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# Bar load-displacement curve model based on statistical damage mechanics

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The load displacement curve model of bolt is of great significance to estimate the ultimate uplift bearing capacity of bolt and analyze the force transmission of bolt. Therefore, it is particularly necessary to establish a high-precision load displacement curve model of bolt. In this paper, the statistical damage theory is introduced to establish a new stiffness degradation model of bolt load displacement curve, and a model which is more consistent with the test data and has higher accuracy in predicting the ultimate uplift capacity is proposed. The influence of model parameters on the model curve is analyzed. It can be concluded that the key to the load displacement curve of the anchor and the prediction of the uplift bearing capacity of the anchor is to determine the statistical random distribution characteristics of the micro element strength of the anchor interface.

## Introduction

Bolt support is widely used in foundation pit, slope, tunnel and roadway engineering. The ultimate uplift bearing capacity and stress characteristics of bolt have always been an important issue concerned by scholars and engineering designers (Li, 1999; Li et al., 2012; Wang et al., 2019; Wang et al., 2021a; Wang et al., 2021b; Zhang et al., 2022). The ultimate pullout bearing capacity of the bolt is usually determined by the pull-out test of the bolt. In order to study the ultimate pullout bearing capacity of bolt, some scholars analyze the ultimate bearing capacity of bolt from the perspective of limit equilibrium and limit analysis (Serrano et al., 2000; As and Co, 2002; Serrano and Olalla, 2002; Wang et al., 2013). Some scholars estimate the ultimate bearing capacity of the anchor rod from the load displacement curve of the anchor rod pull-out test. This method can not only estimate the bearing capacity of the anchor rod, but also describe the load displacement curve, which lays a foundation for further analysis of the stress distribution of the anchor

rod along the rod length (Huang et al., 2014; Zou and Zhang, 2019). The load displacement curve was first used in the estimation of the ultimate bearing capacity of the pile.

Van er veen C (1953) described the load displacement curve of the pile with an exponential function model and predicted the ultimate bearing capacity of the pile. Xu et al. (2000) described the load displacement curve of uplift pile with power function model and predicted the ultimate bearing capacity. Deng and Lu (2002) described the pile load displacement curve with the adjusted hyperbolic model and exponential model, and predicted the ultimate bearing capacity of the pile. Jiang et al. (2009) uses Weibull mathematical model ($y = a - be^{-(cx)^d}$) to describe the load displacement curve of PHC pile, which is better than exponential function and hyperbolic function. However, when the model is applied to the load displacement curve of some anchor bolts, the accuracy of predicting the ultimate uplift bearing capacity is not high, and the improved physical and mechanical mechanism is not clear. Li (2009) believed that the load displacement curves of pile and anchor rod are similar to each other. Ying et al. (2005) applied the exponential model to the load displacement curve of anchor rod. Zhang et al. (2005) described the load displacement curve of the anchor rod through the double broken line relationship, and obtained the expression of the ultimate bearing capacity of the anchor rod through the load transfer analytical method. However, the expression is complex, there are many parameters and the prediction accuracy is low. Sun et al. (2015) proposed a power exponential mixed form bolt load displacement curve model by using the general function model framework, and applied it to two forms of bolt load displacement curve, reflecting the advantages of the improved model in terms of accuracy. However, the improvement of this method is also based on pure mathematical method, which cannot be analyzed from the perspective of mechanical principle.

Based on the exponential function of bolt load displacement curve proposed in reference (Ying et al., 2005), a modified exponential function model is proposed by introducing statistical damage mechanics (Rinaldi et al., 2007; Chen, 2020; Xie et al., 2020). By analyzing the deterioration law of the pullout stiffness of the exponential function model, it is found that the pullout stiffness of the bolt in the model decreases exponentially with the displacement. This law essentially reflects the assumption that the micro element strength of the anchorage interface follows a certain random exponential distribution, but the exponential distribution is not accurate enough to describe the random distribution of micro element strength of the anchorage interface. Therefore, the prediction of the ultimate pullout resistance of the anchor bolt is inaccurate and the deviation between the fitting curve and the test data points is large. In view of this, a modified exponential function bolt load displacement curve model is proposed by using the higher-order statistical distribution of exponential distribution modified Weibull distribution (Lai

et al., 2003) and combined with the principle of statistical damage mechanics, and the accuracy of the modified model is verified by six groups of pull-out test data.

