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On the analytical model for jitter

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Jitter in Internet Protocol (IP) networks refers to the variation in packet arrival times, which can cause delays or disruptions in real-time communications like voice or video calls. Managing jitter is crucial for applications requiring consistent data flow, such as voice over internet protocol, online gaming, and video conferencing, to maintain quality and reliability. Dahmouni et al. derived an analytical model for jitter in an important paper cited by many authors. But some of the fundamental equations in Dahmouni et al. do not appear correct. We correct the equations and study some analytical properties of jitter. Among others, we show that jitter can be expressed as $J = \frac{(\eta+\lambda)^2+\mu^2}{(\eta+\lambda)\mu(\eta+\mu+\lambda)}$, where μ denotes the rate of service times, η denotes the rate of transit times and λ denotes the rate of inter-arrival times. We provide a Maple code to check correctness of our equations.

KEYWORDS

integration, maple, singularity, IP, jitter

1 Introduction

According to Dahmouni et al. (2012a), "IP network planning and design is mostly based on the average delay or loss constraints which can often be easily calculated. Jitter, on the other hand, is much more difficult to evaluate, but it is particularly important to manage the QoS of realtime and interactive services such as VoIP and streaming video". Dahmouni et al. (2012a) derived an important analytical model for jitter. Their results have been so important that their paper has been cited by many authors, see, for example, (Ye et al., 2023; Masli et al., 2022; Chang et al., 2024; Dbira et al., 2016; Dahmouni et al., 2012b; Sreedevi and Rama Rao, 2020; El Amri and Meddeb, 2021; Gore et al., 2022; Husen et al., 2021; Mustapha et al., 2020).

However, some of the fundamental equations (see Equations 8–11 of Dahmouni et al. (2012a)) derived in Dahmouni et al. (2012a) do not appear correct. The aim of this letter is to correct the equations. In the process, we derive an exact expression for jitter when the service times, transit times and inter-arrival times are exponentially distributed (Section 2). We also study some analytical properties of the expression (Section 2). The correctness of our derivation is verified by a Maple code given in Section 3. Some conclusions are provided in Section 4.

2 Analytical derivation

Let S, T and I denote the probability density functions of service, transit and inter-arrival times, respectively. According to Equation 7 of Dahmouni et al. (2012a), the most general formula for jitter is

$$J = \int_0^\infty I(i)\left\{\int_0^\infty S(s)\left[\int_0^i \mid s-x \mid T(x)dx + \mid s-i \mid \int_i^\infty T(x)dx\right]ds\right\}di. \quad (1)$$

Equations 9–11 of Dahmouni et al. (2012a) derive an expression for Equation 1 when $T(x) = \eta \exp(-\eta x)$, $S(s) = \mu \exp(-\mu s)$ and $I(i) = \lambda \exp(-\lambda i)$. Actually, Dahmouni et al. (2012a) stated the following: "Note that this expression (referring to Equation 1) is entirely general. We now assume that the distributions are exponential, namely that

$$T(x) = \eta \exp(-\eta x), \quad S(s) = \mu \exp(-\mu s), \quad I(i) = \lambda \exp(-\lambda i).$$

From these, we can compute Equation 7 and get.

$$J = \eta\mu\lambda(A + B), \tag{2}$$

$$A = \frac{1}{\mu^2}\left(\frac{1}{\lambda\eta} - \frac{1}{\eta\mu}\right), \tag{3}$$

$$B = \frac{1}{\mu^2}\frac{1}{\eta\mu}. \tag{4}$$

from which we get ...". The expression given by Equations 2–4 reduces to $J = \frac{1}{\mu}$.

Proposition 1 in Dahmouni et al. (2012a) supposes that $\eta = \mu - \lambda$, corresponding to an M/M/1 queue, although there is no mention of an M/M/1 queue in Dahmouni et al. (2012a). The condition $\eta = \mu - \lambda$ is mentioned only in Proposition 1 and it is not clear if $\eta = \mu - \lambda$ applies to any result outside of the proposition. The derivations leading to Equations 2–4 do not refer to Proposition 1 and do not appear to suppose any restriction between μ, λ and η. Yet, Equations 2–4 reduces to $J = \frac{1}{\mu}$. We now show that $J = \frac{1}{\mu}$ is incorrect unless $\eta = \mu - \lambda$.

The reminder of this section gives an exact expression for Equation 1 and studies its analytical properties when $T(x) = \eta \exp(-\eta x)$, $S(s) = \mu \exp(-\mu s)$ and $I(i) = \lambda \exp(-\lambda i)$. Note that we can write

