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Energy evolution and damage constitutive model for deep marble under triaxial compression

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Introduction: High ground stress in deep mining operations results in rocks exhibiting mechanical properties that differ from those at shallow depths. This study conducted conventional triaxial compression (CTC) tests to elucidate themechanical behaviors of deep marble under CTC conditions.

Methods: Initially, it analyzed energy evolution of deep marble under CTC conditions and developed a damage variable (*D*) expression. Subsequently, we examined impacts of confining pressure (σ_3) and *D* on dissipated energy (W_d) and Poisson's ratio (μ') and proposed an expression for damage energy dissipation rate (*U*). Finally, based on the first law of thermodynamics, we established a differential equation for energy balance that facilitated the development of a damage constitutive model (DCM) for marble under CTC conditions. This model effectively couples the internal energy and damage within the rock.

Results: The results indicated that: (1) the maximum W_d increased with increasing σ_3 , and the W_d experienced stages of stability, slow growth, steady growth, and slowdown. (2) At a constant σ_3 , the μ' under loading conditions increased with increasing damage level. However, as rock damage intensified in the residual stage, the growth rate of μ' slowed and subsequently stabilized. (3) The proposed model effectively captures the nonlinear characteristics of stress-strain (SS) curves for deep marble under CTC conditions during the elastic, plastic yield, instability failure, and residual stages.

Discussion: This research offers theoretical insights for stability analysis in deep mining activities.

KEYWORDS

rock mechanics, energy dissipation, triaxial compression, damage variable, constitutive model

1 Introduction

As shallow mineral resources are being exhaustively mined, mining operations worldwide are shifting to deeper levels (An and Mu, 2024; Chen et al., 2025). The effects of high ground stress, high ground temperatures, high osmotic pressure, and intense mining disturbances are becoming more pronounced. High stress can deteriorate the mechanical properties of rocks. Therefore, developing an appropriate constitutive model is essential for

understanding their mechanical behavior under complex stress conditions (Zheng Z. et al., 2023; Hu et al., 2025; Bian et al., 2024; Guo and Li, 2023; Feng et al., 2024).Thereby establishing a critical theoretical foundation for stability assessment of surrounding rock in deep underground engineering, geohazard prediction, and multiphysics-coupled numerical simulations.

A damage variable that measures the degradation of the rock's mechanical properties needs to be introduced for constructing a rock DCM (Gu et al., 2025). Subsequently, the damage variable is integrated into the rock's constitutive relations to characterize its mechanical behavior (Li WY. et al., 2023; Wen et al., 2022; Chajed and Singh, 2024; Du et al., 2024; Lu et al., 2024). Research on rock DCMs has yielded considerable outcomes globally. Numerous scholars have recognized the challenge of directly measuring damage variables and have opted to define them using externally measurable parameters. Pan et al. (2022) characterized the rock damage via the phenomenological theory. They developed a DCM for the rock sample containing a single crack under uniaxial compression by combining Weibull distribution with fracture mechanics. Cheng et al. (2025a) proposed a rock statistical damage constitutive model based on quantitative energy conversion Based on CT scanning and digital imaging technique, Wu et al. (2022) proposed a new damage variable using the fracture area projection method to develop an innovative DCM. However, these damage definition methods heavily rely on technological measurements, whose accuracy significantly influences damage calculations, thereby limiting the applicability of the models. Consequently, some researchers have incorporated theories from plastic mechanics, fracture mechanics, and statistics into damage evolution models. Chen et al. (2019) introduced a thermal damage variable to develop a statistical DCM. Zheng F. et al. (2023) introduced the Weibull distribution along with temperature, cushion, and load-coupled damage variables to establish a statistical DCM for high-temperature layered rocks. Given that the damage and failure processes in deep rocks involve continuous energy transformation and dissipation (Yang Jian et al., 2024; Yang K. et al., 2024; Li YQ. et al., 2023; Wang et al., 2023)[,] defining damage variables from an energy perspective and developing constitutive models can fundamentally elucidate the mechanisms of rock damage evolution (Yang J. et al., 2024; Li and Guo, 2023; Lv et al., 2024). Significant advancements have also been made in deriving constitutive models based on energy considerations. Fu et al. (2025) defined a damage variable for layered rocks based on the energy dissipation principle and proposed a DCM. Likewise, Li YB. et al. (2023) and Luo and Wang (2023) developed a novel DCM for rocks under triaxial compression by integrating energy conversion principles with the mechanical properties of rocks. Nevertheless, shortcomings still exist in constitutive models developed from an energy perspective. For example, Cheng et al. (2025b) and Wang et al. (2022); Cheng et al. (2025c) only introduced the concept of energy without establishing a direct mathematical relationship between damage variables and energy. Few studies have directly solved energy differential equations to develop constitutive models.