## Stiffness deterioration law of exponential function model

According to the principle of damage mechanics, accurately describing the change law of nonlinear curve stiffness is an important premise for establishing a more accurate model. Therefore, the key to improve the accuracy of constitutive curve model is to accurately describe and predict the deterioration and change law of curve stiffness. Based on this principle, the load displacement curve model of exponential function bolt is improved. The model built in this paper applies to anchor bar is pulled out and anchor solid is pulled out these two cases.

Ying et al. (2005) proposed an exponential function model for describing the load displacement curve of anchor rod and predicting the ultimate uplift bearing capacity. The expression is as follows.

$$P = P_u \left[ 1 - \exp(-as) \right] \quad (1)$$

Where, $P$ is the bolt load, $P_u$ is the ultimate pullout bearing capacity of the bolt, $a$ is the attenuation coefficient (mm-1), and $s$ is the shear displacement of the bolt under pullout.

The slope of the curve is defined as the uplift stiffness of the anchor rod, hereinafter referred to as the stiffness. The expression of uplift stiffness in the load displacement curve of anchor bolt can be obtained from Eq. 1 as follows.

$$Q(s) = \frac{dP}{ds} = P_u a \exp(-as) \quad (2)$$

From Eq. 2, it can be known that the ratio of stiffness and initial stiffness under a certain displacement is:

$$H = \frac{Q(s)}{Q(s=0)} = \frac{P_u a \exp(-as)}{P_u a} = \exp(-as) \quad (3)$$

Where, the ratio ($H$) of stiffness and initial stiffness under a certain displacement is a dimensionless parameter, which is hereinafter referred to as stiffness deterioration coefficient. It can be seen from the above formula that the stiffness degradation law is described as a degradation law in the form of exponential function.
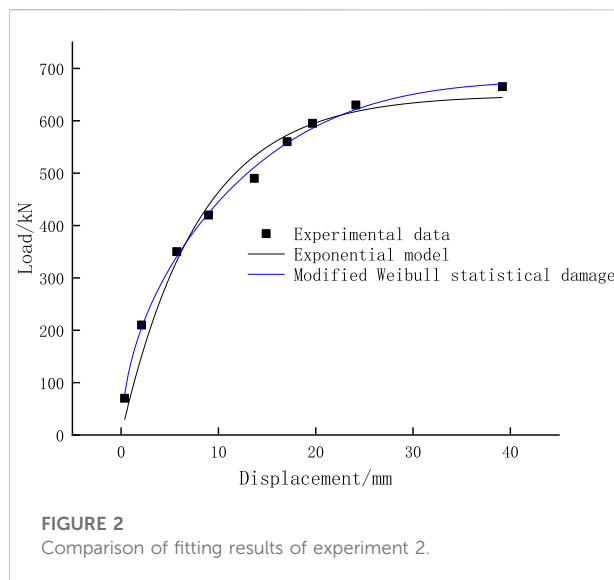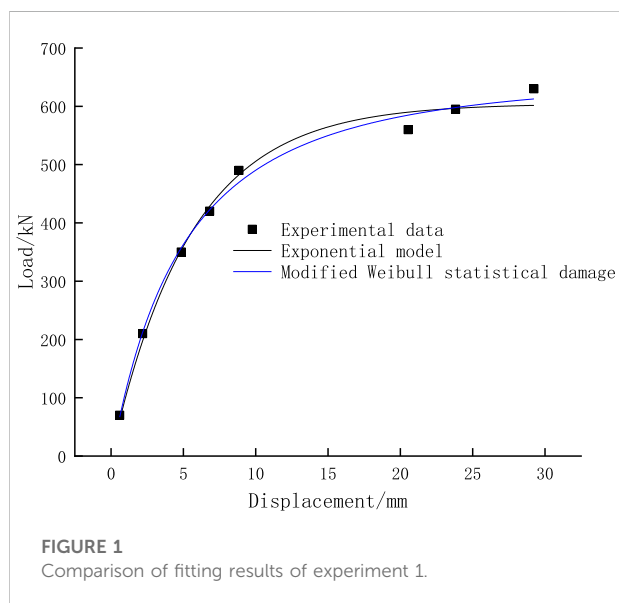
## Stiffness change correction method based on statistical damage mechanics

