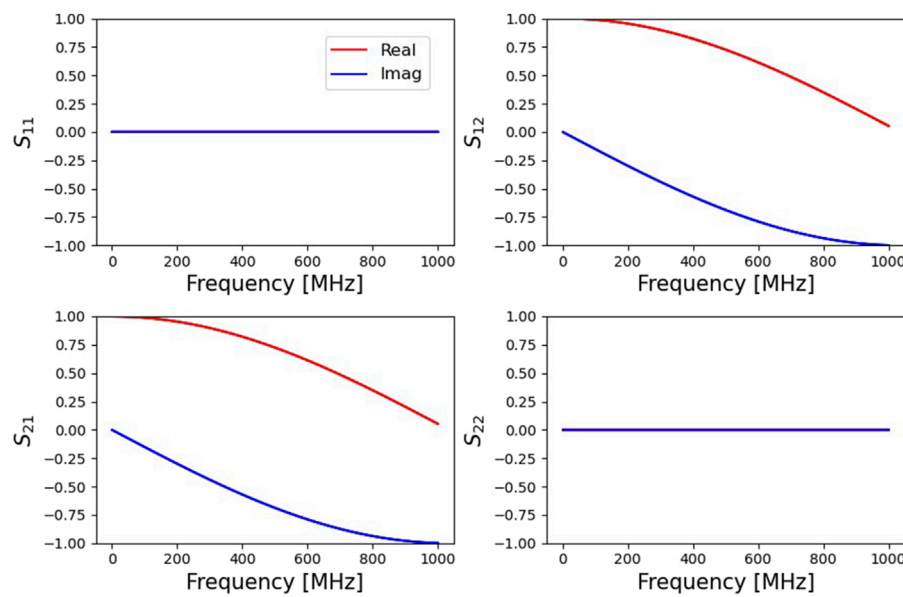


FIGURE 6
S-parameters of the 2 inch, RG401, stem.

$$\begin{bmatrix} S'_{22} & S'_{23} \\ S'_{32} & S'_{33} \end{bmatrix} = \begin{bmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{bmatrix} + \frac{\Gamma_1}{1 - S_{11}\Gamma_1} \begin{bmatrix} S_{21}S_{12} & S_{21}S_{13} \\ S_{31}S_{12} & S_{31}S_{13} \end{bmatrix} \quad (7d)$$

To add more equations and solve all 27 unknowns, three different standards (an open, short, and load) are used, so that there are three different reflection coefficients and nine total measurements. After solving the least-square fit to these 36 equations, the nine balun S-parameters are obtained (the remaining terms being higher order multiples of these terms). The balun S-parameters obtained from this method are also plotted in Figure 9 as solid lines. Compared with the 50 Ω terminator method, it is clear that there is almost no difference between the two methods for this setup. With a reliable matched load, the direct measurement using a 50 Ω terminator is sufficient.

Since the 3-port balun is defined by a 3×3 S-parameter matrix, it must be converted to a 2-port 2×2 S-parameter matrix, to implement the de-embedding method of Section 3.2. Although a 3-port system can be represented by a 2×4 (single line to double line) or 4×2 (double line to single line) T-parameter matrix, the inverse of these matrices (such as a Moore–Penrose pseudoinverse) will also be non-square and can only be applied to one-side of a 2×4 matrix (Frei et al., 2008). Unfortunately, this does not allow for de-embedding, since the transition from a 4×4 matrix to a 2×2 matrix loses information.

To solve this problem, the transition from a single-line (2×2) to a double-line (4×4) balanced system is defined as a transition from a single-line to a mixed system with two modes: “differential” mode and “common” mode (Steer, 2013). The differential mode is what could be considered the expected signal for a dipole, propagating down one of the dipole stems and returning along the other, with the two ports 180° out of phase. In contrast, the common mode propagates down both dipole stems in phase. Since the baluns are designed to reject common modes, the only signal propagating down the stems is assumed to be the differential mode.

This is valid above the balun’s 1 MHz lower operating limit. The 3-port S-parameters of the balun are converted to the multi-mode S-parameters (S_{mm}) of Equation 8 using Equation 9 to represent the single line (l), differential (d), and common (c) S-parameter components.

$$S_{mm \text{ balun}} = \begin{bmatrix} S_{1l} & S_{1d} & S_{1c} \\ S_{dl} & S_{dd} & S_{dc} \\ S_{cl} & S_{cd} & S_{cc} \end{bmatrix} \quad (8)$$

$$S_{1l} = S_{11} \quad (9a)$$

$$S_{1d} = \frac{S_{12} - S_{13}}{\sqrt{2}} \quad (9b)$$

$$S_{1c} = \frac{S_{12} + S_{13}}{\sqrt{2}} \quad (9c)$$

$$S_{dl} = \frac{S_{21} - S_{31}}{\sqrt{2}} \quad (9d)$$

$$S_{dd} = \frac{S_{22} - S_{23} - S_{32} + S_{33}}{\sqrt{2}} \quad (9e)$$

$$S_{dc} = \frac{S_{22} + S_{23} - S_{32} - S_{33}}{\sqrt{2}} \quad (9f)$$

$$S_{cl} = \frac{S_{21} + S_{31}}{\sqrt{2}} \quad (9g)$$

$$S_{cd} = \frac{S_{22} - S_{23} + S_{32} - S_{33}}{\sqrt{2}} \quad (9h)$$

$$S_{cc} = \frac{S_{22} + S_{23} + S_{32} + S_{33}}{\sqrt{2}} \quad (9i)$$

By assuming a negligible common mode, the single line is transitioned directly to the differential mode, at the balun. The S-parameters are reduced to the upper left of Equation 8,

