



Seismic Bearing Capacity of a Strip Footing Over an Embankment of Anisotropic Clay

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The present study adopts lower bound finite element limit analysis technique in association with non-linear optimization to compute the seismic bearing capacity of a strip footing placed over an embankment of anisotropic clay. The influence of seismic loading is incorporated in terms of a horizontal seismic acceleration coefficient. The bearing capacity factor N_c of the strip footing is obtained for various combinations of (i) slope angle, (ii) footing setback distance, (iii) horizontal seismic acceleration coefficient, and (iv) anisotropic strength ratio of clay. The bearing capacity factor of the strip footing increases with the increasing setback distances. However, beyond a certain setback distance, hardly any influence is found. For a particular footing position and constant value of horizontal seismic acceleration coefficient, the magnitude of bearing capacity factor increases with the increase in anisotropic strength ratio. However, the percentage increment in the bearing capacity reduces when the value of anisotropic strength ratio becomes greater than one. In addition, the bearing capacity reduces with the increasing slope angle and horizontal seismic acceleration coefficient. Failure patterns are obtained to understand the failure mechanism.

OPEN ACCESS

Edited by:

Sanjay Shrawan Nimbalkar, University of Technology Sydney, Australia

Reviewed by:

Subhadeep Banerjee, Indian Institute of Technology Madras, India Anindya Pain, Central Building Research Institute (CSIR), India

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Specialty section:

This article was submitted to Transportation and Transit Systems, a section of the journal Frontiers in Built Environment

> Received: 29 June 2019 Accepted: 31 October 2019 Published: 15 November 2019

Citation:

Krishnan K, Halder K and Chakraborty D (2019) Seismic Bearing Capacity of a Strip Footing Over an Embankment of Anisotropic Clay. Front. Built Environ. 5:134. doi: 10.3389/fbuil.2019.00134 Keywords: seismic bearing capacity, embankment, lower bound limit analysis, non-linear optimization, clay anisotropy

INTRODUCTION

In recent times, shortage of construction land compels the people to build structures on the embankment; even on its edge or side face. Construction of shallow foundations for short storeyed buildings and bridge abutments, roadways, and railways on flood plains or riverbanks are the perfect examples of such structures. The presence of slope on both the sides of the footing reduces the bearing capacity of the footing, which significantly enhances the vulnerability of the structures (Georgiadis, 2009; Chakraborty and Kumar, 2013; Chakraborty and Mahesh, 2015; Leshchinsky, 2015; Halder and Chakraborty, 2018; Raj et al., 2018; Halder et al., 2019). The risk and vulnerability of the structures increases further under the seismic loading, generated during an earthquake (Kumar and Mohan Rao, 2003; Choudhury and Subba Rao, 2006; Yang, 2009; Kumar and Chakraborty, 2013; Chakraborty and Mahesh, 2015; Halder et al., 2018; Halder and Chakraborty, 2019). The literature review reveals that although there are many studies available for the computation of the bearing capacity of footing on the embankment under seismic loading, no one has considered the effect of anisotropy of clay on the bearing capacity of a strip footing placed on an embankment and subjected to seismic loading. The effect of anisotropy is very much significant for undrained clay (Jakobson, 1955; Lo, 1965; Bishop, 1966; Duncan and Seed, 1966; Davis and Christian, 1971). Hence, the introduction of anisotropy in numerical analysis increases the accuracy of the analysis towards field condition. Davis and Christian (1971) formulated an anisotropic failure criterion (Equation 2) for cohesive soil in the form of an ellipse by using the anisotropic strength ratio (b/a) which is a function of undrained shear strengths s_{u0} , s_{u45} , and s_{u90} . In the present

analysis, the effect of anisotropy of clay is incorporated using the formulation proposed by Ukritchon and Keawsawasvong (2018) using the yield function of Davis and Christian (1971). From here, throughout the article, DC indicates the yield criterion of Davis and Christian (1971). The advantage of DC yield function is associated with the consideration of three parameters (s_{uo} ,



 s_{u90} , and s_{u45}); whereas, most of the other anisotropic yield criterions are based on two parameters (s_{uo} and s_{u90}). The parameters mentioned above are the undrained shear strengths of clay measured in different orientations (δ): s_{uo} (plane strain compression, $\delta = 0^{\circ}$), s_{u90} (plane strain tension, $\delta = 90^{\circ}$), and s_{u45} (simple shear, $\delta = 45^{\circ}$). It is also indicated by Ukritchon and Keawsawasvong (2018) that the three-parameter model simulates the actual failure envelope in a better way with respect to other models. The influences of other parameters investigated in the study are slope inclination angle (β), footing setback distance (S), and seismic acceleration coefficient (k_h). With the consideration of all the parameters mentioned above, the present study computes the bearing capacity of a strip footing situated over an embankment of anisotropic clay.

