On the mathematical content of financial literacy items

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In light of the growing intricacy of the financial sector, there is a growing imperative for individuals to possess a robust understanding of financial principles and practices. Since the global financial crisis of 2008, a variety of financial education programmes have been introduced. It is therefore not surprising that the state of financial literacy in society is becoming a topic of great interest. For this reason, surveys have been developed to assess the level of financial education in a society. Some of the items used also seem to test mathematical proficiency. This article takes a closer look at selected financial literacy items. In more detail, it considers the mathematical content of these items, the necessity of mathematical thinking for answering the items and ideas for application-oriented teaching in mathematics that can be derived from the items. The analysis shows that a wide range of mathematical topics are addressed in the items. In most cases, there are two possible lines of reasoning to arrive at the answer, one mathematical and one economic. Unless quantification is involved, then a mathematical approach is unavoidable. The items provide a couple of ideas for financial topics that can be integrated into mathematics courses, for example risk considerations. From this perspective, mathematics teaching can be seen to support the idea of financial literacy as a cross-curricular subject.

KEYWORDS

mathematical literacy, financial literacy, financial literacy items, risk, diversification

1 Introduction

Developments such as the shifting of responsibility for retirement savings, rapid changes in the financial markets, new and more complex financial products, mutual funds, student loan debt show the urgent need for financial literacy (Lusardi, 2019). Consequently, financial education is a crucial aspect of economic and personal development in the contemporary era (OECD, 2017; Sole, 2014). But what does it mean to be financially literate? There are a numerous definitions, a commonly used definition is that of the OECD (2024):

> "Financial literacy is knowledge and understanding of financial concepts and risks, as well as the skills and attitudes to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the financial well-being of individuals and society, and to enable participation in economic life." (OECD, 2024, p. 112)

The OECD (2012) has called for financial literacy to be a core part of the school curriculum for over 10 years, as part of a national strategy that starts as early as possible. In the meantime, some states or federal states have even introduced a standalone course on financial literacy, while others see it as a cross-curricular subject, which particularly affects mathematics lessons. What significance and consequences does this have for the learning of mathematics? In some cases, it will undoubtedly be challenging to introduce a new subject such as "Financial Literacy," given the already congested schedules of the students. Regardless of the decision that

is made, in my opinion, mathematics lessons will be required to make a significant contribution. There are certain topics that are fundamental to financial education, yet they must be addressed within the context of mathematics lessons. These include at least basic arithmetic and percentage calculations (OECD, 2022). As stated in Savard and Cavalcante (2024), the extant literature indicates that mathematical competencies play an important role in the development of financial literacy. In this context, it is noteworthy that the OECD (2024) has reported on the relationship between financial literacy and mathematical literacy. The results of the financial literacy survey conducted in 2022 indicated that students who perform better in mathematics (and/or reading) are more likely to be financially literate.

1.1 Financial literacy test items

Since the financial crisis in 2008, there has been an increased focus on financial literacy, leading to the launch of numerous education programmes. Consequently, the level of financial literacy in society has become a topic of great interest. Surveys have been developed to measure the level of financial literacy in a society. In a pioneering paper, Lusardi and Mitchell (2014) published the results of a survey that used just three questions, later known as "The Big Three," to test whether a person was financially illiterate, see [Q1], [Q2], and [Q5]. Similarly, the Financial Scale for Young Adults (Center for Financial Security, 2017) uses five questions to assess an individual's level of financial literacy (Financial Literacy Quiz Score), see [Q1]–[Q5].

Q1. Suppose you had $100 in a savings account and the interest rate was 2% per year. After 5 years, how much do you think you would have in the account if you left the money to grow? (more than $102, same, less than $102)

Q2. Imagine that the interest rate on your savings account was 1% per year and inflation was 2% per year. After 1 year, how much would you be able to buy with the money in this account? (more than today, same, less than today)

Q3. If interest rates rise, what will typically happen to bond prices? (rise, fall, stay the same, no relationship)

Q4. A 15-year mortgage typically requires higher monthly payments than a 30-year mortgage, but the total interest paid over the life of the loan will be less. (True/False)

Q5. Buying a single company's stock usually provides a safer return than a stock mutual fund. (True/False)
(Center for Financial Security, 2017, p. 5)

These items are frequently used by various institutions in different constellations to survey the level of financial literacy in society. For example, the OECD (2022) or the Central Bank of the Republic of Austria regularly conducts surveys that contain these questions in very similar form (Fessler et al., 2020). A closer look at the items might raise the following question: How much mathematical literacy is required to complete these tasks?

In order to elucidate the focus of the present essay, three guiding questions are proposed.

- What is the mathematical content of these financial literacy items?

- To what extent are mathematical arguments necessary for answering these financial literacy items?
- What ideas can be drawn from these financial literacy items for teaching mathematics?

1.2 Economic or mathematical explanations

In the following, I will try to give an answer based exclusively on economic arguments, trying to debunk elements of mathematical literacy or, alternatively, to provide a mathematical argument.