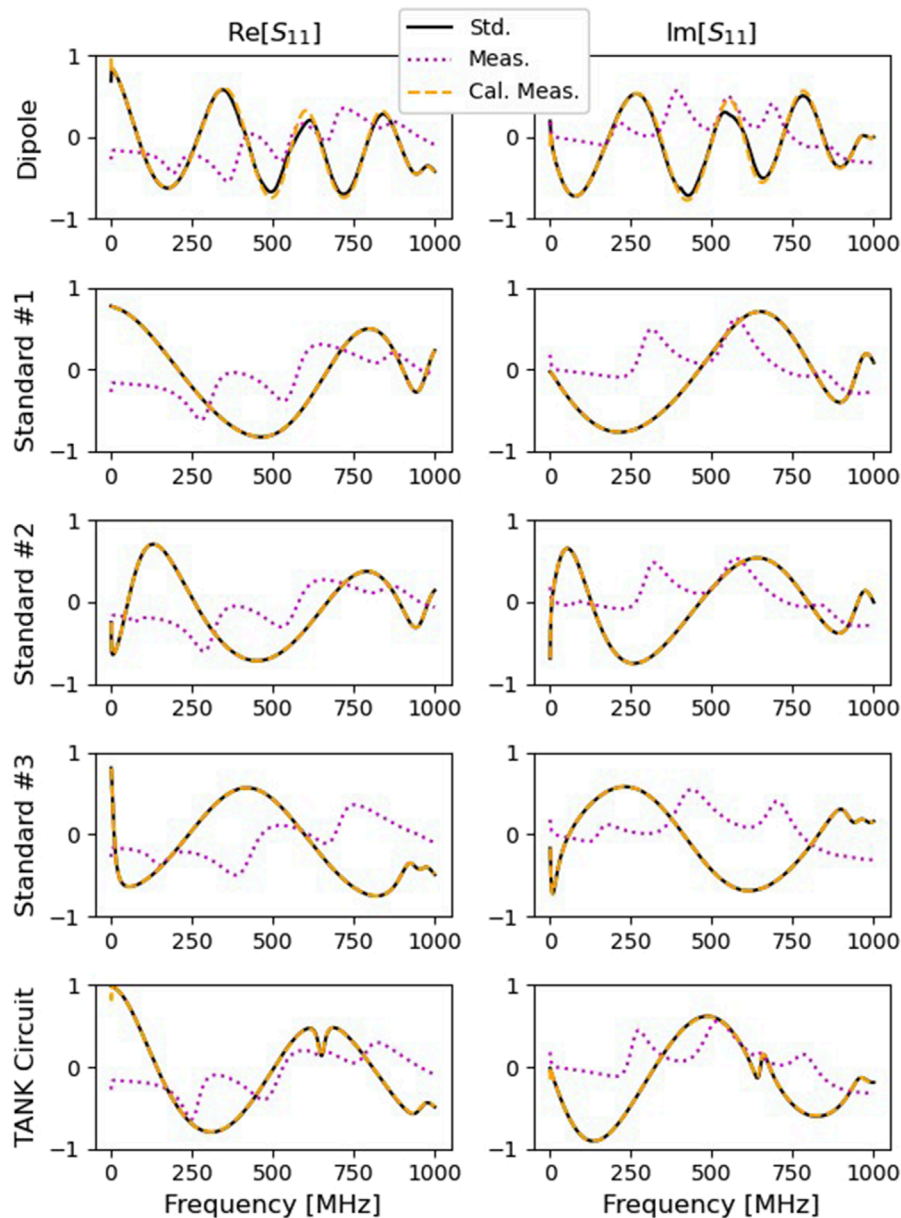


FIGURE 7

Reflection Coefficients (S_{11}) of the dipole, the three RFIV standards, and a reference TANK circuit: The raw measurement (dotted purple), the “true” VNA measurement (solid black), and the result of the least-square fit to the “true” standards (dashed yellow).

resulting in Equation 10 the 2×2 S-parameters of the balun (S_{Bn}) used in the following section.

$$S_{balun\ 2 \times 2} = \begin{bmatrix} S_{11} & S_{1d} \\ S_{d1} & S_{dd} \end{bmatrix} \quad (10)$$

3.2 De-embedding method 1: T-parameter inversion

Once all S-parameters have been acquired, they are converted to T-parameters to perform the necessary linear algebra for de-embedding the dipoles, using Supplementary Equation 11 in

Section VI C of the Supplementary Appendix. Although the S-parameters fully characterize the frequency response of the different components, matrix multiplication between S-parameters of sequential components does not cascade their effect (Caspers, 2010). The total S-parameter can not be formed from its parts. To do this operation, the S-parameters must first be converted to T-parameters, which can be cascaded. Multiplication of the T-parameters from all the components, Equation 11, results in $[T]_{Meas}$, which is equal to the T-parameter of the VNA measurement of the embedded system, performed in Figure 5.

$$[T]_{Meas} = [T]_{Cb1}[T]_{Bn1}[T]_{TL}[T]_{DUT}[T]_{TL}[T]_{Bn2}[T]_{Cb2} \quad (11)$$

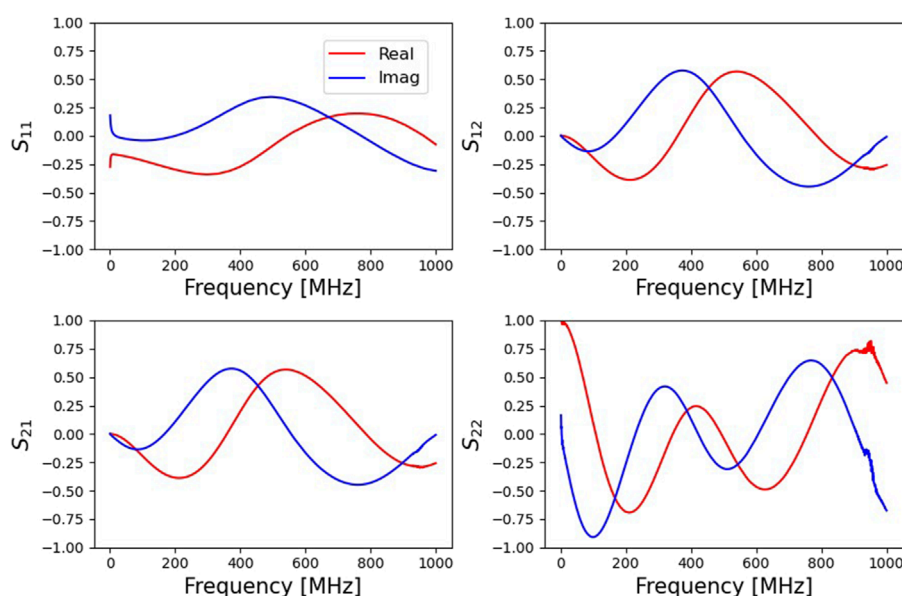


FIGURE 8
S-parameters of the cable and circuit path between the VNA and the VNA Access on the RFIV board.

