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Prospective teachers' beliefs on mathematics during internships a quasi-longitudinal analysis

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Prospective teachers repeatedly emphasize the importance of practical phases during mathematics teacher education. Therefore, in light of the professionalization of teacher education, prospective teachers' beliefs during their internships are examined within a (quasi-)longitudinal study. The purpose of this study is to evaluate key points and desiderata to get a deeper insight into the landscape of prospective teachers' beliefs. The gathered results could be considered in the development of a theoretical framework for supporting courses related to the practical phases. Based on the results of this study, certain categories identified in the TEDS-M study could be confirmed. Moreover, the beliefs concerning the sub-categories of those categories do not change significantly during the practical phases. This stability is also expressed by the result that a dynamic view on mathematics, a constructivist focus on teaching mathematics and a self-active understanding of once own given lessons dominate a static view on mathematics, a transmissive focus on teaching mathematics and a traditional directive understanding of selfconducted teaching Finally, significant correlations between subcategories could be evaluated. Using the obtained results, a draft of a didactically based design supporting the practical phases is formulated.

KEYWORDS

epistemological beliefs, internship, mathematics education, TEDS-M, beliefs change, teacher education

1 Introduction

In mathematics education research, there is a strong assumption that teachers' beliefs potentially influence their classroom performance (Pajares, 1992; Skott, 2015; Eichler et al., 2023). Consequently, students' learning is shaped by those beliefs indirectly via the teachers' classroom practices (Eichler and Erens, 2015; Levin, 2015). In mathematics education research, these beliefs are often treated as personality traits and studied quantitatively using Likert-scale surveys (e.g., Laschke and Blömeke, 2014; Voss et al., 2013). This approach was notably used in the TEDS-M study and its follow-ups, which examined mathematics teachers' professional competences and beliefs about mathematics education (Tatto et al., 2008; Wang and Hsieh, 2014; Yang et al., 2020). As Eichler et al. (2023, p. 1490) state, "[...] investigation of the current status of teachers' beliefs and their development is a crucial topic for mathematics education research."

There is widespread agreement that prospective teachers start their teacher training program with fixed ideas which relate to a variety of aspects relevant to working as a teacher and are based on their own school experiences (Patrick and Pintrich, 2001). Therefore, the question arises if these experiences can be relativized by "strong impacts" (Eichler et al., 2023, p. 1491) like an internship during teacher education.

The investigation, which is presented in this paper, can be seen as a supplement to the studies by Eichler et al. (2023) who examined teachers' beliefs during their first 2 years of

classroom teaching experiences after their studies at university. The research focus is on the practical phases (internships) of prospective teachers during Bachelor and Master programs, which provide (first) practical experiences as a teacher and are accompanied by supporting courses at universities and mentors at schools. Mentors are supervisors who support the prospective teachers during their internships.

This research on beliefs is based on the categories "nature of mathematics," "teaching and learning mathematics," "nature of mathematical achievement in classes" and "the ability to prepare, to conduct and to reflect on mathematics lessons" partly identified by the TEDS-M study (Laschke and Blömeke, 2014).

2 Theoretical background

2.1 The concept of beliefs

Research on mathematics education has increasingly focused on teachers' beliefs over the past few decades as teachers' beliefs are a key affective component of their professional competence (Buchholtz and Vollstedt, 2024; Eichler et al., 2023; Philipp, 2007; Skott, 2015; Thompson, 1992). Dohrmann (2021) highlights Pajares' (1992) assertion that discussions about "teachers' beliefs" always refer to their views on school, teaching, learning, and students (Buchholtz and Vollstedt, 2024). Over 30 years ago, Pajares (1992) described teachers' beliefs as a "messy construct," and this complexity persists today. There is no consensus regarding an explicit definition or "no agreed definition" (Beswick, 2005, p. 39) of beliefs, according to Safrudiannur and Rott (2020, p. 2). The term "teachers' beliefs" appears both in general teacher research (Turner et al., 2009; Fives and Gill, 2014) and subject-didactical studies (Thompson, 1992; Philipp, 2007). In this article, beliefs are defined as "psychologically held understandings, premises, or propositions about the world that are thought to be true" according to Philipp (2007, p. 259). Following this definition, beliefs reflect a person's epistemological stance toward an object - its worldview - incorporating affective attitudes and willingness to act (Grigutsch and Törner, 1998; Buchholtz and Vollstedt, 2024). Unlike knowledge, they depend on individual agreement (Beswick, 2005, 2007). Therefore, beliefs encompass both cognitive and affective dimensions, shaping perception and interaction with the environment (Furinghetti and Pehkonen, 2002). While professional knowledge is widely accepted, beliefs vary significantly among individuals (Beswick, 2005; Philipp, 2007; Buchholtz and Vollstedt, 2024).

In their theory of belief systems, Green (1971) and Philipp (2007) distinguish between central and peripheral views (Eichler et al., 2023). Centralized beliefs are relatively stable and change only over time through training, experiences, or "strong impacts" (Buchholtz and Vollstedt, 2024). However, the factors influencing these changes remain unclear, with research yielding mixed results (Dohrmann, 2021). In contrast, peripheral beliefs are more adaptable and linked to expertise development and may be influenced by students' abilities (Buchholtz and Vollstedt, 2024).

Another approach is to distinguish between beliefs which are hold conscious and those which are not. They are interconnected with each other, influencing one another, with some being more central and firmly established than others (Furinghetti and Pehkonen, 2002). Here we define conceptions [...] as conscious beliefs, i.e. they form a subgroup of beliefs. In the case of conceptions, the cognitive component of beliefs is stressed, whereas in subconscious beliefs the affective component is emphasized. (Pehkonen and Pietilä, 2003, p. 2)

Teacher beliefs are organized into individual belief systems, where different – even contradictory – beliefs can coexist (Furinghetti and Pehkonen, 2002; Leder et al., 2002; Philipp, 2007). For example, a teacher may value conceptual understanding over memorization but still rely on routine exercises in class. This flexibility highlights the complexity of beliefs as both affective and cognitive constructs, influencing teaching practices in varying contexts (Buchholtz and Vollstedt, 2024).

Further, while beliefs are thought to guide teaching actions (Grigutsch and Törner, 1998), research on their consistency with classroom practices is inconclusive (Buchholtz and Vollstedt, 2024). Some studies show alignment (Staub and Stern, 2002; Safrudiannur and Rott, 2017), while others reveal discrepancies (Furinghetti and Pehkonen, 2002; Li and Yu, 2010). These inconsistencies may stem from situational influences, emotional involvement, lack of experience, or external factors like school (Buchholtz and Vollstedt, 2024).

