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CoRe as a tool to evaluate the pedagogical knowledge of future teachers about the fraction as an operator

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Numerous studies have shown that one of the fundamental factors for effective teaching is the possession of an adequate Pedagogical Content Knowledge (PCK). This knowledge enables teachers to effectively translate specific topics for their students and requires strong support during teacher training, as it is developed throughout the professionalization process. This study aims to determine whether the Content Representation (CoRe) instrument, widely used in international research, aligns with the three components defined by the Mathematical Knowledge for Teaching model for assessing PCK: students learning, teaching and curriculum. To this end, the instrument focuses on a specific mathematical concept, the construct fraction as an operator, and was administered to 263 prospective teachers at a Spanish university. The results reveal the reliability of the scale scores in the sample, and confirmatory factor analysis demonstrate a stable underlying structure, showing that the CoRe questions aligns satisfactorily with the three associated components. However, the findings also highlight notable deficiencies in the participants' PCK, especially in understanding students' difficulties and planning effective teaching methodologies. These results emphasize the need to integrate tools like CoRe into teacher education programs to address these gaps and better prepare future teachers to teach complex mathematical concepts, such as fractions as operators.

KEYWORDS

pedagogical content knowledge, mathematical knowledge for teaching, future teachers, content representation, fraction as operator

1 Introduction

Since the early 21st century, teacher education has been a prominent topic in academic debates due to deficiencies identified in teacher's pedagogical and disciplinary knowledge, as well as their impact on student learning (Güler and Çelik, 2018). These shortcomings directly affect teachers' ability to design effective teaching strategies and address students' conceptual errors, both of which are fundamental to achieving meaningful learning outcomes (Darling-Hammond et al., 2017). In this context, improving the quality of initial teacher education has emerged as a global priority.

The debate over the essential knowledge teachers should possess has generated numerous studies and theoretical investigations. In the field of mathematics education, models such as Mathematical Knowledge for Teaching (MKT) by Ball et al. (2008) and Mathematics Teachers' Specialized Knowledge (MTKS) by Carrillo-Yañez et al. (2018) stand out as key proposals.

These models integrate pedagogical and disciplinary knowledge as fundamental pillars of teaching practice.

Pedagogical Content Knowledge (PCK), introduced by Shulman (1986), combines disciplinary and pedagogical knowledge, enabling teachers to transform complex concepts into comprehensible ideas for students. This knowledge is essential in initial teacher education, as it facilitates the anticipation of common errors, the design of meaningful activities, and the adaptation of teaching to meet students' needs (Blömeke and Delaney, 2012).

Pedagogical Content Knowledge (PCK), as introduced by Shulman (1986), represents the integration of disciplinary and pedagogical knowledge, enabling teachers to transform complex concepts into comprehensible ideas for students. This framework is crucial in initial teacher education, as it supports the anticipation of common errors, the design of meaningful activities, and the adaptation of teaching strategies to meet students' needs (Blömeke and Delaney, 2012). According to Li and Copur-Gencturk (2024), PCK also encompasses understanding how students conceptualize specific mathematical topics, the use of effective representations, and the application of pedagogical strategies that enhance learning. These perspectives collectively underscore the foundational role of PCK in equipping teachers to bridge the gap between curriculum goals and students' learning processes, fostering more effective and reflective teaching practices.

This is particularly critical given that, despite theoretical advancements, recent research (Kelcey et al., 2019; Hoth et al., 2022) has identified substantial gaps in teacher training: while prospective teachers often demonstrate a reasonable mastery of disciplinary knowledge (CK), they face considerable limitations in translating this knowledge into effective pedagogical strategies. In this context, Zhang (2015) emphasizes that PCK, particularly in early mathematics, is closely related to the quality of teaching and learning. His study shows that deficits in PCK negatively impact teachers' strategies and their ability to interpret and adapt complex concepts to students' needs.

These shortcomings are particularly evident in the teaching of fractions, specifically in the subconstruct of fractions as operators: a key concept for transitioning to advanced mathematics (Kieren, 1980, 2020). This subconstruct presents significant challenges in both comprehension and teaching, with studies such as those by Depaepe et al. (2015) and Khashan (2014) highlighting gaps in both didactic and disciplinary knowledge among pre-service and in-service teachers. These challenges underscore the need for more comprehensive training approaches and specialized assessment tools to investigate the level of PCK required by teachers to effectively address these deficiencies (Blömeke et al., 2015).

The Content Representation (CoRe) instrument, initially developed in the field of science education (Loughran et al., 2004), has emerged as a promising tool for evaluating PCK across various disciplines. This instrument encourages teachers to identify key concepts within a topic and reflect on critical aspects of teaching, such as learning objectives, student difficulties, and pedagogical strategies. While studies in other areas, such as science or chemistry (Boothe et al., 2023; Forsler et al., 2024; Hume and Berry, 2011), have highlighted its utility in fostering pedagogical reflection, its application in mathematics is still very limited (Maryono et al., 2017; Suripah et al., 2021; Zhang, 2015).

As a preamble to a more extensive study focused on the assessment of prospective teachers' pedagogical knowledge about the content of fractions, in particular, when they act as operators, this study aims to determine whether the Content Representation (CoRe) evaluation instrument aligns with the three components that the MKT mathematical model, taken as a reference, establishes to organize PCK. This model, developed by Ball et al. (2008), defines three components related to knowledge about curriculum, teaching, and student learning, offering a structure that allows for a comprehensive analysis of the pedagogical capacities necessary for teaching mathematics.

This study not only validates the CoRe instrument within the mathematical domain but also provides evidence of its alignment with one of the most well-known theoretical models, the MKT. It positions the CoRe as a key tool in identify critical areas in the initial training of teachers, such as the didactic treatment of fractions as an operator, contributing to the design of more effective training programs.

To address the proposed objective, the remainder of the document is organized as follows: Section 2 provides an overview of the research on teachers' professional knowledge, detailing the pedagogical domain, and its measurement and application in mathematics. Section 3 outlines the research objective and guiding question. Section 4 describes the materials and methods utilized in the study. Section 5 presents the results and their interpretation, incorporating insight from statistical analyses. Finally, Section 6 concludes with a summary of findings contextualized within the framework of recent literature.

2 Related literature

2.1 Teachers' professional knowledge

Teachers' professional knowledge has been conceptualized over time as a multifaceted construct. Despite significant efforts by numerous scholars to explore its various components, there remains no clear consensus on the minimum knowledge base required for effective teaching (Ball et al., 2008; Blömeke and Delaney, 2012; Guerriero, 2017; Shulman, 1987). However, there is widespread agreement on the critical importance of the knowledge that future teachers must acquire. This issue is particularly pronounced in countries like Spain, where access to the teaching profession does not necessarily require a specialized degree, but rather preparation focused on a specific discipline to support professional practice (Torres Rodríguez et al., 2019).