STATEMENT OF THE PROBLEM

The objective of the present study is to determine the bearing capacity factor N_c of a rigid, rough and surface strip footing of width *B* placed over an embankment of anisotropic clay, in the presence of horizontal earthquake acceleration (k_hg) using the lower bound finite element limit analysis method. A non-linear optimization technique based on the second-order cone programming (SOCP) method is used. The

embankment faces are assumed to incline at an angle β with the horizontal axis. The classical bearing capacity equation of Terzaghi (1943) reduces to Equation (1), as there is no presence of frictional property of soil and overburden pressure in the ground.

$$q_u = Q_u/B = s_{uav}N_c \tag{1}$$

where, Q_u is the ultimate collapse load determined from lower bound limit analysis and $s_{uav} = (s_{u0} + s_{u90})$ is the average undrained shear strength of clay.

PROBLEM DOMAIN, MESH DETAILS AND STRESS BOUNDARY CONDITIONS

The problem domain used in the present study is represented by **Figure 1A**. The domain is varied up to a length and depth of 7.5*B*-15*B* in the horizontal and vertical directions. **Figure 1A** also shows the stress boundary conditions employed on different boundary edges. With the absence of any overburden pressure, the normal and shear stresses are zero along the ground surface (MI and JN) and inclined faces (RM and NO) of the embankment. In order to account the footing roughness, an





equation: $|\tau_{xy}| \leq s_{uav}$ is adopted. The semi-infinite and perfectly plastic soil mass is assumed to follow associated flow rule and DC yield criterion. The effect of pseudo-static lateral earthquake force is considered in the study by introducing a lateral shear force ($Q_h = k_h Q_u$) along the footing-soil interface IJ.

The finite element mesh is first created using ABAQUS software¹. Then the coordinates of the mesh are transferred to MATLAB². Figures 1B,C show the ABAQUS generated finite element mesh, which is later used in MATLAB in order to obtain the collapse load of footing. In Figures 1B,C, N_e , E, and N_{disc} denote the number of nodes, elements and discontinuity edges, respectively.

LOWER BOUND FORMULATION

The present study uses the lower bound finite element limit analysis formulation of Sloan (1988). The plane strain lower bound formulation of Sloan (1988) has certain unique qualities such as: (i) it requires only the shear strength parameters for solving any stability problem (ii) it gives safe and conservative solution of the collapse load (iii) the assembly of elements is easier due to the distinct node numbers for each element. As per the lower bound theorem, any statically admissible stress field that satisfies the (i) element equilibrium condition, (ii) stress boundary condition, and (iii) yield criteria, gives a safe solution of the true collapse load. Among the above three, the first two conditions are the equality constraints and the third one is an inequality constraint. In the finite element lower bound limit analysis formulation, the elements are assigned distinct node numbers. Hence, there arises a discontinuity between the edges of shared elements. The discontinuity of the stresses is considered by imposing an additional equality constraint. To solve any stability problem, the above four conditions need to be satisfied.

Equation (2) gives the DC yield condition for anisotropic clay below.

$$F = \sqrt{\left(\frac{\left(\sigma_y - \sigma_x\right)/2 - h}{a}\right)^2 + \left(\frac{\tau_{xy}}{b}\right)^2} = 1$$
(2)

where, σ_x and σ_y are the normal stresses on x and y directions, respectively. $h = \frac{(-s_{u0} + s_{u90})}{2}$, $a = \frac{s_{u0} + s_{u90}}{2}$, and $b = \frac{as_{u45}}{\sqrt{s_{u0}s_{u90}}}$ are

¹ABAQUS 6.14 [Computer software]. Dassault Systemes, Vélizy - Villacoublay.
²MATLAB 8.5 [Computer software]. MathWorks, Massachussets, USA.



the center, major semi-axes, and minor semi-axes of the elliptical yield surface formulated by DC yield criteria. The b/a ratio: the ratio between the minor and major axes, respectively, is used as a factor for anisotropic strength.

The objective function is generated by integrating the normal stresses along the footing-soil interface. The canonical form of the optimization scheme is given in Equation (3). A MATLAB code is written to form the canonical structures of the formulation; whereas, an optimization toolbox MOSEK³ is used to carry out the non-linear optimization.

$$Maximize: c^T \sigma \tag{3}$$

Subjected to:
$$[A_g]{\sigma} = {B_g}$$
 (3a)

$$[A_{1SOCP}]\{\sigma\} + [A_{2SOCP}]\{\xi\} = \{B_{SOCP}\}$$
(3b)

Where A_g and B_g are the global matrix and vector, respectively constituting equality constraints. A_{1SOCP} , A_{2SOCP} , and B_{SOCP} are the yield constraint matrices and vector, respectively. $\{\sigma\}$ is the nodal stress vector and $\{\xi\}$ is the nodal slag variable vector.

