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EDITED BY

Guangzhao Wang,
Yangtze Normal University, China

REVIEWED BY

Xiaoming Zhang,
Hebei University of Technology, China
Weikang Wu,
Shandong University, China

*CORRESPONDENCE

Jiaren Yuan,
jryuan@ncu.edu.cn

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Electrical and magnetic properties of antiferromagnetic semiconductor MnSi_2N_4 monolayer

Dongke Chen^{1,2}, Zhengyu Jiang², Ying Tang², Junlei Zhou², Yuzhou Gu², Jing-Jing He³ and Jiaren Yuan^{1*}

¹School of Physics and Materials Science Nanchang University, Nanchang, China, ²School of Physics and Electronic Engineering, Jiangsu University, Zhenjiang, China, ³College of Information Science and Technology, Nanjing Forestry University, Nanjing, China

Two-dimensional antiferromagnetic semiconductors have triggered significant attention due to their unique physical properties and broad application. Based on first-principles calculations, a novel two-dimensional (2D) antiferromagnetic material MnSi_2N_4 monolayer is predicted. The calculation results show that the two-dimensional MnSi_2N_4 prefers an antiferromagnetic state with a small band gap of 0.26 eV. MnSi_2N_4 has strong antiferromagnetic coupling which can be effectively tuned under strain. Interestingly, the MnSi_2N_4 monolayer exhibits a half-metallic ferromagnetic properties under an external magnetic field, in which the spin-up electronic state displays a metallic property, while the spin-down electronic state exhibits a semiconducting characteristic. Therefore, 100% spin polarization can be achieved. Two-dimensional MnSi_2N_4 monolayer has potential application in the field of high-density information storage and spintronic devices.

KEYWORDS

two-dimensional materials, antiferromagnetic semiconductor, half metals, electronic properties, biaxial strain

Introduction

In 2004, the successfully prepared graphene opened a new era of two-dimensional materials (Novoselov et al., 2004). Subsequently, numerous new systems have already been discovered, greatly promoting the development of the two-dimensional material family. Two-dimensional materials have a wide variety of electronic properties, including metallic, semi-metallic, semiconducting and insulating properties. For example, 1H-MoS₂ is a semiconductor with a direct band gap, 1T phase MoS₂ is a metal, while 1T' phase MoS₂ is semimetal (Hung et al., 2018). In addition, hexagonal boron nitride (h-BN) shows insulating properties (Liu et al., 2003), and graphene is semimetal (Sheng et al., 2019). However, many 2D materials lack intrinsic magnetism, such as graphene and MoS₂, which motivates researchers to induce magnetism through defect engineering, adsorption or insertion of magnetic atoms. However, these schemes are difficult to construct stable

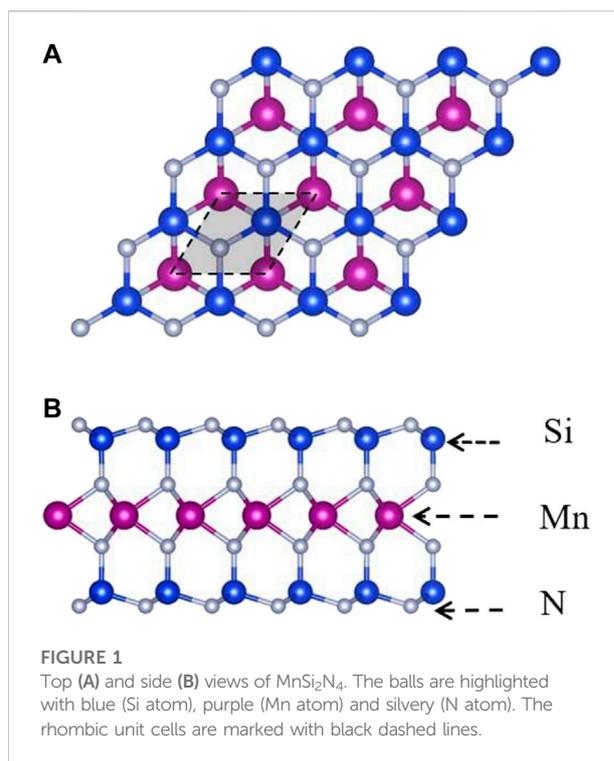
long-range magnetic order. Therefore, two-dimensional intrinsic ferromagnetic materials have aroused tremendous attention.

