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Improving mathematical modeling competencies of mathematics teachers in a technology-supported learning environment

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Mathematical modeling process has an important role in solving problems encountered in daily life. For this reason, this study aims to improve mathematical modeling competencies of mathematics teachers through training in a learning environment prepared with a mathematical modeling approach. In this case study research, the participants were 5 volunteer mathematics teachers working in public schools. The study was designed with a specific process and firstly, teachers were interviewed and their preliminary information about mathematical modeling was obtained. Then, an 11-week mathematical modeling teaching process was carried out in a learning environment prepared with mathematical modeling. It was revealed that the training carried out in a learning environment with today's technology greatly improved the mathematical modeling competencies of mathematics teachers.

KEYWORDS

mathematical modeling process, mathematical modeling competencies, virtual reality, augmented reality, mathematical modeling

Introduction

This study focuses on the development of mathematical modeling competencies of mathematics teachers through mathematical modeling training in a technology-supported learning environment prepared by the mathematical modeling approach. Mathematical modeling competencies, which constitute the focus of the research, constitute a subject area that information societies have given importance in recent years.

The mathematical modeling process has an important role in solving problems encountered in daily life. It has been revealed that mathematical modeling cycles (Blum and Leiß, 2007; Kaiser and Sriraman, 2006) and mathematical modeling competencies (Blum, 2002; Maaß, 2006; Niss et al., 2007) are important in the creation of the mathematical modeling process (Çakmak-Gürel, 2018). This is because mathematical modeling cycles and competencies are the components that enable the behaviors expected to occur in the mathematical modeling process (Haines and Crouch, 2010). Maaß (2006) stated in his study that there should be some competencies and sub-competencies for the mathematical modeling process to take place positively and for individuals to transition. Blum and Kaiser (1997) proposed mathematical modeling competencies for transitions between mathematical modeling processes (as cited in Maaß, 2006). These are as follows:

- The ability to understand the real problem and create a model based on reality,
- Ability to create a mathematical model from a real model,

- Ability to solve mathematical problems with the mathematical model created,
- Ability to interpret mathematical results in real situations, and
- Ability to validate the solution.

In other words, Aydın-Güç (2015) stated that the mathematical modeling process will be completed well if these competencies are maintained and stated that learning environments should be made suitable for the development of these competencies. For this reason, a learning environment was created to develop teachers' mathematical modeling competencies, and its development in the process was examined.

Mathematical modeling has been included in every update of elementary, middle, and high school mathematics curricula. Many national curricula (e.g., Next Generation Science Standards) argue that developing mathematical modeling competencies is critical for students to deeply understand science (Adair et al., 2022). Curricula aiming to provide students with mathematical modeling experiences are implemented through teachers. However, studies show that mathematics teachers do not have sufficient knowledge about mathematical modeling (Bilgili and Çiltaş, 2018; Lingefjärd, 2007; NCTM, 2000; Niss et al., 2007; Superfine and Wagreich, 2010; Tekin and Bukova-Güzel, 2013). For this reason, the learning environment should be enriched to determine and develop the mathematical modeling competencies of mathematics teachers in the mathematical modeling teaching process. In this way, the learning environment will be suitable for the mathematical modeling approach, and it will also save the teaching process from monotony and make it remarkable. Since mathematical modeling is an approach that is intertwined with real life, it is important for teachers to maintain their connections with real life in the learning environment.

Various approaches and applications have been developed to develop mathematical modeling competencies in recent years. When the literature is examined, two approaches are discussed to develop mathematical modeling competencies. These are holistic and micro-level approaches. In the micro-level approach, the learning environment is created according to some sub-competencies, while in the holistic approach, the learning environment is organized according to all competencies.

In the holistic approach, it is accepted that the whole is not equal to the sum of the parts, and it is focused on the fact that individuals should see the picture as a whole beyond acquiring information piece by piece (Babacan, 2014). For this reason, in holistic environments, mathematical modeling situations are given, and it is aimed at individuals gaining experience in all competencies and developing all competencies. In other words, according to the holistic approach, during a mathematical modeling activity, the entire modeling process should be experienced, and all modeling competencies should be employed (Aydın-Güç, 2015).

In this study, a learning environment was created to develop mathematical modeling competencies per the holistic approach and enriched with tools such as virtual reality, augmented reality, concrete materials, and the internet to access real information about the data. The reason for considering the holistic approach is that when a problem encountered in daily life is addressed, the person goes through all the processes and considers all the competencies in the solution phase and, therefore, handles the problem holistically.

In their study on digital tools for mathematical modeling, Schönbrodt et al. (2022) found that the use of technological tools in the learning environment is inevitable for accessing today's problems, as well as providing a large number of activities to be introduced to students. Already in science, industry, and research, real-world problems are addressed and solved with the help of technologies. Therefore, realizing the solution to a daily life problem using digital tools provides integrity in interpreting the relationship between the solution and daily life (Siller and Greefrath, 2010). In this sense, applications such as virtual and augmented reality, which have recently come to the agenda with new and modern technology, have brought a different perspective to educational methods. In addition, technological opportunities should be included in this environment to create an effective learning environment using the constructivist approach (Jonassen et al., 1998). The importance of virtual reality and augmented reality, which are among these technological opportunities, is rapidly increasing because they offer opportunities such as active participation and learning by doing and experiencing in line with the curriculum's expectations.

Virtual reality is an environment where the real world is transferred to the digital environment, consisting of three-dimensional real models that are designed based on individual interaction. Virtual learning environments allow individuals to gain experience in situations that are difficult to reach or do in the real world while concretizing abstract concepts (Bakas and Mikropoulos, 2003). Augmented reality is a derivative of virtual reality (Azuma, 1997). It is defined as a virtual environment in which objects in the real world interact with virtual objects (Zhu et al., 2004).

When studies on the use of virtual reality in education are examined, they are mostly focused on areas such as chemistry education and geography education (Manseur, 2005). In addition, there are studies (Çavaş et al., 2004; Halvorson et al., 2011; Harris and Rea, 2009; Heid and Kretschmer, 2009; Holmberg, 1997; Mishra and Koehler, 2006; Özdinç, 2010) that its use in learning environments increases motivation, social skills and communication and positively affects learning.

Studies comparing augmented reality with traditional methods used in the classroom (Freitas and Campos, 2008; Kerawalla et al., 2006; Lave and Wenger, 1991; Perez-López and Contero, 2013; Squire and Jan, 2007) revealed that augmented reality facilitated students' learning and attracted their attention more. In addition, when the applications of augmented reality studies in the field of education are examined, activities in which technology is associated with daily life stand out (Quarles et al., 2008, cited in Karal and Abdüsselam, 2015).

In this study, in the learning environment created in accordance with the mathematical modeling approach, virtual reality, and augmented reality were included in mathematical modeling activities to attract teachers' attention and increase motivation, to save the lesson from boredom with different methods, and to increase retention by appealing to multiple sensory organs. In addition, in studies evaluating mathematical modeling competencies with a holistic approach, it has been observed that criteria such as whether individuals can complete mathematical modeling steps or whether they have competencies are considered in the evaluation of mathematical modeling competencies. However, the level of realization of these competencies was not detailed. Considering this situation, in this study, the level of teachers' modeling competencies in the learning environment created in accordance with the holistic approach was detailed.

In this study, the effect of the technology-supported learning environment created in accordance with mathematical modeling according to the holistic approach on the development of mathematical modeling competencies of mathematics teachers was examined.

Methods

Qualitative research is a process that requires holistically and realistically revealing events and perceptions using data collection methods such as interviews, observation, and written document analysis (Patton, 2014; Creswell, 2019). In this study, case study design, one of the qualitative approaches, was used.

The participants of this study consisted of 5 mathematics teachers doing postgraduate education in the Department of Mathematics Education of a state university. A total of 16 mathematics teachers enrolled in the "Mathematical Modeling in Mathematics Education-I" postgraduate course were informed about the study before selecting participants. Six mathematics teachers were selected to participate voluntarily, and a voluntary agreement was signed before the process started. Two weeks after the implementation process started, one teacher left the study group voluntarily. The teachers who participated in the study were coded as T1, T2…, and T5, and their names were not included in the ethical rules.

Design of the learning environment

Following the identification of the research problem, the literature on what can be done to find an answer to this problem was examined, and it was concluded that for teachers to develop mathematical modeling competencies, they should have experiences that will help them go through all the steps in the mathematical modeling process. In this context, it was believed that the approach that would provide such an environment for acquiring mathematical modeling competencies was the holistic approach, and the learning environment was designed with the holistic approach in mind.

In learning environments created according to the holistic approach, all modeling competencies are tried to be developed simultaneously with mathematical modeling activities. For this reason, the learning environment was designed by taking into account the conditions that should be provided in learning environments for developing mathematical modeling competencies, which were revealed in detail as a result of the literature review (Anhalt and Cortez, 2016; Aydın-Güç, 2015; Biccard and Wessels, 2011; Blomhøj and Jensen, 2003; Brand, 2014; Çakmak-Gürel, 2018; Galbraith and Stillman, 2001; Ji, 2012; Kaiser and Grünewald, 2015; Kaiser and Stender, 2013; Maaß, 2006). These situations

i Information about the model, modeling, mathematical model, mathematical modeling, process, and competencies,
ii Group work,
iii Participants frequently encounter mathematical modeling activities, and
iv The learning environment is suitable for learning mathematical modeling.