For reasons such as the distinctions between mathematics graduates and trained teachers, organizations like the NCTM (National Council of Teachers of Mathematics (NCTM), 1991), proposed in the early 1990s that teacher training programs should integrate the development of the disciplinary knowledge with elements such as the didactic transposition of specific mathematical content for student learning. As noted by Lappan and Theule-Lubienski (1992, cited in Godino et al., 1999), an exclusively mathematical or broadly psycho-pedagogical training approach is insufficient due to the cognitive and didactic complexity of specific mathematical concepts and methods.

Among the various models that consolidate the knowledge base a teacher should possess, Shulman's (1986) seminal work stands out. He introduced Pedagogical Content Knowledge (PCK) as a distinct category of pedagogical expertise, emerging from the intersection of Content or Subject Knowledge (CK) and general Pedagogical Knowledge (PK) (see Figure 1). This innovative framework provided a foundational perspective on the integration of subject matter and pedagogy, which has since shaped the field of teacher education research. As Shulman (1986, p.9) stated, "PCK is the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations-in a word, the ways of representing and formulating the subject that make it comprehensible to others." This, as the author highlighted, distinguishes the pedagogue from the specialist.

This conceptual model of teacher education has been extensively utilized over the past two decades. Since its inception, PCK has become a central focus of research in teacher education across various disciplines (Keller et al., 2017; Sakaria et al., 2023; Star, 2023). However, despite its acknowledge theoretical and practical significance, teachers' professional knowledge, particularly PCK, has remained an elusive concept. Challenges persist in its conceptualization and measurement largely due to the ambiguous boundaries defining each knowledge category (Fauskanger, 2015).

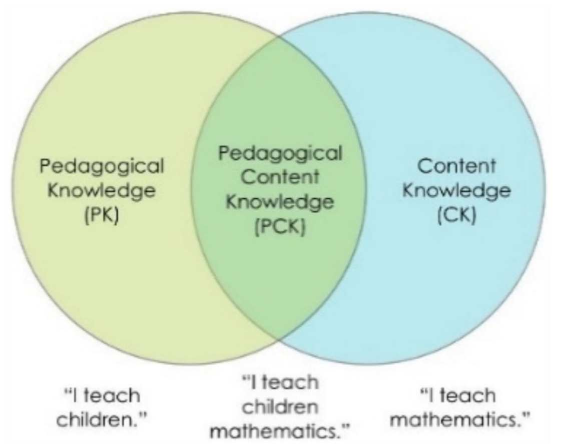

FIGURE 1
Teacher's professional knowledge model. Adapted from Shulman (1986, p.7).

In fact, Shulman (1987), a year after his initial proposal, presented a more comprehensive version of his framework by refining the boundaries and renaming some of the categories established earlier. This revised framework identified seven essential knowledge domains for teachers. The first four (General pedagogical knowledge, Knowledge of learners and their characteristics, Knowledge of educational context and Knowledge of educational ends, purposes and values) were classified as general dimensions of teaching knowledge. The remaining three (Content knowledge, Curricular knowledge and Pedagogical content knowledge) were defined as content-specific dimensions. Together, these categories formed what Shulman referred to as the "missing paradigm" in teaching research, offering a structured approach to understanding the multifaceted nature of teachers' professional knowledge (see Figure 2).

In summary, this lack of consensus on defining a clear structure has led to the development of various models outlining the essential knowledge base for teaching across disciplines (Ball et al., 2008; Hill et al., 2008; Starkey et al., 2023). This ambiguity has also resulted in differing interpretations regarding the relationship between knowledge categories. Some scholars view CK as a component within PCK (Eraut, 1994; Grossman, 1990); others question whether PCK can be theoretically and empirically distinguished from CK, given that teaching decisions inherently involve both subject matter and pedagogical considerations (Baumert et al., 2010; Bednarz and Proulx, 2009). Conversely, some researchers argue that CK and PCK are two independent yet correlated dimensions, suggesting that teachers with higher CK tend to possess more developed and integrated PCK (Llinares, 1995; Krauss et al., 2008; Rozenszajn and Yarden, 2014). Notably, there is a shared understanding that elevated levels of PCK among teachers contribute to improved student learning outcomes (Kunter et al., 2013; Juhler and Haland, 2016).

Recent studies continue to explore these relationships. For instance, Kleickmann et al. (2013) examined how teacher education

- General pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter
- Knowledge of learners and their characteristics
- Knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures
- Knowledge of educational ends, purposes, and values, and their philosophical and historical grounds
- Content knowledge
- Curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers
- Pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding

(Shulman, 1987, p. 8)

FIGURE 2
Shulman's major categories of teacher knowledge (Ball et al., 2008, p. 391).

influences the development of CK and PCK, highlighting the impact of structural differences in teacher education programs on these knowledge domains. Additionally, Güler and Çelik (2018) investigated the relationship between CK and PCK in the context of algebra teaching among elementary mathematics teacher candidates, emphasizing that a solid CK foundation contributes significantly to the development of PCK and effective teaching practices.

Building on these foundations, Blömeke et al. (2015) delved into the multifaceted nature of teachers' professional competence, emphasizing the need for comprehensive assessment tools that capture the dynamic aspects of PCK. Similarly, Forsler et al. (2024) examined through the CoRe the development of PCK in sustainable development education, highlighting the intricate interplay between content knowledge and pedagogical strategies.

These ongoing discussions underscore the complexity of delineating and interrelating CK and PCK, reinforcing the importance of comprehensive teacher education programs that address both knowledge domains to enhance teaching efficacy and student learning.

2.2 Components and measurement of pedagogical content knowledge

At that time, Shulman (1986) identified only two components: (1) knowledge about learners' misconceptions and their impact on learning, and (2) knowledge of the instructional conditions necessary to address and transform those initial concepts. However, this limited definition was soon challenged by other authors, sparking a debate about its components. Some scholars proposed incorporating additional aspects, such as curricular content (Grossman, 1990), beliefs (Friederichsen et al., 2011) or emotions (Zembylas, 2010). Others suggested a more comprehensive framework, as seen in the work of Van Driel et al. (1998) and Park and Oliver (2008), who identified nine components: (a) the purpose of teaching the subject, (b) students' difficulties, (c) the curriculum, (d) instructional strategies and content representations, (e) instructional resources or materials, (f) assessment, (g) the subject to be taught, (h) the teaching context, and (i) the didactics of the discipline.

Currently, the version most widely recognized across disciplines consists of four main components (Abell, 2008; Magnusson et al., 1999), which incorporate various elements. Specifically, these components are (a) knowledge about teaching strategies for specific content, (b) knowledge of students' understanding of the content, (c) knowledge of methods to assess the content, and (d) knowledge of the goals and objectives for teaching the content within the curriculum.

The lack of a clear structure in this sense means that not all scholars agree on the same components within PCK or how they are related. However, despite these differences, there is consensus regarding the dynamic nature of PCK (Abell, 2007, 2008). PCK is understood as knowledge that develops as future teachers gain experience through various opportunities encountered throughout their professional careers, including during their training. Moreover, recognizing the components that comprise PCK is essential for its characterization and assessment. Just as there are different types of PCK and structures of its components, the instruments used to measure it are also varied.