The concept of epistemological beliefs is another approach to categorizing beliefs. It is important to note that epistemological beliefs can be conceptualized in a variety of ways and are not universally defined (Pajares, 1992; Philipp, 2007). Known as "individual representations about knowledge and knowing" (Mason and Bromme, 2010, p. 1), they are developmental and cognitive psychological models of beliefs about knowledge and how knowledge is acquired. The origins of belief research (e.g., Kitchener and King, 1981; Perry, 1968; Schommer, 1990) have led to epistemological beliefs being viewed as domain unspecific (Schommer-Aikins and Duell, 2013). Recent approaches, however, interpret beliefs as partially crossdomain and partially domain-specific (Buehl et al., 2002; Hofer, 2000; Schommer-Aikins, 2002). As Muis et al. (2006) argue in their "theory of integrated domains in epistemology," learners acquire domainspecific epistemological beliefs as soon as they enter the educational system (Urhahne and Kremer, 2023, p. 100). In recent years, several domain-specific epistemological studies have been conducted, especially in science (Conley et al., 2004; Urhahne and Hopf, 2004), but also in mathematics (Ernest, 1989; Grigutsch et al., 1998; Rott et al., 2015; Schreck et al., 2023).

In mathematics didactics, there is broad consensus on differentiating profession-related beliefs (Buchholtz and Vollstedt, 2024). Besides the model of Ernest (1989), which proposes three views of mathematics, the theoretical framework of Grigutsch et al. (1998) can be used to analyze prospective teachers' beliefs about the nature (structure) of mathematics. Through a questionnaire with 77 items, Grigutsch et al. (1998) examined the beliefs of mathematics teachers in Germany. Their study identified four distinct aspects that categorize teachers' belief clusters about the nature of mathematics. These aspects are referred to as a teacher's "mathematical world view" and can be defined as follows by Eichler et al. (2023, p. 1493):

• The formalist aspect represents beliefs implying that (school) mathematics is characterized by a strongly logical and formal approach.

- The schema aspect represents beliefs implying that (school) mathematics is a set of calculation rules and procedures to apply to routine tasks.
- The process-oriented aspect represents beliefs implying that (school) mathematics is a creative activity consisting of problem-solving using different and individual ways.
- The application-oriented aspect represents beliefs implying that the utility of mathematics for real-world problems is the main aspect of the nature of (school) mathematics.

Grigutsch et al. (1998) identified a positive correlation between the formalist and schema aspects, as well as between the processoriented and application-oriented aspects of teachers' beliefs. Based on these findings, they characterized the first two aspects as representing the static nature of mathematics, while the latter two reflect its dynamic nature (Viholainen et al., 2014).

Additionally, to measure beliefs about the structure of mathematics, studies have also been conducted on the acquisition of mathematical knowledge (constructivist vs. transmissive). These research approaches differ between transmission-oriented beliefs, where students are seen as passive learners (e.g., "Students learn best by attending to the teacher's explanations"), and constructivist beliefs, which emphasize on constructive learning (e.g., "Teachers should encourage students to find their own solutions to mathematical problems even if they are inefficient") (Buchholtz and Vollstedt, 2024, p. 3; Peterson et al., 1989; Staub and Stern, 2002; Laschke and Blömeke, 2014; Voss et al., 2013). While the impact of teacher beliefs on student achievement remains unclear, dynamic beliefs and constructivist approaches are linked to iterative, procedural mathematics instruction (Reusser et al., 2011).

An example may be found in the TEDS-M study (e.g., Laschke and Blömeke, 2014), which analyzed (in addition to beliefs about the structure of mathematics) such beliefs of mathematics secondary 1 teachers and reported the following main findings (cf. Eikmeyer, 2021, p. 148):

- Teachers may simultaneously hold constructivist and transmission-oriented beliefs about teaching and learning mathematics.
- Students whose teachers' beliefs on teaching can be interpreted as mainly constructivistic work more frequently with challenging text tasks and perform better after one school year than students whose teachers' beliefs are more on the transmissive pole of the spectrum.
- The amount of mathematics in teacher training has a significant impact on prospective teachers' beliefs regarding the structure of mathematics. Therefore, beliefs may change during teacher training.
- The majority of teachers prefer a construction-oriented approach to teaching.

The COACTIV study found a significant link between constructivist beliefs and both teaching quality and student learning outcomes (Voss et al., 2013). However, earlier studies showed that many in-service teachers held transmission-oriented beliefs, leading to less challenging, mistake-averse instruction (Dubberke et al., 2008). While Voss et al. (2013) identified a strong negative correlation between constructivist and transmissive beliefs, they suggest these beliefs are not fundamentally contradictory. Instead, an optimal balance between them may enhance teaching, with transmissive elements are benefitial in certain contexts or for specific student groups (Buchholtz and Vollstedt, 2024).

Accordingly, these findings regarding beliefs have motivated research projects to evaluate a potential change in beliefs or to stimulate a change with specific interventions. This research focus suggests that beliefs can be difficult to change (Liljedahl et al., 2012). To change (prospective) teachers' beliefs, a long-term, strong impact, such as the first teaching experience, is required (e.g., Eichler et al., 2023). Another possibility for adapting those concepts is the influence of the teacher trainers' beliefs in mathematics and mathematics didactics (Steinmann and Oser, 2012). However, central beliefs are hard to change (stable) even if they are faced with such a strong impact (Eichler et al., 2023, p. 1493).

As reported by Eichler et al. (2023, p. 1494) only a limited number of studies have examined the influence of practical experience on (prospective) teachers' beliefs, particularly when they begin teaching. Based on teachers' metaphors, Alger (2009) categorized beliefs in a quasi-long-term study. As a result of her research, teacher-centered teaching approaches increase after entering school but decrease in the long run, while student-centered beliefs increase ("Konstanzer Wanne," Hascher, 2012, p. 93; cf. Erens and Eichler, 2019, p. 346). Furthermore, teacher trainings do not sufficiently expand prospective mathematics teachers' epistemological beliefs without targeted interventions (Rott, 2020). According to Eichler et al. (2023), teachers' central beliefs changed significantly after their first practical experience. However, no general pattern of beliefs change was identified.

2.2 Internships—an important part of teacher training

Internships are designed to familiarize prospective teachers with their future professional reality and to make them aware of the demands that accompany it. In detail, students should be encouraged to develop their own lesson plans, should implement them independently at school, and afterwards, they should evaluate them. A mentor, who is an experienced teacher, supervises (and evaluates) all three components of the practical training.

Prospective teachers have high expectations on internships concerning their preparation for their future autonomous teaching (Hascher and Hagenauer, 2016 and the literature cited there). These desires can be explained as follows (Hascher, 2011, p. 9):

- 1. The desire to confirm subjectively assumed "professional inclination and suitability."
- 2. The desire for a successful synthesis of theory and practice in academic teacher training to confirm the fundamental nature and significance of this approach.
- 3. A desire to develop prospective teachers' professional competence through practical experience with the assistance of experienced teachers.

Unfortunately, Hascher (2011) provides contrary findings to these desiderata.

However, the importance of practical school training in students' education is widely shared by prospective teachers. An extension of the practical training in the study of mathematics is frequently desired and demanded by prospective teachers (Hascher, 2011). For instance, prospective teachers think that school practices in teacher training are not focused enough. Further, they believe that they do not have enough lessons to teach.