In the form of a holistic approach. To create a learning environment suitable for developing mathematical modeling competencies, a holistic approach was adopted. When the literature is examined, it is seen that the learning environment is prepared in accordance with the holistic approach

i Theoretical knowledge orientated,
ii Free mathematical modeling activities focused, and
iii Focused on following the steps of mathematical modeling.

In this study, all three situations were considered while creating a learning environment suitable for developing mathematical modeling competencies (Aydın-Güç and Baki, 2016).

While addressing *the theoretical knowledge-oriented* item in the holistic approach-based learning environment, theoretical knowledge about the mathematical model and modeling process was given first. After the completion of the theoretical knowledge process, a solution plan with the process of "analyzing the problem, creating a conceptual model, mathematization, creating a mathematical model, creating a mathematical solution, interpretation, and communication, and reflection" *focused on following the steps of mathematical modeling* was given with the activity as an instruction. With this approach, it was aimed to support all steps of the mathematical modeling process. Then, *free mathematical modeling activities* without any instructions for the mathematical modeling process were presented to the participants, and the researcher gave hints when necessary. While working on the mathematical modeling activities without instructions, the teachers were expected to follow the guiding steps and direct their solutions.

In the holistic approach, teachers should solve real-life problems in which they can see and experience mathematical modeling competencies, and all steps are available. For this reason, no specific topic or outcome was determined during the activity design, and activities suitable for modeling competencies were selected for teachers. The researcher's instructions (Table 1) were used more frequently, especially in the first activities, to avoid skipping any competence. Otherwise, a step may be skipped in the process; this may lead to deficiencies in the development of mathematical modeling competencies, which is the focus of the study. Şen-Zeytun (2013) also states that the mathematical modeling process is negatively affected by *disorganized/unsystematic working and writing* factors.

Instructions for revealing mathematical modeling competencies developed by Blum and Kaiser (1997) and transferred by Maaß (2006) were prepared (Table 1).

While creating the instructions, care was taken to ensure that they would help to reveal mathematical modeling competencies and provide experiences for these competencies. The instructions prepared by Aydın-Güç (2015) in accordance with mathematical modeling sub-competencies were revised in accordance with mathematical modeling competencies and used in this study. Sub-competencies are not included in Table 1 because of the focus on mathematical modeling competencies in the learning environment created for teachers. The instructions prepared by the researcher were finalized based on the opinions of two experts who have studied in the field of mathematical modeling. While taking expert opinions, first, mathematical modeling competencies were presented to the experts, and then, the experts expressed the situations expected from teachers based on these competencies. For example, for the competency of "Understanding the real problem

TABLE 1 Mathematical modeling competencies and related instructions.

Mathematical modeling competencies that teachers should have	Instructions used by the researcher in the process
A. Ability to understand the real problem and create a model based on reality	What do we know about the problem? What are the possible variables to be used in the process? What would be the first model you would create for a solution?
B. Ability to create a mathematical model from a real model	Is your first model suitable for the problem? Can it be detailed (table, graph, etc.)?
C. Ability to solve mathematical problems with the mathematical model created	Can a problem similar to this problem be created? Can we solve it in a similar way?
D. Ability to interpret mathematical results in real situations	What is the usefulness of this model in daily life? Can it be generalized?
E. Ability to verify the solution	Can the model be tested to be correct? Can a different model be created in addition to this model?

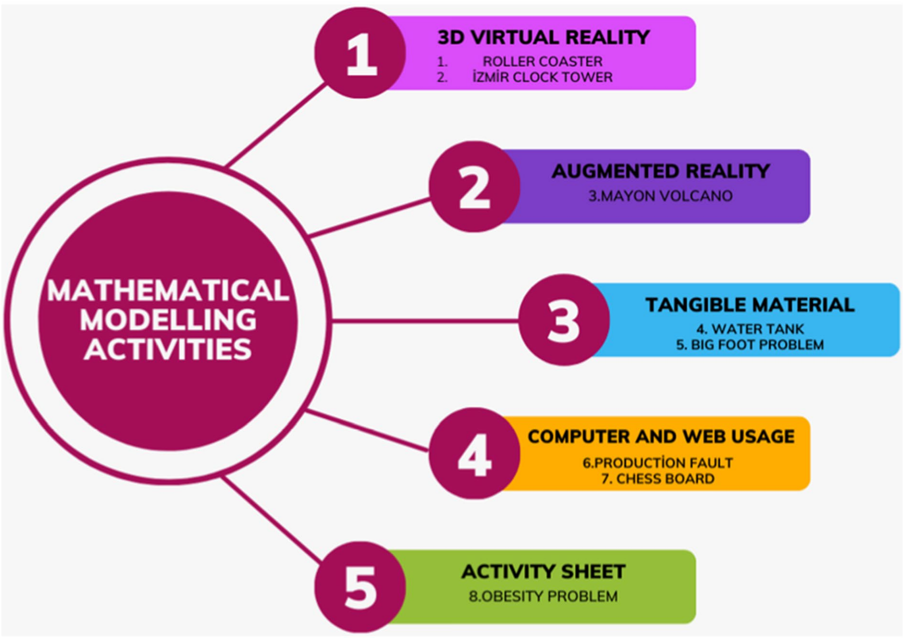


FIGURE 1
Mathematical modeling activities applied in the learning environment.

and creating a model based on reality," the expectations from the teachers were stated as "Being able to clearly express what is desired in the problem, being able to determine what is given and what is desired, being able to present the first situation they think of for the solution (being able to express a representation of the situation)." Then, the experts approved the instructions' suitability, and changes were requested. After the feedback, the instructions were finalized.

In short, the learning environment was enriched with augmented reality, virtual reality, and concrete material supports; it was prepared as an environment in which the researcher was a guide and based on a holistic approach.

In the learning environment prepared in accordance with the mathematical modeling approach, teachers sometimes solved the activities individually and sometimes as group work. After they were given enough time to analyze the activities and reach solutions, the solutions were discussed, and a discussion environment was created with the teachers. In addition, in the learning environment, teachers could use the internet and consult an expert to do the necessary research.

Event design

While designing the activity, mathematical modeling competencies that should be present in teachers were not addressed separately, but all competencies were considered in each activity due to the holistic approach.

According to the holistic approach, individuals should work with mathematical modeling activities selected from real-world problems that will provide a basis for them to go through all the steps in the mathematical modeling process (Aydın-Güç, 2015). For this reason, the activities were selected in a way that helped teachers discover the modeling competencies they should have or reveal the existing ones (Figure 1).

TABLE 2 Mathematical modeling activities and objectives used in the learning environment prepared in accordance with the mathematical modeling approach.

Event	Purpose
Big foot problem (BFP)	It aimed to determine how competent teachers without experience were in solving the first activity.
Production fault (PF)	This activity, which cannot be concluded with simple mathematical operations, is aimed at determining the appropriate variables and reaching a linear model by using the SPSS program, which teachers are already familiar with.
Roller coaster (RC)	With 3D glasses, teachers were given real-life experience due to the nature of modeling, and then it was aimed that they could interpret these graphics in daily life by drawing graphics and designing a roller coaster.
Izmir clock tower (ICT)	It is aimed for teachers using cardboard glasses to reach solutions with original methods.
Chess board (CB)	Using the problem situation given as an old rumor, it was aimed at the teachers to arrive at a generalization and interpret it.
Water tank (WT)	It aimed to draw height-volume graphs for four different tank models, which were given only as pictures. In addition, they were asked to draw the same graph for two different water bottle models brought to the classroom as materials, and then the accuracy of the graphs was tested by filling them with water.
Obesity problem (OP)	After the information given about obesity, which is one of today's health problems, it was aimed that the teachers make a generalization and interpret it.
Mayon volcano (MV)	It aims to predict a future event by interpreting a situation using real data.

In each activity in Table 2, the intended situation was clearly expressed, and technology support for concrete material, 3D virtual reality, and augmented reality was used in the learning environment. Using these in the activities enriched the learning environment and provided positive reflections by the teachers.

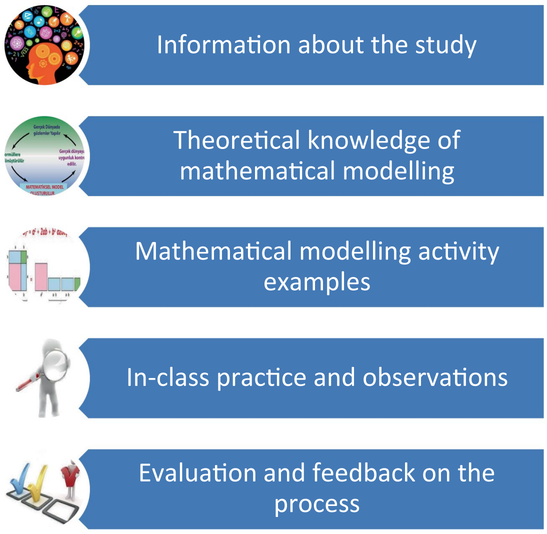
It is difficult to determine the teacher competencies expected to emerge in the mathematical modeling process only by examining the solutions of the activity. Although many competencies revealed in the mathematical modeling process affect the solution process, only mathematical models are involved. For this reason, in determining "mathematical modeling competencies," the observation data were analyzed by the researcher and an expert who has studied in the field of mathematical modeling, taking into account the situations given in Appendix 1. In addition, all activities used are given in Appendix 2.