According to Mermin-Wagner theory, the long-range magnetic order is predicted to be unstable in 2D material and can be easily destroyed by thermal fluctuations (Mermin and Wagner, 1966). Until 2017, the magnetism in the two-dimensional material CrI_3 at the monolayer limit was observed experimentally (Gong et al., 2017; Huang et al., 2017). Hereafter, more 2D magnetic materials have been found, such as Fe_3GeTe_2 (Xian et al., 2022), FePS_3 (Lee et al., 2016) and VSe_2 (Bonilla et al., 2018). Two-dimensional magnetic materials possess a wide variety of excellent physical properties. For instance, monolayer magnetic metal materials have been widely used as electrodes in electronic devices, such as Fe_3GeTe_2 based van der Waals tunnel junctions (O'Hara et al., 2018). Furthermore, magnetic tunnel junction with antiferromagnetic semiconductor CrI_3 tunnel barrier has been reported to possess a giant magnetoresistance effect due to the significant difference of energy band in the ferromagnetic and antiferromagnetic states (Song et al., 2018), which has achieved a huge breakthrough in spintronic devices. Hence antiferromagnetic semiconductor materials have become a hot research topic because of their novel band characteristics. However, such materials are very rare, the prediction of new antiferromagnetic semiconductor materials becomes the key to the development of spintronic devices.

In this paper, the electronic structure and magnetic properties of monolayer MnSi_2N_4 are explored based on first-principles calculations. The results demonstrate that 2D MnSi_2N_4 is a stable antiferromagnetic semiconductor in which the ground state is an antiferromagnetic state. The large magnetic exchange parameter indicates a strong antiferromagnetic coupling between the magnetic Mn atoms. When an external magnetic field is applied, the MnSi_2N_4 monolayer turns into a half-metal with a magnetic state transition from an antiferromagnetic state to a ferromagnetic state. In which the spin-up electronic state displays a metallic nature, while the spin-down electronic state exhibits a semiconducting feature. Therefore, the MnSi_2N_4 monolayer has great application prospects in spintronics and nanosensors.

Computational details

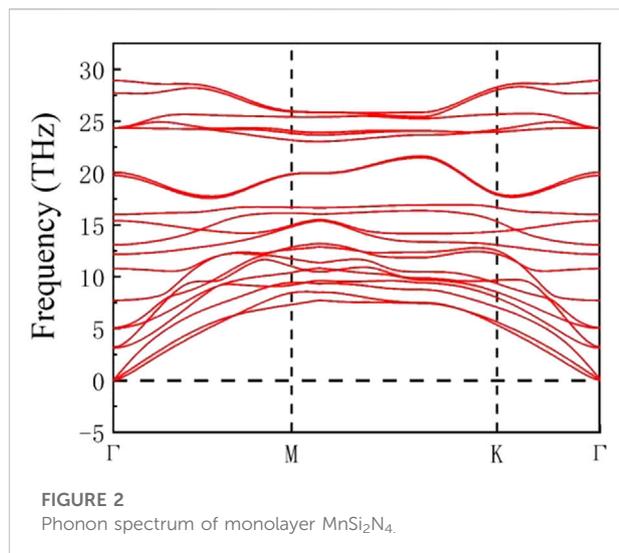
All calculations were conducted using the Vienna *ab initio* simulation package (VASP) (Kresse and Furthmüller, 1996; Kresse and Joubert, 1999). The projection plane wave (PAW) method was adopted to describe the interaction between ions and electrons (Blöchl, 1994). The cutoff energy is set as 500 eV. The generalized gradient approximation (GGA) of the form



Perdew–Burke–Ernzerhof (PBE) was employed to describe the exchange correlation (Perdew et al., 1996). The convergence criteria for electronic iteration and ionic relaxation were 10^{-6} eV and 0.001 eV/Å, respectively. An 18 Å vacuum layer was added in the out plane direction of the monolayer MnSi_2N_4 to eliminate interlayer interactions. The Brillouin zone was sampled with a $13 \times 13 \times 1$ k-point mesh. Due to the strong correlation effect of Mn atoms, the DFT + U method proposed by Dudarev et al. (Dudarev et al., 1998) was adopted, and the effective parameter U_{eff} was set to 3.9 eV (Wang et al., 2006; Jain et al., 2011; Ling and Mizuno, 2012; Togo and Tanaka, 2015). The phonon spectrum of monolayer MnSi_2N_4 was calculated by the PHONONPY software (Togo et al., 2015) using a 5×5 supercell.