According to the principle of strain equivalence (Voyiadjis and Kattan, 2016; Xu et al., 2017; Chen et al., 2018a; Chen et al., 2018b), the damage factor can generally be defined as:

**TABLE 1 Overview of cite experiment.**

| Name | Project | Anchor diameter/mm | Anchor bar | Type of anchoring agent | Stratum |
|---|---|---|---|---|---|
| Experiment 1[(Sun et al., 2015)] | Anti floating anchor rod of a club in Shenzhen A1 | 150 | 2Φ32 Screw thread steel | cement mortar | Gravel sand cohesive soil and medium coarse sand mixed clay |
| Experiment 2[(Sun et al., 2015)] | Anti floating anchor rod of a club in Shenzhen A2 | 150 | 2Φ32 Screw thread steel | cement mortar | Gravel sand cohesive soil and medium coarse sand mixed clay |
| Experiment 3[(Wang and Guo, 2016)] | Indoor simulation experiment | - | 30 mm Diameter anchor bar | cement mortar | - |
| Experiment 4[(Liu et al., 2011)] | Rock bolt foundation in a mountainous area | 150 | 42 mm Diameter reinforced anchor rod | cement mortar/Fine aggregate concrete | Rock stratum |
| Experiment 5[(Zhang and Tang, 2002)] | Grouting anchor rod for foundation pit of a high-rise building M3 | 130 | 2Φ32 Threaded bar anchor | cement mortar | The anchorage section is hard clay |
| Experiment 6[(Jia et al., 2017)] | Indoor simulation experiment | 75 | Unthreaded anchor bar | cement mortar | Hard rock stratum simulated by rough steel pipe |

Note: "-" in the Table indicates the situation not described in the cited literature.



FIGURE 1
Comparison of fitting results of experiment 1.



FIGURE 2
Comparison of fitting results of experiment 2.