When the relationship between the designed activities and teacher competencies is examined, it is understood that teacher competencies can be identified and observed in all activities. Information about this is given below in Table 3.

Data collection process and data collection tools

In this study, an 11-week mathematical modeling teaching process was carried out in a learning environment prepared in accordance with mathematical modeling to develop the mathematical modeling competencies of mathematics teachers. The process started with giving theoretical information about mathematical modeling and continued with the implementation of a *mathematical modeling activity* in the learning environment every week. After each activity, the teachers filled out *activity evaluation forms*. While the activities were analyzed and solutions were developed by the teachers, the researcher observed them and evaluated their competencies based on *the criteria* (Bilgili, 2022) established separately for each activity. The whole process was recorded with a video camera to increase the reliability of the data, and the competency assessments were also examined by an expert in the field of mathematical modeling other than the researcher. Considering the data collection process, this study was based on behaviorist theory because the observable behaviors of teachers were focused on.

As a result of the pilot study, it was decided that the data collection process should be conducted in this way. The pilot study aimed to shape the data collection tools, identify deficiencies in the process, and gain experience. For this purpose, the pilot study was conducted with 16 prospective mathematics teachers who took the undergraduate mathematical modeling course in the spring semester of a state university, and the 11-week training process is as follows:



The study group consisted of senior students because the mathematical modeling course was included in the undergraduate program, and in-class observations could be made within the scope of the teaching practice course.

The fact that the pilot study group consisted of 16 people made individual determinations difficult, so it was decided to work with fewer people in the main study. The participants of the main study were five mathematics teachers doing graduate education at a state university. While determining the participants, 16 mathematics teachers who took the "Mathematical Modeling in Mathematics Education-I" graduate course came together and were informed about the study. Six mathematics teachers were selected to participate voluntarily, and a voluntary agreement was signed before the process

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TABLE 3 The relationship between mathematical modeling activities and teacher competencies.

Activities	Expected contingencies for modeling competencies
Big foot problem (BFP)	• A. Ability to understand the real problem and create a model based on reality Expressing the need for a data set Identifying appropriate variables (such as foot length, height, and gender) Creating an exemplary model (such as thinking that it is a man's foot at first glance) • B. Ability to create a mathematical model from a real model Mathematical support of the initial model • C. Ability to solve mathematical problems with the mathematical model created Talking about whether similar problems can be solved with the mathematical model created • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Testing the accuracy of the model created
Production fault (PF)	• A. Ability to understand the real problem and create a model based on reality Search for relationships between explicitly given variables Predicting the series to be produced defectively (model building) • B. Ability to create a mathematical model from a real model Mathematical support of the initial model • C. Ability to solve mathematical problems with the mathematical model created Talking about whether similar problems can be solved with the mathematical model created • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Testing the accuracy of the model for each case
Roller coaster (RC)	• A. Ability to understand the real problem and create a model based on reality Expressing the given and desired situations in the problem Drawing an estimated roller coaster (graph) • B. Ability to create a mathematical model from a real model Edit the first model according to the conditions given in the question • C. Ability to solve mathematical problems with the mathematical model created Talking about designs similar to this problem that can be created under certain conditions • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Discussion on the accuracy of the model by comparing the graphs created by the groups

(Continued)

TABLE 3 (Continued)

Activities	Expected contingencies for modeling competencies
Izmir clock tower (ICT)	• A. Ability to understand the real problem and create a model based on reality Expressing desired states without any numerical data in the problem Specify an estimated length • B. Ability to create a mathematical model from a real model Reflecting on the initial guess and supporting it mathematically • C. Ability to solve mathematical problems with the mathematical model created Talking about whether the mathematical model created can be used in solving similar problems • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Testing for which situations the model will be correct
Chess board (CB)	• A. Ability to understand the real problem and create a model based on reality Expressing the given and desired situations in the problem Developing a predictive model (in the form of a formula suitable for solution) • B. Ability to create a mathematical model from a real model Mathematically support or recreate the initial model • C. Ability to solve mathematical problems with the mathematical model created Talking about whether the mathematical model created can be used in solving similar problems • D. Ability to interpret mathematical results in real situations Talking about the usability of the model outside the narration mentioned in the question, interpreting mathematical results in non-mathematical contexts, and generalizing the model • E. Ability to verify the solution Testing the accuracy of the model
Water tank (WT)	• A. Ability to understand the real problem and create a model based on reality Expressing the desired states in this problem without numerical data Discussion on possible graphic shapes (first model) • B. Ability to create a mathematical model from a real model Mathematically organize the first model created • C. Ability to solve mathematical problems with the mathematical model created Testing the mathematical models on concrete material and talking about whether similar problems can be solved with these models • D. Ability to interpret mathematical results in real situations Talk about the usability of the model in daily life • E. Ability to verify the solution Talking about the situations in which the model will be correct

(Continued)

TABLE 3 (Continued)

Activities	Expected contingencies for modeling competencies
Obesity problem (OP)	• A. Ability to understand the real problem and create a model based on reality Expressing the given and desired situations in the problem Creating a model that will reach the goal with a selected exercise • B. Ability to create a mathematical model from a real model Test the model for different exercises • C. Ability to solve mathematical problems with the mathematical model created Talking about the models created for different exercises, relating them to daily life • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Testing the accuracy of the generalized model
Mayon volcano (MV)	• A. Ability to understand the real problem and create a model based on reality Search for relationships between explicitly given variables Predicting the year of the explosion (modeling) • B. Ability to create a mathematical model from a real model Mathematical support of the initial model • C. Ability to solve mathematical problems with the mathematical model created Talking about whether solutions to similar problems can be created with the mathematical model created • D. Ability to interpret mathematical results in real situations Interpret mathematical results in non-mathematical contexts and generalize the model • E. Ability to verify the solution Testing for which situations the model is correct

started. Two weeks after the implementation process started, one teacher voluntarily left the study group. The teachers who participated in the study were coded as T1, T2…, and T5 and their names were not included in the ethical rules.

As a result of the pilot study conducted with pre-service teachers, it was decided to conduct the actual study with teachers. Because it was determined that the modeling competencies of pre-service teachers increased in the learning environment prepared in accordance with mathematical modeling. In addition, as a result of the preliminary interview form with teachers, it was seen that none of them had any prior knowledge about mathematical modeling; therefore, it was decided that it would be better to conduct the actual study with teachers.

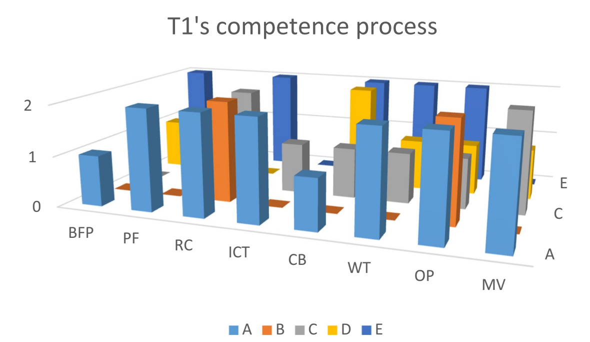
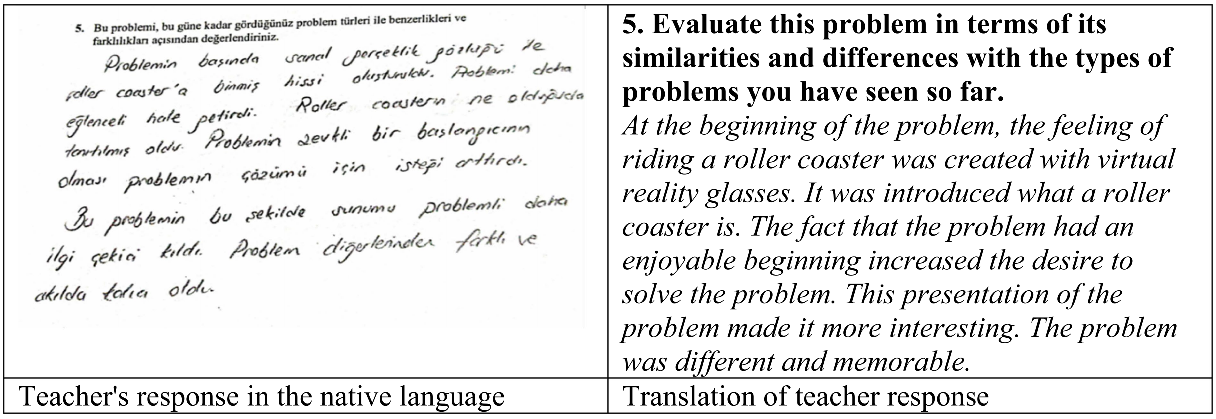


FIGURE 2
Competence process of teacher coded T1.