$$\begin{aligned}
J =& \int_0^\infty I(i)\int_0^\infty S(s)\int_0^i |s-x|\, T(x)dx \\
&+ \int_0^\infty I(i)\int_0^\infty S(s)\,|s-i|\int_i^\infty T(x)dxdsdi \\
=& \int_0^\infty \lambda\exp(-\lambda i)\int_0^\infty \mu\exp(-\mu s)\int_0^i |s-x|\,\eta\exp(-\eta x)dxdsdi \\
&+ \int_0^\infty \lambda\exp(-\lambda i)\int_0^\infty \mu\exp(-\mu s)\,|s-i|\int_i^\infty \eta\exp(-\eta x)dxdsdi \\
=& \int_0^\infty \lambda\exp(-\lambda i)\int_0^i \mu\exp(-\mu s)\int_0^s (s-x)\eta\exp(-\eta x)dxdsdi \\
&+ \int_0^\infty \lambda\exp(-\lambda i)\int_0^i \mu\exp(-\mu s)\int_s^i (x-s)\eta\exp(-\eta x)dxdsdi \\
&+ \int_0^\infty \lambda\exp(-\lambda i)\int_i^\infty \mu\exp(-\mu s)\int_0^i (s-x)\eta\exp(-\eta x)dxdsdi \\
&+ \int_0^\infty \lambda\exp(-\lambda i)\int_0^\infty \mu\exp(-\mu s)\,|s-i|\,\exp(-\eta i)dsdi \\
=& \lambda\mu\eta\int_0^\infty \exp(-\lambda i)\int_0^i \exp(-\mu s)\int_0^s (s-x)\exp(-\eta x)dxdsdi \\
&+ \lambda\mu\eta\int_0^\infty \exp(-\lambda i)\int_0^i \exp(-\mu s)\int_s^i (x-s)\exp(-\eta x)dxdsdi \\
&+ \lambda\mu\eta\int_0^\infty \exp(-\lambda i)\int_i^\infty \exp(-\mu s)\int_0^i (s-x)\exp(-\eta x)dxdsdi \\
&+ \lambda\mu\int_0^\infty \exp(-\lambda i)\int_0^i \exp(-\mu s)(i-s)\exp(-\eta i)dsdi \\
&+ \lambda\mu\int_0^\infty \exp(-\lambda i)\int_i^\infty \exp(-\mu s)(s-i)\exp(-\eta i)dsdi \\
=& \lambda\mu\eta I_1 + \lambda\mu\eta I_2 + \lambda\mu\eta I_3 + \lambda\mu I_4 + \lambda\mu I_5,
\end{aligned} \tag{5}$$

say. Some simple calculations show that

$$I_1 = \frac{1}{\lambda(\eta\lambda^2 + 2\eta\lambda\mu + \mu^2\eta + \lambda^3 + 3\lambda^2\mu + 3\lambda\mu^2 + \mu^3)}, \tag{6}$$

$$I_2 = \frac{1}{\lambda(\eta^3 + 3\eta^2\lambda + \eta^2\mu + 3\eta\lambda^2 + 2\eta\lambda\mu + \lambda^3 + \lambda^2\mu)}, \tag{7}$$

$$I_3 = \frac{\lambda + 2\mu}{\mu^2(\eta + \lambda + \mu)(\lambda + \mu)^2}, \tag{8}$$

$$I_4 = \frac{1}{(\lambda + \eta)^2(\eta + \lambda + \mu)} \tag{9}$$

and

$$I_5 = \frac{1}{(\eta + \lambda + \mu)\mu^2}. \tag{10}$$

Substituting Equations 6–10 into Equation 5 and simplifying, we obtain

$$J = \frac{(\eta + \lambda)^2 + \mu^2}{(\eta + \lambda)\mu(\eta + \mu + \lambda)}. \tag{11}$$

If $\eta = \mu - \lambda$, corresponding to an M/M/1 queue, then Equation 11 reduces to $J = \frac{1}{\mu}$, the expression given in Dahmouni et al. (2012a). Equation 11 can be useful for general queues not just an M/M/1 queue.

Some properties of Equation 11 are

$$\lim_{\eta\to\infty} J = \frac{1}{\mu},$$

$$\lim_{\lambda\to\infty} J = \frac{1}{\mu},$$

$$\lim_{\mu\to\infty} J = \frac{1}{\eta + \lambda},$$

$$\lim_{\eta\to 0} J = \frac{\lambda^2 + \mu^2}{\lambda\mu(\lambda + \mu)},$$

$$\lim_{\lambda\to 0} J = \frac{\eta^2 + \mu^2}{\eta\mu(\eta + \mu)}$$

and

$$\lim_{\mu\to 0} J = \infty.$$

Let $a = \eta + \lambda$. We can then write Equation 11 as

$$J = \frac{a^2 + \mu^2}{a\mu(a + \mu)}. \tag{12}$$

Note that

$$\lim_{a\to\infty} J = \frac{1}{\mu}$$

and

$$\lim_{a\to 0} J = \infty.$$

The partial derivatives of Equation 12 with respect to a and μ are

$$\frac{\partial J}{\partial a} = \frac{a^2 - 2a\mu - \mu^2}{a^2(a + \mu)^2}, \tag{13}$$

$$\frac{\partial J}{\partial \mu} = \frac{a^2 + 2a\mu - \mu^2}{\mu^2(a + \mu)^2}, \tag{14}$$

$$\frac{\partial^2 J}{\partial a^2} = \frac{-2a^3 + 6a^2\mu + 6a\mu^2 + 2\mu^3}{a^3(\mu + a)^3},$$

$$\frac{\partial^2 J}{\partial \mu^2} = \frac{-2a^3 - 6a^2\mu - 6a\mu^2 + 2\mu^3}{\mu^3(\mu + a)^3}$$

and

$$\frac{\partial^2 J}{\partial a\partial \mu} = -\frac{4}{(\mu + a)^3}.$$

Moreover,

$$\frac{\partial^2 J}{\partial a^2}\frac{\partial^2 J}{\partial \mu^2} - \left(\frac{\partial^2 J}{\partial a\partial \mu}\right)^2 = \frac{4a^6 - 36a^4\mu^2 - 96a^3\mu^3 - 36a^2\mu^4 + 4\mu^6}{a^3(\mu + a)^6\mu^3}.$$