Commonly used tools to collect data on PCK include questionnaires with closed or open-ended questions, written assignments or reflections, classroom observations of pre-service teachers, and semi-structured interviews (Vergara and Cofre, 2014). Among the instruments proposed to distinguish and evaluate the internal composition of PCK are the CoRes (Content Representation) and PaP-eRs (Pedagogical and Professional experience repertories) developed by Loughran et al. (2004), which facilitate reflection on teaching planning and pedagogical and professional experiences. In this context, it is essential to examine findings from studies that explore the components of PCK using CoRe's.

2.3 Applications of CoRe in teacher training PCK

The Content Representation (CoRe), developed by Loughran et al. (2004) in science education, is a powerful tool for exploring and improving teaching strategies. By organizing content around specific guiding questions, CoRe helps teachers reflect on critical aspects of teaching, such as learning objectives, student challenges, and effective pedagogical strategies. Its adaptability across various disciplines underscores its significance, particularly in shaping teacher education and professional development. However, while extensively studied and applied in science education, its implementation in mathematics remains comparatively limited.

Hume and Berry (2011) highlighted the transformative potential of CoRe in teacher education, particularly for pre-service teachers. By engaging educators in the construction of CoRe frameworks, they demonstrated how this tool fosters a deeper understanding of pedagogical decision-making, making teaching strategies more explicit and aligned with student needs. Also, Hobbs and Porsch (2022) highlighted CoRe's value in collaborative settings, where teachers working outside their subject areas used it to share strategies and insights. This approach fostered richer discussions and enhanced pedagogical content knowledge (PCK), further underscoring CoRe's significance in teacher education.

Remaining in science education, Maurício and Valente (2024) expanded on this by integrating CoRe frameworks with the Lesson Study (LS) methodology in the context of teaching science to sixth-grade students. Their work focused on helping pre-service teachers connect theoretical principles with classroom practice while addressing common student challenges in understanding scientific concepts. This combination promoted collaboration and reflective practices among educators, emphasizing CoRe's capacity to guide teachers in analyzing their practices and iteratively refining them, significantly contributing to professional growth.

Similarly, Boothe et al. (2023) illustrated CoRe's utility in higher education by examining how it supports the alignment of instructors' pedagogical content knowledge (PCK) with students' conceptual needs in complex topics such as organic acid–base chemistry. By addressing student misconceptions and ensuring clarity in instructional adjustments, CoRe helped bridge the gap between foundational concepts and advanced applications, reinforcing its importance in improving teaching efficacy.

Forsler et al. (2024) also expanded the application of CoRe by integrating it with video-based reflection, demonstrating how it enables educators to connect disciplinary knowledge with real-world challenges, such as sustainability education. This iterative approach helps teachers refine their practices and adapt to diverse contexts, contributing to their professional growth.

Despite these promising findings, the application of CoRe in mathematics lacks the depth and breadth observed in other fields. Maryono et al. (2017) applied CoRe to the teaching of systems of linear equations and found that teachers often struggled to align their pedagogical strategies with the conceptual and procedural demands of the topic. This highlights the need for detailed guidance in the design of CoRe and ongoing support to effectively adapt its implementation.

In terms of the internal structure of CoRe, Zhang (2015) focused on teaching foundational mathematics concepts by analyzing CoRe implementation across three main components: "What" (foundational mathematical knowledge), "Who" (understanding how young learners approach mathematics), and "How" (mathematics-specific teaching methods). The findings from Zhang's study indicated that while increased competence in these areas correlates with improved teaching skills and student outcomes, only 4% of teachers in the study demonstrated a high level of pedagogical content knowledge (PCK). This underscores the critical need for targeted support in the "Who" and "How" components to foster meaningful growth in teaching practices.

Similarly, Suripah et al. (2021) evaluated CoRe development in mathematics instruction, identifying strengths in content importance (84.4%) and didactic objectives (81.3%). However, teachers scored lower (71.9%) in understanding student learning difficulties, emphasizing a key area for improvement. These findings align with broader calls for structured frameworks like CoRe to address specific challenges in mathematics teaching and enhance reflective practices among educators.

In summary, while the use of CoRe in mathematics education is less established than in science or other fields such as physics or technology integration, the existing studies highlight its potential to support teacher reflection, bridge the gap between theoretical understanding and classroom practice, and improve pedagogical strategies. These findings establish a robust foundation for future research and emphasize the necessity of extending its application to further advance mathematics education.

2.4 Professional knowledge and pedagogical content knowledge in mathematics

In the field of mathematics education, three large-scale studies have established the empirical research foundation for understanding teachers' CK and PCK and their relationship to student learning outcomes (Depaepe et al., 2015). These are: (1) the MKT (Mathematical Knowledge for Teaching) study, (2) the COACTIV (professional competence of teachers, COgnitively ACTIVating instruction, and development of students' mathematical literacy) study, and (3) the TEDS-M (Teacher Education and Development Study in Mathematics) study.

Although all three studies focus on mathematics education research and teacher preparation, each has a distinct emphasis. The MKT study primarily addresses mathematical knowledge specific to teaching. The COACTIV study examines teaching practices and development of students' mathematical literacy. The TEDS-M takes an international perspective on teacher education and development. Among these, the MKT study by Ball et al. (2008) provides a widely recognized theoretical model within mathematics education, outlining the essential knowledge needed by mathematics teachers.

More recently, a newer model, MTSK (Mathematics Teacher's Specialized Knowledge) proposed by Carrillo-Yañez et al. (2018), emphasizes aspects such as teacher emotions. Despite the availability of the MTSK model, the MKT framework remains the preferred model in studies on mathematics teacher education because it focuses on the specific mathematical knowledge that teachers need to develop for effective instruction, starting from their formative years (Groth and Meletiou-Mavrotheris, 2018). Notably, both models identify the same three components within PCK, differing from most disciplines that recognize four components. They also share the premise that a teacher's PCK is essential for improving teaching quality. Figure 3 illustrates the structure of MKT model for further analysis.

As shown in Figure 3, the MKT model delineates two main domains of knowledge: (1) Subject matter knowledge (SMK) and (2) Pedagogical content knowledge (PCK). The model is considered an integrated PCK framework because it merges mathematical knowledge and pedagogical knowledge to support effective mathematics teaching. These domains are further divided into specific components, as detailed below.

On the one hand, SMK comprises: Common Content Knowledge (CCK) or mathematical knowledge common to other professions; Specialized Content Knowledge (SCK) or mathematical knowledge specific to teaching mathematics; and Horizon content knowledge, knowledge connecting mathematical ideas across the curriculum, but is not considered a standalone component, as remains unclear whether it belongs to subject knowledge or another component.