Findings

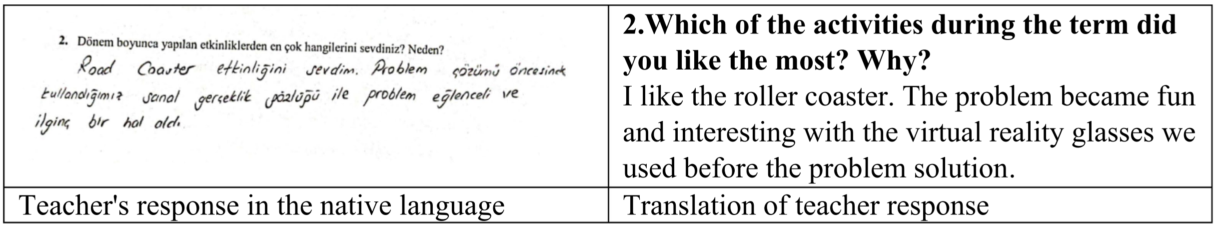
Before starting the implementation process, preliminary interviews were conducted with the teachers, and it was determined that they had no knowledge of mathematical modeling. Therefore, teachers obtained all their knowledge about mathematical modeling in the learning environment created.

The development of teachers' mathematical modeling competencies in the learning environment prepared in accordance with the mathematical modeling approach is given in the tables below for each teacher. The process is also exemplified by the development of an activity (Figure 2).



5. Bu problemi, bu güne kadar gördüğünüz problem türleri ile benzerlikleri ve farklılıkları açısından değerlendiriniz. Problemin başında sanal gerçeklik gözlüğü ile roller coaster'a binmiş hissi oluşturuldu. Problemi daha eğlenceli hale getirdi. Roller coaster'ın ne olduğu da tanıtılmış oldu. Problemin zevkli bir başlangıcının olması problemin çözümü için isteği arttırdı. Bu problemin bu şekilde sunumu problemi daha ilgi çekici kıldı. Problem diğerlerinden farklı ve akılda kalıcı oldu.	**5. Evaluate this problem in terms of its similarities and differences with the types of problems you have seen so far.** *At the beginning of the problem, the feeling of riding a roller coaster was created with virtual reality glasses. It was introduced what a roller coaster is. The fact that the problem had an enjoyable beginning increased the desire to solve the problem. This presentation of the problem made it more interesting. The problem was different and memorable.*
Teacher's response in the native language	Translation of teacher response

FIGURE 3
T1's answer to the 5th question in the post-activity evaluation form.



2. Dönem boyunca yapılan etkinliklerden en çok hangilerini sevdiniz? Neden? Road Coaster etkinliğini sevdim. Problem çözümü öncesinde kullandığımız sanal gerçeklik gözlüğü ile problem eğlenceli ve ilginç bir hal aldı.	**2.Which of the activities during the term did you like the most? Why?** I like the roller coaster. The problem became fun and interesting with the virtual reality glasses we used before the problem solution.
Teacher's response in the native language	Translation of teacher response

FIGURE 4
T1's answer to the 2nd general evaluation question.

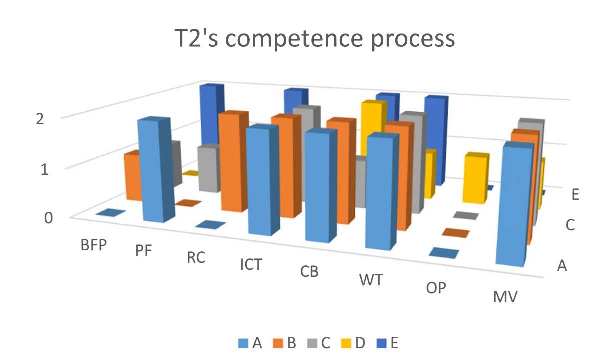


FIGURE 5
Competence process of teacher coded T2.

Activity-based competence change process of the teacher-coded T1

When the competency process of T1 is analyzed, it is seen that he was not evaluated as Criterion 0 in any activity for competency A. However, competency B was evaluated as Criterion 0 in all activities except Roller Coaster and Obesity Problem activities. When T1's activity evaluation forms are examined, it can be said that the teacher developed a positive attitude, especially for the roller coaster activity (Figure 3).

The answer to T1 to the 2nd question in the general evaluation form given to the teachers after the education-training process is as follows (Figure 4).

When the responses of T1 are analyzed above, it is seen that the teacher's positive attitude toward the roller coaster activity is reflected positively in almost all competencies. It was evaluated as criterion 0 only in competence D and as criterion 2 in all other competencies. In addition, for the Obesity Problem activity, T1 stated in the activity evaluation form that he fully understood the problem and believed he could solve it. Considering the Big Foot Problem activity, competence A was evaluated as Criterion 1. Competence E was evaluated as Criterion 2, and other competencies were evaluated as Criterion 0. S1 answered the Big Foot Problem activity as his least favorite activity among the activities carried out during the semester, which is one of the general evaluation questions. Namely, the only activity in which competency C was also evaluated as Criterion 0 was the Big Foot Problem activity. When the competence process of T1 is analyzed, it is seen that she could not exhibit much competence in the İzmir Clock Tower activity. When the teacher's answers to the evaluation form after the İzmir Clock Tower activity are analyzed, it is observed that there is no negative situation. In general, when T1's competence table is analyzed, it is seen that competence B did not emerge except for two activities (Figure 5).

Activity-based competence change process of teacher-coded T2

When the competence process of T2 is examined, it is seen that the teacher exhibited behaviors toward all Chessboard and Water Tank activities but did not exhibit much behavior toward competencies in

2. Dönem boyunca yapılan etkinliklerden en çok hangilerini sevdiniz? Neden? Satranç ile ilgili olan etkinliği sevdim. Hikaye çok güzeldi. Sayıların mükemmelliği ancak bu kadar güzel farkettirilirdi. Satrancın tarihsel gelişiminden bahsetmesi ilgi çekici.	**2.Which of the activities during the term did you like the most? Why?** I liked the activity about chess. The story was beautiful. The perfection of numbers could only be realised so well. It was interesting to talk about the historical development of chess.
3. Dönem boyunca yapılan etkinliklerden en az hangilerini sevdiniz? Neden? Üretim hatası etkinliği. Bilgisayar programı kullanmak benim sıkıldığım bir yönü. Çocukların da sıkılacağını düşünüyorum.	**3.Which of the activities during the term did you like the least? Why?** Production Fault activity. Using computer programmes was an aspect I was bored with. I think children will be bored too.
Teacher's response in the native language	Translation of teacher response

FIGURE 6
T2's answers to general evaluation questions 2 and 3.

2. Bu soruyu çözdükten sonra neler öğrendiniz? Dikkatli okuyup, anlarak gerektiğini farkettim. Çünkü çözümümüz yanlış anladığımız için uzadı.	**2.What did you learn after solving this question?** I realised that you need to read and understand carefully. Because our solution was prolonged because we misunderstood.
Teacher's response in the native language	Translation of teacher response

FIGURE 7
T2's answer to the 2nd question in the evaluation form after the obesity problem activity.

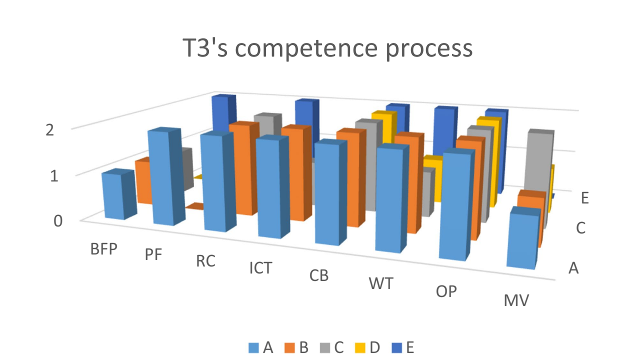


FIGURE 8
Competence process of teacher coded T3.

Production Fault and Obesity Problem activities. When T2's answers to the 2nd and 3rd questions from the general evaluation questions are examined, it is observed that that he identified Chessboard as his favorite activity and Production Fault as his least favorite activity (Figure 6).

In the Obesity Problem activity, which was solved as group work, T2 had problems solving with his group mates and stated that they misunderstood the problem at the beginning. The problems in understanding the problem at the beginning affected the whole process, and in this activity, T2 exhibited behavior only at Criterion 1 level of competence D.

When the competency table of the teacher-coded T2 is examined, it is seen that the teacher exhibited behaviors toward all competencies. It can be deduced from Figure 7 that the least demonstrated competence is competence E (Figure 8).

Activity-based competence change process of teacher-coded T3

When the competence process of T3 was analyzed, it was observed that the competence evaluation situations in the first activity, Big Foot Problem and Production Fault, and the last activity, Mayon Volcano, were different from the others. Namely, T3, which was evaluated as Criterion 2 in the activities in general, was mostly evaluated as Criterion 0 and Criterion 1 in the Big Foot Problem, Production Fault, and Mayon Volcano activities. T3's answers to the general evaluation questions are as follows (Figure 9).