Based on the theoretical framework of fundamental thermodynamic laws, this study establishes the quantitative coupling relationship between damage evolution and energy

dissipation by dynamically embedding the damage variable D into the thermodynamic equilibrium equations under the constraint of energy conservation principles. This approach not only elucidates the intrinsic mechanisms of progressive rock failure from the perspective of energy-driven dynamics but also achieves rigorous mathematical derivation of the DCM. First, the energy evolution of deep marble under CTC conditions was analyzed using experimental data, and a damage variable (D) expression was proposed. Subsequently, the impacts of σ_3 and D on W_d and μ were examined, and a mathematical expression for the U was developed. Finally, the deep marble system was treated as a thermodynamic system, and a differential equation for energy balance was established based on the principle of energy conservation under static assumptions. The coupled equations were solved to determine rock damage, strain, and stress. resulting in the development of a DCM for rocks under CTC conditions. This model was validated using experimental data. Research on rock constitutive models provides critical theoretical support for stability analysis of surrounding rock in deep engineering projects, geohazard prediction, and multiphysicscoupled numerical simulations, while advancing the transition of rock mechanics from empirical estimation to multiscale precision prediction.

2 Experiment preparation and procedure

2.1 Sample preparation

The marble was sourced from Liyang, Hunan Province, China. The sample composition includes the percentage of calcite (CaCO₃) content (98%), dolomite (CaMg (CO₃) ₂) content (1.5%), and trace quartz (SiO2). It has a fine and smooth texture with uniformly small particles. The marble was processed into cylindrical samples measuring 50 mm in diameter and 100 mm in height with a parallelism tolerance of $\pm 0.3\%$. Samples meeting the experimental accuracy were selected using wave speed testing and density calculations, as illustrated in Figure 1.

2.2 Experimental procedure

The testing equipment employed the MTS815 triaxial rock testing system at Central South University, as depicted in Figure 2. Based on the deep geological environment where the marble is located, the confining pressure levels for the conventional triaxial tests were set at 10 MPa, 20 MPa, 30 MPa, 40 MPa, and 50 MPa, respectively.

The experimental procedure is outlined as follows:.

- (1) The $\sigma_2 = \sigma_3$ was gradually increased to 10 MPa, 20 MPa, 30 MPa, 40 MPa, and 50 MPa, respectively, at a loading rate of 0.1 MPa/s in a stress-controlled manner under hydrostatic pressure conditions.
- (2) σ₃ was maintained for 10 s. Subsequently, the axial pressure was gradually increased until the rock sample failed. The stress path is presented in Figure 3.



The SS curves for rock samples under various σ_3 are presented in Figure 4a, where the x-axis represents axial strain (ε_1) and the y-axis indicates axial stress (σ_1). As shown in Figure 4b, the microscopic characteristics of the rock samples when the peripheral pressure of the marble is 30 MPa, when the rock samples are damaged. The main rupture surface is dominated by along-crystal fracture, the angularity of the rupture surface is relatively sharp, high and low, in the shape of a step, the crystal fracture forms a confluence, the tendency of the fracture is randomly distributed without an obvious pattern, and under high magnification, it can be seen that the rupture surface has an obvious shear damage feature, and locally there is a tensile rupture surface developed at the same time, which exhibits a shear and tensile composite damage feature.