Nevertheless, it has been observed that the "subject-didactic tools" which are taught at the university, are not adequately addressed in the actual teaching of lessons (Eikmeyer, 2021). Moreover, this discrepancy may be further compounded by some instructions given by the supervising teachers ("practice shock"). Due to Hascher and Hagenauer (2016, pp. 15-16), prospective teachers showed a tendency to teacher-centered instruction instead of learner-oriented practices. Didactical concepts are substituted by "tips and tricks" received from their mentors. Many different reasons for this behavior can be found in the literature: quality of the internships, qualifications of the mentors and – another point – the lack of consideration of prospective teachers' individual characteristics (Hascher and Hagenauer, 2016, and the literature cited there). Therefore, during internships, adaption efforts dominate over subject and context specific learning processes (Hascher, 2012, p. 90; König and Rothland, 2018).

Consequently, innovative yet didactically approved ideas in lesson planning may be quickly replaced by tried and tested, traditional teaching elements (Eikmeyer, 2021, p. 62). On the contrary, Reichenauer's (2023) study of professionalization research in teacher education in Austria shows that traditional views of learning and teaching are declining while constructivist views are on the rise. Anyhow, Weyland and Wittmann (2011, p. 51 sqq.) highlight the relationship between the theoretical framework of school practice phases and the behavior of professional teachers. Their model of educational professionalism is based on three reference systems, namely "science, practice, and person" (ibid., p. 52). In summary, the accompanying courses play an important role in this specific phase of teacher training to prevent this development.

Analyzing a teacher's role, two sides of a coin appear. Teacher acting in a pedagogically professional manner namely has on the one hand a generalizable theoretical reflection knowledge and, on the other hand, a practical action-related and case-specific experiential knowledge (ibid.). By observing and analyzing educational system processes in German-speaking countries, Hascher (2012, p. 92) illustrates a close connection between practical work experience and science-based training. Thus, teachers' actions reflect a dual habitus: scientific-reflective and routine-practical. This duality should be discussed in the internships including the accompanying courses.

For the professional development in teacher education Hascher (2012, p. 93) emphasizes the reflection on one's own actions, too. The traditional form of reflection, namely debriefings after lessons, fails to meet these expectations (ibid.), so meta-debriefings could be considered during internships within accompanying courses. This could be an opportunity to transform unconscious beliefs into conscious ones (Subsection 2.1).

At the University of Vienna, a subject-related school internship (7 ECTS) is required in the fifth semester of the Bachelor's degree program for secondary education in mathematics. In the related Master's program, the third semester is reserved for the practical phase (15 ECTS). In both cases, attendance at an accompanying seminar is compulsory. Mathematical teacher training at the University of Vienna has not induced already a standardized framework of the accompanying courses during internship. In most cases, the process involves an individualized mix of practical school needs and specific

didactic intervention options. Prospective teachers discuss and reflect on their teaching experiences from all three phases (planning, implementation, and reflection on the lesson). These three components of the practical school training program are supervised (and evaluated) by a mentor. As a result of prospective teachers' reports, didactic problems can be identified (e.g., heterogeneity, performance assessment, varied methods, use of technology) and possible solutions can be suggested. As part of their work assignments, prospective teachers are required to describe the school's situation (school profile, focus, time structure, special features, social environment, class composition, etc.), conduct observation tasks (teachers, teaching prospective teachers, students or entire classes) and create detailed lesson plans (factual analysis, subject-didactic analysis, tasks, reflection on "special" teaching moments, etc.).

2.3 Project motivation and description

Due to the discussion in Subsection 2.1 there is a huge diversity of interpretations of beliefs (stable and instable, conscious and unconscious, epistemological, focusing on learning and teaching mathematics, etc.). The main assumption of this paper is that an internship can change prospective teachers' beliefs because the experience of an internship can be interpreted as a "strong impact" (Subsection 2.1). This assumption is supported by the prospective teachers' desires regarding the internships (Subsection 2.2).

Unfortunately, the related literature provides no specific, accurate indications regarding which general pattern of beliefs change can be evaluated caused by internships as a realization of a strong impact. Therefore, to cover up all discussed kinds of beliefs (Subsection 2.1), an extensive questionnaire was developed. For this reason, the TEDS-M study (Laschke and Blömeke, 2014) serves as a foundation. It is important to mention that in advance of this analysis, no assumptions on the directions of the possible changes of beliefs are assumed.

In detail, this project is designed to get empirical insights into students' beliefs regarding

- the nature of mathematics,
- teaching and learning mathematics,
- the nature of mathematical achievements (in classes),
- the ability to prepare, conduct, and reflect on mathematics lessons and
- one's own school practice.

The categories mentioned above, except for the last one, can be found in detail in the questionnaire of the TEDS-M study (Laschke and Blömeke, 2014). As part of his study, Eikmeyer (2021) included the school practice perceived during internships. This aspect is modified within this study: the main interest is situated in how the prospective teachers reflect upon their own teaching experiences during their internships. In common, the focus is on eliciting the mathematical beliefs and views of prospective teachers of mathematics regarding epistemology and school practice during internships.

In addition, the focus on students' beliefs during their internships contributes to professionalism research in teacher training, which examines the individuality of learning processes (Hascher and Hagenauer, 2016, p. 16). Considering the lack of studies that can be used to make statements regarding the conditions under which it is possible to contribute to professionalization through practical school training (Eichler et al., 2023, p. 1494), this article is intended to provide further information on this subject. The insights into the beliefs of prospective teachers are intended to deepen and expand the discussion of such possible conditions.

In summary, this study does not focus on any increase in competence but instead evaluating students' beliefs and their potential to change as a result of the anticipated "strong impact" of practical experiences (Eichler et al., 2023). In the view of the lecturers who supervise the accompanying courses, empirical research into the effectiveness of internships in teacher training is also important on a meta-level (Eikmeyer, 2021). "Effectiveness" is defined in the following manner:

- changes
- expansion and/or
- strengthening of mathematical beliefs,

which are related to the categories mentioned above. The scope of this investigation lies at the micro level of the research field "internship," and from this an assumption is to gain hints for developing a theoretical framework for the accompanying courses (meso level).

3 Research questions

Due to the introduction (Section 1), and the theoretical background (Section 2), three research questions are formulated:

- 1. Which factors of the categories given by the TEDS-M study can be exploratively evaluated due to prospective teachers' beliefs at the beginning of their internship?
- 2. Do the evaluated beliefs of prospective teachers change significantly after attending the internships?
- 3. Which correlations between the identified subcategories (of the categories mentioned in Subsection 2.3) do exist? Do these correlations change due to attending the internships?

4 Methodology

4.1 Sample and data collection

At the University of Vienna, both the Bachelor's and Master's mathematics programs for secondary school teachers involve a practical school training (Subsection 2.2). The study is conducted by recruiting prospective teachers who attended the mentioned accompanying courses related to their internships in 2023/24 (three semesters), within the Bachelor as well as in the Master program (Subsection 2.2). The pre-test was carried out in the first lessons of the courses, the post-test in the final lessons. The participation was voluntary. As a result, the distribution of the 186 evaluated questionnaires is shown in Table 1 according to the study phase (Bachelor/Master) and the phase at which they were collected (pre-survey/post-survey). Within this sample, only 31 people filled out the questionnaire in both phases (pre- and post-test). This group can be separated into six Bachelor and 25 Master students.