When T3's answers are examined, it is seen that her favorite activity is the Big Foot Problem, and her least favorite activity is the Obesity Problem. When the competency evaluations of the teacher are examined, it is observed that all competencies are evaluated as Criterion 2 in the activity of Obesity Problem, which is the least favorite activity. It is believed that the activity solution process for the obesity problem was carried out as group work and that the people in the group positively affected each other. That is to say, the fact that the teachers in the other group had a majority of Criterion 0 s in the Obesity Problem competencies is also associated with the fact that the people in the group

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Teacher's response in the native language	Translation of teacher response
2. Dönem boyunca yapılan etkinliklerden en çok hangilerini sevdiniz? Neden? Yapılan etkinliklerden en çok ayakkabı izinden bu kişinin kim olacağının bulunması etkinliğini beğendim. Bu etkinlik matematiksel modelleme üzerine ilk yaptığımız etkinliklerdendi. O nedenle dikkatimi çok çekti. Ayrıca çözümünde kişilerin ayak uzunluğu ve ayakkabı numaralarından yararlanarak regresyon analizinden tahmin yürütülmesi farklı bir çözüm yoluydu.	**2.Which of the activities during the term did you like the most? Why?** Among the activities, I liked the activity of finding out who this person would be from the shoe print the most. This activity was one of the first activities we did on mathematical modelling. Therefore, it attracted my attention a lot. In addition, it was a different way to make predictions from regression analysis by using the foot length and shoe size of the people in the solution.
3. Dönem boyunca yapılan etkinliklerden en az hangilerini sevdiniz? Neden? Genel anlamda tüm etkinlikleri beğendim ama birini seçmek gerekiyorsa obezite etkinliği diğer etkinliklere göre daha az beğendiğim etkinliktir diyebilirim. Çünkü diğer etkinlikler bana daha açık uçlu soru gibi geldi ve obezite etkinliğinde sanki net bir cevap var gibiydi.	**3.Which of the activities during the term did you like the least? Why?** In general, I liked all the activities, but if I have to choose one, I can say that the obesity activity is the activity that I liked less than the other activities. Because the other activities seemed more open-ended questions to me. In the obesity problem, it seemed like there was a clear answer.

FIGURE 9
T3's answers to general evaluation questions 2 and 3.

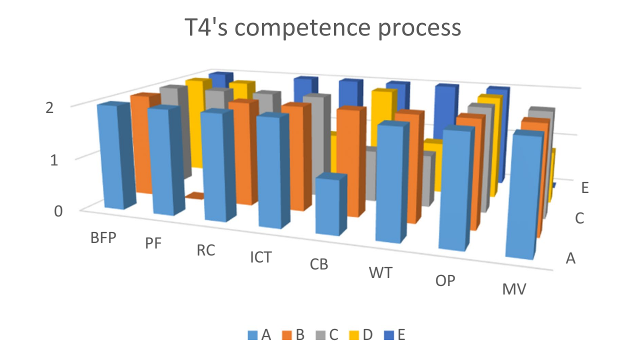

FIGURE 10
Competence process of teacher coded T4.

affected each other negatively. In the Production Error activity, the competency evaluations negatively reflected the fact that the teacher did not put forward too many ideas. Because the behavior expected from the teacher and the expression of his/her ideas could not be observed. In addition to the Obesity Problem activity, T3 was evaluated as Criterion 2 for all competencies in the Chessboard activity. The fact that the teacher coded T3 exhibited behavior toward all competencies for the Chessboard activity, which requires summing in sequences, suggests that the fact that the subject of sequences is included in the high school curriculum and that T3 is a high school mathematics teacher may be related to the fact that the subject of sequences is included in the high school curriculum and that T3 is a high school mathematics teacher (Figure 10).

Activity-based competence change process of teacher-coded T4

When the competence process table of the teacher-coded T4 is analyzed, it shows that she was evaluated as Criterion 0 in the activities of Production Fault and Mayon Volcano; in the other activities, she was mostly evaluated as Criterion 2. The teacher's active participation in the mathematical modeling learning process, expression of ideas, and exhibition of the expected behaviors reflected positively on her competence assessment process (Figure 11).

When T4's answers to the general evaluation questions are analyzed, it is seen that he liked the İzmir Clock Tower and Obesity Problem activities more; he liked the Big Foot Problem activity less. T4 stated that the activities with group work were more productive, played an active role in group work, and exhibited competent behaviors. In the answers given by T4 to the evaluation form after the Izmir Clock Tower activity, which he stated was one of his favorite activities, it was stated that the use of virtual reality glasses was both interesting and helpful in the solution to the different side of the activity (Figure 12).

Activity-based competence change process of teacher-coded T5

When the competence process form of the teacher-coded T5 is examined, it is seen that the competence evaluation criteria of the first activity, the Big Foot Problem activity, and the Obesity Problem

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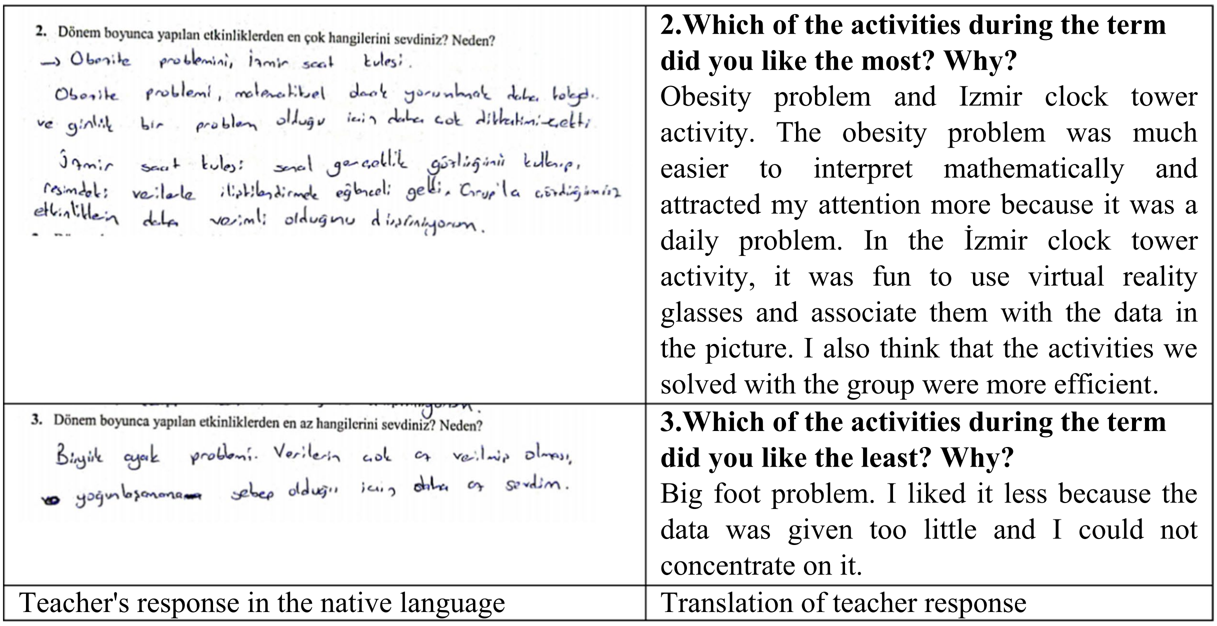

2. Dönem boyunca yapılan etkinliklerden en çok hangilerini sevdiniz? Neden? → Obezite problemini, İzmir saat kulesi. Obezite problemi, matematiksel olarak yorumlamak daha kolaydı ve günlük bir problem olduğu için daha çok dikkatimi çekti. İzmir saat kulesi: sanal gerçeklik gözlüğünü kullanıp, resimdeki verilerle ilişkilendirmek eğlenceli geldi. Grup'la çözdüğümüz etkinliklerin daha verimli olduğunu düşünüyorum.	**2.Which of the activities during the term did you like the most? Why?** Obesity problem and Izmir clock tower activity. The obesity problem was much easier to interpret mathematically and attracted my attention more because it was a daily problem. In the İzmir clock tower activity, it was fun to use virtual reality glasses and associate them with the data in the picture. I also think that the activities we solved with the group were more efficient.
3. Dönem boyunca yapılan etkinliklerden en az hangilerini sevdiniz? Neden? Büyük ayak problemi. Verilerin çok az verilmiş olması, yoğunlaşamamama sebep olduğu için daha az sevdim.	**3.Which of the activities during the term did you like the least? Why?** Big foot problem. I liked it less because the data was given too little and I could not concentrate on it.
Teacher's response in the native language	Translation of teacher response

FIGURE 11
T4's answers to general evaluation questions 2 and 3.