3 Energy evolution of samples under CTC conditions

3.1 Energy calculation based on experimental data

The generation and propagation of microcracks in rock consume energy. Thus, rock deformation involves complex energy transformations. The amount of energy dissipated reflects the degree of reduction in the rock's original strength. As depicted in Figure 5, the areas enclosed by OABCDE, OABCN, CMN, CDEM, and PQM represent the total work done by external forces (*W*), the dissipated energy before the peak, the stored elastic strain energy before the peak, the dissipated energy after the peak, and the residual strain energy after the peak, respectively.

According to the energy conservation principle, the rock sample can be considered a closed system where the W is equal to the sum of the elastic energy stored in the rock and the total energy released (Yan et al., 2023; Feng et al., 2023), as expressed in Equation 1:

$$W = W_e + W_d \tag{1}$$

Where W represents the total work done by external forces on the rock sample under any strain ε , W_e corresponds to the recoverable elastic energy stored by the rock in this strain state, and W_d characterizes the energy dissipated by the rock due to the evolution of the damage.

Based on relevant research findings (Zhang et al., 2023; Liu and Gu, 2024), the W and W_e under CTC conditions can be expressed as:

$$W = \int_{0}^{\varepsilon_{1}} \sigma_{1} d\varepsilon_{1} + \int_{0}^{\varepsilon_{2}} \sigma_{2} d\varepsilon_{2} + \int_{0}^{\varepsilon_{3}} \sigma_{3} d\varepsilon_{3}$$
(2)
$$V_{e} = \frac{\left[\sigma_{1}^{2} + \sigma_{2}^{2} + \sigma_{3}^{2} - 2\overline{u} \times (\sigma_{1} \times \sigma_{2} + \sigma_{1} \times \sigma_{3} + \sigma_{2} \times \sigma_{3})\right]}{2E_{u}}$$

where σ_1 , σ_2 , and σ_3 represent the maximum, intermediate, and minimum principal stresses of the rock sample, respectively; ε_1 , ε_2 , and ε_3 are the corresponding strains; $\overline{\mu}$ and E_u denote the Poisson's ratio and elastic modulus during the linear elastic phase.

Under conventional triaxial conditions, the intermediate and minimum principal stresses of the rock sample satisfy:

$$\sigma_2 = \sigma_3$$

Equation 2 can be simplified to:

W

$$W_{e} = \frac{\lfloor \sigma_{1}^{2} + 2\sigma_{3}^{2} - 2\overline{u} \times (2\sigma_{1}\sigma_{3} + \sigma_{3}^{2}) \rfloor}{2E_{u}}$$
(3)

Based on Equation 1, the W_d can be expressed as:

$$W_d = W - W_e \tag{4}$$

A combination of Equations 1–3 allows for the calculation of the W_d of the rock sample under CTC conditions.

3.2 Analysis of energy evolution

The evolution of W, W_e , and W_d for marble sample under CTC conditions at varying σ_3 was derived from experimental data, as illustrated in Figure 6.

Experimental analysis reveals that the enhancement effect of confining pressure (σ_3) significantly governs the energy evolution mechanisms in rocks. As the confining pressure increases from 10 MPa to 50 MPa, the total energy density input by external loads exhibits nonlinear growth characteristics, reaching 0.965 MJ/m³, 1.859 MJ/m³, 3.615 MJ/m³, 8.478 MJ/m³, and 9.275 MJ/m³ respectively. This progression aligns with the exponential evolution trend of total energy (W) under elevated confining pressures. Energy partitioning features demonstrate: 1) The maximum stored elastic energy (W_e) shows a stepwise increase with confining pressure gradients, achieving peak values of 0.345 MJ/m³, 0.387 MJ/m³, 0.406 MJ/m³, 0.434 MJ/m³, and 0.626 MJ/m³ at corresponding pressure levels; 2) The dissipated energy during failure (W_d) displays accelerated growth, with measured values of 0.939 MJ/m³, 1.795 MJ/m³, 3.518 MJ/m³, 8.313 MJ/m³, and 8.920 MJ/m³. These energy evolution patterns indicate that confining pressure not only enhances the energy storage capacity of rock media (manifested by an 81.4% increase in W_{e}) but also promotes energy accumulation through restrained crack propagation, ultimately resulting in up to 8.5-fold intensification of energy release (W_d) at failure. This



energy conversion mechanism fundamentally explains the energydriven nature of engineering disasters in rock masses under deep high-stress environments.