On the other hand, PCK is subdivided into: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of content and curriculum. These three strands explicitly highlight the various dimensions and connections between curricular content knowledge, learning and teaching (Williams et al., 2012). Given that this type of knowledge will be the focus of investigation in this paper, a comprehensive description of its components is deemed essential, incorporating the contributions of its original proponents. First, KCS Combines knowledge of mathematics with knowledge of students' difficulties, questions, motivations, and expectations regarding specific content. This includes understanding common errors and anticipating students' misconceptions. Second, KCT integrates knowledge about teaching with mathematics. It involves making instructional decisions, selecting methods, and evaluating the advantages and disadvantages of the various representations used in teaching specific mathematical ideas. Finally, Knowledge of Content and Curriculum, focuses on deep knowledge of the mathematical content to be taught, including its objectives within the mathematics curriculum. This encompasses understanding the didactic goals, topics, and specific content for each educational level to ensure coherence with educational standards. Similar to the Horizon Content Knowledge, this component's boundaries are not well-defined and may overlap with KCT or span both primary domains.

In essence, the MKT model refines Shulman (1986, 1987) framework specifically for mathematics education. Shulman's original categories are incorporated into the MKT structure as follows:

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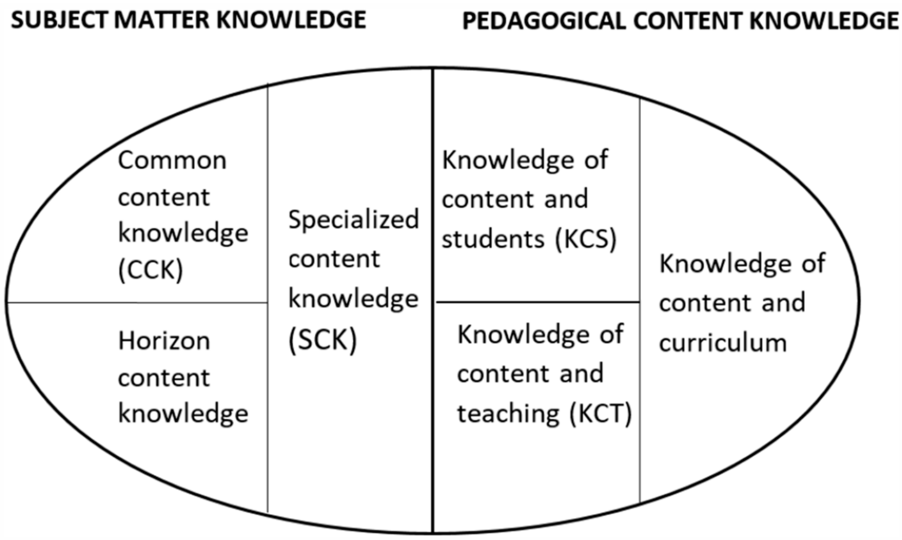

FIGURE 3
The common representation of MKT (Ball et al., 2008, p. 403).

- Content Knowledge and Pedagogical Content Knowledge align with the two major domains of knowledge: SMK and PCK.
- Curricular Knowledge is embedded within PCK as one of its components due to its ambiguous boundaries.

Since the PCK domain of the MKT model encompasses Shulman's essential PCK components through its three subdivisions, the MKT framework is considered an ideal structure for exploring the PCK of future teachers. Its strong foundation in early theoretical principles makes it particularly relevant for mathematics teacher education research.

3 Objective and research question

As part of a larger study aimed at conducting an exploratory evaluation of the initial training that prospective primary school teachers receive on didactic content knowledge, specifically in relation to fractions and the concept of to the fraction as an operator, this article focuses on the following specific objective and research question:

Objective 1 (O1): To determine whether the selected evaluation instrument, the CoRe, aligns with the three components established by the MKT model as part of a teacher's pedagogical content knowledge: curriculum, teaching and student learning.

Research Question 1 (Q1): Does the CoRe align with the structure defined by the MKT model for evaluating the components of PCK?

4 Methodology

This section provides a detailed description of the study participants, the design and implementation of the research instrument, as well as the procedures for data collection and analysis aimed at achieving the research objective.

TABLE 1 Distribution of respondents by sex.

Sex	Grade	Participants
Male	3rd	31
	4th	32
Female	3rd	115
	4th	85

4.1 Sample

The study involved a total of 263 pre-service teachers of both sexes (23.95% of male, as detail in Table 1) from 8 intact groups selected out of 11 existing groups within the Primary Education Teacher's Degree program at the University of Valencia, Spain. These participants were students enrolled in the third and fourth grade of the program during the 2021–2022 academic year. The sample was selected randomly and representatively, ensuring a confidence level of 95% with a margin of error of 8%. Specifically, the sample comprised 146 participants from the third year out of a total population of 419 and 117 participants from the fourth year out of a total population of 504. The typical age of participants was 22 years (mean = 22.4, range = 20–25 years).

It is important to note that none of the participants had professional teaching experience at the time of the study. Therefore, the information gathered reflects their initial training, unaffected by maturation gained through practical experience in the profession. Additionally, although the sample was one of convenience, the participants did not exhibit characteristics that differentiated them from the broader student population. The only notable differences between the academic years were that the 4th year students had completed a specific subject in Arithmetic Didactics and had an additional month and a half of internship in a school setting. These differences could potentially influence the results of the questionnaire.

4.2 Instrument

The selected assessment tool is the Content Representation (CoRe) introduced by Loughran et al. (2004). This instrument originates from specialized literature on PCK and has gained significant prominence in research on science education, while also proving valuable in other disciplines.

The CoRe aims to assist teacher in identify key aspects of content, establishing connections and relationships between concepts, and designing activities that facilitate student learning. In essence, this instrument was developed to encourage teachers (both pre-service and in-service) to reflect on various aspects of their professional knowledge and instructional practices, thereby providing insight into their concerns and needs.

Specifically, the CoRe focuses on three key aspects: (1) organizing and structuring content by identifying and determining the sequence in which concepts will be taught, as well as their connections and relationships, (2) representing content in ways that are visually appealing, engaging and meaningful for students, and (3) guiding teaching practices and designing activities aligned with the significance of the specific content. These three dimensions, as noted by Williams et al. (2012), explicitly outline the different aspects and interconnection between curricular content knowledge, learning and teaching. Moreover, they align with the components of PCK in the MKT model proposed by Ball et al. (2008): Knowledge of Content and Curriculum (KCC), Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT), respectively.

It is essential to note, however, that the specific design and content of the CoRe questionnaire may vary depending on the study or research context in which it is employed. According to Kind (2009), while the CoRe provides a comprehensive overview of teaching approaches and the rationale behind instructional decisions, it can subsequently be revised and adapted as a pedagogical and collaborative tool for lessons planning.

In all cases, the CoRe instrument begins with a situation or problem specific to the discipline, followed by a series of questions designed to explore the teacher's understanding of how to teach and present the selected mathematical content.

In this study, we adopted the version of the CoRe developed by Verdugo-Perona (2017) for use with pre-service science teachers. This version was adapted to focus on a specific content area, fractions, and id detailed in Figure 4.