In order to substantiate the statements in the items [Q1] and [Q2], you must be familiar with interest rates and the principle of upcounting and discounting, respectively. In other words, a dollar today is worth more than a dollar tomorrow. Economically, interest, the size of which is determined by the interest rate, can be seen as an incentive for savers to lend their money to the bank. It therefore makes sense that the money invested will increase. But determining the size of the amount is genuine mathematical literacy, there is no getting around it. With two of these items, 2% of $100 is to be determined and then the respondent must be able to argue for himself that if another 2% of this new amount is added and the whole thing happens three more times, one will have more than $102 in the account. The same applies to the item about inflation [Q2], which involves discounting. A mathematical remark, better distractors for the item [Q1] would be those used by the OECD (2022) for den *financial knowledge score*: more than $110, same, less than $110.

With regard to the item on the behavior of the price of a bond in relation to interest rate changes [Q3], it is important to clarify some technical terms. First of all, what is a bond? When a government or a company needs money, it can issue a bond. This is the equivalent of a loan from many investors. The issuer of the bond agrees to repay the borrowed money after a certain period of time. Investors receive regular payments, known as interest or coupon, for their investment. When interest rates increase, bond prices decrease. This can be explained by the fact that if investors own bonds with an interest rate of 2% and the market interest rate rises to a level above 2% p.a., for example to 3% p.a., then the investors will attempt to sell their bonds and invest their money at the market interest rate. According to the principle of supply and demand, this will cause the price of the bond to fall (many sellers, few buyers). For the mathematical argument, in other words for quantification, let us consider a bond of $1,000 with a maturity of 10 years and an interest rate of 2% p.a. (the annual coupon payment is therefore $20). In this consideration this should mean that the first coupon payment is due after one year, the second coupon payment after two years and so on. If the market interest rate amounts 1% due to discounting the price of the bond is:

$$\frac{20}{1{,}01} + \frac{20}{1{,}01^2} + \frac{20}{1{,}01^3} + \ldots + \frac{1020}{1{,}01^{10}} \approx 1095\,\$$$

If the interest rate is 3%, the price falls because the interest rate is in the denominator

$$\frac{20}{1{,}03} + \frac{20}{1{,}03^2} + \frac{20}{1{,}03^3} + \ldots + \frac{1020}{1{,}03^{10}} \approx 915\,\$$$

The change in one quantity (market interest rate) is related to changes in the bond price. To determine the exact value of the bond, mathematics is needed again.

As in [Q1], the focus in [Q4] is again on calculating interest rates. However, the question regarding mortgages is slightly more complex. Typically, a mortgage is repaid to the financial institution in monthly payments. Interest must be paid for borrowing, and the interest rate determines how much this is. For the loan to be repaid at all, the monthly payments must exceed the monthly interest. To repay the initial principal of the loan in a short period of time, the monthly payments must be higher to cover the initial principal compared to a long-term mortgage. As for the total interest statement, an economic argument can be made that you have to pay interest for the time you borrow money. If you borrow money for a longer period, then you have to pay more. This attempted economic explanation is on the borderline of mathematical literacy, as functional relationships are argued here with larger and smaller ratios. From a mathematical point of view or in order to obtain precise values, respectively, one must consider the amortization calculation formula, in a simple case (interest period is equal to the payment period), as shown below. Express the monthly payments A explicitly (P initial principal, r interest rate per period, n total number of payments):

$$A = P \cdot \frac{r(1+r)^n}{(1+r)^n - 1} = P \cdot r \frac{1}{1 - \dfrac{1}{(1+r)^n}}$$

Since r is greater than zero, $\dfrac{1}{(1+r)^n}$ is less than 1. Using a very informal line of mathematical arguments, we see that the larger n (r and P fix), the smaller $\dfrac{1}{(1+r)^n}$, the bigger $1 - \dfrac{1}{(1+r)^n}$, the smaller $\dfrac{1}{1 - \dfrac{1}{(1+r)^n}}$, hence, the smaller are the monthly payments A. To calculate the sum of all interest accruing, analogous arguments are needed.

The item [Q5] contributes to the principle of diversification. Simply, do not put all your eggs in one basket. Even Shakespeare was already familiar with this principle, as you can read in *The Merchant of Venice*:

> "Believe me, no. I thank my fortune for it,
> My ventures are not in one bottom trusted,
> Nor to one place; nor is my whole estate.
> Upon the fortune of this present year:
> Therefore, my merchandise makes me not sad."
> (Shakespeare, 2014, p. 10)

If I invest all my money in one stock, I risk losing everything if it performs poorly. If I spread my money across many stocks, I can avoid losing all my invested money if a few stocks perform poorly. Mathematically seen, this is an extreme value problem. In the financial market, one way to measure risk is to look at price fluctuations, which can be measured using the standard deviation. For two stocks A and B, this risk of the portfolio with these two stocks can be expressed as follows:

$$\sigma_P(a,b) = \sqrt{a^2 \sigma_a^2 + b^2 \sigma_b^2 + 2ab\sigma_a\sigma_b\rho_{ab}}$$

where a is the proportion of his money that is invested in stock A, the risk of stock A (standard deviation of the returns) is denoted by σ_a and ρ_{ab} represents the correlation of the two stocks (analogous denominations for stock B). Now, under the condition $a + b = 1$, you want to find the distribution of the proportions a and b of your money that minimizes the risk. For example, even if both stocks have the same risk value ($\sigma_a = \sigma_b$) and are uncorrelated ($\rho_{ab} = 0$), you will minimize your risk by dividing your money equally between the two stocks. The minimum of $\widehat{\sigma_P}(a) = a^2 \cdot \sigma_a^2 + (1-a)^2 \cdot \sigma_a^2$ is at $a = b = 0.5$.