The analysis revealed that the energy evolution can be categorized into five stages:

- (A) Compaction Stage: W, W_e , and W_d all increased slowly with increasing σ_1 and ε_1 . This suggested that during the compaction stage, the marble sample experienced energy dissipation as a result of microcrack closure.
- (B) Linear elastic stage: W and W_e exhibited a distinct upward concave growth at an increasing rate, while W_d remained relatively constant. This indicated that as σ_1 increased, the micro-cracks within the marble gradually closed, resulting in all input energy being stored as elastic energy with negligible loss.
- (C) Plastic yield stage: As σ_1 increased, the SS curve showed a nonlinear growth. *W*, *W*_e, and *W*_d all increased gradually with increasing σ_1 . However, the rate of increase in *W*_d began to accelerate while the rate of increase in *W*_e decreased. A detailed analysis revealed that micro-cracks started to initiate and expand within the marble sample, resulting in most of the input energy being converted into *W*_d.
- (D) Instability failure stage: The micro-cracks within the marble sample began to interconnect, forming a macroscopic fracture surface. W_e was rapidly converted into W_d , which was manifested by the increase in W and W_d , as well as a gradual decrease in W_e .
- (E) Residual strength stage: As the macroscopic fracture surface developed, the strength of the sample diminished. The rate of increase in W and W_d slowed down, while W_e continued to decline.



4 Development of a DCM based on energy balance

4.1 Fundamental equations for the model

4.1.1 Definition of the damage variable

Previous analysis of energy evolution revealed that the proportion of W_d reflects the transition of rock damage from intact to complete failure. Therefore, a new method was introduced to define the damage variable in terms of cumulative dissipated energy.

$$D = \frac{W_d}{W_{dm}} \tag{5}$$

where *D* represents the damage variable; W_d denotes the dissipated energy for any given strain ε ; and W_{dm} is the cumulative dissipated energy required for complete failure of the sample.

D = 0 indicates that the rock is intact and undamaged, while D = 1 indicates that the rock completely fails. However, the rock under CTC conditions still retains some W_e and load-bearing capacity in the residual stage. Therefore, a correction factor y related to residual strength is introduced and defined as:

$$y = \frac{\sigma_c - \sigma_r}{\sigma_c} \tag{6}$$

where σ_c represents the peak compressive strength of the rock under CTC conditions, and σ_r denotes the residual strength.

Modify the damage variable D, as defined in Equation 5, based on Equation 6.

$$D = \left(1 - \frac{\sigma_r}{\sigma_c}\right) \frac{W_d}{W_{dm}} \tag{7}$$

Numerous scholars frequently employ the degree of degradation in the elastic modulus to characterize the damage of rock masses (Liu et al., 2024). This study followed this approach and calculated the elastic modulus under damaged conditions via Equations 7. As shown in Equation 8:

$$E = E_0 (1 - D)$$
(8)

where *E* represents the elastic modulus after damage, and E_0 denotes the initial elastic modulus.

4.1.2 Assumptions on strain

It is assumed that both the axial and circumferential strains consist of two components: elastic strain ε_e and plastic strain ε_p (Zhou et al., 2019). The expression for the strain is as follows:

$$\varepsilon = \varepsilon_e + \varepsilon_p \tag{9}$$

where ε signifies the strain tensor; ε_e and ε_p denote the elastic and plastic strain tensors, respectively.

When differentiating the axial and circumferential strains, it is assumed that the strain increment includes both elastic and plastic components, as expressed in Equation 10.

$$d\varepsilon = d\varepsilon_e + d\varepsilon_p \tag{10}$$

where d denotes the differential operator.

4.1.3 Equation for SS relationship

According to Hooke's Law, the SS equation for rocks under CTC conditions can be expressed as:

$$\sigma = (1 - \gamma D) M_0 \varepsilon_e \tag{11}$$

where σ represents the stress tensor under CTC conditions; M_0 is the initial stiffness matrix for the undamaged rock; and γ is the correction factor, as illustrated in Equation 6.