Setting (Equation 13) to zero, we obtain $a = (1 + \sqrt{2})\mu$. Setting (Equation 14) to zero, we obtain $a = (\sqrt{2} - 1)\mu$.

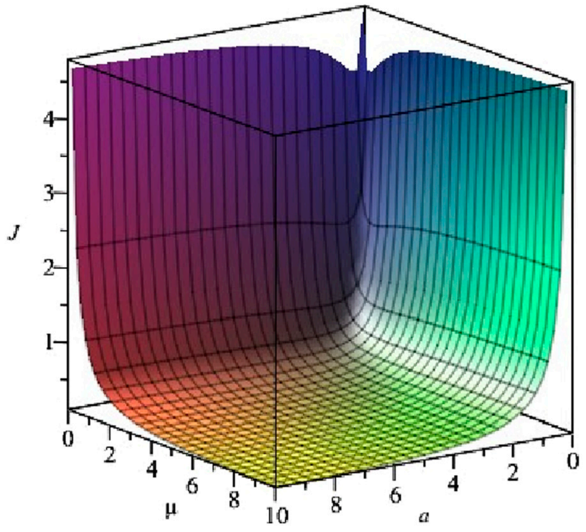

FIGURE 1
Plot of Equation 12 versus a and μ.

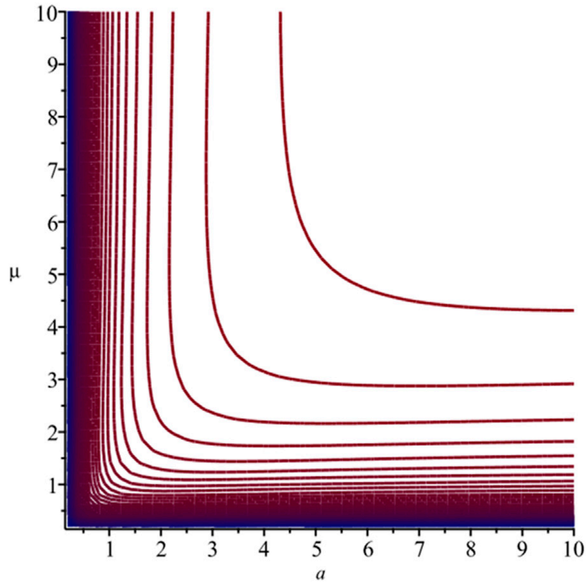

FIGURE 2
Contours of Equation 12 versus a and μ.

Plots of the jitter in Equation 12 versus a and μ are shown in Figures 1, 2. We see that jitter is a decreasing function of a for $a < (1 + \sqrt{2})\mu$, an increasing function of a for $a > (1 + \sqrt{2})\mu$ and reaches a minimum with respect to a when $a = (1 + \sqrt{2})\mu$. The minimum of jitter at $a = (1 + \sqrt{2})\mu$ is $J = \frac{2}{(1+\sqrt{2})\mu}$, which is a decreasing function of μ. We also see that jitter is a decreasing function of μ for $a < (\sqrt{2} - 1)\mu$, an increasing function of μ for $a > (\sqrt{2} - 1)\mu$ and reaches a minimum with respect to μ when $a = (\sqrt{2} - 1)\mu$. The minimum of jitter at $a = (\sqrt{2} - 1)\mu$ is $J = \frac{2}{\mu}$, which is a decreasing function of μ. Further, there is a singularity at $(a, \mu) = (0, 0)$.

3 Derivation using maple

The execution of the following code in Maple gives the same answer as in Equation 11.

```
assume(mu>0);
assume(lambda>0);
assume(eta>0);
t1:=int(mu*exp(-mu*s)*abs(s-y)*exp(-
eta*y), s = 0 ... infinity);
tt1:=int(t1*lambda*exp(-lambda*y), y =
0 ... infinity);
t2:=int(abs(s-x)*eta*exp(-eta*x), x=0...y);
t3:=int(mu*exp(-mu*s)*t2, s = 0 ... infinity);
t4:=int(lambda*exp(-lambda*y)*t3, y =
0 ... infinity);
final:=simplify(t4+tt1);
```

4 Conclusion

We have derived an exact expression for jitter when the service times, transit times and inter-arrival times are exponentially distributed. The correctness of this expression has been checked by a Maple code.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

SN: Writing – review and editing, Writing – original draft. AB: Writing – original draft, Writing – review and editing.

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Conflict of interest

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