1.3 Ideas for teaching—benefits for both areas of literacy

As we have seen, each of these items can be viewed mathematically, addresses different mathematical content areas and requires different mathematical processes. In my opinion, these mathematical approaches can be seen as a benefit for both areas of knowledge. For example, Savard et al. (2020) showed that a deeper, more measurement-theoretical view of money leads to a better conceptual understanding of money.

A similar argument can be made with regard to the risk assessment in a financial context, as exemplified by the calculation of the risk associated with an investment in a stock (Dorner, 2019). In this case, at the initial stage, considerations are made regarding the object under examination, namely the share. What is the nature of this entity? What is the mechanism by which a share generates profits? What are the potential sources of risk when investing in a share? Answering these questions clarifies the key concepts of a share. A capital gain is realized when a stock is sold for a price in excess of its purchase price. However, it is important to understand that the stock in question is subject to fluctuations in value, and thus the value may decrease. This then gives rise to further questions regarding the appropriate measure to be employed and the inclusion of historical data or share prices. The question thus arises as to whether all fluctuations should be measured, or only those in a downward direction. It would be beneficial to ascertain whether historical data should be weighted differently. It is unlikely that events from a more distant point in time will exert a significant influence on current events. A variety of statistical habits of mind that improve understanding of fundamental concept in statistics can be worked through in this teaching suggestion. According to Burrill and Biehler (2011), one should use real data, build intuitions, begin with a graph, explore alternate representations of data, investigate and explore before introducing formulas and use student projects and experiments to engage students in doing statistics. When analysing risk, several of the above-mentioned requirements are met by looking at real share prices (by the day, hour, minute, etc.), presenting them in charts, calculating measures (e.g., standard deviations) and modifying formulas by weighting historical data differently. Further discussion could focus on the concept of

chance and the question of whether share prices can be regarded as random.

The principle of up- and discounting does not only have to be illustrated using savings books. At the upper secondary level, this can also be addressed through the examination of bonds, as outlined in item [Q3], or through the utilization of loans, whereby the interest rate is implemented into a spreadsheet programme using random numbers [see Dorner (2022)]. According to Pournara (2009), spreadsheets provide an effective way to learn the mathematics of annuities. Additionally, risk considerations can be incorporated into both aforementioned cases.

2 Discussion

Each of the items listed above can be considered mathematically, and as has been shown, the mathematical explorations have extensive and rich mathematical content and methods. These range from percentage calculations, functional dependencies and limit value consideration to extreme value problems. As these items are used in serval surveys, they also show the importance of this content in the area of financial literacy. This subsequently indicates how rooted mathematics is in the world of finance. In light of the aforementioned context, it is unsurprising that mathematical abilities are a significant contributing factor in the advancement of financial literacy (Savard and Cavalcante, 2024; OECD, 2017). Nevertheless, it has been shown that an appropriate answer can also be found exclusively with economic arguments. Except in the case of quantification, where mathematical techniques are essential, for example for calculating interest rates. Nevertheless, the teaching of mathematics can make a contribution to financial literacy and supports the idea of financial literacy as a cross-curricular subject. Even if it is introduced as a separate subject, some financial skills are so deeply mathematical that they have to be taught in mathematics classes.

In addition, the content addressed in the items can serve as a source of ideas for application-oriented teaching in mathematics. This is in line with Sole (2021), who claims that financial topics can be easily integrated into mathematics courses. One area that has been identified is that of risk considerations, which can be addressed in a number of ways. For good implementations in mathematics lessons, more concrete didactic preparations are needed, such as those on mathematical structures of simple and compound interest by Cavalcante et al. (2024).

It should be noted, however, that although the items considered are frequently used, they represent only a small number overall. Consequently, there are many more items related to financial literacy. For example, the PISA study contains items that do not allow mathematical considerations, e.g., when it comes to internet security and fraud [see, for instance, item "Bank error," OECD (2024), p. 222].

3 Conclusion

The selected financial literacy items are replete with mathematical content. In instances where quantitative considerations are necessary, recourse to mathematics is inevitable. Furthermore, the items provide guidance on financial topics that can be integrated into mathematics lessons.

Mathematics teachers should use this wealth of mathematical content and methods in the world of finance for their mathematics lessons. In this way, students can be offered up-to-date and application-oriented mathematics lessons that also provide an answer to the question of what mathematics is needed for.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

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Conflict of interest

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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