The expression for the M_0 is given by:

$$M_{0} = \frac{E_{0}(1-\mu)}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1 & \frac{\mu}{1-\mu} & \frac{\mu}{1-\mu} \\ 1 & \frac{\mu}{1-\mu} \\ \text{sym} & 1 \end{bmatrix}$$
(12)

where μ is the Poisson's ratio.

In CTC tests, the *W* includes two components: one from σ_1 and the other from intermediate and minimum principal stresses. According to the stress conditions of the rock, it is known that $\sigma_2 = \sigma_3$.

By combining Equations 11, 12, the SS relationship for the rock under CTC condition can be simplified as Equation 13:

$$\begin{cases} \sigma_1 = \eta \varepsilon_{1e} + 2\beta \varepsilon_{3e} \\ \sigma_2 = \sigma_3 = \beta \varepsilon_{1e} + (\eta + \beta) \varepsilon_{3e} \end{cases}$$
(13)

where η and β as shown in Equation 14

$$\eta = \frac{(1 - \gamma D)E_0(1 - \mu)}{(1 + \mu)(1 - 2\mu)} \quad \beta = \frac{(1 - \gamma D)E_0\mu}{(1 + \mu)(1 - 2\mu)}$$
(14)

4.2 Damage evolution equation under CTC conditions

For any infinitesimal element of rock, it is assumed that the compression process involves only energy exchange with the surroundings. The incremental form of the law of conservation of energy is expressed as (Qu et al., 2023):

$$dW = dW_e + dW_d \tag{15}$$

Regarding the work done by an infinitesimal element, the differential equation for energy balance under triaxial stress conditions comprises three components:



(a) SS curves obtained from CTC tests. (b) Scanned image of rupture surface electron microscope at 30 MPa circumferential pressure.



(1) Differential form of W

Under conventional triaxial compression conditions, the differential form of work done by external forces can be expressed as:

$$dW = \sigma d\varepsilon = \sigma_1 d\varepsilon_1 - 2\sigma_3 d\varepsilon_3 \tag{16}$$

where ε represents the strain tensor, and σ denotes the stress tensor.

(2) Increment of W_e

Under CTC conditions, the W_e stored in the rock can be expressed as:

$$W_e = \int_0^{\varepsilon_e} M_0 (1 - \gamma D) \varepsilon_e d\varepsilon_e = \frac{1}{2} M_0 (1 - \gamma D) \varepsilon_e^2$$
(17)

By differentiating Equation 17, the increment of W_e can be obtained:

$$dW_e = (1 - \gamma D)M_0\varepsilon_e d\varepsilon_e - \frac{1}{2}\gamma\varepsilon_e^2 M_0 dD$$

= $\sigma d\varepsilon_e - \frac{1}{2}\gamma\varepsilon_e^2 M_0 dD$ (18)

(3) Increment of W_d

According to thermodynamic principles, D is closely related to W_d . Analogous to temperature and entropy, we introduced a variable: the damage energy dissipation rate U, which represents the energy required for a one-unit increase in the D as the rock transitions from an intact state to failure. The W_d can be expressed as the product of the U and the D:

$$dW_d = UdD \tag{19}$$

where U denotes the damage energy dissipation rate, with a unit of MJ/m³. Converting Equation 19 to Equation 20

$$U = \frac{dW_d}{dD} \tag{20}$$

Substituting Equations 16, 18 and 19 into differential equation for energy balance Equation 15 yields:

$$dD = \frac{\sigma(d\varepsilon - d\varepsilon_e)}{U - \frac{1}{2}\varepsilon_e^T M_0 \varepsilon_e} = \frac{\sigma d\varepsilon_p}{U - \frac{1}{2}\varepsilon_e^T M_0 \varepsilon_e}$$
(21)



10

8

 $W_{\rm d}$ (MJ/m³)

2

0

0.0

FIGURE 7

0.2

Relationship between ε_1 and W_d

0.4



Equation 21 is a universal expression. To characterize the damage evolution of specific rock materials, further determination of the plastic strain increment and U is necessary.