$$D = 1 - \frac{Q(s)}{Q(s=0)} \qquad (4)$$

From the above formulas, the load displacement curve expression of anchor rod with damage factor can be obtained.
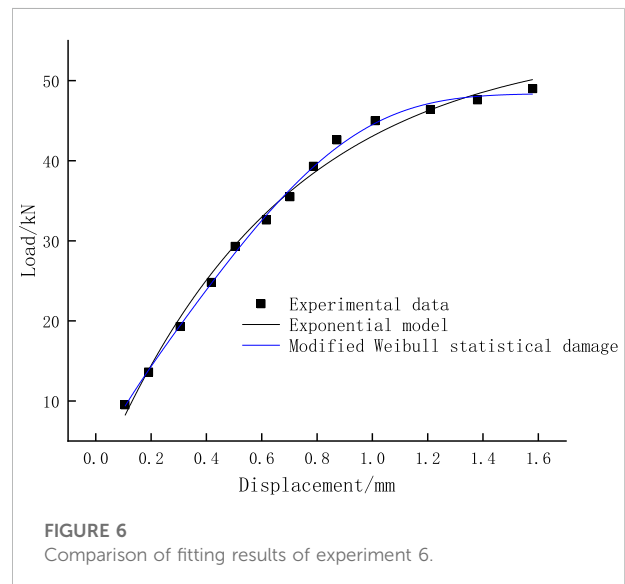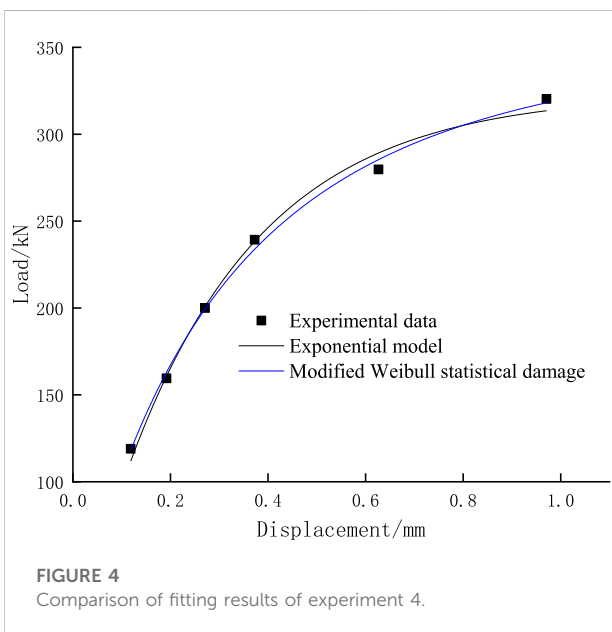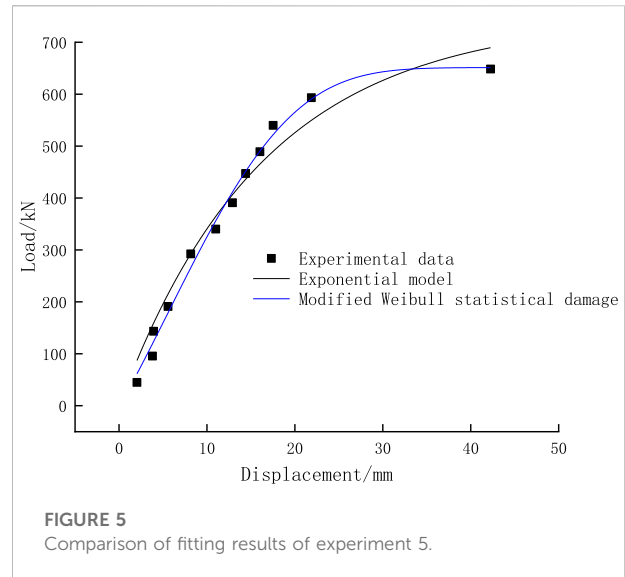
$$P = P_u D = P_u (1 - H) \qquad (5)$$

Statistical damage mechanics considers the heterogeneity of the medium from the microscopic point of view, and considers that the micro damage of the medium is caused by the failure of the internal micro elements after reaching a certain strength. Therefore, statistical damage mechanics is used to reflect the essential law of curve stiffness degradation.

It is assumed that the micro element strength random distribution characteristics of the anchor bolt anchorage interface obey the modified Weibull distribution (Sun et al., 2015), and the displacement is taken as the strength criterion parameter.

Firstly, the probability density function expression of the modified Weibull function is as follows.

$$f(x) = a_w x^{b_w-1} (b_w + c_w x) \exp\left[c_w x - a_w x^{b_w} \exp(c_w x)\right] \qquad (6)$$

Where, $a_w$, $b_w$, $c_w$ are the fitting parameters describing the statistical distribution form respectively. The modified Weibull statistical distribution is more accurate than ordinary Weibull in describing material life estimation. It is

FIGURE 3
Comparison of fitting results of experiment 3.



FIGURE 5
Comparison of fitting results of experiment 5.



FIGURE 4
Comparison of fitting results of experiment 4.