4.2 Test instruments

As part of the (quasi-)longitudinal design, a questionnaire on beliefs from the TEDS-M study was adapted (Subsection 2.3), which was used in both phases of the study. Additionally, the post-survey contains 28 items from Eikmeyer (2021) that had been slightly modified by the authors. This paper is a continuation of Woltron (2020) and Woltron and Götz (2023), which examined teachers' and students' beliefs about the nature of mathematics.

The questionnaire consists of 74 closed items that are scored on a six-point Likert scale: 1 means "do not agree at all," 2 means "disagree," 3 means "rather disagree," 4 means "rather agree," 5 means "agree" and 6 means "fully agree." Furthermore, two open questions were asked about experiences and their influence on participants' beliefs, but their evaluation is not discussed in this paper. The usual demographic data (age, high school grades in mathematics, second subject) were also collected, but are not included in this report.

It is possible to summarize or divide the 74 closed items into five categories:

- 1. Nature of Mathematics (NM): The items of this category include questions that explore how prospective teachers perceive mathematics as a subject (e.g., mathematics as formal, structural, procedural, or applied) (Tatto et al., 2008, p. 43).
- Beliefs about Learning Mathematics (LM): This category includes questions about the appropriateness of particular instructional activities, questions about students' cognition processes, and questions about the purposes of mathematics as a school subject (ibid.).
- 3. Beliefs about Mathematics Achievement (MA): This category contains items that explore beliefs about if mathematics ability may develop or whether it is considered a fixed characteristic (ibid.).
- 4. Beliefs about Preparedness for Teaching Mathematics (PT): The fourth area of beliefs concerns the extent to which future teachers perceive that their teacher preparation has given them the capacity to manage the central tasks of teaching and to meet the demands of practice in general (ibid.).
- 5. My perceptions of my internship teaching-experiences (EXP): The related items focus on the participants' experiences of their lessons given during their internship. Following this, the mentor's instructions or teaching strategies, which are also observed by the participants, are not targeted within this category (Eikmeyer, 2021).

A pilot study (Woltron and Götz, 2023) was used to refine some of the given items. For category 5, it was emphasized more strongly that the items relate to the prospective teachers' own mathematics lessons to prevent biases in evaluated beliefs due to adaption processes to mentors' school practice (cf. König and Rothland, 2018). In Table 2,

TABLE 1 Sample group.

	Pre-survey	Post-survey	Total
Bachelor	43	28	71
Master	76	39	115
Total	119	67	186

the weighting of the categories is shown based on the number of items of each category. In the category EXP, the items were naturally only asked after the school practice had been completed (Eikmeyer, 2021, p. 208 sqq.).

4.3 Data analysis

To identify and interpret subcategories (factors) within each category, a principal component analysis (PCA) was performed. Based on the strength of the loading (rotated component matrix), items were assigned to the individual factors. Only factors with eigenvalues ≥ 1 were considered (Guttman, 1954; Kaiser, 1960).

To assess the degree of agreement or rejection within the subcategories, the respective individual arithmetic means and medians were calculated. In order to obtain the mean (arithmetic/median) for each subcategory, the total mean of all participants' means was evaluated.

To get a deeper insight into prospective teachers' beliefs, single items also have been analyzed descriptively (arithmetic mean, median, standard deviation). The interest of this study focuses on items which are strongly agreed or disagreed, respectively, or on such items which provoke divers responses.

For the arithmetic means the ordinal scaled data are interpreted as quasi-metric (Laschke and Blömeke, 2014; Eikmeyer, 2021). This part of the analysis involves the comparison of the central tendencies obtained using the Mann–Whitney-U-test for unpaired samples in the quasi-longitudinal design. For the small, paired sample group the Wilcoxon-test is applied (longitudinal design).

Additionally, correlations between the individual subcategories were examined. Using Kendall-Tau τ correlation coefficients for ordinal scaled data, the subjects' total scores for the individual factors are calculated.

SPSS-28 and Excel were used for statistical analyses.

5 Results

The reported results of the PCA with forced factors refer to the pre-survey of the categories NM, LM, MA, and PT and unite both Bachelor and Master students' responses (Weygandt, 2021, p. 214 sqq.). In detail, the following factors according to the different categories are given (including the number of the items involved and an anchor example):

NM_{Dynamic}: Problem-solving and applied mathematics (six items, "Students have the opportunity to explore and experiment with many aspects of mathematics.")

TABLE 2 Item distribution.

Categories	Total number of items
NM	12
LM	14
MA	7
PT	13
EXP	28

NM_{Static}: Mathematics as a formal system with fixed procedures given by authorities (six items, "Mathematics means learning by heart, remembering and applying.")

LM_{Constr}: Learning math through active learning: constructivist view (nine items, "Time used to investigate why a solution to a mathematical problem works is time well spent.")

LM_{Trans}: Learning math through teacher direction: transmissive view (five items, "Students learn mathematics best by attending to the teacher's explanations.")

MA: Mathematic achievement as a fixed ability (seven items, "Some people are good in mathematics and some are not.")

PT: Conviction to be prepared for teaching mathematics (13 items, "I believe my teacher training has prepared me to challenge students to think critically.")

The internal consistency differs from Cronbach's $\alpha = 0.629$ to $\alpha = 0.909$ (cf. Tatto, 2013, p. 170). A slight decrease in the internal consistency of the other categories might be caused by the addition of one category (EXP) and the extension of the questionnaire. The total variation explained has a range from 33.824 to 49.58%.

For the PCA of the category EXP, which was naturally only a part of the post-survey, considering the findings of Eikmeyer (2021, p. 195), two factors are determined: self-active (EXP_{Self-Act}) vs. the traditional-directive (EXP_{Trad-Dir}) view:

- 1. EXP_{Self-Act}: self-active (16 items, "As a teacher, I encourage each student to be self-active in my lessons.," $\alpha = 0.859$)
- 2. EXP_{Trad-Dir}: traditional-directive (11 items, "As a teacher, I predetermine calculation methods and strategies," $\alpha = 0.678$)

One item of the subcategory $\text{EXP}_{\text{Trad-Dir}}$ was neglected for a better internal consistency. The proportion of the total variation explained is 31.403%.

In Table 3, the descriptive statistics for each subcategory are presented. The examined sample contains both sub-groups, Bachelor and Master students, for a quasi-longitudinal analysis. To justify this decision, the results of these sample groups for each subcategory are compared (Mann–Whitney-U-test). There were no statistically significant differences between the tested sub-groups, except for the factor LM_{Constr} within the pre-survey. The effect size is small (r < 0.2; Cohen, 1988, p. 79 sqq.). The interpretation in the right column relates to the arithmetic means, not to the medians.

As can be seen in the third and the fifth column of Table 3, there are only small differences between the pre- and post-survey means. They are not statistically significant (Mann–Whitney-U-test). Therefore, no significant differences within the quasi-longitudinal comparison of mean values occur.