By inverse matrix operation, the inverse T-parameters of the components are manipulated, as shown in Equation 12, to isolate the T-parameters of the “Device Under Test” (T_{DUT}). In this case, it is the dipoles and medium (plasma or vacuum) between the dipoles. The resulting T_{DUT} is then transformed back to S-parameters by Supplementary Equation 12 in Section VI C of the Supplementary Appendix.

$$[T]_{DUT} = [T]_{TL}^{-1} [T]_{Bn1}^{-1} [T]_{Cb1}^{-1} [T]_{Meas} [T]_{Cb2}^{-1} [T]_{Bn2}^{-1} [T]_{TL}^{-1} \quad (12)$$

3.3 De-embedding method 2: virtual standards

In an attempt to incorporate all of the balun S-parameters, the transition from a single line system to a double line system is modeled in the python S-parameter simulation package *scikit-rf* (Arsenovic et al., 2022) and calibrated using the SOLT method with virtual standards (Keysight, 2024). Section 3.2 demonstrated a method for de-embedding the dipole system, but this is limited to 2×2 matrices and cannot incorporate the balun’s full 3×3 S-parameters without reducing it to the differential mode (Salter and Alexander, 1991). In contrast, this section uses the full 3×3 S-parameters of the balun, Equation 6, to incorporate both the balun’s common mode and differential mode in the calculations. *scikit-rf* simulates the different components of the embedded system (stems, balun, and cabling) using their measured S-parameters as inputs. By connecting the nodes of each component and given a known DUT, *scikit-rf* simulates the entire system presented in Figure 5 and diagrammed in Figure 10a. *scikit-rf* does the same transfer matrix calculations as Section 3.2, in the background of the program. However, the node connections allow for the user to

simply transition from a 2×2 matrix to a 3×3 matrix, as long as all of the nodes are connected.

A virtual SOLT calibration scheme is applied to this simulation, to move the calibration plane from the VNA, to the dipoles. Ideal virtual S-parameters for a short, open, and load are created as 1-port network elements in the *scikit-rf* simulation space, while the through is modeled as a direct node-to-node connection. These are then applied as loads at the ends of the simulated cable, balun, and stems to obtain their simulated VNA responses, as diagrammed in Figures 10a, b. The “Virtual Measured” embedded standards are fit to the “Virtual Standards,” using the class *skrfSOLT*, to obtain a calibration that reduces the measurement to the blue dashed line box in Figure 5. The error terms of this calibration are then applied to future real VNA measurements of the embedded dipoles (using the *apply_cal* function from the *skrfSOLT* object) to obtain the de-embedded S-parameters of the dipoles.

The nodal connections between the *scikit-rf* networks for this procedure are diagrammed in Figure 10. For the reflection standards (short, open, load), the following impedances are used to create their 2×2 S-parameter matrices: Matched Load: $Z = 50 \, \Omega$, Short: $Z = 0 \, \Omega$, and Open: $Z = 10^{30} \, \Omega$. The embedded standards are connected to the nodal ends of the stems where the dipoles would be located, as shown in Figure 10b. The ideal 2×2 networks form a connection between the simulated ports and ground. For the thru standard, the stem nodes are directly connected, as are the ideal nodes between Port 1 and Port 2, as shown in Figure 10c. It should be noted that this corresponds to connecting the same stems, respective to each RFIV box. Looking at Figure 2, this is equivalent to connecting the diagonally opposite stems, rather than the mirror opposite stems, between the RFIV boxes. Reversing this configuration, by connecting to the mirror opposite port will switch the polarity of S_{21} and S_{12} compared with the method presented in Section 3.2. The de-embedded 4×4 DUT, Figure 10a, is equivalent to a 2-port network,