0.8

 ε_1 (%)

1.0

1.2

1.4

4.3 Expression for U

Despite Equation 21, it is essential to provide an expression for the *U*. Based on the methods for calculating W_d (Equations 3, 4) and the definition of the *D* (Equation 7), *U* is derived through experimental data.

Figure 7 illustrates the W_d versus ε_1 curves for rocks under different σ_3 . Figure 7 revealed that the variation patterns of W_d are consistent under different σ_3 , and the maximum W_d increased as the σ_3 increased. The evolution of W_d experienced four stages with increasing axial strain: stability, slow growth, steady growth, and slowdown.

A combination of Equations 2, 4, 7 enables the calculation of the *D* during the conventional triaxial loading process. The resulting relationship between *D* and ε_1 is depicted in Figure 8.

Figure 8 illustrated that the damage evolution process can be categorized into three stages: stability, accelerated growth, and slow growth. The marble sample experienced a sharp increase in *D* during the plastic yield and instability failure stages. As it entered the residual stress stage, the growth rate of damage diminished.

Figure 9 illustrates the relationship between W_d and D, where the x-axis and y-axis represents the D and W_d , respectively.

A clear positive correlation was identified between W_d and D. Moreover, growth rate of W_d varied under different σ_3 , and a higher σ_3 led to a greater growth rate. This behavior can be attributed to the inherent density of marble. In deep and high confining pressure environments, the W_d absorbed per unit damage remained stable and uniform as the σ_1 increased.

Figure 9 showed that W_d is significantly influenced by both the D and σ_3 , with higher σ_3 leading to greater W_d . Based on this observation, a model was constructed to characterize the magnitude





of the U:

$$U = U_0 + K\sigma_3 D \tag{22}$$

where U_0 represents the damage energy dissipation rate at zero σ_3 , which characterizes the inherent properties of the rock material; Kdenotes the influence coefficient of σ_3 and D on the U.

The U values under different σ_3 are summarized in Table 1.

The approach for determining the U_0 and K of σ_3 and D in Equation 22 is as follows.

The W_d corresponding to each D of the rock during the CTC was computed using Equations 2–4, 7. A scatter plot was generated with W_d on the vertical axis and D on the horizontal axis.

Based on the scatter distribution characteristics of Wd versus D in Figure 9, Pearson correlation analysis was used to show a significant positive linear correlation (r = 0.92, p < 0.001). To

TABLE 1 Summary of U values under various σ_3 .

| | - | | | | | | |
|-----------------------|-------|-------|-------|-------|-------|--|--|
| $\sigma_{ m 3}$ (MPa) | 10 | 20 | 30 | 40 | 50 | | |
| U_0 | 0.005 | 0.083 | 0.127 | 0.156 | 0.174 | | |
| K | 0.136 | 0.159 | 0.208 | 0.533 | 0.756 | | |



further quantify the relationship, we used OriginPro 2022 to fit a least-squares linear regression to the experimental data to obtain the corresponding functional expression. The slope of the line represents the value of $K\sigma_3$. Dividing this value by the corresponding σ_3 yields the *K* of *D* and σ_3 on *U*. The *y*-intercept of the line indicates the U_0 .

4.4 Expression for Poisson's ratio

According to the theory of elasticity, the standard μ does not accurately represent the relationship between lateral strain and axial strain after the peak stress. Therefore, this study introduced a new Poisson's ratio, denoted as μ' , to characterize the relationship between lateral and axial strains throughout the entire SS process of the rock.

$$\mu' = \frac{\varepsilon_3}{\varepsilon_1} \tag{23}$$

 μ' was computed for deep marble under different σ_3 using Equation 23. The evolution curves of μ' in relation to the *D* and ε_1 are illustrated in Figures 10, 11, respectively, where the x-axis indicates the *D* or ε_1 , while the *y*-axis represents the μ' .

A thorough analysis revealed that σ_3 and D are significant factors influencing the evolution of μ' . The σ_3 suppressed the increases in circumferential strain and μ' of the rock. When the σ_3 was held constant, μ' increased with increasing D. As rock damage in the residual stage intensified, the growth rate of μ' decreased and subsequently stabilized.