VARIATION OF BEARING CAPACITY FACTOR N_C FOR DIFFERENT K_H , β , S/B, AND B/A VALUES

The variation of bearing capacity factor N_c with respect to k_h is determined for various b/a ratios ranging from 0.8 to 1.2. The **Figures 2–4** show the results obtained for different β (=10°, 20°, 30° , and 40°) and S/B (0, 2, and 4) values. The results indicate that as the footing moves away from the slope edge, the bearing capacity increases. However, after a particular setback distance of footing, there is no further increment in the value of bearing capacity factor. The reason behind this is the reduction in the influence of plastic zone over the slope when setback distance is increased. Similar results are also obtained in Chakraborty and Kumar (2015) and Halder and Chakraborty (2018). Results also illustrate that the bearing capacity increases with the increase in anisotropic strength ratio (b/a). It is noted that the increment in N_c value is higher for b/a < 1. However, the increment becomes constant when b/a > 1. For a slope of $\beta = 30^{\circ}$, S/B = 0, and k_h = 0, the increase in N_c value is 57.81% with the increase in the value of b/a from 0.8 to 1.0. Whereas, when b/a ratio is increased from 1.0 to 1.2, the percentage increase in the N_c value is reduced to 9.41%. The decrease in N_c value with the increase in β also is evident from the results. For example, when *b/a* ratio is reduced

³MOSEK ApS version 8.0 [Computer software]. MOSEK, Copenhagen, Denmark.







FIGURE 6 | Failure contours for S/B = 4 and b/a = 1 with (A) $\beta = 30^{\circ}$, $k_h = 0$; (B) $\beta = 40^{\circ}$, $k_h = 0$; (C) $\beta = 30^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; and failure contours for S/B = 4 and b/a = 0.8 with (E) $\beta = 30^{\circ}$, $k_h = 0$; (F) $\beta = 40^{\circ}$, $k_h = 0$; (G) $\beta = 30^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$, $k_h = 0.1$; (D) $\beta = 40^{\circ}$

from 1.0 to 0.8 for a footing with S/B = 0, the reduction in N_c value becomes higher from 13.75 to 45.43% for the increase in β value from 10 to 40°, respectively. In the same way when setback

distance is varied from S = 0B-4B for a slope angle of 30° , the magnitude of N_c value reduces from 36.63 to 5.85% for the reduction in b/a ratio from 1.0 to 0.8.



FIGURE 7 | (A) Comparison of seismic bearing capacity factor N_c of Chakraborty and Mahesh (2015) and the present study for S/B = 0; (B) Comparison of N_c on an horizontal ground surface for different anisotropic strength ratio (*b/a*).

TABLE 1 | Comparison of available experimental results with the present study.

References	Measured		<i>b/a</i> from Davis and Christian	Nc	
	s _{u0} /s _{u90}	s _{u45} /s _{u90}	(1971)	Davis and Christian (1971)	Present study
Jakobson (1955)	1.03	1.13	1.10	5.30	5.30
Lo (1965)	0.75	0.85	0.98	5.11	5.13
Duncan and Seed (1966)	0.87	0.74	0.80	4.84	4.86
Bishop (1966)	1.56	0.78	0.62	4.58	4.60

FAILURE PATTERNS

The state of stress at any point with respect to collapse is determined in terms of F as given in Equation (2). As per the DC failure criterion, the soil fails or enters plastic state when $F \geq 1$. When F < 1, the soil will remain in a nonplastic state. The failure mechanism of an embankment for S/B = 0 and S/B = 4 are shown in Figures 5A-H, 6A-H, respectively. It is evident from Figure 5 that when S/B = 0, the failure surface extends to the slope face easily. It is also clear that when b/a ratio is reduced from 1.0 to 0.8, the failure surface extends to a greater depth. The influence of seismic force is well-understood from the Figures 5C,D,G,H. With the consideration of earthquake loading, the failure surface shifts more toward the slope face in the direction of seismic force. From Figures 6A-D, it is seen that the plastic zone is developed locally within the horizontal surface when b/a = 1 (isotropic condition). However, when the b/a ratio is decreased to 0.8 as in Figures 6E-H, additional to the local failure due to the footing on horizontal surface, the plastic zone is developed in the slope as well.