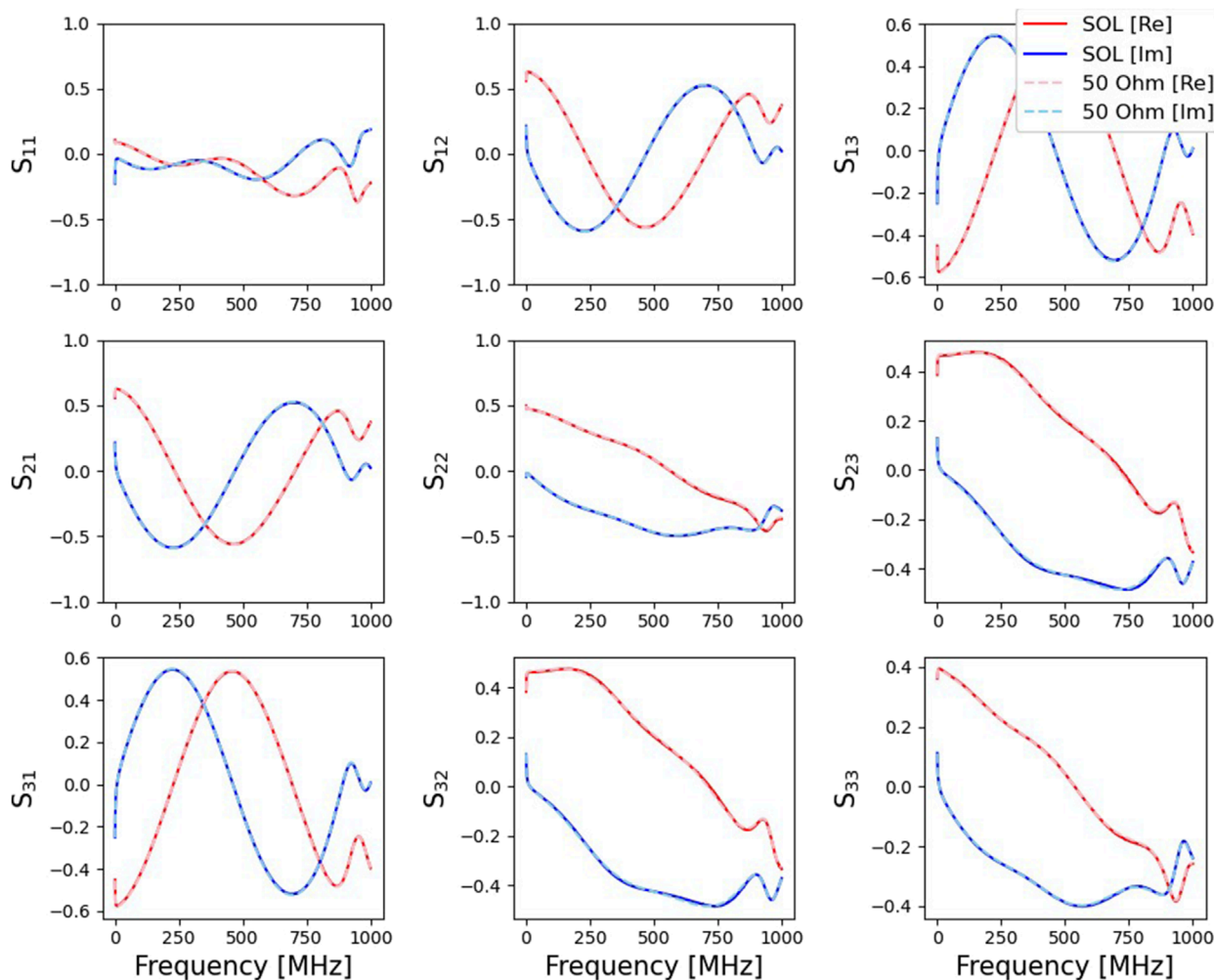


FIGURE 9
The full 3×3 S-parameters of the balun: using the 50Ω standard (dashed) and the SOL standards (solid).

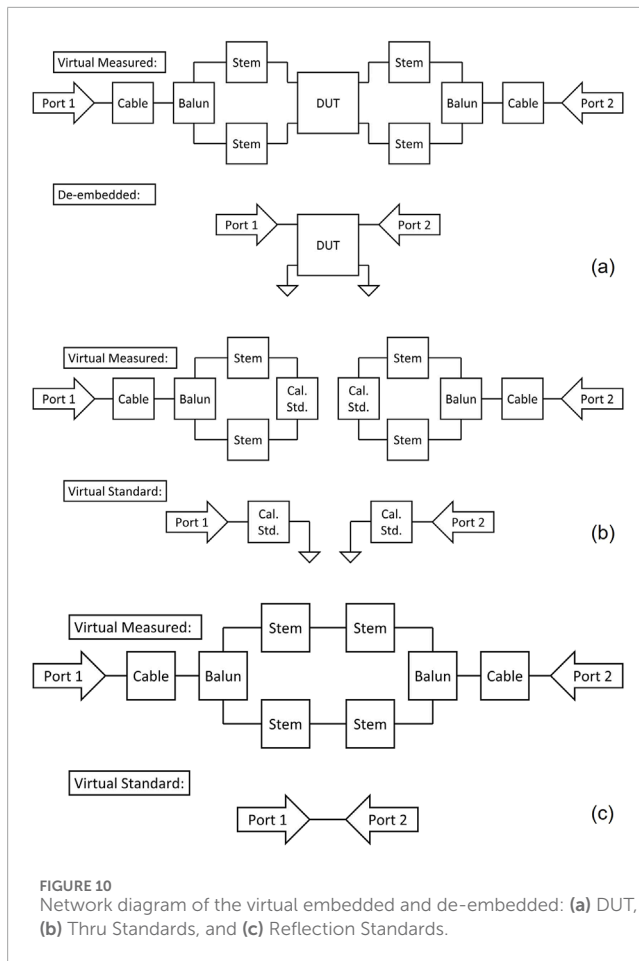
since the calibration standards have been configured to make two of the stems grounded. This is consistent with the real balun circuit, diagrammed in Figure 4.

3.4 Finite element simulation

To validate the dipole calibration, the measurements are compared with S-parameters calculated from a frequency domain simulation, run in the finite element software: “COMSOL Multiphysics.” (COMSOL AB, 2023) The configuration of the simulation space is shown in Figure 11a. This simulation models: the dipole elements, the stems, the support ring, and the RFIV boxes. Previous work by Brooks et al. (2023), on plasma impedance probes with monopoles, demonstrated a maximum of 20% error between simulations and measurements when the surrounding conductive elements were not taken into consideration. Inclusion of the coaxial stems allows the system to be excited by uniform Coaxial Lumped Ports at the base of the stems. This provides an easily identifiable calibration plane, compared with exciting the system at the dipoles.

The two Coaxial Lumped Ports at the base of a pair of stems excite the differential signal on a single dipole. Due to the two-line nature of the dipoles, the voltages and currents at the ports are exported and converted into S-parameters, rather than exporting the S-parameters directly from the simulation. To excite the system, one dipole has voltage values applied to its ports while the other dipole has a zero-current boundary condition, satisfying the Z-parameter definition (Steer, 2013). To satisfy the 180° phase difference for the differential signal, one of the ports on the active dipole is set to 1V while the other is set to $-1V$. The outer conductors of the stems are connected to the RFIV box, as shown in Figure 11b, so that both stems share a common ground reference. From COMSOL's post-processing, the voltage and current measurements are obtained as a function of frequency for all four ports.