Results and discussion

Similar to the two-dimensional MoSi_2N_4 , the monolayer MnSi_2N_4 is a two-dimensional material with a hexagonal lattice structure and D_{3h} point group as shown in Figure 1. MnSi_2N_4 monolayer consists of seven atomic layers stacked with the order N-Si-N-Mn-N-Si-N, which can be regarded as a 1H-phase MnN_2 triple-layer sandwiched between two buckled N-Si layers. The lattice constant of unit cell is 2.88 Å, the bond length between Mn and N atoms is 2.02 Å and the bond length between Si and N atoms is 1.74 Å.



The cohesive energy of the monolayer MnSi₂N₄ was evaluated to confirm the stability of monolayer MnSi₂N₄ using the equation:

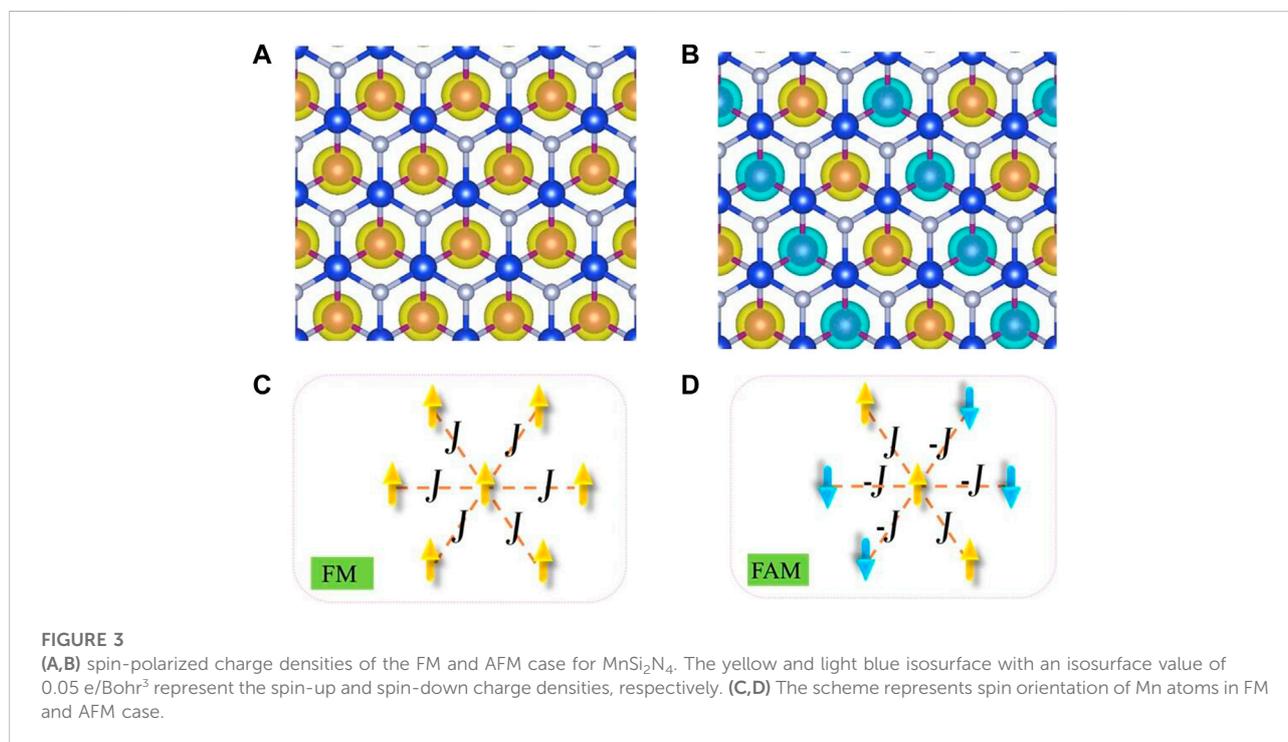
$$E_{\text{coh}} = (E_{\text{MnSi}_2\text{N}_4} - E_{\text{Mn}} - 2E_{\text{Si}} - 4E_{\text{N}})/7 \quad (1)$$