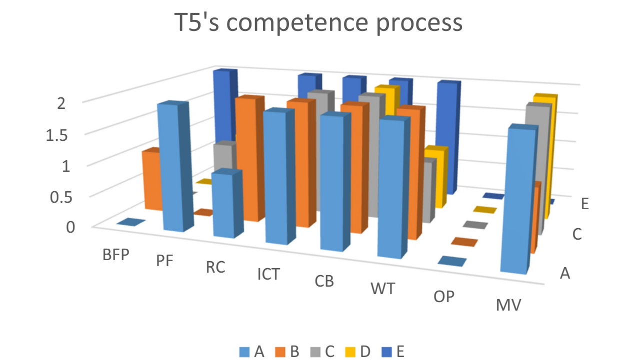

FIGURE 12
Competence process of teacher coded T5.

activity are generally Criterion 0. In the teacher's answers to the general evaluation questions, although all competencies were evaluated as Criterion 0, she said she liked the Obesity Problem activity the most. Notably, the teacher expressed the Izmir Clock Tower activity, in which competencies A, B, C, and E were evaluated as Criterion 2 and competency D as Criterion 1, as her least favorite activity (Figure 13).

It is seen that there are differences between the answers given by T5 and his behaviors toward competencies in the activity process. Namely, in the Obesity Problem activity, which he expressed as his favorite activity, he could not exhibit any behavior toward any competence. During the activity solution process, which was carried out as group work, he and his group mate stated that they misunderstood the activity and could not reach the result. However, the subject of the activity and its processing suggest that it attracted the teacher's attention.

- Below, it is exemplified how the teacher competencies were put forward in the solution process of the İzmir Clock Tower activity and how they were evaluated according to the criteria in Appendix 1.

İzmir Clock Tower

Before the İzmir Clock Tower activity was given to the teachers, cardboard glasses were distributed to the teachers and they were asked to set it up. The teachers, who set it up in a short time, stated that they thought that their students would not have difficulty in setting it up and that they would enjoy it very much. Afterwards, the teachers were asked to download the necessary application to their phones to be used in the activity solution and the activity papers were given to the teachers. In this activity, which has a visual of Izmir Clock Tower, the teachers were asked to estimate the height of the tower without any numerical data (Figure 14).

> *T5: We are asked for the height of the tower. But there is nothing we can process.*
>
> *T1: Actually, there are trees and people next to it. They can be data for us.*
>
> *T2: I saw it live. It is the same with these glasses. I wonder how many times my height is, it's like 30 metres.*
>
> *T5: I think there is 35 metres.*
>
> *T3: I counted the tiles on the floor and there are 30 tiles. So that makes 9 metres. I will make a Pythagorean guess from the slope. I think the length of the tower is close to 20 metres.*

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2. Dönem boyunca yapılan etkinliklerden en çok hangilerini sevdiniz? Neden? → Obezite problemi Çünkü günlük hayata en yakın olan etkinlikti. Birebir herkesin yaşadığı bir sorun olması, ve çözümünün mantıksal süreçlerde formülize ederek hesaplanabilir olması.	**2.Which of the activities during the term did you like the most? Why?** Obesity problem. Because it was the activity closest to daily life. The fact that it is a problem experienced by everyone and that the solution can be formulated and calculated in logical processes.
3. Dönem boyunca yapılan etkinliklerden en az hangilerini sevdiniz? Neden? → İzmir Saat Kulesi problemi Bende ilgi uyandırmadı ve çözüm yollarına ulaşmak için elimizde hiçbir sayısal veri yoktu.	**3.Which of the activities during the term did you like the least? Why?** Izmir clock tower. It didn't arouse my interest and we didn't have any numerical data to find solutions.
Teacher's response in the native language	Translation of teacher response

FIGURE 13
T5's answers to general evaluation questions 2 and 3.

FIGURE 14
Cardboard experiences of teachers in İzmir clock tower activity.

> *T4: I thought of the tower as 3 parts. If the first part is 4-5 metres, the second part is the same. The last one is as much as the sum of these, a little more than half of them. I think the height of the tower is around 20-25 metres.*
>
> *T1: I also think it is around 20 metres.*

Considering what the teachers expressed, all of them were evaluated as Criterion 2. Because what is expected from the teachers for competence A is to express the given and desired situations and to specify an estimated length. As can be understood from this dialog, the teachers revealed the expected situations for competency A.

> *T2: (towards the researcher) Can I approach the tower with these glasses?*
>
> *A: You can try.*
>
> *T2: Oh yes, yes. Wait a minute, I am 1.74 tall. I cannot climb the stairs naturally. But I come from downstairs to this side of the door (showing the visual on the activity sheet). If I take it a little higher, if I think I am climbing the stairs, I can think 1.90 above the ground. Wait a minute. There is a man next to the tree. If I compare his height with the tree, let me take the man as 1,80 on average. The tree is 6 times the man. The tower is twice the height of the tree. Then the tower is 12 times the height of the man. Then the tower is about 25 metres tall (Figures 15, 16).*
>
> *T4: I counted the tiles near the door (using cardboard glasses). There are 6 tiles on top of each other. If one of them is about 50 cm, the door would be 3 metres. The door (by measuring with a ruler) corresponds to 1 cm. The whole tower is 6.5 cm. Then the tower is 19.5 metres. So my answer is about 20 metres.*

FIGURE 15
T2's Solution process of İzmir clock tower activity.

FIGURE 17
T3 and T5's use of cardboard in the solution process of İzmir clock tower activity.

FIGURE 16
T4's activity solution with cardboard.

> *T3: Well, it is not possible to comment on the paper. I can count the tiles on the ground with the glasses. I mean, I can determine how far I am from the tower. For example, I can find out how many metres there is between that tree and the tower by counting the tiles. There are 30 tiles on the ground. If I take a tile 30 cm, that's about 9 metres. Then I thought I could make Pythagoras by thinking about the top of the tower, which is between 18 and 20 metres. Closer, I mean 20 metres (Figure 17).*
>
> *T5: I mean, when I look with glasses, the shadows on the ground attract my attention. I think I can compare the shadow lengths of the tree and the tower, but I need to walk a little bit to see the whole shadow. I proceed by counting the tiles on the ground. There are about 12 tiles in the shadow of the tree. Well, not exactly 12. It's more like 12. The shadow of the tower goes beyond these tiles. It didn't work with the shadow. Let me compare the height of the tower with the height of the tree. Let me see how many times the tree is next to it. 2.5 times. The tree is about 5 metres tall. And the tower is six times the height of the tree. Then the tower is about 30 metres.*
>
> *T1: When I examine the visual, I want to say 20 metres.*

As can be seen, in the process of solving the activity, the teachers reflected on the first prediction and tried to support their predictions with mathematical operations. For this reason, in this activity, all teachers except T1 were evaluated as Criterion 2. T1, on the other hand, did not make any mathematical reflection to support his prediction expressed in competence A and expressed his answer with an estimated expression. However, what is expected from teachers for competence B is to reflect on the first guess and support it mathematically. The teacher coded T1 did not make any mathematical comments in this activity, and for this reason, she was evaluated as Criterion 0.

In the process of creating a mathematical model in the İzmir Clock Tower activity, all teachers except the teacher coded T1 created a mathematical model. In this process, they solved the mathematical model they created by using mathematical knowledge. Following the teachers who stated that the problem situation and solution process were interesting, teacher coded T4 associated the problem with similar problems without any instructions.

> *T4: During moving, we always think whether this item will pass through this door or I will buy a cover for the table when I am out, what the dimensions of the table can be. I think it is the same logic. We are guessing after all.*

After this statement of the teacher coded T4, other teachers expressed their own opinions without any directive.

> *T1: It is used wherever there is estimation, this is true. In other words, when we buy a gift for someone, we actually guess the height and weight and say that this will fit him/her.*
>
> *T3: Yes, that's right, I thought of that too.*

> *T2: Or we say whether the products we buy will fit in this cupboard or not and we actually estimate its size.*
>
> *T5: Could it be, for example, we found a cardigan, don't guess who this person might be. After all, we take into account the dimensions of both the cardigan and the person.*
>
> *S3: I think it is possible.*

Considering the dialog above, all teachers except the teacher coded T3 associated the problem with similar problems. Teacher coded T1 could not perform a mathematical solution since she could not create a mathematical model beforehand. However, teachers coded T2, T4 and T5 both made mathematical solutions and associated the problem with similar problems. For this reason, teachers coded T1 and T3 were evaluated as Criterion 1 and the others as Criterion 2.

In the İzmir Clock Tower activity, teachers were able to interpret mathematical results in non-mathematical contexts. Namely, while estimating the height of the tower, they started from the visuals they saw on the activity sheet and with the coardbord. They interpreted these visuals in non-mathematical contexts. In addition, the following dialog was held with the teachers in order to generalize the solutions developed for a special case.

> *A: Do you think the models you created are valid only for the given situation or can they be valid for different situations?*
>
> *T5: I mean, mine is valid for this situation. Because I compared the height of the tree here with the height of the tower. I mean, how can I generalise this.*
>
> *T2: Exactly the same is true for me. I proportioned the height of the man next to the tree with the tree and the tower with the tree. So they are all specific to this image.*
>
> *T4: I mean, the models we created here are specific things, teacher. I think a general model cannot be created, I mean for all of our answers.*
>
> *T3: Exactly.*

Considering the dialog above, it was seen that teachers could interpret mathematical results in non-mathematical contexts, but they could not generalise the solutions developed for a specific situation. However, it is known from the previous competency findings that the teacher coded T1 could not create a mathematical model and could not develop any interpretation. Therefore, teacher coded T1 was evaluated as Criterion 0 and the others as Criterion 1.