FIGURE 6
Comparison of fitting results of experiment 6.
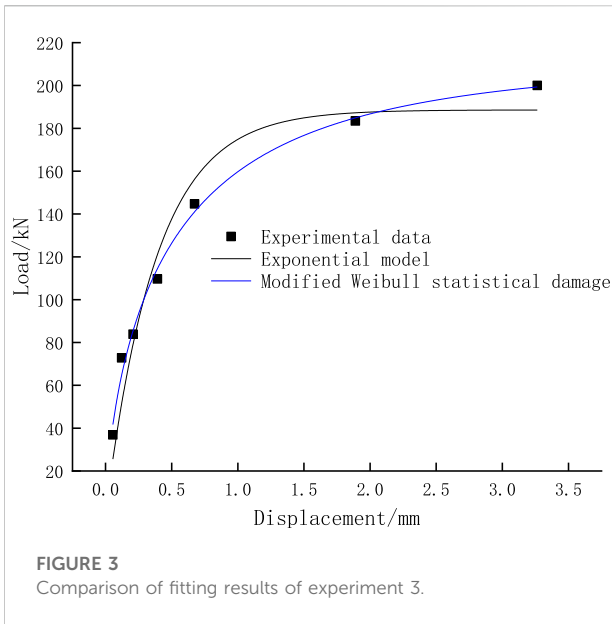
also one of the common Weibull statistical distributions in the field of Applied Mathematics. In order to more accurately describe the deterioration characteristics of stiffness, it is assumed that the random distribution characteristics of micro element strength at the anchorage interface obey the modified Weibull distribution. When $c_w = 0$, the modified Weibull distribution degenerated to the ordinary Weibull distribution; When $c_w = 0$ and $b_w = 1$, the modified Weibull distribution degenerates to a negative exponential distribution.

Assuming that the failure occurs when the shear displacement of the interface micro element reaches a certain value, the number of failure of the anchor bolt anchoring interface micro element under a certain displacement can be expressed by modifying the probability distribution density function of Weibull statistical distribution:

$$n = \int_0^u Nf(u)du = \int_0^u Na_w u^{b_w-1}(b_w + c_w u)\exp\left[c_w u - a_w u^{b_w}\exp(c_w u)\right]du$$
$$= N - N\exp\left\{-a_w u^{b_w}\exp(c_w u)\right\}$$
(7)

Where, $n$ is the number of damaged micro elements, $N$ is the total number of micro elements, where the dimension ($a_w$) is

TABLE 2 Summary of parameters of each experiment model.

| Model name | Model parameters of experiment 1 | Model parameters of experiment 2 | Model parameters of experiment 3 | Model parameters of experiment 4 | Model parameters of experiment 5 | Model parameters of experiment 6 |
|---|---|---|---|---|---|---|
| Index | $Pu$=604.68 $a$=0.181 | $Pu$=649.11 $a$=0.13 | $Pu$=188.59 $a$=2.62 | $Pu$=323.55 $a$=3.58 | $Pu$=746.40 $a$=0.06096 | $Pu$=55.16 $a$=1.52 |
| Modified Weibull | $Pu$=645.41 $a$=0.19 $b$=0.95 $c$=-0.02 | $Pu$=673.63 $a$=0.22 $b$=0.57 $c$=0.03 | $Pu$=217.66 $a$=1.41 $b$=0.65 $c$=-0.07 | $Pu$=373.43 $a$=2.74 $b$=0.91 $c$=-0.34 | $Pu$=651.27 $a$=0.04371 $b$=1.05 $c$=0.035 | $Pu$=48.37 $a$=0.69 $b$=0.58 $c$=1.30 |
| Actual ultimate bearing capacity/kN | 665.5 | 683 | 219 | 360.8 | 648.3 | 50.7 |

mm$^{-1}$, and the dimension ($c_w$) is mm$^{-1}$, and $b_w$ is a dimensionless parameter.

The damage factor is defined as by the proportion of the number of damaged micro elements to the total number of micro elements (Chen et al., 2018b):

$$D = \frac{n}{N} \tag{8}$$

By combining the above two formulas

$$D = 1 - \exp\left\{ -a_w u^{b_w} \exp(c_w u) \right\} \tag{9}$$

By substituting Eq. 9 into Eq. 5, the modified load displacement curve expression of anchor rod is as follows:

$$P = P_u D = P_u \left[ 1 - \exp\left\{ -a_w u^{b_w} \exp(c_w u) \right\} \right] \tag{10}$$

## Engineering case verification and parameter analysis

The bolt pull-out test in reference (Zhang and Tang, 2002; Rinaldi et al., 2007; Liu et al., 2011; Wang and Guo, 2016; Jia et al., 2017) is recorded as experiment 1~experiment 6 respectively. The overview of each experiment is shown in Table 1.