TABLE 2 Parameter values and goodness of fit under different σ_3 .

| $\sigma_{ m 3}$ (MPa) | | а | b | R ² |
|-----------------------|------|-------|---------|----------------|
| 10 | 0.23 | 12.57 | -7.73 | 0.997 |
| 20 | 0.23 | 12.87 | -11.01 | 0.999 |
| 30 | 0.23 | 11.87 | -9.68 | 0.998 |
| 40 | 0.23 | 31.39 | -51.76 | 0.996 |
| 50 | 0.23 | 44.82 | -106.05 | 0.999 |

Figure 11 indicated that μ' exhibits an approximately quadratic relationship with the *D*. To emphasize the influence of σ_3 on μ' , a binomial function was employed to define μ' :

$$\mu' = \mu + b\sigma_3 D + cD^2 \tag{24}$$

where μ represents the Poisson's ratio for intact rock (when the damage variable is 0); *b* denotes the combined influence coefficient of the *D* and σ_3 ; and *c* signifies the influence coefficient of the *D*.

Additionally, we employed the least squares method to ascertain the parameters in Equation 24, with the fitting results depicted by the solid line in Figure 11. The average goodness of fit (R^2) was 0.995, which indicated that the fitting results effectively reflect the relationship among μ' , σ_3 , and *D*. The parameter values and goodness of fit under different σ_3 are summarized in Table 2.

4.5 Expression for plastic strain

Based on the strain assumptions of Equation 9 and the principle of strain equivalence, the expressions related to the damage variable were employed to characterize the relationships between elastic and



plastic strains and macroscopic strain, as shown in Equation 25:

$$\varepsilon_{1e} = (1 - D)\varepsilon_1$$

$$\varepsilon_{3e} = (1 - D)\varepsilon_3$$
(25)

Equation 26 was derived by differentiating the axial elastic strain ε_{1e} and the circumferential elastic strain ε_{3e} with respect to *D*.

$$d\varepsilon_{1e} = (1-D)d\varepsilon_1 - \varepsilon_1 dD \tag{26}$$

Substituting Equation 26 into Equation 10 yields the expression for axial plastic strain:

$$d\varepsilon_{1p} = d\varepsilon_1 - (1 - D)d\varepsilon_1 + \varepsilon_1 dD = Dd\varepsilon_1 + \varepsilon_1 dD$$
(27)

Equation 24 defines the expression for the μ' , which characterizes the mathematical relationship between circumferential and axial strains. According to this assumption, the circumferential strain can be expressed as Equation 28:

$$\varepsilon_3 = \mu' \varepsilon_1 = (\mu + b\sigma_3 D + cD^2)\varepsilon_1 \tag{28}$$

The increment of circumferential strain can be expressed as Equation 29:

$$d\varepsilon_3 = d(\mu'\varepsilon_1) = (\mu + b\sigma_3 D + cD^2)d\varepsilon_1 + (b\sigma_3 + 2cD)\varepsilon_1 dD$$
(29)

Similarly, the increment of circumferential plastic strain can be expressed as:

$$d\varepsilon_{3p} = Dd\varepsilon_3 + \varepsilon_3 dD = \left(\mu + b\sigma_3 D + cD^2\right) d\varepsilon_1 + \left[\mu + (b+1)\sigma_3 D + 3cD^2\right]\varepsilon_1 dD$$
(30)

4.6 Solution of the constitutive model

In CTC tests, the constant σ_3 results in a stress increment of 0 (i.e., $d\sigma_3 = 0$). Thus, differentiating the circumferential stress under CTC conditions yields:

$$(\eta + \beta)d\varepsilon_{3e} + \beta d\varepsilon_{1e} - (\gamma \eta \varepsilon_{3e} + \gamma \beta \varepsilon_{1e} + \gamma \beta \varepsilon_{3e})dD = 0$$
(31)

Equations 10, 19, 27, 30, 31 are combined and represented in a matrix form, as expressed in Equation 32.