For the interpretation of the arithmetic means an equidistance scale is used which is derived from the Likert-scale range 1 to 6. Accordingly, the following classification is applied:

- 1 to 1.8: strong disagreement
- 1.8 to 2.7: disagreement
- 2.7 to 3.5: weak disagreement

	N Pre/Post	Arithmetic mean Pre/Post	Standard deviation Pre/Post	Median Pre/Post	Interpretation
$\mathrm{NM}_{\mathrm{Dynamic}}$	118/66	5.04/4.93	0.51/0.54	5/5	Agreement
NM _{Static}	117/64	3.64/3.67	0.67/0.55	3.5/4	Weak agreement
LM _{Constr}	BA 42, MA 73/65	BA 5.25, MA 5.08/5.07	BA 0.38, MA 0.47/0.49	BA 5.5, MA 5/5	Strong agreement, agreement/agreement
LM _{Trans}	117/65	2.65/2.71	0.64/0.58	3/3	Disagreement/weak disagreement
MA	116/63	2.37/2.43	0.61/0.59	2/2	Disagreement
РТ	115/60	3.90/3.94	0.82/0.93	4/4	Weak agreement

TABLE 3 Descriptive statistics: comparison of Bachelor and Master students simultaneously within the pre- and post-survey.

- 3.5 to 4.3: weak agreement
- 4.3 to 5.2: agreement
- 5.2 to 6: strong agreement

The paired sample groups of Bachelor and Master probands together also contain no significant differences within all sub-categories (Wilcoxon test). Due to the small sample sizes, a separate analysis of each group was not realized.

In Table 3 a huge difference between the agreements concerning NM_{Dynamic} and NM_{Static} can be observed. A Wilcoxon test for paired samples comparing the arithmetic means of each participant concerning his or her agreement or disagreement on each subcategory confirms the descriptive statistics for pre- (N = 117) and the post-survey (N = 64). The participants agree significantly stronger with NM_{Dynamic} then with NM_{Static} before as well as after their internship (p < 0.001). The effect size is large in either case ($r_{Pre} = 0.83, r_{Post} = 0.81$).

Another huge difference can be observed between the subcategories LM_{Constr} and LM_{Trans} (Table 3). Due to the divided evaluation of Bachelor and Master students within the pre-survey, these subgroups are compared also separately now. A Wilcoxon test for paired samples indicates a significant difference (p < 0.001) for the Bachelor students (N = 42) as well as for the Master students (N = 73). There is a large effect size in both cases (r = 0.87). For the post survey, a significant difference occurs, too (p < 0.001), with the same effect size r = 0.87.

Analogous to Table 3, a descriptive analysis of the category EXP in the post survey is provided in Table 4. Once again, for the subcategory EXP_{Trad-Dir} a significant difference with a small effect size (r = 0.25) between the Bachelor and the Master sample group can be observed (Mann–Whitney-U-test, p < 0.05). The interpretation of the right column refers to the arithmetic means.

Table 4 shows a remarkable difference between the state of agreement between EXP_{Self-Act} and EXP_{Trad-Dir} in favor of EXP_{Self-Act}. A Wilcoxon test compared the arithmetic means of each participant concerning both subcategories (using a paired sample including Bachelor and Master students with N = 64). It indicated a highly significant difference (p < 0.001). The same holds true when Bachelor and Master students are analyzed separately. For all measured differences, a large effect size was observed ($r_{Bachelor} = 0.68$; $r_{Master} = 0.76$; $r_{Together} = 0.63$).

Now items which are warmly welcomed by prospective teachers are discussed. What does this mean? The average agreement of these items is beyond 5.2 (arithmetic mean) and equal to six (median), respectively. These determinations can be interpreted as "strong agreement." Items which fulfil these conditions in the pre- as well as in the post-survey are reported. The item, which means the opposite of "Students' understanding is not as important as the correct solution to a task or mathematical problem", holds these conditions for the Bachelor and the Master students together in both surveys (pre: mean 5.38; median 6 /post: 5.50; 6). Another item in this group is "In maths, it is not only important for my students to find the right solution, but also to understand why this solution is correct." (5.54; 6/5.57; 6). The participants (Bachelor and Master students together) strongly disagree with the items "Boys are generally more gifted in maths than girls." (1.66; 1/1.78; 1) and "To be good at maths, you just have to memorize all the formulas." (1.74; 2 /1.71; 2). Opinions are divided on the items "Based on my teacher training, I believe that I am prepared to use computers and the internet as supporting materials in math lessons." (4.12; 4/ 3.92; 4) and "With my teacher training, I am prepared to provide meaningful information about students' learning progress to parents." (2.93; 2/3.05; 3). Therefore, the average evaluation of the prospective teachers (in the Bachelor and Master program) concerning these items is indifferent. Consequently, the standard deviation is high in both cases: in the pre-survey it is 1.46 for the first item and 1.49 for the second one. In the post-survey, the values are 1.41 and 1.58, respectively. These are the highest standard deviation values in the evaluation. The range of the answers is equal to the maximum span from 1 to 6.

Significant correlations between the subcategories shown in the pre-survey are summarized in Table 5. For this evaluation, the Bachelor and Master students together are considered (N = 119) (Weygandt, 2021, p. 214 sqq.). One medium ($0.3 < |\tau| < 0.5$) effect size and five small ($0.1 < |\tau| < 0.3$) ones appear (Cohen, 1988, p. 79 sqq.).

For the post-survey, Table 6 includes the evaluated significant correlations. Both, the number of subcategories and the number of significant correlations between those categories increased in comparison to the pre-survey. These correlations can be separated into four medium and five small ones. In detail, if only the subcategories of the pre-survey are considered, the number decreases. Once again, the Bachelor and Master students are observed together (N = 67) (Weygandt, 2021, p. 214 sqq.).

Due to the significant differences between the means of the Bachelor and Master students concerning one subcategory within the pre-survey as well as in the post-survey correlations are calculated once again separately for each sample group (Tables 7–10). Three medium and one weak correlation in the pre-survey among the Bachelor students occur (Table 7). Additionally, three medium correlations in the pre-survey among the Master students can also be observed (Table 8). In the post-survey, there is one strong correlation between NM_{Static} and EXP_{Trad-Dir} and one strong correlation between PT and EXP_{Self-Act} within the Bachelor sample group. Furthermore, four medium correlations are found (Table 9). Finally, Table 10 shows five medium correlations among the Master students in the post-survey.

TABLE 4 Descriptive statistics: post-survey of Bachelor and Master students simultaneously.

	Number of items	N Post	Arithmetic mean Post	Standard deviation Post	Median Post	Interpretation
EXP _{Self-Act}	16	65	4.53	0.50	5	Agreement
EXP _{Trad-Dir}	11	64	BA 3.81 MA 3.99	BA 0.55 MA 0.33	BA 4.00 MA 4.5	Weak agreement

TABLE 5 Pre-survey correlations of subcategories using Kendall-Tau τ (** p < 0.01 highly significant; * p < 0.05 significant, N = 119).

Sub-categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM _{Trans}	MA	PT
NM _{Dynamic}	1.000		0.480**			
NM _{Static}		1.000		0.207**		0.160*
LM _{Constr}			1.000	-0.212**	-0.272**	0.170*
LM _{Trans}				1.000		
MA					1.000	
РТ						1.000

TABLE 6 Post-survey correlations of subcategories using Kendall-Tau (** p < 0.01 highly significant; * p < 0.05 significant, N = 67).