The voltages (V) and currents (I) are first converted into Z-parameters and then transformed into S-parameters. The differential Z-parameters are expressed in Equation 13 and Equation 14 (Steer, 2013; Bockelman and Eisenstadt, 1995). The subscript of the voltage and current refers to dipole A or B and stem 1 or 2, where dipole A is the active dipole. Since the components of the



simulation are passive, the system is reciprocal: $Z_{21} = Z_{12}$. Since the simulation is symmetrical: $Z_{11} = Z_{22}$ (Steer, 2013). The Z-parameters are converted to S-parameters with Supplementary Equation 13, in Section VI C of the Supplementary Appendix. Due to the double-line nature of 50Ω stems, their combined characteristic impedance is 100Ω (Steer, 2013). Finally, the S-parameter of the dipoles are de-embedded from the stems using Equation 12, without the balun and cable terms.

$$Z_{11} = \frac{(V_{A1} - V_{A2})}{\frac{1}{2}(I_{A1} - I_{A2})} \quad (13)$$

$$Z_{21} = \frac{(V_{B1} - V_{B2})}{\frac{1}{2}(I_{A1} - I_{A2})} \quad (14)$$

To reduce the required number of mesh nodes and computation time, the metal volume is removed from the simulation and impedance boundary conditions applied to their surfaces. The white space in Figure 11c, shows the contour map of the electric field results and highlights the absence of the metal volumes. Most of the current travels on the surface of a conducting material, but COMSOL also simulates the skin depth for any field that would have entered the volume at higher frequencies. This skin depth is based on the material properties of the given surface. The dielectric behind the Lumped Ports is also removed as it creates a non-realistic path for the fields to travel between the outer and inner conductor, by going around the backside of the port.

The outer surface of the cylindrical simulation volume is a Scattering Boundary, to minimize the reflection of waves back towards the dipoles. The cylinder has a diameter of 1 m and a length of 0.75 m. At this size, increasing the chamber has no effect on the simulation results. The Scattering Boundary works best when the incident waves are normal to the surface, so the cylinder is aligned parallel with the axis of the dipole. At the ends of the cylinder, the flat surface, with a surface normal parallel to the dipoles, has minimal effect, since the transmitted power approaches zero as the wave vector becomes parallel with the dipole (Balanis, 2015).

The main volume of the simulation has a mesh size that ranges from 0.5 mm to 55 mm. Additional mesh refinement was applied to areas with increased field strength. Particularly, the transmission line of the stem and the corner edges of the dipole and stem, shown in Figure 11. These regions have a maximum mesh size of 0.5 mm.

4 Results

First, the de-embedding method is compared to a directly measured known load, validating the method independent of analytical models or simulations. A lossy transmission line cable is used as this known load due to its well defined characteristics. As a sanity check, the de-embedded cable is also compared to the analytical model of a cable, Equation 2.

Second, the mutual impedance of the dipoles are validated by comparing the de-embedded dipole measurement with the finite element simulation, described in Section 3.4. The two different de-embedding methods from Section 3.2 and Section 3.3 are also compared. Although errors exist in all of the validation approaches, reasonable agreement is demonstrated between the measurements, simulation, and analytical model.

4.1 Cable: de-embedded, direct measurement, and analytical model

To embed the cables, two 4 ft long RG316 cables are attached to the baluns as loads for two separate configuration: a transmission configuration and a reflection configuration. For the transmission configuration, each cable is strung between the baluns, connecting the same respective ports of the two RFIV boxes. This means that in Figure 2, the cables are strung diagonally between the RFIV boxes. The network diagram is the same as Figure 10c, but with the stems removed. For the reflection configuration, one cable connects the two ports on each RFIV box. In this case, the cable would take the place of the “Cal. Std.” in Figure 10b, but with the stems removed.

The resulting transmission configuration measurements, in Figure 12, match a single cable, directly measured by the VNA. The RG316 cable is modelled using the published velocity factor and loss data, as described in Section 3.1.2. The resulting reflection configuration measurements show a characteristic cable load, but on the S_{11} . This is the load that would be observed if a cable were measured on a two-port system, but then converted to a differential mode. The full S-parameters of an analytical cable model and directly measured cable are converted to a single differential