In the İzmir Clock Tower activity, except for one of the teachers (T1), the others presented mathematical solutions. In order to reflect on these solutions, the researcher presented a directive and all teachers were given the right to speak to express their ideas.

> *A: You developed models close to each other to solve the problem. You listened to each other's solutions. Which one do you think was the most useful?*
>
> *T4: In general, everyone proportioned the tree to the person next to it and from there to the tower. I counted the tiles using the glasses (cardboard), estimated the length of the door and proportioned it to the tower. I think mine would be more useful and enjoyable for the student. Because the use of glasses is also enjoyable for the student.*
>
> *T3: I also reached a solution with glasses and used Pythagoras.*
>
> *T5: Actually, it depends on the lesson we use the activity. For example, if we are explaining Pythagoras, S3's solution is useful, if we are explaining ratio and proportion, the other one is useful.*
>
> *T1: Yes, I agree with S5.*
>
> *T2: As long as the student reaches the solution, no matter which way.*

Considering the dialog above, it is seen that T4 and T5 explained the useful solutions for the problem with their justifications. In this process, although the other teachers were given the right to speak again by the researcher, it was determined that they did not justify their ideas; therefore, teachers coded T4 and T5 were evaluated as Criterion 2 and the other teachers were evaluated as Criterion 0.

Discussion and conclusion

When the first competency, "Competency of Understanding the Real Problem and Creating a Model Based on Reality (Competency A)," of the teachers involved in the learning environment was examined, it was seen that the number of teachers who were evaluated as Criterion 0 in the Big Foot Problem activity, which was the first activity they encountered, was high. After the activity evaluation form was applied to the teachers after the activity and the class discussion, it was concluded that the teachers' evaluation of each other's answers affected the process positively. In other words, class discussions supported teachers in making sense of the situations expected from the activity. Maaß (2006) emphasized that classroom discussions are important for developing mathematical modeling competencies. In the following activities, Production Fault, Roller Coaster, Izmir Clock Tower, Water Tank, and Mayon Volcano, competency A was found to be at a very good level for all teachers. The reason for this is the class discussion after the solution in the first week and the instructions given for the activities. In this study, a holistic approach was exhibited, and teachers were not given information about mathematical modeling competencies, but they were made to gain competencies by experiencing the process with instructions. As it can be understood from here, teachers can exhibit A1 competence when they encounter the relevant instructions. In the 5th activity, CB activity, although there was no Criterion 0, it was observed that the number of teachers evaluated as Criterion 1 was higher than the number of teachers evaluated as Criterion 2. It is believed that the reason for this is that the activity is mathematically associated with a geometric sequence, and teachers started the question with this judgment. Although the teachers were shown a video based on the rumor in the question before starting the activity and were allowed to do the research they wanted from the internet, most of the teachers were insufficient in the model-building step. The same situation was also observed for the

Obesity Problem activity. This shows how the prejudice against mathematical modeling activities affects the process. Studies on the effect of these prejudices on the process are similar to this result (Busse and Kaiser, 2003; Galbraith and Stillman, 2001; Özturan-Sağırlı, 2010; Taşpınar-Şener, 2017; Urhan and Dost, 2016; Şen-Zeytun, 2013; Yenilmez and Dereli, 2009). Galbraith and Stillman (2001) stated that the activity's content can be negative and positive and that this situation may distract the individual from solving the activity. In this sense, Van den Heuvel-Panhuizen (1999) states that the negative process will disappear in problems related to the contexts in which individuals know themselves; that is, when they can imagine themselves in that problem situation. In the activities in which virtual reality was used, such as the Roller Coaster and Izmir Clock Tower activities, it was observed that A competence was at higher levels. Because virtual reality gives individuals the chance to feel themselves in the situation, in this context, it is believed that technological support in learning environments is important.

Although specific criteria were determined for competence A in each activity, in general terms, teachers are expected to "express what is given and desired about the problem, determine the possible variables to be used in the process, and create a model (first model) for the solution." In the Big Foot Problem activity, teachers were mostly unable to express many ideas about the problem because it was the first activity and did not contain any numerical data. In the following activities, teachers realized what was expected from the activity, analyzed the data related to the problem, and started using expressions per the determined criteria. This shows that the application carried out in the learning environment prepared with a holistic approach positively affected the development of competence. In other words, it suggests that the learning environment created is effective in the development of A competence.

When the teachers' "Competency of Constructing a Mathematical Model from a Real Model (Competency B)" was examined, it was seen that it was generally evaluated as Criterion 2 after the first activity and the second activity. It was observed that none of the teachers performed as expected in the PF activity. The reason for this is believed to be the fact that the data in the activity were presented with a computer program, and the teachers did not have sufficient knowledge about this program. It is noteworthy that all teachers were evaluated only in the roller coaster activity, as Criterion 2. The reason for this is believed to be that the activity was supported with virtual reality goggles and, as Aydın-Güç (2015) stated, to make teachers aware of real-life contexts that require exponential modeling. Virtual reality goggles facilitated teachers' interpretation of the real context, contributing to the first model created in Competency A being supported by real-world perception and revealing a mathematical model. Similarly, virtual reality goggles were used in the ICT activity, and only one teacher was evaluated as Criterion 0, while the other four teachers were evaluated as Criterion 2. The comment about virtual reality is believed to be valid for this activity. For the teacher who was evaluated as Criterion 0, he made predictions only by using the virtual reality goggles, but he did not support these predictions mathematically. It was observed that the reason for this was that the teacher found the virtual reality glasses remarkable and did not hear the instructions. Namely, the teacher in question listened to the answers of other teachers and tried to interpret their answers using the virtual reality glasses. At this stage, it is obvious that the virtual reality goggles caused a distraction for the teacher, who only developed assumptions and made predictions. Wrzesien and Raya (2010), Patera et al. (2008), and Özdemir (2017) reveal findings in their studies that the use of virtual affected the development of competence. In other words, it suggests that the learning environment created is effective in the development of A competence.

When competence B was analyzed for the whole process, it was seen that a general picture was not presented. In the first two activities, Big Foot Problem and Production Fault, it was observed that the evaluations made as Criterion 0 and Criterion 1 were in the majority. Although it was observed that they decreased in the following weeks, they increased again in the last two activities, Obesity Problem and Mayon Volcano. The fact that group work was done in the Obesity Problem activity shows that teachers in the same group negatively affected each other. The fact that Yıldırım and Selvi (2018) mentioned the negative effect of group work in their study also supports this situation. It is believed that the same situation is in question for the Obesity Problem activity. In the Mayon Volcano activity, the number of volcanic eruptions starting from the 1600s was given, and students were asked to develop a mathematical model based on the last 100 years. The fact that there were too many eruptions in the given data group forced the teachers to overlook the fact that they should create a model with data from the last century. Teachers who tried to create a model by taking into account all the data could not create a model, and this situation was reflected negatively in the evaluations. In other words, it is believed that negative reflections emerged in the evaluation of this competence due to the structure of the activity. Maaß (2006) also states that individuals have difficulty creating a model when they do not understand what is expressed. However, when the performance development of individual teachers in the process of competence B is taken into consideration, a positive graph stands out except for the teacher-coded T1. This suggests that the learning environment created positively affected the development of B competence.

When the "Competence of Solving Mathematical Problems with the Created Mathematical Model (Competence C)" was examined, it was seen that the competence criteria showed an up-and-down situation as the process progressed. At the end of the process, all teachers were evaluated as Criterion 2. In general, it is noticeable that there are very few evaluations, such as Criterion 0, and most Criterion 1 evaluations are made. Especially in the activities of the production fault and water tank, the excess of the evaluations made by Criterion 1 draws attention. In the previous competence B, all teachers were evaluated as Criterion 0 in the Production Fault activity. This is because none of them could present a mathematical model. For this competency, the fact that they could not create a model themselves in the Production Fault activity shows that it is difficult to perform mathematical operations on the model.

For this competency, in which mathematization is at the forefront, it is believed that teachers' reaching the solution creates the perception of completion of the activity. For this reason, while Criterion 0 and Criterion 1 evaluations were high in the activities in the first 2 weeks, it was observed that Criterion 2 increased with the instructions given in the following weeks. It was determined by Brand (2014), Çakmak-Gürel (2018), Çiltaş (2011), Aydın-Güç (2015), Ji (2012), and Kertil (2008) that teachers who were initially unable to perform competency C, in which mathematization is at the forefront, achieved success in this competency as a result of their participation in the learning environment. In addition to these, Biccard and Wessels (2011) and Kaiser and Brand (2015) revealed in their studies that the fact that individuals have received training in modeling has a positive effect on their competence in solving mathematical problems. It is believed that the learning environment created in this study contributed positively to the development of C competence.