Where $E_{\text{MnSi}_2\text{N}_4}$ represents the energy of MnSi₂N₄, E_{Mn} , E_{Si} and E_{N} represent the energy of isolated single Mn, Si and N atoms, respectively. The calculated results show that the cohesive energy of MnSi₂N₄ is -5.03 eV/atom which is comparable to the value of

MoS₂ monolayer (-5.12 eV/atom) (Canton-Vitoria et al., 2020) and MoSi₂N₄ (-8.46 eV/atom) (Bafekry et al., 2021). We also calculated the phonon spectrum to check the stability, and there is no imaginary phonon frequency throughout the Brillouin zone, indicating that the structure is dynamically stable. Consequently, the MnSi₂N₄ monolayer has excellent stability and thus may be experimentally prepared in Figure 2.

The magnetic properties of monolayer MnSi₂N₄ were investigated. We first determined the ground-state magnetic ordering with two possible magnetic order ferromagnetic (FM) and antiferromagnetic (AFM) states. The total energies of the AFM and FM phases of MnSi₂N₄ are -218.650 eV and -217.658 eV, respectively. The energy of the AFM state is lower than that of the FM state, hence MnSi₂N₄ has an AFM ground state. The AFM order in monolayer MnSi₂N₄ sourced from the superexchange interactions between two magnetic atoms bridged by nonmetal atoms, following the Goodenough-Kanamori rules (Goodenough, 1955; Kanamori, 1959). In this case, the net magnetic moment is zero and the four Mn atoms in the supercell have an antiparallel magnetic state along with the same value of magnetic moments (3.05 μ_B). The spin-polarized charge density and the schematic diagram for FM and AFM order are plotted in Figure 3. The spin-polarized charge density map shows that Mn atoms possess a high spin-polarized charge density, while spin-polarization of N atoms is tiny with small magnetic moments (0.05 μ_B).

The electronic properties of AFM states are investigated to further explore potential applications of MnSi₂N₄. The electronic band structure and density of states (TDOS) are calculated as



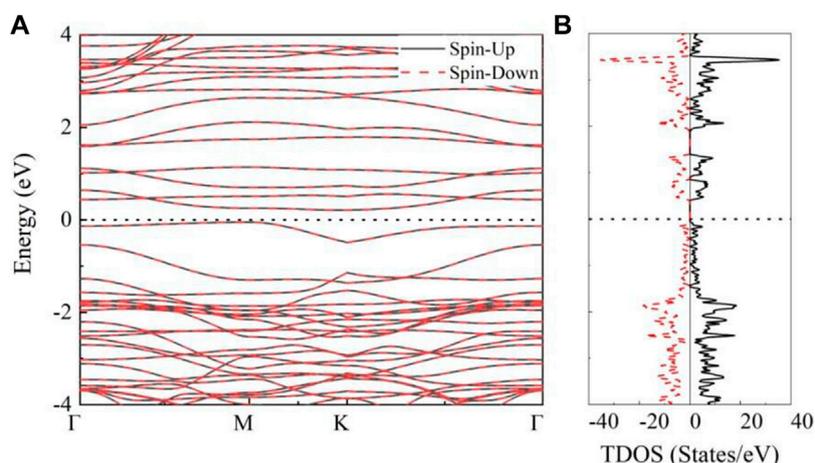


FIGURE 4

(A) The band structures and (B) total density of states (TDOS) of MnSi_2N_4 monolayer in the AFM states, the spin-up states and spin-down states are represented by the black lines and red dotted lines, respectively.