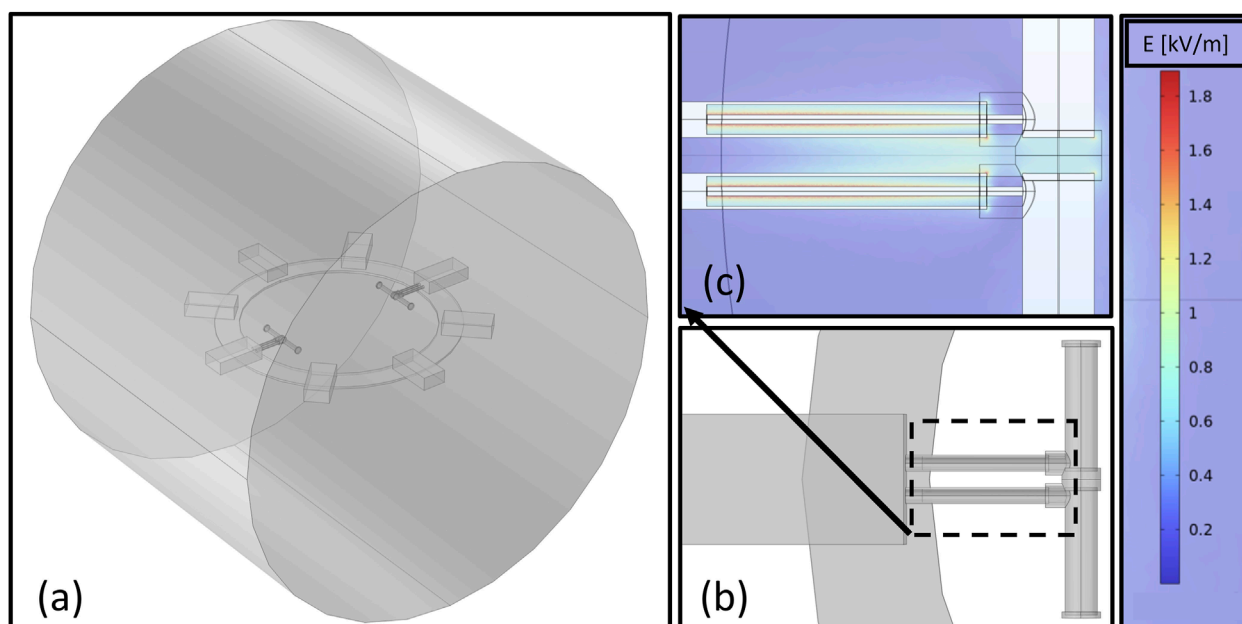


FIGURE 11

Mutual Impedance Simulation Configuration: (a) Isometric view of simulation space with two dipoles on the ring frame and adjacent RFIV boxes. (b) Top down view of dipole, stem, and RFIV box. (c) Countour plot of the E-field along the stems and between the dipole elements.

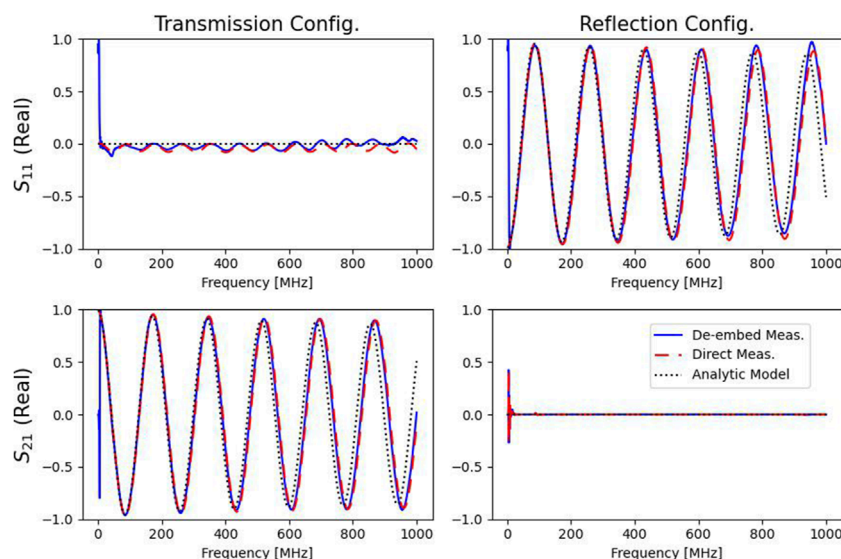


FIGURE 12

S-parameters of a 4 ft cable in the reflection and transmission configuration: de-embedded by the virtual standard method (blue solid), directly measured with the VNA (red dashed), and analytically modelled (black dotted).

reflection coefficient, using Equation 15a (Konstroffer, 1999).

$$S_{11diff} = \frac{A}{B} \quad (15a)$$

$$A = (2S_{11} - S_{21})(1 - S_{22} - S_{12}) + (1 - S_{11} - S_{21})(1 + S_{22} - 2S_{12}) \quad (15b)$$

$$B = (2 - S_{21})(1 - S_{22} - S_{12}) + (1 - S_{11} - S_{21})(1 + S_{22}) \quad (15c)$$

Since the directly measured cables almost exactly match the de-embedded cables, for both the transmission and reflection configurations, these measurements validate the de-embedding process up to the stems' ports on the RFIV boxes. The analytical cable model closely replicates the directly measured cable, however, there is a phase delay that is equivalent to adding an extra 2 cm electrical length to the 4 ft long cable. There is also a ripple in the S_{11} of the transmission configuration for the

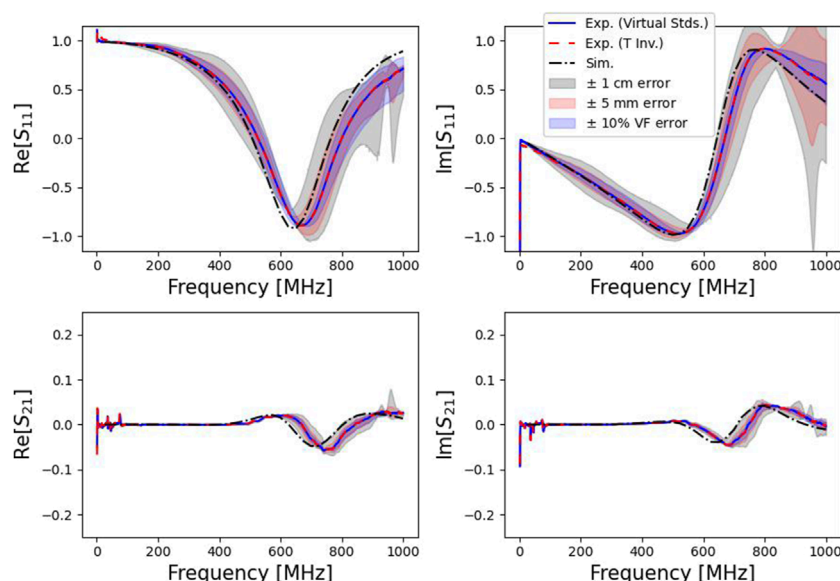


FIGURE 13
S-parameters of simulated and de-embedded dipoles, using both de-embedding methods. Includes shaded regions identifying the spread of the sensitivity analyses.

directly measured and de-embedded cable. The analytical cable model assumes a perfect impedance match between the VNA and cable, which results in a zero S_{11} . The ripples suggest an impedance mismatch at the connections between the measured cable and the VNA calibration plane. Despite these discrepancies, the directly measured cable and de-embedded cable, for both configurations, all closely agree. This independently validates the de-embedding method.