In particular, the knowledge intended to be assessed in the broader study on which this work is based focuses on the concept of fractions as an operator (Kieren, 1980), due to its insufficient treatment in textbooks (Kieren, 2020; Ríos, 2007) and the challenges it poses for students (González del Olmo, 2015; Rueda Seguro, 2018; Tsai and Li, 2016). For this reason, one of the explicit contents in the initial situation, which is

GRADE: ______

SEX: MALE___ FEMALE_____

AGE: ______

DEGREE SPECIALTY: ______________

STARTING SITUATION
Fractions are studied in 4th, 5th and 6th grade of Primary School. Some of the contents of this topic included in the relevant curriculum are:
- Concept of fraction as a division of natural numbers. Relationship between fractions and decimals.
- Graphic representation of proper (for example, 1/2) and improper fractions (for example, 5/3).
- Meaning and utility of fractional and decimal numbers in personal and social contexts (commercial invoices, sales, taxes, etc.). Solving everyday problems involving fractions.
-Calculation of the product of a fraction by another number, either natural or fractional.

QUESTIONS:

Q1. What would you try to get students to learn about this particular situation (objectives)?

Q2. Why do you think it is important for students to learn what has been stated above (relevance of the topic or situation)?

Q3. Do you know the possible learning difficulties of children or their alternative ideas about this situation? Justify your answer.

Q4. Do you know the difficulties or limitations in the teaching about the mentioned aspects? Justify your answer.

Q5. What teaching methodology would you use to obtain greater learning from the students in the case presented? What specific activities would you propose?

Q6. How would you evaluate if the students have really achieved the objectives set at the beginning?

FIGURE 4
CoRe instrument for the analysis of future teachers' PCK on fractions.

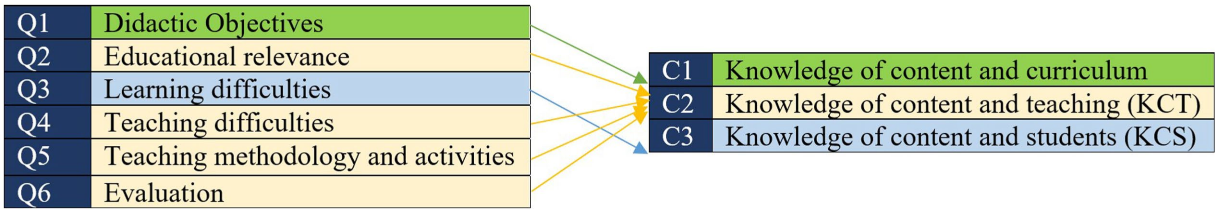

FIGURE 5
Relationship between the CoRe instrument and the Pedagogical Content Knowledge components.

addressed through the six questions of the questionnaire, is the "Calculation of the product of a fraction by another number, either natural or fractional." However, to avoid limiting the instrument to this specific content and to foster the development of a broader understanding of fractions, related concepts have also been included. These concepts were drawn from the 4th, 5th, and 6th grades of Primary Education, in accordance with Royal Decree 126/2014 of February 28 (2014), which established the basic curriculum for Primary Education at that time. These additional topics include the concept of fractions, the interpretation of a fraction as the division of two natural numbers, its decimal representation, the graphical representation of proper and improper fractions, their meaning and utility in social contexts, and the resolution of everyday problems involving fractions. However, it is important to note that this study does not aim to evaluate the acquisition of mathematical knowledge, as this lies outside the scope of the present research.

The CoRe instrument includes six questions focusing on curricular content knowledge, teaching and learning related specific knowledge (see Figure 4).

Regarding Q1 (didactic objectives), the aim is to gain a clear understanding of what students are expected to achieve and comprehend by the end of the instructional process. Q2 addressed educational relevance, seeking to identify the reasons that justify the importance of teaching fractions to students. Q3 focuses on student difficulties, aiming to explore the participant's knowledge and understanding of common conceptual errors among children regarding fractions, as well as their potential questions, expectations and motivations, to be support their learning success. Q4 revolves around teaching difficulties, seeking to gather insights into the challenges educators face when teaching fractions. This includes issues ranging from a lack of resources and classroom diversity to teachers' ability to explain abstract or complex concepts effectively. While related, Q5 delves into methodology and teaching activities, concentrating on the teacher's knowledge of methods, strategies, and procedures for delivering instruction on specific content. Finally, Q6 addresses the assessment process, aiming to evaluate the respondent's understanding and knowledge of assessment strategies to measure student learning outcomes effectively in the context of fractions.

As can be seen, these six questions provided not only a comprehensive overview of the teaching practice related the initial scenario and the knowledge encompassed within the three components of the PCK domain in the MKT model, but also offer a basis for linking each question to the respective components of the MKT model as described in the literature by Ball et al. (2008) (see Figure 5). This alignment represents the central objective of our work.

First, Q1, which focuses on teaching objectives, can be linked to C3 (Knowledge of Content and Curriculum), as it gathers information about teachers' specific knowledge regarding teaching standards and curricular objectives when planning instruction. Second, Q2, Q4, Q5, and Q6, which address the educational relevance of content, teaching difficulties, methodology and assessment, can be associated to C2 (Knowledge of Content and Teaching). These questions collectively capture knowledge about the most appropriate instructional decisions, ranging from justifying the relevance of teaching a particular content to assessing it in students. Third, Q3, which focuses on students' difficulties in learning, aligns with C1 (Knowledge of Content and Students). This question pertains to the knowledge teachers need regarding common errors made by students in relation to the mathematical content, as well as their preparation and response to address these difficulties effectively.

4.3 Research variables

As detailed in the instrument, six questions form the foundation of our assessment tool and define the six research variables; Didactic objectives (Q1), Educational relevance (Q2), Learning difficulties (Q3), Teaching difficulties (Q4), Teaching methodology and activities (Q5), and Evaluation (Q6). However, each of these variables also encompasses multiple associated concepts that are evaluated. Consequently, while Q1 to Q6 are the primary variables, each is calculated based on several sub-variables, enabling a more detailed and exhaustive analysis. The identification of these sub-variables was guided by expert input as well the iterative categorization and analysis of the participants' answers to the questions.

In total, 53 sub-variables were identified, and the scoring for each was stablished as follows: 0 if the response was incorrect, 0.5 if the response was correct but incomplete, and 1 if the response was correct and complete. Notably, blank responses were not permitted. The evaluation of each response was conducted based consensus among experts, achieving a high degree of inter-rater agreement (always exceeding 0.8) after three iterative correction cycles involving the three researchers. The iterative analysis conducted by each researcher for every participant's responses enabled a comprehensive evaluation of the data, thereby strengthening the study's validity and ensuring robust findings.

Using the quantification, the value of the six research variables was calculated as the average of their respective sub-variables, rescaled from 0 to 10, following (Equation 1):

$$Q_i = \frac{\sum_{j=1}^{k} p_{ij}}{1 \cdot k} \cdot 10, \quad (1)$$

where k is the number of sub-variables of question i, 1 is the maximum score value, and 10 is the rescaled value.

Similarly, the overall questionnaire result, QT, considering all six questions as a whole, was calculated as the average of the six variables, scaled from 0 to 10. Further details are provided in Supplementary Table I.

As shown in Table 2, Q5 consisted of seven sub-variables, and its rescaling followed the equation: $Q_5 = \frac{\sum_{j=1}^{7} P_{5j}}{1 \cdot 7} \cdot 10$.