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\left[\mu + b\sigma_3 D + cD^2\right]d\epsilon_1} & 0 & -\frac{\left[\mu + (b+1)\sigma_3 D + 3cD^2\right]\epsilon_1}{\left[\mu + b\sigma_3 D + cD^2\right]d\epsilon_1} \\ \frac{1}{D} & 0 & 0 & 0 & -\frac{\epsilon_1}{D} \\ -\sigma_1 & 0 & -2\sigma_3 & 0 & U_0 + K\sigma_3 D \\ 0 & \beta & 0 & \eta + \beta & -\gamma\eta\epsilon_{3e} - \gamma\beta\epsilon_{1e} - \gamma\beta\epsilon_{3e} \end{bmatrix} \begin{bmatrix} d\epsilon_1 \\ d\epsilon_2 \\ d\epsilon_3 \\ dD \end{bmatrix} = \begin{bmatrix} d\epsilon_1 \\ d\epsilon_1 \\ d\epsilon_3 \\ dD \end{bmatrix} = \begin{bmatrix} d\epsilon_1 \\ d\epsilon_1 \\ d\epsilon_1 \\ d\epsilon_3 \\ dD \end{bmatrix}$$
(32)

Applying Gaussian elimination to Equation 32 yields Equation 33

$$\frac{dD}{d\varepsilon_1} = \frac{\sigma_1 + D\sigma_1 \left(1 - \frac{1}{D}\right) 2\sigma_3 d\varepsilon_1^2 (cD^2 + b\sigma_3 D + a)}{U_0 - \sigma_1 \varepsilon_1 + DK\sigma_3 + 2\varepsilon_1 \sigma_3 (3cD^2 + \sigma_3 (b+1)D) + a}$$
(33)

5 Results and analysis

Equation 21 was incorporated into the SS constitutive relation, which was subsequently iterated using the Euler method to derive the complete SS curves for rock under various σ_3 , as illustrated in Figure 12, These curves were then compared with experimental data for marble.

The established equilibrium differential equations in this study quantitatively account for the effects of σ_3 and damage on the *U* and μ '. Therefore, the proposed constitutive model effectively captures the evolution of complete SS curves for deep marble under CTC conditions during the elastic, plastic yield, instability failure, and residual strength stages. Furthermore, the curves resulting from this constitutive model are continuous and closely align with actual data.

6 Conclusion

This study analyzed the energy evolution of marble under CTC conditions and proposed a new expression for the damage variable from the perspective of energy dissipation. The expressions for U and μ' were established based on the data derived from CTC tests. A DCM was established using the differential equation for energy balance, and the model parameters were calibrated. The following conclusions are drawn:

- The W_d curve experienced stages of stability, slow growth, steady growth, and slowdown. The maximum W_d increased with rising σ₃.
- (2) The increase in σ₃ inhibited the growth of rock damage. When the σ₃ was held constant, μ' increased with the degree of damage. As rock damage in the residual stage intensified, the growth rate of μ' decreased and subsequently stabilized.
- (3) The energy release rate U demonstrates a significant positive correlation with confining pressure σ₃, revealing that under high confining pressure conditions, rock exhibits enhanced energy dissipation characteristics at equivalent damage levels: As σ₃ increases, the dissipated energy W_d released during failure intensifies markedly, thereby driving systematic growth in U values. This correlation validates the physical mechanism whereby confinement effects promote energy accumulation through suppressing disordered crack propagation.
- (4) A comparison between the experimental data and model predictions indicated that the proposed constitutive model accurately captures the elastic stage, plastic yield stage, instability failure stage, and the nonlinear characteristics of the SS curves for rock under residual strength.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

Author contributions

TH: Conceptualization, Validation, Formal Analysis, Methodology, Writing – original draft, Writing – review and editing, Data curation, Software. ZZ: Supervision, Conceptualization, Writing – review and editing, Project administration. YZ: Writing – review and editing, Validation, Conceptualization, Methodology. JZ: Methodology, Writing – review and editing, Investigation, Software, Validation.

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Conflict of interest

Author YZ was employed by China Construction Eighth Engineering Division Corp., Ltd.

The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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