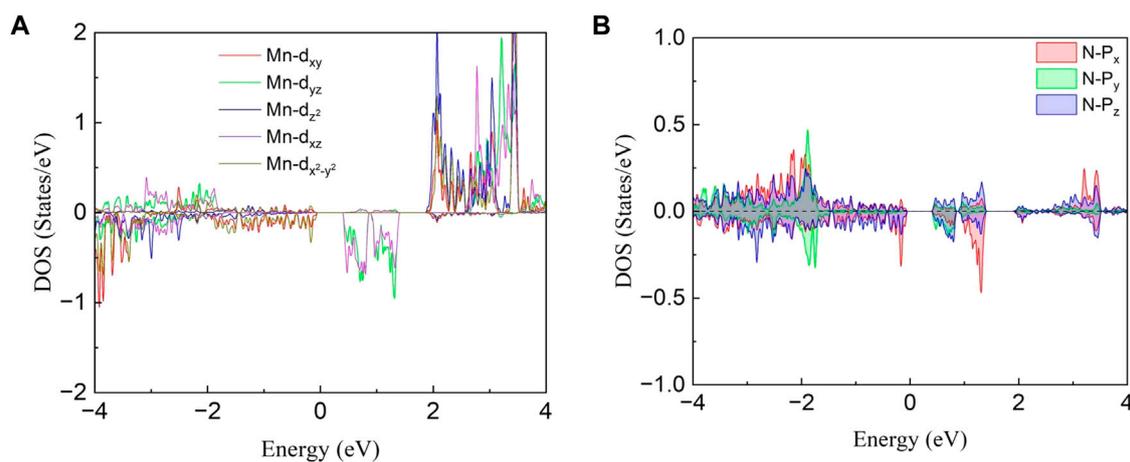


FIGURE 5

Projected density of states (PDOS) for (A) Mn-*d* orbital and (B) N-*p* orbital.

illustrated in Figure 4. It is clear that MnSi_2N_4 exhibits indirect semiconducting property without band cross Fermi level, which is different from nonmagnetic direct bandgap semiconductor MoSi_2N_4 monolayer (Yuan et al., 2022). The conduction band minimum (CBM) and the valence band maximum (VBM) are located at K point and M point, respectively. The band gap is small (0.26 eV). The bands are degenerate and the TDOS is symmetrical for spin-up and spin-down states. Furthermore, no states exist near the Fermi level along with a small energy gap.

The projected density of states (PDOS) for Mn atom and the nearest neighbor N atom are depicted in Figure 5 to better

analyze orbit contribution for electron structure and magnetic properties. One can notice that the density of states for the five 3*d* orbitals are all asymmetric as shown in Figure 5A, indicating the large spin splitting for an isolated Mn atom. The magnetic moment ($3.04 \mu_B/\text{Mn}$) is mainly dominated by the spin-up (majority-spin) states of *d* orbitals which is much more than the spin-down electron. For the N atom, the difference in PDOS between the spin-up and spin-down states is not obvious, resulting in a smaller magnetic moment. In addition, the DOS mainly comes from Mn-*d* and N-*p* in the energy range from 0.5 to 1.5 eV indicating that the hybridizations between the N-*p* and Mn-*d* orbitals are strong.

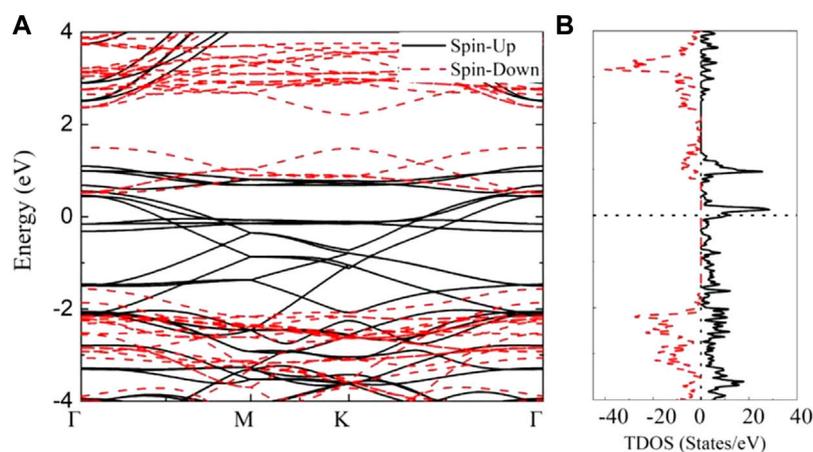


FIGURE 6

(A) The band structures and (B) total density of states (TDOS) of MnSi_2N_4 in the FM states, the spin-up states and spin-down states are represented by the black lines and red dotted lines, respectively.

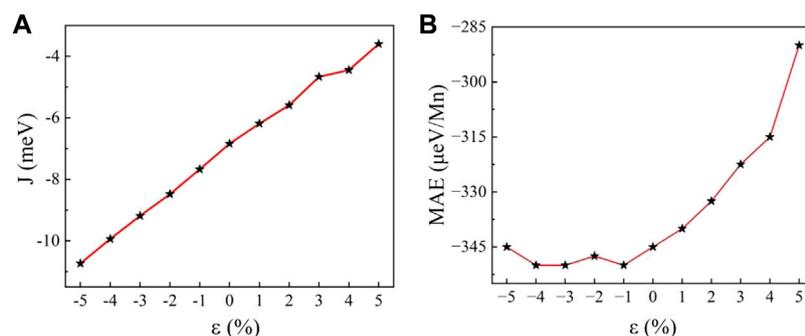


FIGURE 7

(A) Magnetic exchange parameter J and (B) magnetocrystalline anisotropy energy E_{MAE} as a function of strain.