Notably, for each of the questions (Q1 to Q6), a sub-variable was established to account for general or imprecise responses. This allowed identification of participants who provide related to each variable but failed to contribute substantive information. Specifically, in Q5, Q5.6 captured this aspect when related to methodology, and Q5.7 did so when related to activities.

TABLE 2 Sub-variables for Q5 (Teaching methodology and activities).

Variable	Sub-variables
Q5 Teaching methodology and activities	Q5.1 General instructional approach (active, constructivist, etc.)
	Q5.2 Arguments for/against certain methodologies
	Q5.3 Organization: roles of teacher and students, environment, etc.
	Q5.4 Types of tasks (observing, experimenting, discussing)
	Q5.5 Concrete activities associated with stated objectives.
	Q5.6 General and vague methodology
	Q5.7 General and vague activities

4.4 Data collection and analysis process

To access participants, permissions were first obtained from the instructors of each group. Data collection occurred 1 month after the beginning of the academic term, specifically in October 2021. Participants, were informed a few days in advance, and their voluntary participation was requested, with approximately 85–90% of the officially enrolled members of each group agreeing to participate. A researcher distributed paper copies of the CoRe instrument (as shown in Figure 5) and read the instructions aloud, allowing 55 min for task completion. No personal information was requested beyond grade, gender, age, and degree specialization, ensuring complete anonymity of the questionnaires.

Once collected, participant responses were evaluated according to the variables, sub-variables, and criteria described earlier. To determine whether the instrument effectively measured the three components of PCK in the MKT model, a Confirmatory Factor Analysis (CFA) was conducted (Brown, 2015). Prior to the CFA, the following analyses were performed:

1 An inferential study was carried out at a with a 95% confidence level to determine whether significant differences existed based on grade or sex. Given the non-normal distribution of the research variables (Kolmogorov–Smirnov test), the Mann–Whitney U test was used to compare medians (U; *p*-value).

2 A study about the reliability of the scale scores in our study sample (Frías-Navarro, 2022). Although Cronbach's alpha (α) is a commonly used coefficient for internal consistency, our study utilized McDonald's omega coefficient (ω) due to violations of the continuity assumption underlying Cronbach's alpha and the three-level scoring system of our instrument (incorrect, partially correct, correct). An acceptable omega reliability value ranges from 0.70 to 0.90 (Campo-Arias and Oviedo, 2008), although values exceeding 0.65 may be acceptable in certain contexts (Katz, 2006). If lower values were observed, the standard error of measurement (S_e) was calculated using the formula (Equation 2):

$$S_e = standard\ deviation \cdot \sqrt{1 - \omega} = 0.245 \quad (2)$$

This equation represents the standard deviation of measurement errors and can be interpreted as the variation in a subject's empirical scores across repeated test applications. As the distribution of empirical scores broadens, the reliability coefficient of the test diminishes.

3 An exploratory factor analysis was conducted in accordance with Cohen et al. (2007), adhering to the assumptions of Bartlett's test of sphericity (1950) (*p*-value <0.05) and Kaiser-Meyer-Olkin (KMO) index (KMO > 0.7). If these assumptions were met, factor extraction was performed using minimum residuals, as maximum likelihood estimation is inappropriate for non-normally distributed data. For factor rotation, oblique rotations were applied, with Simplimax identified as the most effective method (Kiers, 1994; Ferrando and Anguiano-Carrasco, 2010). Finally, parallel analysis (Horn, 1965) was used to determine the number of factors by selecting those with eigenvalues exceeding those obtained by chance.

All data exploration and analysis were conducted using the open-source statistical software JAMOVI (The JAMOVI Project, 2022).

5 Results

5.1 Inferential study

Firstly, we analyzed whether there were differences by grade, also taking sex into account, to determine if the two grades should be treated as independent samples. Table 3 shows significant differences by grade (U = 6,742; *p*-value = 0.003), but not significant differences by sex (U = 6,258; *p*-value = 0.936). The results confirm our hypothesis, making it necessary to perform the subsequent analysis differentiating between grades.

Figure 6 illustrates that while differences in the overall questionnaire scale exist, they are not substantial, with an average value difference of 0.3 points. This suggests that the Didactics of Arithmetic subject completed by 4th year students and their additional practice hours did not significantly enhance their knowledge.

Finally, we evaluated potential sex differences within each grade. The results indicate no significant differences for either third-year (U = 1735; *p*-value = 0.821) or fourth-year (U = 1,259; *p*-value = 0.536, respectively) students.

TABLE 3 Differentiation of scores by question according to sex and grade U(*p*-value).

	Sex	Grade
Q1	6,255 (0.932)	8,135 (0.505)
Q2	6,013 (0.530)	8,153 (0.465)
Q3	6,092 (0.666)	8,316 (0.688)
Q4	6,190 (0.812)	7,563 (0.068)
Q5	5,937 (0.475)	6,460 (<0.001)
Q6	5,883 (0.399)	7,381 (0.044)
Global	6,258 (0.936)	6,742 (0.003)

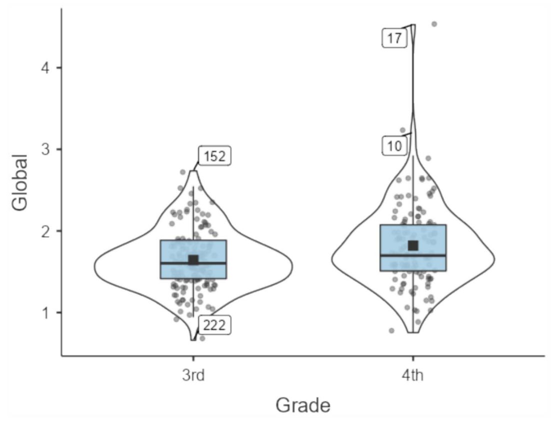

FIGURE 6
Global scores by grade.

5.2 Reliability of the scale scores

Before assessing the internal consistency of the instrument, it is important to examine the overall scores and the difficulty index of the items. This can be analyzed through the weighted response by grade (Table 4), where the index represents success rather than difficulty.

Table 4 reveals that third- year students achieved higher scores in Q1 (Didactic Objectives) and Q2 (Educational Relevance), while in the fourth-year students excelled in Q1 (Didactic Objectives) and Q5 (Teaching methodology and activities). However, both groups exhibited generally low scores variables. Scores ranged from a minimum of 0 to a maximum of 6 points, resulting in a maximum success rate of 6 points out of 10. The 25th percentile reflected a success rate just above 0.9, while the 75th percentile did not exceed 3 points.

Regarding the internal consistency of the questionnaire, reliability is not an inherent property of the questionnaire but of the scale scores applied to a particular sample (Thompson and Vacha-Haase, 2000). For both grade, the reliability, assessed using McDonals's ω, was acceptable (Table 5). For third year students, the standard deviation was 0. 367, ω = 0.444, and the standard error of measurement (Se) = 0.245. For fourth-year students, the standard deviation was 0.516, ω = 0.573, and Se = 0.337.

5.3 Exploratory factor analysis

After examining the inter-item correlation matrix, exploratory factor analysis was conducted (Table 6). In both grades, two factors were identified accounting for over 44% of the cumulative variance. Although the KMO value exceeded 0.6 (but

TABLE 4 Item difficulty indices by grade.