4.2 Dipole: measurements, calibration methods, and simulation

The dipoles de-embedded in air are compared with the finite element simulations and presented in Figure 13. The reflection coefficients (S_{11}) have excellent agreement between 100 MHz and 500 MHz. Below 100 MHz, there is significant noise in the measured transmission signal (S_{21} and S_{21}), relative to the magnitude of the antenna resonance. With increasing frequency, the measured data shows a shift up in frequency, like the phase delay found in Figure 12. The dipole resonance of the simulation occurs at 645 MHz and the resonance of the measured data occurs at 665 MHz. This equates to a difference of 20 MHz, a 3% shift down from the de-embedded dipoles, and an additional electrical length of 1 cm.

The plasma density is determined from the plasma resonance, so this shift in frequency, rather than any resulting error in magnitude, is the primary concern for the mutual impedance probes. The plasma density (n_e) is a function of the squared plasma frequency (ω_p), as shown in Equation 16 (electron mass: m_e , electron charge: e , and free-space permittivity: ϵ_0). So the 3% frequency error equates to a 6% lower plasma

density error.

$$n_e = \frac{\omega_p^2 m_e \epsilon_0}{e^2} \quad (16)$$

By assuming an electrical length delay of 1.0 cm for the measurement, the maximum difference between the S-parameter magnitudes is halved. This electrical delay would represent an unaccounted for additional cable length in the system. However, the drawback to assuming this electrical length delay is that, below 400 MHz, the average difference between the simulation and de-embedded S_{11} magnitudes would increase by a factor of four.

A sensitivity analysis was performed on the calibrated system's final results. The artificial error was introduced as a phase shift in the VNA's calibration plane for each measurement in the de-embedding process. The sensitivity study looked at the final results of the de-embedded dipoles, if the VNA calibration plane was off by ± 1 cm or ± 5 mm. The phase shifts were applied to each measurement involved in the process: the full system measurement, the true measurement of the RFIV standards, the measurement of the RFIV standards at the end of the cables, the true measurement of balun standards, and the through measurement of the balun with standards attached at the third port. Every combination of error (e.g., +1 cm, 0 cm, and -1 cm) at each measurement step was calculated and the spread of the possible range is plotted in Figure 13 as the grey (± 1 cm) and red (± 5 mm) shaded backgrounds. In addition, the variation in the stem's constants, provided by the data sheet (Author anonymous, 2013), were analyzed by applying a range of errors from -10% to 10%, and looking at the range of resulting values. Since the stems are only 5 cm, the attenuation has almost no effect at this length, changing the magnitude by at most 0.1%. The dominant stem constant is the velocity factor. Its $\pm 10\%$ variation is plotted in Figure 13 as a shaded blue background.

From this analysis, it is clear that with a large enough variation in the calibration plane, the difference between the simulated and experimental data could be accounted for. Although the variation in the stem velocity factor can also account for similar changes, more than a 10% variation is needed to account for the difference seen in this data. Although the data sheet (Pasternack Inc., 2013) does not provide confidence values for the data, a $\pm 10\%$ error results in permittivity values (1.7–2.5) that are outside the published range for Teflon (2.0–2.1) (Ehrlich, 1953; Riddle et al., 2003; Krupka et al., 1998; Burger et al., 2018). In addition, we measured and calculated the VF of the RG401 cable, using the method described in Supplementary Appendix Section VI B, and obtained a VF = 67.9, a 0.4% difference from the data sheet (Pasternack Inc., 2013). From this analysis, the suspected sources of error, between the experimental and simulated/analytical data, are the calibration standards, which could shift the calibration plane, and will be discussed further in Section 5.1. For this reason and the relatively small phase delay, we accept the error between the experimental and simulated data.

The two de-embedding methods are both applied to the measured data, for comparison in Figure 13. Both methods have almost the exact same result, but there is a deviation in S_{11} below 100 MHz. The Virtual Standards align with the simulation data. However, the T-parameter inversion produces an S-parameter above one, which is an unrealistic condition for a passive system, such as this. By comparison, the Virtual Standards method has better agreement with the simulated S_{11} than the T-parameter inversion. Since the primary difference between the two methods is that the Virtual Standards method uses the three-port model of the balun (incorporating the common and differential mode) while the T-parameter method uses the two-port model (discarding the common mode), this difference is assumed to be from the lack of consideration for the common mode.

Finally, the calibrated S-parameters are converted to Z-parameters using Supplementary Equation 13 and plotted in Figure 14. The Z_{11} is the self-impedance and Z_{21} is the mutual-impedance. Although there is a slight difference in the S-parameters of the two calibration methods, shown in Figure 13, the difference between the calibration methods can be most clearly seen in the Z_{11} of Figure 14. Below 100 MHz, the Virtual Standards method (three-port balun model) follows the simulations predictions, while the T-Inversion method (two-port balun model) deviates from the simulation with an opposite sign to the simulation. In particular, this negative real impedance by the T-parameter Inversion method is not physically possible for our passive system. It would imply that power was being added by the system. For the Z_{21} , the noise found in the S_{21} becomes more prominent, with no agreement below 150 MHz.

To find the plasma density, the model for self-impedance from Balmain (Balmain, 1964) is fit to the self-impedance, or an adapted version of the Balmain model is fitted to the mutual impedance. More complex analytical solutions to the mutual-impedance measurement have also previously been implemented (Storey et al., 1969; Pottelette et al., 1975; Chasseriaux et al., 1972; Debie et al., 1976). From these equations, the plasma resonance is a function of the plasma density, magnetic field, distance between the dipoles,

collision frequency, sheath thickness, and the angle between the dipole and the magnetic field. The derivation of the mutual impedance model and sample plasma measurements goes beyond the scope of this work, which is focused on calibrating the dipole instrument.

5 Discussion

The calibration method is validated by the previous plots, despite slight errors between simulated and measured data. The major validating piece of evidence, is the agreement between the de-embedded and directly measured cables. Despite potential error in the VNA calibration, the de-embedding method returns the same result as the direct measurement.