Furthermore, the ground antiferromagnetic states will transition to ferromagnetic states under an external magnetic field. The band structure and density of states for FM state are depicted in Figure 6. It is clear that the spin-up and spin-down energy bands are not degenerate. The spin-polarized states can be noticed around the Fermi level. In the spin-up channel, several flat bands near the Fermi level exist and the others band display dispersion along Γ -M and K- Γ , which behave as a metal. But for the spin-down channel, a direct bandgap of 2.07 eV is observed with the VBM and CBM located at Γ point. Hence, MnSi_2N_4 monolayer behaves half-metallic properties in the FM case. The spin polarization is obvious with an asymmetric density of states distribution for spin-up and spin-down states as depicted in Figure 6B. One can find that a peak of spin-up

states can be seen near the Fermi level and a large bandgap exists in the spin-down states, which further confirms the metallic behavior for the spin-up states and semiconducting property for spin-down states, respectively. Therefore, the 2D MnSi_2N_4 in FM state is a half-metal with 100% spin polarization.

Strain is an effective means of manipulating electronic structure and magnetic properties which is widely utilized to modulate the electronic structure and magnetic properties of monolayer system. In this paper, the strain is defined as $\varepsilon = (a - a_0)/a_0$, where a_0 is the relaxed lattice constant in the equilibrium state. The magnetic moment of the Mn atom remains about $3 \mu_B$ per unit cell under strain. The effective spin Hamiltonian based on the Heisenberg model can be expressed as

$$H = - \sum_{\langle i,j \rangle} J_{ij} \mu_i \mu_j \quad (2)$$

where J_{ij} is the magnetic exchange parameter, and μ_i/μ_j is the magnetic moment at nearest neighbor sites i and j , respectively (Kan et al., 2013). The magnetic exchange parameter J is a significant parameter, which can be evaluated by calculating the total energy of the system in different magnetic states. For the FM case, the total energy can be written as $E_{\text{FM}} = E_0 - 3J|\mu|^2$, where E_0 represents the total energy without spin polarization. For the AFM case, the total energy can be expressed as $E_{\text{AFM}} = E_0 + J|\mu|^2$. Thus, the exchange parameter can be extracted by $J = (E_{\text{AFM}} - E_{\text{FM}})/4|\mu|^2$. According to Figure 7A, although the value of J increases nearly linearly with biaxial strain, the energy difference between AFM case and FM case remains positive over the range of applied biaxial strain, indicating that MnSi_2N_4 behaves as AFM phase and no transition from AFM to FM phase is observed. Furthermore, the magnetic exchange parameter J increases with tensile strain and decreases with compressive strain. According to this trend, an extremely large tensile strain may be needed to turn the AFM to the FM ordering.

To identify the easy axis of MnSi_2N_4 , we computed the magnetic anisotropy energy (MAE). The MAE of the magnetic crystal is defined as $E_{\text{MAE}} = E_{\text{in}} - E_{\text{out}}$ (Webster and Yan, 2018), that is, the energy difference between the in-plane (E_{in}) and out-of-plane (E_{out}) of MnSi_2N_4 . For the strain-free monolayer MnSi_2N_4 , the MAE is $-345 \mu\text{eV}/\text{Mn}$ atom, indicating that the easy axis of MnSi_2N_4 prefers in-plane and the spin of the Mn atoms is arranged parallel to the basal plane. The MAE of monolayer MnSi_2N_4 is mainly derived from Mn atoms since Mn atoms have relatively stronger spin-orbit coupling than other atoms. The MAE is depicted as a function of strain in Figure 7B. When the structure is compressed, this value fluctuates around $-350 \mu\text{eV}/\text{Mn}$, hence the effect of compressive strain on MAE is not obvious. While MAE increases significantly with increasing tensile strain. MAE changes from $-345 \mu\text{eV}/\text{Mn}$ to $-290 \mu\text{eV}/\text{Mn}$ under the 5% tensile strain.