		Q1	Q2	Q3	Q4	Q5	Q6	Global
Mdn (RIC)	3°	2.08 (1.25)	2.00 (1.00)	1.00 (0.50)	0.91 (0.00)	1.43 (0.71)	1.25 (0.00)	1.60 (0.47)
	4°	2.50 (1.25)	2.00 (1.00)	1.00 (0.50)	0.91 (0.45)	2.14 (1.43)	1.25 (1.25)	1.70 (0.56)
Min (Max)	3°	0.00 (5.00)	1.00 (6.00)	0.50 (3.00)	0.00 (2.27)	0.71 (4.29)	0.00 (3.75)	0.66 (2.74)
	4°	0.00 (5.00)	1.00 (6.00)	0.00 (4.00)	0.00 (4.55)	0.71 (5.71)	0.00 (5.00)	0.76 (4.53)
SW (p-val)	3°	0.97 (0.003)	0.78 (<0.001)	0.82 (<0.001)	0.80 (<0.001)	0.88 (<0.001)	0.82 (<0.001)	0.98 (0.036)
	4°	0.97 (0.007)	0.67 (<0.001)	0.80 (<0.001)	0.72 (<0.001)	0.88 (<0.001)	0.88 (<0.001)	0.92 (<0.001)
Q_1	3°	1.67	2.00	1.00	0.91	1.43	1.25	1.42
	4°	1.67	2.00	1.00	0.91	1.43	1.25	1.51
Q_2	3°	2.08	2.00	1.00	0.91	1.43	1.25	1.60
	4°	2.50	2.00	1.00	0.91	2.14	1.25	1.70
Q_3	3°	2.92	3.00	1.50	0.91	2.14	1.25	1.89
	4°	2.92	3.00	1.50	1.36	2.86	2.50	2.08

TABLE 5 Considerations on the reliability of scale scores in the sample.

Grade	ω McDonald's	ED Global	S_e
3rd	0.444	0.367	0.245
4th	0.573	0.516	0.337

TABLE 6 Factors, suppositions, and cumulative variance for exploratory factorial analysis.

Grade	Factorial rotation	Factor 1	Factor 2	KMO	Barlett (p-value)	%accumulated variance
3rd	Simplimax	Q1, Q2, Q3	Q4, Q5, Q6	0.614	33.9 (0.003)	44.2%
4th	Simplimax	Q2, Q3	Q1, Q4, Q5, Q6	0.644	<0.001	48.9%

TABLE 7 Confirmatory factor analysis results.

Grade	χ^2	g.l.	Parsimony adj.	p-value	CFI	TLI	Comp. adj	RMSEA	SRMR	Abs adj.	IC90% RMSEA
3rd	3.93	8	Yes	0.86	1	1.38	Yes	0.00	0.03	Yes	(0.00, 0.05)
4th	7.17	8	Yes	0.52	1	1.04	Yes	0.00	0.04	Yes	(0.00, 0.01)

not 0.7), Bartlett's test indicated that factor analysis was feasible (p-value <0.05). Minimum residuals were used for factor extraction due to non-normal data distribution, and Simplimax was applied for oblique rotations (Kiers, 1994; Ferrando and Anguiano-Carrasco, 2010). Parallel analysis (Horn, 1965) confirmed the factors, retaining those with eigenvalues greater than expected by chance.

The results indicate two PCK components (Factor 1 and Factor 2) rather than the three outlined in the MKT model. The six variables are distributed as follows:

For third year:

- o Factor 1. Didactic objectives (Q1), Educational relevance (Q2), Learning difficulties (Q3).
- o Factor 2. Teaching difficulties (Q4), Teaching methodology and activities (Q5), and Evaluation (Q6).

For fourth year:

- o Factor 1. Educational relevance (Q2) and Learning difficulties (Q3).
- o Factor 2. Didactic objectives (Q1), Teaching difficulties (Q4), Teaching methodology and activities (Q5), and Evaluation (Q6).

5.4 Confirmatory factor analysis

Unlike exploratory factor analysis, confirmatory factor analysis assumes that the researcher is capable of hypothesizing the structure of the data in advance, preferably based on a well-established theoretical framework. In this study, the theoretical foundation is the PCK model as conceptualized in the MKT framework. The goal of confirmatory factor analysis is to verify empirically whether the hypothesized structure aligns with the observed data, which constitutes the central focus of this paper.

Confirmatory factor analysis requires the existence of a clearly articulated theory that serves as the basis for the developing of a model. The empirical analysis then tests whether the model adequately fits the data. A good model fit requires that the parameters comprising the model demonstrate both the expected direction and statistical significance.

Refering back to Ball et al.'s (2008) framework for the MKT, PCK is divided into three components of content-specific knowledge: (a) Student thinking (KCS), (b) Instructional strategies (KCT), and (c) Curriculum (KCC). Thus, our six variables are subdivided into:

1 Knowledge of content and students (KCS): Learning difficulties (Q3).
2 Knowledge of content and teaching (KCT): Educational relevance (Q2), Teaching difficulties (Q4), Teaching methodology and activities (Q5), and Evaluation (Q6).
3 Knowledge of content and curriculum (KCC): Didactic objectives (Q1).

To assess whether the proposed model adequately fits the data, several statistical measures need to be evaluated, not just the p-value. One key statistic is the Root Mean Square Error of Approximation (RMSEA), which measures the degree of variance unexplained by the model relative to the degrees of freedom. An RMSEA value below 0.05 is considered indicative of a good model fit, provided that the 90% confidence interval (C.I.) for the RMSEA lies between 0 and 0.05. Additionally, other fit indices such as the Comparative Fit Index (CFI) must exceed 0.95 to support the adequacy of the model. It is advisable to present these indices alongside the chi-square statistic (χ^2), the degrees of freedom, and the associated likelihood to provide a comprehensive evaluation of the model fit.

As shown in Table 7, although the p-value of the model does not indicate a perfect fit, all other fit indices support the validity of the model. For the third year: $\chi^2 = 3.93$, CFI = 1, Tucker-Lewis Index (TLI) = 1.38, Standardized Root Mean Square Residual (SRMR) = 0.026, and RMSEA = 0.0. For the fourth year: $\chi^2 = 7.17$, CFI = 1, TLI = 1.04, SRMR = 0.038, and RMSEA = 0.0. Based on these results, it can be concluded that the proposed model demonstrates a satisfactory fit to the data, validating its appropriateness.

Based on these findings, the CoRe instrument is validated as a reliable tool for assessing PCK within the framework of the MKT model, effectively aligning its questions with the model's three components. Our findings suggest that the CoRe instrument could be implemented in teacher education programs, specifically in courses dedicated to mathematics education, to assess participants' Pedagogical Content Knowledge (PCK) both before and after completing these courses. This implementation would allow the course development to focus on addressing the deficiencies or challenges identified in the initial questionnaire. The topics that could be assessed are diverse, including, for instance, the specific case of fractions as an operator.