Sub- categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM_{Trans}	MA	PT	$EXP_{Self-Act}$	$EXP_{Trad-Dir}$
NM _{Dynamic}	1.000	-0.208*	0.319**		-0.226*		0.265**	
NM _{Static}		1.000						0.330**
LM _{Constr}			1.000		-0.371**		0.271*	
LM _{Trans}				1.000	0.252*			
MA					1.000			
РТ						1.000	0.438**	
EXP _{Self-Act}							1.000	
EXP _{Trad-Dir}								1.000

6 Discussion

In this section, the received results (Section 5) are compared with results from chosen literature.

6.1 (Sub-)categories defined by PCA

As a result of PCA with forced factors, factors related to various categories of beliefs which are similar to those of the TEDS-M study can be identified (first research question, Section 3). The factor NM_{Dvnamic} can be identified with the respective category of the TEDS-M-study "strukture [sic!] of math as process of inquiry" and the factor NM_{Static} with the category "strukture [sic!] of math as rules and procedures" (Laschke and Blömeke, 2014, p. 112 sqq.). This dualism is also reflected in Grigutsch et al. (1998). The static view (Formalism aspect and Scheme orientation) focuses on the products of mathematics (e.g., definitions, theorems), while the dynamic view (Application and Process character) focusses on the processes which result in these products (Grigutsch et al., 1998). This distinction is continued in school practice $(EXP_{Self-Act} and EXP_{Trad-Dir})$ as well as in the view of teaching and learning mathematics (LM_{Constr} and LM_{Trans}), which is perceived as either traditional-directive, respectively, transmissive (teacher-centered) or as self-active, respectively, constructivistic (student-centered) (cf. Weißeno et al., 2013).

6.2 Evaluation of the subcategories by prospective secondary teachers

The small not significant differences between the pre- and post-evaluation for each of the sub-categories (Table 3, quasilongitudinal) are in line with Depping et al. (2021), given the assumption that the subcategory PT is related to self-efficacy of the prospective teachers. The same holds true for the paired sample groups (Bachelor and Master students, longitudinal design). Therefore, the second research question (Section 3) must be negated. A reason for the small paired sample group is that many questionnaires are not fully answered and are therefore excluded from the analysis. A related aspect could be the large extent of the questionnaire. Therefore, probands who completed the pre-survey will not (fully) complete the post-survey due to their experience.

Grigutsch et al. (1998, p. 36) found that the average teacher's view is based on ideas about the process and application aspect of maths ($NM_{Dynamic}$) and those for the schematic aspect (NM_{Static}) tended to be rejected. The authors appreciated these findings. The results in this study are in line with those of Grigutsch et al. (1998). The subcategory $NM_{Dynamic}$ (problem-solving and applied mathematics) is positive evaluated by the students ("agreement") whereas the subcategory NM_{Static} (Mathematics as a formal system with fixed procedures given by authorities) is only weakly

Sub-categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM _{Trans}	MA	PT
NM _{Dynamic}	1.000	0.258*	0.458**			
NM _{Static}		1.000		0.436**		
LM _{Constr}			1.000			0.406**
LM _{Trans}				1.000		
MA					1.000	
РТ						1.000

TABLE 7 Bachelor students' pre-survey correlations of subcategories using Kendall-Tau (** p < 0.01 highly significant; * p < 0.05 significant, N = 43).

TABLE 8 Master students' pre-survey correlations of subcategories using Kendall-Tau (** p < 0.01 highly significant; * p < 0.05 significant, N = 76).

Sub-categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM _{Trans}	MA	РТ
NM _{Dynamic}	1.000		0.487**			
NM _{Static}		1.000				
LM _{Constr}			1.000	-0.355**	-0.383**	
LM _{Trans}				1.000		
MA					1.000	
РТ						1.000

TABLE 9 Bachelor students' post-survey correlations of subcategories using Kendall-Tau (** *p* < 0.01 highly significant; * *p* < 0.05 significant, *N* = 28).

Sub- categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM_{Trans}	MA	PT	$EXP_{Self\text{-}Act}$	$EXP_{Trad-Dir}$
NM _{Dynamic}	1.000		0.337*					-0.356*
NM _{Static}		1.000						0.529**
LM _{Constr}			1.000			0.362*		
LM _{Trans}				1.000	0.407*			
MA					1.000			
РТ						1.000	0.551**	
EXP _{Self-Act}							1.000	
EXP _{Trad-Dir}								1.000

TABLE 10 Master students' post-survey correlations of subcategories using Kendall-Tau (** p < 0.01 highly significant; * p < 0.05 significant, N = 39).

Sub- categories	$NM_Dynamic$	NM_{Static}	LM _{Constr}	LM_{Trans}	MA	PT	$EXP_{Self\text{-}Act}$	$EXP_{Trad-Dir}$
NM _{Dynamic}	1.000		0.323*		-0.365**		0.329*	
NM _{Static}		1.000						
LM _{Constr}			1.000		-0.484**			
LM _{Trans}				1.000				
MA					1.000			
PT						1.000	0.382**	
EXP _{Self-Act}							1.000	
$\mathrm{EXP}_{\mathrm{Trad-Dir}}$								1.000

supported (Table 3). The difference is highly significant with a large effect size.

Eichler et al. (2023, p. 1499) report a different result concerning the state of agreement or disagreement within the categories of the mathematical world view of teacher trainees, but similar to the views of prospective teachers: The majority of prospective teachers tend to favor processoriented beliefs and to reject schema-oriented beliefs as characteristics [...]. The majority of teacher trainees tend to favor beliefs representing the aspect formalism partly combined with a strong refusal of process-oriented beliefs [...]. The difference between a static and a dynamic view of mathematics can also be found within the subcategories of LM. So, there is a continuation in the evaluation by the participants. In other words: their view on NM is transferred to their perception of LM. The static view of mathematics (NM_{Static}) of students can be continued as transmissive view (LM_{Trans}) of mathematics education of prospective teachers (Weingarten, 2015, p. 40). The same context holds true for $NM_{Dynamic}$ and LM_{Constr} .

In detail, the constructivist view on mathematics education (LM_{Constr}) received a (strong) agreement by the students. In contrast, the transmissive view (LM_{Trans}) is (weakly) rejected (Table 3). The difference is also highly significant with a large effect size.

The highly significant difference between the acceptations of $EXP_{Self-Act}$ (self-active) and $EXP_{Trad-Dir}$ (traditional-directive) illustrated in Table 4 can also be related to the difference between $NM_{Dynamic}$ and NM_{Static} . The category "My perceptions of my internship teaching-experiences" (EXP) can be explained as realization of the category LM. This is in line with Eichler et al. (2023, p. 1492):

Fives and Buehl (2012) state that the function of beliefs as a filter is particularly relevant in teacher education since prospective teachers' beliefs 'shape what and how they learn'. Secondly, [...] teachers' beliefs seem to inform their classroom practice (Davis et al., 2019; Skott, 2015) and, finally, could potentially have an influence on students' learning (Eichler and Erens, 2015).