Figures 1–6, it can be seen that the modified model in this paper is more consistent with the experimental data than the exponential function model. The modified model is more accurate in describing the random distribution characteristics of micro element strength at the anchorage interface, and can more accurately represent the law of the deterioration of bolt pullout stiffness with displacement, so it can be more consistent with the experiment data of load displacement curve in pullout experiment. It can be seen from Table 2; Figure 7 that in terms of the accuracy of predicting the ultimate bearing capacity, the absolute error range of the modified Weibull model is 1.34–20.9 kN, and the relative error rate is 0.456%–4.82%. The absolute error range of the exponential function model is 4.46–98.1 kN, and the relative error rate is 5.22%–16.1%. The error of the modified model is basically within 5%, which is obviously better than the exponential function model. This is because the modified model can more accurately reflect the macro deterioration law of bolt pullout stiffness, so as to predict the load (ultimate pullout capacity) under the "asymptote" limit state more accurately.

Taking the model parameters of the experiment as an example, the control variable method is used for parameter analysis. The influence of each parameter is shown in Figure 8.

It can be seen from Figure 8 that when the ultimate pullout force of anchor bolt is certain, the three model

FIGURE 7
Absolute error and relative error.
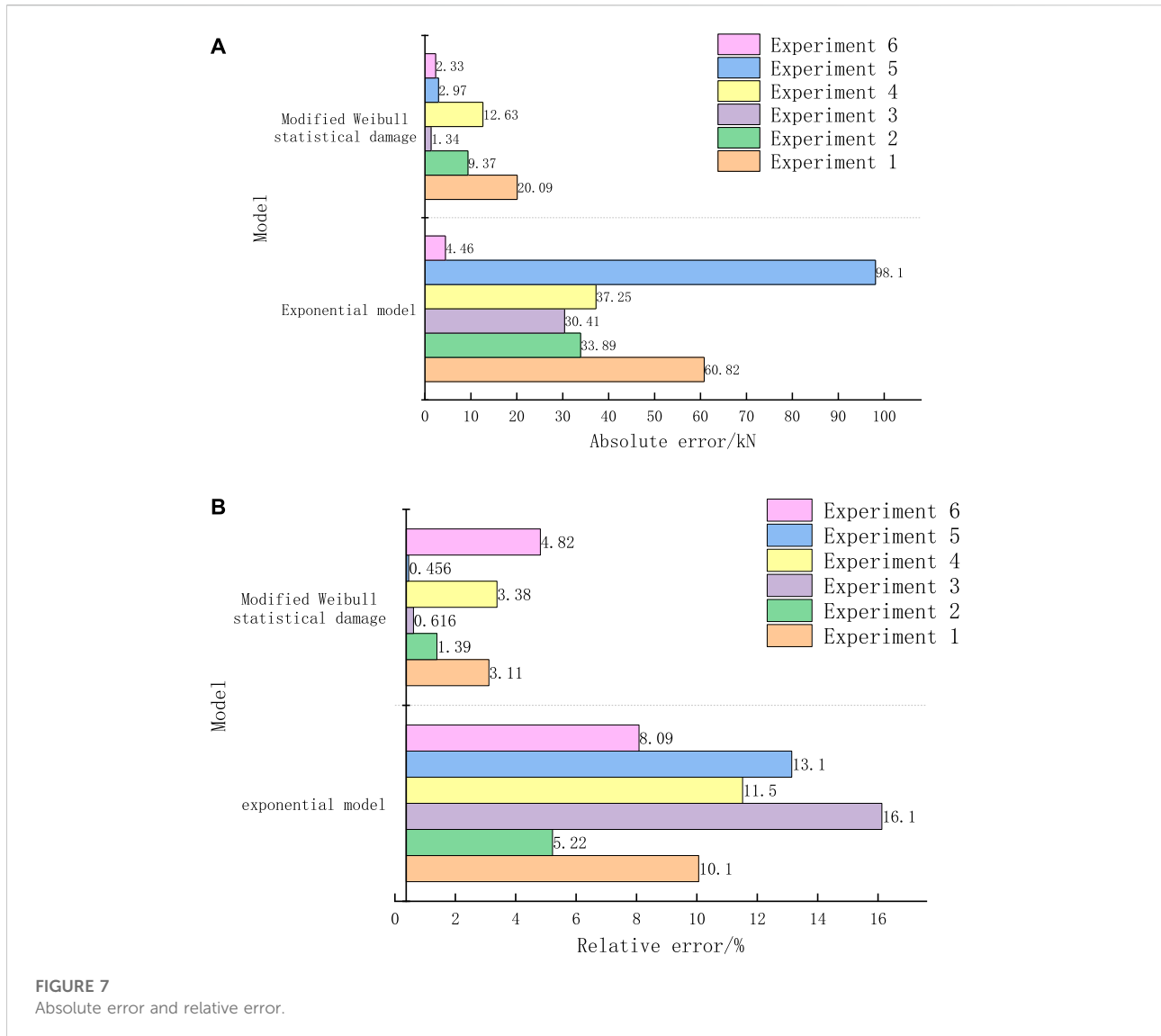
parameters have a great impact on the slope of load displacement curve. Here, it is proved that it is important to accurately describe the slope change of load displacement curve. With the increase of the value $a$, the rate of the load reaching the ultimate uplift load increases with the increase of displacement, and the trend of the damage evolution curve reaching the maximum damage is also synchronized with it. With the increase of the value $b$, the rate of load reaching the ultimate uplift load increases with the increase of displacement, and the trend of damage evolution curve reaching the maximum value of damage is also synchronized with it. Parameters $a$ and $b$ are important parameters that affect the micro element strength