6 Discussion

Ensuring that prospective teachers develop robust Pedagogical Content Knowledge (PCK) is crucial to improving mathematics instruction and ultimately fostering students' success. This study sought to contribute to that goal by validating the usefulness of Content Representation (CoRe) as an instrument aligned with the three elements of Mathematical Knowledge for Teaching (MKT) proposed by Ball et al. (2008): Knowledge of Content and Curriculum (KCC), Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT). Although CoRe has been used extensively in science education (Hume and Berry, 2011; Boothe et al., 2023), its application in mathematics, specifically in the domain of fractions, has been more limited. Through this research, we captured critical aspects of PCK, such as preservice teachers' awareness of student misconceptions, their strategies for teaching and representing content, and their ability to plan effective learning activities and assessments.

The findings highlight that, despite CoRe's strengths, significant gaps persist in the PCK of future teachers. Notably, participants demonstrated difficulties in anticipating students' errors and selecting instructional approaches that cater to diverse learning needs—aligning with Copur-Gencturk and Li (2023) and Li and Copur-Gencturk (2024), who emphasize that not all components of PCK evolve uniformly. Understanding how students think about mathematics, particularly complex fraction concepts, appears especially resistant to quick improvement and benefits substantially from structured reflection on real teaching scenarios (Zolfaghari et al., 2021). In this regard, our results extend observations by Suripah et al. (2021), who found that preservice teachers using CoRe can become more attuned to possible student misconceptions, yet still struggle to devise methods that effectively address those challenges in practice. Similarly, Zhang (2015) underscores the need to situate reflection on content representation within a broader framework of professional development; while CoRe can initiate critical thinking about fraction content, teachers may require ongoing support to translate those insights into durable classroom practices.

In our study, fourth-year participants scored slightly higher on items related to methodology and classroom activities, suggesting that added exposure to school-based practicum and coursework can enhance preservice teachers' PCK. However, the overall low scores underscore the need for more deliberate interventions early in teacher training. This resonates with Tröbst et al. (2019), who argue that well-structured instruction on fractions not only reinforces content knowledge but also enhances the pedagogical capacity to integrate conceptual and procedural views—a critical skill for ensuring that students develop a nuanced understanding of fractions.

Persistent difficulties with fractions as operators have been noted in Spain for over a decade (Gómez and Gutiérrez-Gutiérrez, 2014; Castro-Rodríguez and Rico, 2021), and our data suggest that these cannot be attributed solely to a lack of content mastery. Instead, they point to an underdeveloped capacity for diagnosing and addressing children's misunderstandings, which becomes evident when teachers attempt to enact curriculum objectives in real classrooms (Rodríguez Rojas and Navarrete Rojas, 2020; Páez et al., 2023). CoRe holds promise in helping future teachers reflect on these challenges by encouraging them to make explicit links between curriculum goals, pedagogical strategies, and common student errors (Suripah et al., 2021). Yet, as Zhang (2015) observes, one-shot exposures to content representation activities may not suffice; preservice teachers typically require repeated cycles of practice and reflection to progress beyond initial conceptual stages.

Several broader implications emerge. First, teacher education programs could more systematically incorporate CoRe alongside other reflective practices—such as video-based lesson analyses or collaborative lesson design—to support novice teachers in pinpointing and tackling persistent student misconceptions. Second, aligning coursework on fractions with field experiences can facilitate the connection between theory and practice, reinforcing preservice teachers' confidence in identifying fraction subconstructs (Cramer et al., 2002). Third, the cyclical nature of goal-setting observed in our data supports Verdugo-Perona (2017) view that novice teachers iteratively refine their objectives and methods as they gain more exposure to classroom complexities.

Future research could focus on how CoRe, together with emerging frameworks for fraction learning, might be adapted to various cultural and educational contexts, as recommended by Forsler et al. (2024). More longitudinal studies, following the model of Li and Copur-Gencturk (2024), may also shed light on the factors that facilitate or hinder the ongoing development of PCK. Insights from such work could guide teacher education programs in devising targeted interventions that specifically address fraction misconceptions, whole-number biases (Ni and Zhou, 2005), and the multifaceted nature of rational numbers (Ni, 2001).

Overall, our findings validate CoRe as a potentially powerful tool for diagnosing and advancing PCK, especially regarding fractions—a topic widely recognized for its conceptual and didactic complexities. By integrating CoRe into teacher education curricula and complementing it with practical experiences, structured reflection, and theoretical knowledge about fraction subconstructs (Kieren, 2020), educators might be better equipped to foster enduring mathematical understanding in their future classrooms. This effort would extend beyond isolated lessons or generic plans, reflecting the sustained cycle of analysis and action that Suripah et al. (2021) and Zhang (2015) identify as key to enhancing preservice teachers' effectiveness in mathematics teaching.

7 Conclusion

This study validates the CoRe (Content Representation) instrument as aligned with the components of Ball et al.'s (2008) MKT model for Pedagogical Content Knowledge (PCK) through confirmatory factor analysis. Administered to 263 pre-service teachers in their third and fourth years of study, the CoRe demonstrates its potential as a robust tool for assessing and fostering PCK within teacher education programs.

The findings reveal the dynamic and evolving nature of PCK development, with variations in performance across academic years and questionnaire items. However, the overall low scores underscore the pressing need for more effective training interventions, particularly in mathematical topics such as fractions, where misconceptions and instructional gaps persist. These challenges align with studies by Kelcey et al. (2019) and Hoth et al. (2022), who emphasize that pre-service teachers often struggle to transform conceptual understanding into sound instructional strategies. As suggested by Li and Copur-Gencturk (2024), teacher education and professional development programs should focus on how teachers can leverage their practice to enhance their knowledge and skills. Providing time and space for reflection and the analysis of their teaching experiences can be a cost-effective strategy for professional growth. Likewise, integrating tools such as CoRe into teacher education curricula can foster reflective practices and strengthen connections among curriculum design, pedagogical methods, and assessment.

In mathematics education, the CoRe is particularly valuable for improving fraction instruction. It supports lesson planning, anticipates common student misconceptions, and encourages the use of diverse representations (numerical, graphical, and manipulative). Additionally, it facilitates the design of contextualized activities and promotes collaborative exchanges among teachers, thereby enabling the adoption of effective pedagogical strategies.

Altogether, these findings suggest that the CoRe serves both as a reflective and practical tool for transforming disciplinary knowledge into effective teaching practices. Its systematic integration into teacher preparation programs has the potential to enhance PCK development, address the challenges of contemporary classrooms, and ultimately contribute to long-term improvements in student learning outcomes.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Ethics statement

Ethical review and approval was not required for the study on human participants in accordance with the local legislation and institutional requirements. Written informed consent from the participants was not required to participate in this study in accordance with the national legislation and the institutional requirements.

Author contributions

DH-T: Conceptualization, Data curation, Investigation, Visualization, Writing – review & editing. MS: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Writing – original draft, Writing – review & editing. CG-F: Conceptualization, Investigation, Methodology, Supervision, Validation, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Supplementary material

The Supplementary material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/feduc.2025.1274102/full#supplementary-material

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