5.1 Sources of error

For this calibration procedure, four sources of error were identified that can be mitigated:

- 1) The common mode error below 100 MHz, in the conventional T-parameter inversion method, is corrected by incorporating the full balun S-parameters in the Virtual Standards method. By using the balun three-port model, in the Virtual Standards method, information from the balun's common mode is conserved. The common mode is otherwise discarded when the balun is represented as a two-port model, in the T-parameter inversion method, with the balun's single line as one port and the differential mode as the second port. Although the balun's reported operational frequency extends down to 1 MHz, the common mode clearly begins to become noticeable at higher frequencies. By simply implementing the Virtual Standards method, this low frequency error is removed.
- 2) The low frequency noise in S_{21} of Figure 14, from our experience, can be reduced by increasing the power of the VNA signal. Evidence for this can be drawn from Figure 13, where there is far greater noise for the transmission signal compared with the reflection signal, over the low frequency range. As the received power decreases with distance, the receiving antenna experiences greater noise relative to the signal magnitude. These tests were conducted with a signal power of 0 dBm, however many VNAs go up to 5 and 10 dBm.
- 3) The BNC connections were found to cause a variable calibration. BNC connections are not a fixed fit, but allow play for the cables to turn. Due to small capacitive differences in the contact surfaces, when turned, the impedance changes with movement. This can form a ripple in the calibrated magnitude. Since SMAs are screwed on and form a static contact, they were substituted for all the BNC connections, successfully removing the ripple effect in the calibrated VNA magnitude.
- 4) The differences between the measured and modelled data suggest that there is still an issue with the VNA calibration. The analytical cable model, in Figure 12 has a delay from the directly measured cable. Also, the S_{11} of the directly measured cable has ripples indicative of an impedance mismatch between

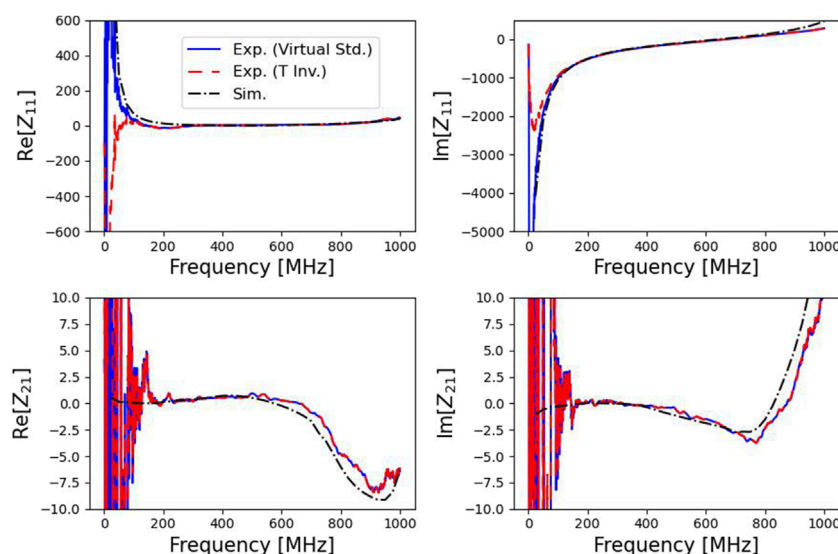


FIGURE 14
Z-parameters of simulated and de-embedded dipoles, using both de-embedding methods.

the VNA and the cable. The difference between the cable model and directly measured cable is the greatest argument for the fault lying with the VNA since the directly measured cable does not utilize the de-embedding process. At first we suspected the attachments used to calibrate the SMA cable with BNC standards, discussed in Section 3.1.1. However, when tested, the same cable length was measured at the BNC calibration plane as at the SMA calibration plane, after accounting for the BNC-SMA adapter's change in length. This led us to suspect the calibration standards are slightly off. Even with this validation, we suggest using SMA calibration standards to perform the SMA calibration measurements and reduce a possible source of error.

6 Conclusion

This work has presented an *in-situ* calibration method for de-embedding two dipoles located in a vacuum chamber. The intended application has been for mutual impedance plasma density measurements and plasma impedance tomography measurements. The *in-situ* calibration standards account for cable heating and cable travel, two significant factors in plasma experiments that may require continued recalibration, while the instruments are inaccessible in the vacuum chamber. Two de-embedding methods were presented: 1) the Virtual Standards method and 2) the T-parameter inversion method. Both agree with one-another almost exactly except below 100 MHz, where the Virtual Standards have better agreement with the dipole simulation. It is assumed that the Virtual Standards account for the common mode in the balun, which becomes more prominent near the balun's lower frequency limit of 1 MHz.

The de-embedding method was validated by comparing it with a direct measurement of a 4 ft length of cable. There was very

little discrepancy between the directly measured cables and the de-embedded cables. Comparison of the directly measured cables, de-embedded cables, and ideal transmission line model, showed a phase shift equivalent to an electrical length of 2 cm, between the ideal and measured values. The dipole de-embedding was compared with COMSOL simulations, which also had a phase delay, but only for an electrical delay of 1 cm. This error equates to a 6% lower plasma density measurement. The measurements and models suggest that the phase shift error is associated with the VNA calibration standards, while the calibration method is still valid. Other sources of error included: the variable capacitance of the BNC connections and the VNA's signal power, which can both be mitigated.

This calibration method gives the ability to remotely calibrate an array of dipoles without performing any through measurements. The presented method has several advantages over other dipole calibration methods: 1) Plasma measurements (including magnetized plasma) can be performed without uncertainty related to temperature and bending effects on the connecting cables, which lead to errors at higher frequencies. 2) Balun common mode errors at low frequencies are significantly reduced. 3) Calibration effort and time is greatly reduced for a multiprobe tomographic setup. Future work will consist of: First, demonstrating mutual impedance plasma measurements at frequencies in the 10s–100s of MHz range, now capable with this calibration method. Second, perform a 2D plasma tomographic measurement and inversion with the 8-probe array, shown in Figure 2.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

MP: Writing–original draft, Writing–review and editing. ET: Writing–review and editing. GG: Writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

The Supplementary Material for this article can be found online at: <https://www.frontiersin.org/articles/10.3389/fspas.2025.1519446/full#supplementary-material>

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