6.3 Correlations of the subcategories

In this subsection, only (highly) significant correlations are reported.

In the post-survey, there are correlations between NM_{Dynamic} and LM_{Constr}, LM_{Constr} and EXP_{Self-Act} as well as NM_{Dynamic} and EXP_{Self-Act} (Table 6). Therefore, this is in line with the assumptions, that a dynamic view of the nature of mathematics (NM_{Dynamic}) is a positive predictor for establishing a constructivist view in mathematical education (LM_{Constr}). Such beliefs on teaching and learning of mathematics result in a perception of encouraging students to selfactiveness (EXP_{Self-Act}) within their own teaching practice. Looking closer to the correlation between $NM_{Dynamic}$ and LM_{Constp} the correlation in the pre-survey (Table 5) is stronger than in the postsurvey (Table 6). This could be explained with experiences made within their internship. The same development can be interpreted by analyzing the connections between $\mathrm{NM}_{\mathrm{Static}}$ (static view of mathematics) and LM_{Trans} (transmissive view of mathematics education). The weak correlation observed in the pre- survey vanishes in the post survey. In line with this process, there is no significant correlation between $LM_{\mbox{\tiny Trans}}$ and $EXP_{\mbox{\tiny Trad-Dir}}$ (traditional-directive perception of mathematical practice) in the post survey (Table 6). However, in the post-survey there is a medium correlation between NM_{Static} and EXP_{Trad-Dir} (Table 6).

It can bet stated that there are decreasing correlations between the sub-categories $\rm NM_{Dynamic}$ and $\rm LM_{Constr}$ due to the internship. From a lower level, this holds true also for $\rm NM_{Static}$ and $\rm LM_{Trans}$. Additionally, a static view of mathematics ($\rm NM_{Static}$) correlates stronger with the perceptions of the internship experiences ($\rm EXP_{Trad-Dir}$) than it does with the transmissive interpretation of learning mathematics ($\rm LM_{Trans}$). Furthermore, a dynamic view on mathematics ($\rm NM_{Dynamic}$) also correlates weaker with $\rm EXP_{Self-Act}$ than $\rm NM_{Static}$ with $\rm EXP_{Trad-Dir}$.

Therefore, the connection between a modern view on mathematics and the perception of modern mathematical practice is not as strong as the link between a traditional view on mathematics and the perception of traditional mathematical practices.

Finally, there is a weak correlation between LM_{Constr} and $EXP_{Self-Act}$ in the post-survey but not for LM_{Trans} and $EXP_{Trad-Dir}$.

Considering the category MA, a negative correlation between MA and LM_{Constr} in the pre-survey (Table 5) is observed, which increases (absolutely) in the post survey (Table 6). This phenomenon is easy to explain: a constructivist view contrasts the belief in fixed mathematics competences of the students. Vice versa, there is a positive correlation between MA and LM_{Trans} only (!) in the post-survey (Table 6), which is in line with the just reported result. Being convinced of "fixed mindsets" (Chestnut et al., 2018) concerning students' mathematics abilities encourages teachers to present elaborated recipes which should only be learned by heart to be executed by the students. Furthermore, another negative correlation between MA and $NM_{Dvnamic}$ (Table 6) emphasizes these theses: a dynamic view on mathematics is not compatible with the conviction described above about "fixed mindsets." Note, that the number of significant correlations between MA and other subcategories increases in the post-survey (Tables 5, 6). This can be explained by the students' experiences made within their internship. These insides hold also true if Bachelor and Master students are analyzed separately (Tables 7–10).

A quite different impression is gained within the correlations of the category PT. The two weak positive correlations between PT and NM_{Static} and PT and LM_{Constr} within the pre-survey (Table 5) vanish within the post-survey (Table 6). Prospective teachers who are convinced to be well prepared for teaching practice by their university training emphasize the static elements of mathematics (Table 5). The other positive correlation between PT and LM_{Constr} (constructivist view on mathematics education) cannot be reconciled with the first one. This contradiction vanishes after the absolved internship (Table 6). A medium correlation occurs in the post survey between PT and EXP_{Self-Act} (Table 6). Therefore, this correlation replaces the former one between PT and LM_{Constr} – practical experiences made within the internship substitute a theoretical view on mathematics education.

The contradiction mentioned above does not occur in the analysis of the separated sample groups of Bachelor and Master students (Tables 7–10). The medium positive correlation between PT and LM_{Constr} can be evaluated only within the Bachelor students' group, as well as in the pre- as in the post-survey (weaker than in the pre-survey), but not in the Master students' group. The medium correlation between PT and EXP_{Self-Act} within the entire participants' group can also be found within the Bachelor students' sample group (strong) and the Master students' sample group (medium). This finding can be caused by the more elaborated experiences of the Master students in comparison with the Bachelor students.

In the literature there are no similar results related to the correlation analysis in this study. In Grigutsch et al. (1998) or Weygandt (2021), the reported correlations are limited to mathematical world views and other aspects which are not investigated in this survey.

6.4 Effect sizes and correlations

To get an overview about the reported results in the Subsections 6.2 and 6.3 and to answer the third research question (Section 3),

Figure 1 shows the relevant connections between the subcategories NM, LM and EXP (see Tables 3–6). The horizontal arrows indicate the significant correlation coefficients τ (above for the pre-survey; underneath for the post-survey), the vertical greater-than signs (meaning higher agreement to the subcategories in the first row in Figure 1 and Tables 3, 4) are labelled with the effect sizes *r* of the significant differences between the respective sub-categories (left side for the pre-survey; right side for the post-survey). Note that there are no measurements in the pre-survey for the subcategories EXP_{Self-Act} and EXP_{Trad-Dir} (indicated by hyphens in Figure 1). The results are derived from the entire participant group (Bachelor and Master students together). In the post-survey, there are no significant correlations between NM_{Static} and LM_{Trans} and between LM_{Trans} and EXP_{Trad-Dir}, this holds true also for the sample groups of the Bachelor students and of the Master students analyzed separately (Tables 9, 10).

6.5 Analysis of single items

As an appendix, the results gained from the analysis of single items are discussed. This procedure is in line with comparable investigations (e. g., Weygandt, 2021, p. 237).

Both items with the strongest agreement, namely the opposite of "Students' understanding is not as important as the correct solution to a task or mathematical problem." and "In maths, it is not only important for my students to find the right solution, but also to understand why this solution is correct.," belong to LM_{Constr}.

The item "To be good at maths, you just have to memorize all the formulas." is strongly rejected by the prospective teachers. It belongs to LM_{Trans} .

These results are in line with the findings shown in Figure 1 and in Table 1: the prospective teachers' constructivist beliefs dominate the static ones.

The highest standard deviations can be found within the items "Based on my teacher training, I believe that I am prepared to use computers and the internet as supporting materials in math lessons." and "With my teacher training, I am prepared to provide meaningful information about students' learning progress to parents." Both items belong to the factor preparedness for teaching (PT). One possible conclusion is that the participants have differing views on their recent preparedness as teachers.