COMPARISONS

The values of N_c calculated from the present study for isotropic condition (b/a = 1) are compared with the results of Chakraborty and Mahesh (2015) as shown in Figure 7A. The present results are slightly higher than that reported by Chakraborty and Mahesh (2015). It is due to the use of different optimization technique in the present study. Chakraborty and Mahesh (2015) determined the seismic bearing capacity of an embankment of isotropic soil using lower bound limit analysis with linear optimization. The present study is carried out with the utilization of lower bound limit analysis in combination with non-linear optimization. However, to verify the results obtained for the anisotropic condition, the N_c values of strip footing are determined for b/a ranging from 0.5 to 1.2 over a horizontal ground surface ($\beta = 0^{\circ}$) using the DC yield criterion as per Ukritchon and Keawsawasvong (2018). The present results match closely with the results of Ukritchon and Keawsawasvong (2018) as shown in Figure 7B. Additionally, the comparative study of Davis and Christian (1971) with the experimental results of Jakobson (1955), Lo (1965), Duncan and Seed (1966), and Bishop (1966) is considered to validate the numerical precision of the present study. Davis and Christian (1971) calculated the N_c values using the b/a ratios corresponding to the experimental shear strength results of Jakobson (1955), Lo (1965), Bishop (1966), and Duncan and Seed (1966). The N_c values of the present work are compared with the N_c values of Davis and Christian (1971) for different b/a ratios in Table 1. The comparison shows that the present study match well with the calculated Nc values of Davis and Christian (1971).

CONCLUSIONS

By incorporating lower bound finite element limit analysis with second-order conic optimization technique, the seismic bearing capacity of a strip footing over an embankment of anisotropic clay is determined. The variation of bearing capacity factor N_c is presented as a function of slope angle (β), setback distance of footing (*S*), seismic coefficient (k_h), and anisotropic strength ratio (b/a). Design charts are proposed based on the results of N_c . The design charts can be used by the practicing engineers to analyse the seismic bearing capacity of a strip footing over an embankment of anisotropic clay using the anisotropic strength ratio (b/a).

It is found that the bearing capacity factor N_c decreases with the increase in the value of β and k_h . The trend of the reduction in the value of N_c with the increase in k_h for a b/a value greater than or equal to one is almost linear. However, when b/a < 1, the trend of the reduction in the value of N_c becomes non-linear.

- It is also evident from the results that the influence of slope angle is higher for footings with lesser setback distances. When *S* value is reduced from 4*B* to 0*B*, for an embankment of $\beta = 40^{\circ}$, b/a = 1.2 and $k_h = 0$, there is 31.95% decrease in the N_c value; whereas, for $\beta = 20^{\circ}$, the decrease is just 6.3%.
- The results show that there is larger reduction in the N_c values for b/a < 1 due to the fact that when b/a is lesser than 1, the minor axis (b) of elliptical yield surface becomes larger

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than the major axis(a), making the direction of anisotropy to change. For that reason, whenever the anisotropic strength ratio is lesser than 1, the strength reduces greatly.

• When anisotropic condition is coupled with earthquake force, the N_c values of strip footing situated over an embankment reduces drastically. Hence, more emphasis on anisotropy needs to be given while analyzing the seismic bearing capacity of strip footing over an embankment.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

KK developed the codes in MATLAB for carrying out all the numerical simulations required in the present study and prepared the initial draft. KH and DC proposed the project. KH modified the initial draft. DC supervised all the works carried out in this project and given a final shape of the paper.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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NOTATIONS

а	= major elliptical semi-axis			
$[A_g]$	= global equality matrix			
$[A_{1SOCP}]$	= non-linear yield matrix with nodal stresses			
$[A_{2SOCP}]$	= non-linear yield matrix with conic constants			
$\{B_g\}$	= global equality vector			
$\{B_{SOCP}\}$	= global yield vector			
b	= minor elliptical semi-axis			
b/a	= anisotropic strength ratio			
В	= footing width			
С	= objective function vector			
Ε	= Number of elements			
F	= elliptical yield function			
g	= acceleration due to gravity			
h	= centre of elliptical yield surface			
k_h	= coefficient of seismic acceleration			
N_c	= bearing capacity factor associated with soil cohesion			
Ne	= number of nodes			
N _{disc}	=number of discontinuity edges			
q_u	= ultimate bearing capacity			
Q_u	= ultimate collapse load			
Q_h	= lateral force on footing			
S	= setback distance of the footing from slope edge			
Suo	= undrained shear strength using plane strain compression $\delta = 0^{\circ}$			
<i>s</i> _{<i>u</i>90}	= undrained shear strength using plane strain tension $\delta = 90^{\circ}$			
<i>s</i> _{<i>u</i>45}	= undrained shear strength using simple shear $\delta = 45^{\circ}$			
s _{uav}	= average undrained shear strength			
β	= slope inclination angle			
σ_n	= normal stress at any point			
σ_x	= normal stress in the x direction			
σ_y	= normal stress in the y direction			
τ_{xy}	= shear stress in the x-y plane			
σ	= nodal stress vector			
γ	= unit weight of soil			
δ	= orientation of measurement			
ξ	= conic constants vector			