Conclusion

The electronic and magnetic properties of monolayer MnSi_2N_4 are explored based on first-principles calculations. Monolayer MnSi_2N_4 is an intrinsic antiferromagnetic semiconductor with a small indirect band gap (0.26 eV). The MnSi_2N_4 has strong antiferromagnetic coupling along with strong in-plane magnetocrystalline anisotropy energy

($-345 \mu\text{eV}/\text{Mn}$). Furthermore, the MnSi_2N_4 monolayer exhibits half-metallic properties with a metallic spin-up state and a semiconducting spin-down state. The effect of biaxial strain on magnetism is also investigated. The magnetic exchange parameter J and MAE increase with biaxial tensile strain. The tunable magnetic properties may enrich the 2D antiferromagnets community and stimulate potential applications in spintronic devices.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

DC and ZJ: Conceptualization, methodology, software; JY and J-JH: Investigation, Formal analysis, Visualization; JZ and YG: Formal analysis, Data Curation; DC and YT: Writing—original draft preparation; JY: Supervision, Writing—review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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References

- Bafekry, A., Faraji, M., Stampfl, C., Abdolhosseini Sarsari, I., Abdollahzadeh Ziabari, A., Hieu, N. N., et al. (2021). Band-gap engineering, magnetic behavior and Dirac-semimetal character in the MoSi₂N₄ nanoribbon with armchair and zigzag edges. *J. Phys. D. Appl. Phys.* 55 (3), 035301. doi:10.1088/1361-6463/ac2cab
- Blöchl, P. E. (1994). Projector augmented-wave method. *Phys. Rev. B* 50 (24), 17953–17979. doi:10.1103/PhysRevB.50.17953
- Bonilla, M., Kolekar, S., Ma, Y., Diaz, H. C., Kalappattil, V., Das, R., et al. (2018). Strong room-temperature ferromagnetism in VSe₂ monolayers on van der Waals substrates. *Nat. Nanotechnol.* 13, 289–293. doi:10.1038/s41565-018-0063-9
- Canton-Vitoria, R., Sayed-Ahmad-Baraza, Y., Humbert, B., Arenal, R., Ewels, C., and Tagmatarchis, N. (2020). Pyrene coating transition metal disulfides as protection from photooxidation and environmental aging. *Nanomaterials* 10 (2), 363. doi:10.3390/nano10020363
- Dudarev, S. L., Botton, G. A., Savrasov, S. Y., Humphreys, C. J., and Sutton, A. P. (1998). Electron-energy-loss spectra and the structural stability of nickel oxide: An LSDA+U study. *Phys. Rev. B* 57 (3), 1505–1509. doi:10.1103/PhysRevB.57.1505
- Gong, C., Li, L., Li, Z., Ji, H., Stern, A., Xia, Y., et al. (2017). Discovery of intrinsic ferromagnetism in two-dimensional van der Waals crystals. *Nature* 546 (7657), 265–269. doi:10.1038/nature22060
- Goodenough, J. B. (1955). Theory of the role of covalence in the perovskite-type manganites [La, M (II)] MnO₃. *Phys. Rev.* 100 (2), 564–573. doi:10.1103/PhysRev.100.564
- Huang, B., Clark, G., Navarro-Moratalla, E., Klein, D. R., Cheng, R., Seyler, K. L., et al. (2017). Layer-dependent ferromagnetism in a van der Waals crystal down to the monolayer limit. *Nature* 546 (7657), 270–273. doi:10.1038/nature22391
- Hung, N. T., Nugraha, A. R. T., and Saito, R. (2018). Two-dimensional MoS₂ electromechanical actuators. *J. Phys. D. Appl. Phys.* 51 (7), 075306. doi:10.1088/1361-6463/aaa68f
- Jain, A., Hautier, G., Moore, C. J., Ping Ong, S., Fischer, C. C., Mueller, T., et al. (2011). A high-throughput infrastructure for density functional theory calculations. *Comput. Mat. Sci.* 50 (8), 2295–2310. doi:10.1016/j.commatsci.2011.02.023