7 Conclusion and limitations

Due to the conducted PCA's, most of the belief-factors and sub-categories could be reproduced, which are well known from literature (e.g., Grigutsch et al., 1998; Laschke and Blömeke, 2014; Eikmeyer, 2021; Woltron and Götz, 2023).

Furthermore, the investigation shows that beliefs of prospective teachers did not change as a result of absolving their internships (Tables 3, 4). Therefore, absolving an internship cannot be interpreted as a "strong impact" on changing beliefs. This is true for each of the analyzed subcategories. Based on these results, the first research question must be answered in the negative (Section 3). Therefore, the high expectations of prospective teachers concerning their internships during their studies (Hascher, 2011; Hascher, 2012) do not imply a change of beliefs which one could suppose. One reason for this stability could be a missing reflection of the prospective teachers concerning their beliefs. Müller-Hill (2015) points out a possible stimulus for targeted focusing on beliefs: she suggests explicitly addressing students' beliefs about the "nature of science" (this corresponds to NM) and consciously reflect on them (see also McComas et al., 2020). In addition, beliefs on LM can be indirectly influenced through discussing subjectspecific didactical concepts.

This stability of beliefs can also be derived from Figure 1. A dynamic view on mathematics, a constructivist focus on teaching mathematics and a self-active understanding of once own given lessons dominate a static view on mathematics, a transmissive focus on teaching mathematics and a traditional directive understanding of self-conducted teaching. This finding can be seen as the main result of this study. The significant correlations in Figure 1 underline this impression: the correlations between NM_{Dynamic} and LM_{Constr} and EXP_{Self-Act} [dynamic constructivist view on mathematics (education)] are stronger than those between NM_{Static} and LM_{Trans} and EXP_{Trad-Dir} [static transmissive view on mathematics (education)].

There are some limitations of this study. A quasi-longitudinal study was carried out in more detail than the longitudinal study because of the small size of the paired sample group. The reason for this small sample size can be interpreted as a "modified respond-shift bias." Those who are familiar with the questionnaire are more likely to fail to complete the post-survey.



Furthermore, Likert scales always harbor the risk of favoring responses that correspond to social desirability (Kreitchmann et al., 2019). Predefined (fictitious) contexts can minimize this problem, but the related qualitative analysis is very time-consuming, poses challenges in comparability and researcher interpretation and is not practicable for large sample sizes (cf. Safrudiannur and Rott, 2020). For example, teachers of low-achieving students tend to prioritize static beliefs, while those teaching high-achievers favor dynamic beliefs (Safrudiannur and Rott, 2019). Q-studies are a potential method for this type of research, but they are also only particularly suitable for small respondent groups (Buchholtz and Vollstedt, 2024).

Hascher (2012, p. 94) states (translated by the authors): "[...] it should be noted that the results are often based on self-assessments." This also applies to this study.

Given the complexity of beliefs and measurement difficulties, mixed-methods research combining qualitative and quantitative approaches appears promising for developing more comprehensive assessment tools (Buchholtz and Vollstedt, 2024).

8 Outlook

Due to Hascher and Hagenauer (2016), one suggestion is to extend the demanded openness towards theory of pre-service teachers to subject didactic concepts (e.g., fundamental ideas, basic mental models, etc.) and towards their beliefs which are evaluated in this study. In order to make prospective teachers aware of certain subject didactic concepts that they should apply during their practical lessons in the internship, these concepts can be prepared for topic-specific completion. This instruction should be provided in the accompanying courses. For example, if a prospective teacher has to prepare a lesson on fractions, he or she should be familiar with basic mental models like fractions as quasi-cardinal numbers (to add two fractions with the same denominator).

Following the "Mathematical Teacher Specialized Knowledge Model" by Carrillo-Yañez et al. (2018), the related "Pedagogical Content Knowledge" (e.g., basic mental models in the given example) is based on beliefs "on Maths Teaching and Learning" as well as on beliefs "on Maths." Therefore, prospective teachers should be made aware of their beliefs. For this purpose, Müller-Hill (2015) and McComas et al. (2020) present an explicitreflexive approach for eliciting beliefs. To realize this approach, prospective teachers should evaluate statements like the items which are used in the questionnaire (reflexive). Due to the evaluated influence of beliefs on teaching practice (Eichler et al., 2023), explicit phases, in which the theoretical concepts of beliefs and their effects on the practical instructions are discussed, should be a part of the accompanying courses. In detail, different realizations of beliefs concerning NM, LM and so on should be presented and reflected.

The results show that the manifestations of prospective teachers' beliefs concerning the nature of mathematics and teaching and learning mathematics are in line with recent didactical trends like a constructivist view on learning mathematics. However, there could be a bias between the given answers and the school practice the prospective teachers will realize ("social desirability"). The suggestion is to emphasize already acquired didactical concepts by presenting options to realize them related to concrete mathematical topics in the accompanying courses of the internships (see above). As a consequence of this procedure, the gap between the prospective teachers' beliefs and basic didactical concepts would be reduced.

A model is developed (Figure 2) to outline a theoretical framework for accompanying courses for internships. The left-hand ellipse "Beliefs" in Figure 2 contains the subcategories of (epistemological) beliefs identified in this study. Some didactic concepts in the right-hand ellipse are given as examples to enrich the input of the practical phases. Both factors influence mathematics teaching and therefore they should be addressed in the course as well



as within mentor training (e.g., Kunter et al., 2013). A special qualification is mandatory for such mentors, who supervise the prospective teachers. Commonly, these people are teachers with a great amount of experience.

Results of beliefs research (e.g., Eichler and Erens, 2015) and the use of subject-specific didactic concepts in the classroom can lead to relativize or reinforce beliefs of one's own teaching or those of the mentors. The two influencing factors are of different nature. Due to the prospective teachers' previous school and academic experiences, the participants complete their practical phases with certain internalized beliefs (black arrow in Figure 2 from "Beliefs" to "Internships"). Subject-specific didactic concepts, on the other hand, must be actively learnt during their studies, and their implementation in the classroom is also part of university teacher training. This means that those concepts do not necessarily have an influence on their lesson planning. This is symbolized by the dashed arrow in Figure 2. Appropriate didactical advice from the mentors may be necessary and helpful. The extent to which subject-specific didactic concepts are incorporated into students' individual school practice must remain open (arrow with question mark in Figure 2, see the panel discussion "Bridging the researchpractice gap" at CERME13, for instance). The requirement to explicitly address subject-specific didactic concepts in accompanying courses during the internships is derived from the observation that the prospective teachers' beliefs about different contexts (categories) do not change during the practical phases (Table 3).

Overall, the need to dovetail prospective teachers' identified beliefs with corresponding subject-specific didactic concepts in the accompanying courses of the internships of the teacher training program is obvious.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Ethics statement

Ethical approval was not required for the studies involving humans because we conducted a survey of students in accordance

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with the ethical guidelines of the University of Vienna. Students participated in the survey voluntarily, and all results were anonymized and stored or deleted in accordance with the University's data protection guidelines. The studies were conducted in accordance with the local legislation and institutional requirements. The participants provided their written informed consent to participate in this study.

Author contributions

FW: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing. SG: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

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The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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