When the teachers' "Interpretation of Mathematical Results in Real Situations (Competency D)" was analyzed, it was seen that the evaluations as Criterion 2 were predominant in the Roller Coaster and Chessboard activities. In the first activity, the Big Foot Problem activity, the excess of the evaluations made as Criterion 0 is striking. For this competency, in which teachers are expected to interpret mathematical results in non-mathematical contexts and generalize the solutions they developed for a special situation, it is among the important results that teachers generally made interpretations in non-mathematical contexts but did not generalize the solutions. For this reason, when the process is examined in general terms, the excess of the evaluations made as Criterion 1 stands out. Firstly, when the criterion of interpreting mathematical results in non-mathematical contexts was considered, it was observed that teachers made interpretations without the need for instructions after the first activity. For the İzmir Clock Tower and Mayon Volcano activities in which virtual reality and augmented reality were used in the activities, all of the teachers exhibited competence in interpretation in real situations. This suggests that today's technology used in the created learning environment provides positive contributions to teachers in order to associate and interpret the problem with real life. Özdemir (2017), who already identified the advantages of using virtual reality in education, revealed that students can better interpret mathematical concepts and adapt them to real life thanks to sensory experiences. Considering that the interpretation of solutions in real life is more in these activities, it can be interpreted that this effect is due to technological support. In the Chessboard activity, in which all teachers were evaluated as Criterion 2, it is believed that the association of mathematical results with the real situation and the generalization of the solution are due to the activity. In terms of generalizing the solutions they developed for a special situation, it is noticeable that in almost every activity, teachers did not make a statement about generalizing after finding the solution. Although the importance of generalization was mentioned in the class discussions after each solution and instruction were given, this criterion of D competence could not be achieved much. Çiltaş (2015) stated in his study that individuals did not generalize due to the perception that reaching the solution completed the process. It is believed that there is a similar situation in this study. In addition, in the study conducted by Aydın-Güç (2015), it was determined that the sub-competency of interpreting mathematical results in non-mathematical contexts (E1) increased, while the sub-competency of generalizing solutions developed for a special situation did not increase. On the other hand, Çakmak-Gürel (2018) found that both sub-competencies increased after the mathematical modeling teaching process. It is believed that the similar result with Aydın-Güç (2015) and the different result with Çakmak-Gürel (2018) is due to the learning environments created. Kaiser and Brand (2015) emphasized the importance of the learning environment they designed and stated that the designed learning environment made a great difference in D proficiency. In this study, it was observed that the learning environment created had a positive effect on interpreting mathematical results in non-mathematical contexts; however, it did not provide the desired effect in generalizing the solutions developed for a special situation.

When the "Solution Verification Competency (Competency E)" was analyzed, it was seen that a general picture did not emerge. It is noticeable that the teachers who were evaluated as Criterion 2 in the Big Foot Problem activity they encountered for the first time were all evaluated as Criterion 0 at the end of the teaching process. In this competency, in which the criteria of reflecting on the solutions and determining the most appropriate model were taken into consideration, differences were determined in the evaluations of the teachers depending on the nature of the activity and their active participation in individual and group work. Although reflections were made on all solutions found in class discussions in the learning environment and instructions were frequently used in almost every activity, no clear situation emerged in the competency evaluations. This is believed to be because teachers are not at a sufficient level in making critical evaluations. Because in the literature, it is found that individuals are weak in making critical evaluations despite having experience in mathematical modeling (Aydın-Güç, 2015; Ji, 2012; Tekin-Dede and Yılmaz, 2013).

On the other hand, many studies in the literature show that individuals believe in the accuracy of their solutions and do not feel the need to revise them and do not check the accuracy of the solutions and calculation errors for the problem situation (e.g., Blum and Borromeo Ferri, 2009; Blum and Leiß, 2007; Galbraith and Stillman, 2001; Maaß, 2006; Şen-Zeytun, 2013). In their studies, Şen-Zeytun (2013) and Blum and Borromeo Ferri (2009) state that this situation is that students think that the instructor is responsible for verifying the solutions. In other words, the idea that the instructor is the one who should give feedback on whether the solution to the problem is correct or incorrect negatively affects reflections on solutions.

Considering the competence of reflecting on solutions, the positive effect of the learning environment created in the general process cannot be mentioned. When the general table is analyzed, the fact that all teachers were evaluated as Criterion 0 in the last activity, the Mayon Volcano activity, showed that the effectiveness of the environment in terms of competency E resulted negatively. In this learning environment created with a holistic approach, teachers were not given theoretical information about competencies, and it was expected that the existing competence would emerge with the instructions used in the process. This suggests that theoretical knowledge should support this situation in the learning environment. Theoretical information about mathematical modeling cycles was given, and the verification of the model, which is one of the modeling steps, was emphasized during this process.

Nevertheless, at the end of the process, it was observed that the expected level of E competence was not reached. Although the modeling cycle was taken into consideration in the solution process, E competence did not emerge. This constitutes an answer to Aydın-Güç (2015) question of whether following a modeling cycle is effective in the emergence of E3 subcompetence of E competence. As a result, it is believed that the emergence of this situation is due to either the learning environment created or the lack of theoretical information about the expected competencies and modeling activities in modeling education. Huang (2011) also stated that since the development of individuals' modeling competencies is a complex and challenging process, longer and more modeling practices should be carried out in the designed learning environment.

It is also believed that the structure of the activity was effective in the emergence of competence E, as facilitated by the given instructions. Specifically, competence E was observed to develop in all teachers during the Big Foot Problem, Roller Coaster, Water Tank, and Chess Board activities. It can be said that this is an indication that the structure of the activity is effective in the emergence of E competence.

Aydın-Güç (2015) also reached a similar conclusion. In other activities, teachers did not respond to the instructions directing them to think of different solutions, although they were directed to different solutions. It is believed that the reason for this is that the teachers already expressed different solutions from each other in the class discussion, that the process of creating a model is difficult and time-consuming, and that they think that the solution they have found is sufficient. In his study, Dowlath (2008) states that although pre-service teachers know that there are different solutions to solve mathematical modeling activities, they use the solution they are most used to. In this study, it could not be observed that the learning environment clearly contributed positively to the E efficacy of the teachers. Competencies emerge depending on the nature of the activity.

To put forward a general conclusion for all these competencies, it was seen that the learning environment created had a positive effect on the development of competencies A, B, and C; it had a partially positive effect on competence D, although not much; it did not have a positive effect on competence E. The reason for this is believed to be that the teachers have problem-solving skills both in their own student lives and in their teaching lives, and they train students with this skill. Namely, problem-solving skills include understanding the problem, planning the solution, and implementing and evaluating the solution. At the end of this process, individuals are satisfied with finding a clear solution to the problem. However, the mathematical modeling process is different from the problem-solving process. In the mathematical modeling process, the steps of interpreting and verifying mathematical results in real contexts show that finding the problem's solution is insufficient. For this reason, it is believed to have a negative impact on the emergence of D and E competencies, which require verifying the solution, reflecting on the solution, and thinking about different solutions, unlike problem-solving steps. It is believed that this resistance is not surprising despite the instructions since teachers' experiences are mostly based on finding a single and clear result based on problem-solving throughout their lives.

When the roller coaster and Izmir Clock Tower activities using virtual reality in the learning environment, the Mayon Volcano using augmented reality, and water tank activities using concrete materials were examined in general, it was determined that Criterion 2 evaluations were predominant, especially in A, B and C competencies. This situation is believed to be due to the positive effect of the materials used. Since virtual reality is a technological support that allows individuals to be in any place or to experience an event, it is believed that it enables individuals to experience the problem situation better in competence A. The demand of the teachers, who internalized the problem situation thanks to the virtual experiences, to recreate these experiences while mathematizing enabled them to progress the process correctly, which was reflected positively in competence B. It was determined by Gül et al. (2020) that the transfer of mathematical relationships and processes to virtual reality facilitates analytical understanding and produces positive results for mathematization, similar to this study's findings. In addition, it was also observed that watching a video containing the scenario of the problem before proceeding to the solution process of the Chessboard activity positively affected the development of competence. Most mathematics educators state that materials effectively develop students' mathematical thinking (Kamii et al., 2001). In parallel with this, it was seen that the use of both technological and concrete materials had positive results on competencies. For this reason, it is believed that such materials should be used in the learning environments created.

The focus was on the development of mathematical modeling competencies of mathematics teachers through mathematical modeling training in a learning environment prepared in accordance with the mathematical modeling approach. In general, the factors affecting teachers' mathematical modeling competencies in this learning environment can be listed as follows:

- Structure of events
- Group work
- Class discussions
- Researcher guidelines
- Technological support
- Tangible material

Moreover, it is believed that many factors, such as teachers' reading-comprehension competencies, their level of participation in the lesson, and their beliefs that they can do it themselves, affect mathematical modeling competencies. Considering all these situations, it can be said that mathematical modeling competencies are a complex structure affected by many factors.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material. Further inquiries can be directed to the corresponding author.

Ethics statement

The studies involving humans were approved by the Atatürk University, Educational Sciences Unit Ethics Committee. The studies were conducted in accordance with the local legislation and institutional requirements. The participants provided their written informed consent to participate in this study. Written informed consent was obtained from the individual(s) for the publication of any potentially identifiable images or data included in this article.

Author contributions

SB: Writing – original draft, Writing – review & editing. AÇ: Data curation, Validation, Writing – review & editing.

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Supplementary material

The Supplementary material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/feduc.2025.1509652/full#supplementary-material

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