distribution at the anchorage interface and the change rate of anchor pullout stiffness. With the decrease of the value $C$, the tail slope of load displacement curve and damage evolution curve decreases to a certain extent. At this time, the damage evolution rate decreases, which is mainly reflected in the slowing down of the degradation rate of bolt pullout stiffness. Parameters $a$, $b$ and $c$ make the bolt load displacement curve adapt to the more complex variation law of uplift stiffness. In the meso upper layer, the three parameters can more accurately describe the random distribution characteristics of micro element strength at the anchorage interface, reflecting the meso damage process of the bolt.
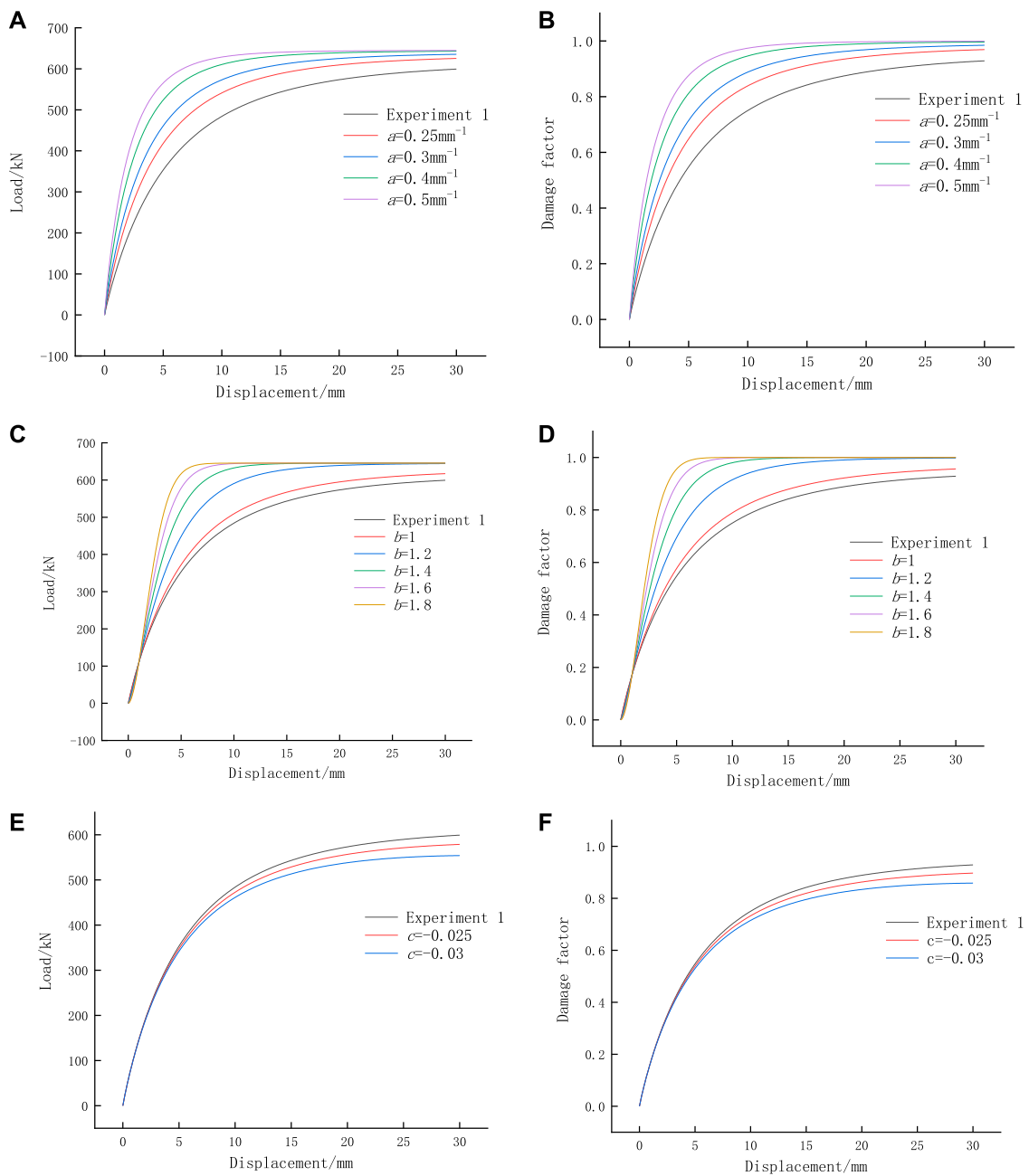
**FIGURE 8**
Influence of model parameters.

## Conclusion

(1) Based on the principle of statistical damage mechanics, a correction method of load displacement curve is proposed in this paper. Through the comparison of cases and experiments, the modified model can more accurately describe the load displacement curve of anchor rod and predict the ultimate uplift bearing

capacity of anchor rod, which has certain engineering application value.

(2) Through the damage mechanics analysis, the random distribution characteristics of micro element strength at the anchorage interface are assumed to be the modified Weibull distribution, which better optimizes the accuracy of the function model method in estimating the uplift bearing capacity of anchor bolts.

(3) Through parameter analysis, it is found that the modified Weibull statistical damage mechanics method can better adapt to the complex load displacement curve. Compared with the damage evolution curve, describing the damage characteristics of the anchorage interface can well characterize the stress change process of the anchor.

## Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

## Author contributions

ZY and SC: Writing of the whole manuscript and data processing. SC: Technical guidance for the entire manuscript and the production of tables. JS: Some drawing and table making and manuscript typesetting. YZ: Language editing and polishing of the entire manuscript. LL: Draw some Figures in the manuscript and data processing. YY: Guide the revision and verification of the manuscript.

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## Conflict of interest

Author ZY was employed by Guangdong Hualu Transport Technology Co., Ltd.

The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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