- Kan, M., Zhou, J., Sun, Q., Kawazoe, Y., and Jena, P. (2013). The intrinsic ferromagnetism in a MnO₂ monolayer. *J. Phys. Chem. Lett.* 4 (20), 3382–3386. doi:10.1021/jz4017848
- Kanamori, J. (1959). Superexchange interaction and symmetry properties of electron orbitals. *J. Phys. Chem. Solids* 10 (2-3), 87–98. doi:10.1016/0022-3697(59)90061-7
- Kresse, G., and Furthmüller, J. (1996). Efficient iterative schemes for *ab initio* total-energy calculations using a plane-wave basis set. *Phys. Rev. B* 54 (16), 11169–11186. doi:10.1103/PhysRevB.54.11169
- Kresse, G., and Joubert, D. (1999). From ultrasoft pseudopotentials to the projector augmented-wave method. *Phys. Rev. B* 59 (3), 1758–1775. doi:10.1103/PhysRevB.59.1758
- Lee, J. U., Lee, S., Ryoo, J. H., Kang, S., Kim, T. Y., Kim, P., et al. (2016). Ising-type magnetic ordering in atomically thin FePS₃. *Nano Lett.* 16 (12), 7433–7438. doi:10.1021/acs.nanolett.6b03052
- Ling, C., and Mizuno, F. (2012). Capture lithium in aMnO₂: Insights from first principles. *Chem. Mat.* 24 (20), 3943–3951. doi:10.1021/cm302347j
- Liu, L., Feng, Y. P., and Shen, Z. X. (2003). Structural and electronic properties of h-BN. *Phys. Rev. B* 68 (10), 104102. doi:10.1103/PhysRevB.68.104102
- Mermin, N. D., and Wagner, H. (1966). Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic Heisenberg models. *Phys. Rev. Lett.* 17 (22), 1133–1136. doi:10.1103/PhysRevLett.17.1133
- Novoselov, K. S., Geim, A. K., Morozov, S. V., Jiang, D., Zhang, Y., Dubonos, S. V., et al. (2004). Electric field effect in atomically thin carbon films. *Science* 306 (5696), 666–669. doi:10.1126/science.1102896
- O'Hara, D. J., Zhu, T., Trout, A. H., Ahmed, A. S., Luo, Y. K., Lee, C. H., et al. (2018). Room temperature intrinsic ferromagnetism in epitaxial manganese selenide films in the monolayer limit. *Nano Lett.* 18 (5), 3125–3131. doi:10.1021/acs.nanolett.8b00683
- Perdew, J. P., Burke, K., and Ernzerhof, M. (1996). Generalized gradient approximation made simple. *Phys. Rev. Lett.* 77 (18), 3865–3868. doi:10.1103/PhysRevLett.77.3865
- Sheng, X. L., Chen, C., Liu, H., Chen, Z., Yu, Z. M., Zhao, Y., et al. (2019). Two-dimensional second-order topological insulator in graphdiyne. *Phys. Rev. Lett.* 123 (25), 256402. doi:10.1103/PhysRevLett.123.256402
- Song, T. C., Cai, X., Tu, M. W. Y., Zhang, X., Huang, B., Wilson, N. P., et al. (2018). Giant tunneling magnetoresistance in spin-filter van der Waals heterostructures. *Science* 360, 1214–1218. doi:10.1126/science.aar4851
- Togo, A., and Tanaka, I. (2015). First principles phonon calculations in materials science. *Scr. Mat.* 108, 1–5. doi:10.1016/j.scriptamat.2015.07.021
- Wang, L., Maxisch, T., and Ceder, G. (2006). Oxidation energies of transition metal oxides within theGGA+Uframework. *Phys. Rev. B* 73 (19), 195107. doi:10.1103/PhysRevB.73.195107
- Webster, L., and Yan, J. A. (2018). Strain-tunable magnetic anisotropy in monolayer CrCl₃, CrBr₃, and CrI₃. *Phys. Rev. B* 98 (14), 144411. doi:10.1103/PhysRevB.98.144411
- Xian, J. J., Wang, C., Nie, J. H., Li, R., Han, M., Lin, J., et al. (2022). Spin mapping of intralayer antiferromagnetism and field-induced spin reorientation in monolayer CrTe₂. *Nat. Commun.* 13, 257. doi:10.1038/s41467-021-27834-z
- Yuan, J., Wei, Q., Sun, M., Yan, X., Cai, Y., Shen, L., et al. (2022). Protected valley states and generation of valley- and spin-polarized current in monolayer MA₂Z₄. *Phys. Rev. B* 105 (19), 195151. doi:10.1103/PhysRevB.105.195151