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RECEIVED 29 November 2024

ACCEPTED 01 September 2025

PUBLISHED 09 October 2025

CITATION

Castillo D, Carrión J, Chamba C,
Jiménez-Gaona Y, Rodríguez-Álvarez MJ and
Lakshminarayanan V (2025) Didactic
strategies for conceptual understanding and
motivation in university mathematics: a
systematic review.
Front. Educ. 10:1536470.
doi: 10.3389/feduc.2025.1536470

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Didactic strategies for conceptual understanding and motivation in university mathematics: a systematic review

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The conceptual understanding and motivation for learning and teaching math constitute a challenge for didactic research at all levels of education. However, it is essential in higher education levels like the university, where achieving advanced reasoning and connecting the nature of math with professional applications is important. This systematic literature review analyzes peer-reviewed research published between 2000 and 2023, focusing on didactic and instructional strategies applied that enhance comprehension and engagement in undergraduate and higher mathematics education. Following PRISMA guidelines, we get a final analysis of 30 studies where the semiotic representations and gamification strategies are considered key strategies to achieve conceptual and motivated understanding of math in the context of higher education. Semiotic methods from Duval's theoretical framework emphasize the coordination of symbolic, graphical, and algebraic registers to promote deep conceptual learning. As an active learning method, gamification is highly effective for enhancing student engagement and motivation, helping students overcome their apprehension toward mathematics. While most studies explored these strategies independently, this review identifies gaps in integrative approaches. It highlights the need for further research on their combined impact, especially when representational depth is aligned with motivational design. The 2000–2023 window captures the consolidation of semiotic frameworks and the expansion of ICT and gamification in higher mathematics education.

KEYWORDS

teaching math, strategies, didactic, mathematics, education, VOSviewer, gamification, semiotic representations

1 Introduction

Mathematics, as constructed over millennia of history beginning with the earliest ancient civilizations, has been fundamental to human development. Undoubtedly, modern civilized society would be impossible without mathematics (Boyer and Merzbach, 2019).

In our daily lives, we use math: when we go to the store, when we buy groceries, when we go traveling, when we count the points in our games, etc. It means that math can be considered

an essential element in our lives. However, when we are learning math, along the entire academic path, for many students, it is generally perceived as a complex subject with a lot of complexity to understand, and often disconnected from real-life problems and applications.

As Dan Meyer pointed out (Klaassen, 2023), this negative perception is often reinforced by traditional and passive instructional methods emphasizing procedural knowledge over connecting mathematical content with students' realities and cognitive development. The challenges and difficulties with math are more visible in higher education because students are expected to perform advanced mathematical reasoning while often lacking foundational conceptual tools.

In this context, over the last few years, there has been continuous research focused on different strategies and innovative pedagogical methodologies to achieve a better conceptual understanding of mathematics while maintaining a sustained engagement with the students. Multiple studies indicate that middle and university level students struggle to apply mathematical knowledge to real-world contexts due to insufficient integration of innovative pedagogical strategies in classroom instruction (Strömberg et al., 2014; Brozo and Crain, 2018).

Therefore, as AlAli et al. (2023) and Wardat et al. (2023) suggest, promoting the development of the students' mathematical thinking skills through contextually meaningful and cognitively engaging strategies is fundamental. In that way, learners could develop the ability and competence to visualize and solve problems with accurate reasoning and efficiency, instead of following merely rules or procedures.

In higher education mathematics, students often experience difficulty connecting abstract concepts to real-world cases, which undermines persistence and confidence. Over the last two decades, research and the need to cultivate a deep understanding of math with engagement and motivation, has inspired a diverse set of strategies and innovations, including: semiotic approaches, integration of information and communication technologies (ICTs) (Hoyles and Noss, 2003), immersive learning environments (Amable Vivanco-Galvan et al., 2018), STEAM-based learning (Perignat and Katz-Buonincontro, 2019; Lakshminarayanan and McBride, 2015), collaborative projects, gamification (Tashtoush et al., 2023; Milovanovic et al., 2021; Jiménez-Gaona et al., 2019), virtual and augmented reality tools (Rodríguez, 2022; Villacís Macías et al., 2022), and recently the use of Artificial Intelligence (AI) based applications (Wardat et al., 2023). These approaches inspire exploration, reasoning, and social interaction over the traditional frontal and procedural instruction that still predominates in many university classrooms.

In this sense, this systematic review focuses on exploring and identifying how these didactic strategies are deployed in university mathematics education, aiming to promote two core elements: conceptual understanding and student motivation. The methodology of this study was a systematic review of the literature; therefore, the following databases were used as sources of information: Scopus, PubMed, Web of Science, Science Direct, IEEE Xplore, and Google Scholar.

By examining studies from 2000 onwards under PRISMA guidelines, this review answers the following guiding question:

- What are the most frequently studied didactic strategies for improving conceptual understanding and student motivation in higher education mathematics?

The results and findings of this review are concentrated on two strategies: semiotic representations, which facilitate the conceptual

processing of math through symbolic, graphical, and algebraic coordination (Duval, 2006); and gamification, which enhances engagement and reduces math anxiety through instructional designs based on games.

The structure of the paper is as follows: Section 2 presents the theoretical framework concerning teaching theory and learning theory, semiotic representations, and motivation and educational technology centered on gamification. Section 3 presents the review methodology and selection criteria. Section 4 discusses the review's findings and results regarding semiotic and gamification strategies. Section 5 provides conclusions, limitations, and implications for practice and future research.

2 Theoretical framework

2.1 Teaching and learning theory

Learning theory (Bada and Olusegun, 2015) describes how students receive, process, and retain knowledge during learning. Learning strategies encompass the thoughts and behaviors that help students acquire new information and integrate it with their existing knowledge (Yip, 2012). Cognitive, emotional, and environmental influences, as well as prior experience, all play a role in how understanding, or worldview, is acquired or changed and knowledge and skills are retained (te Braak et al., 2022).

A key teacher competency in this regard is the ability to collect information on students' learning progress, make diagnostic inferences, and respond through an ongoing, interactive process (Chapman, 2013). In particular, teachers must be able to recognize and understand students' difficulties, infer a broad range of strengths and weaknesses, provide targeted feedback, and design appropriate tasks to foster students' mathematical thinking (Dorier and Mass, 2020).

2.2 Semiotic representations in mathematics learning

Duval (1995, 1999, 2006, 2017) proposed the theory of semiotic representation, which enounces that the understanding and comprehension of mathematical concepts depend on the ability to coordinate and express multiple and different semiotic registers, which means the capacity to express math in various forms, such as verbal, graphical, symbolic, and tabular forms.

This theory is particularly significant in calculus, geometry, and algebra, where translation between registers allows for the acquisition of a deep understanding, e.g., from a function graph to its symbolic expression. Misalignment between semiotic representations is frequently linked to conceptual misunderstanding and fragmented knowledge, reinforcing the need for pedagogical strategies that explicitly design activities to develop this coordination.

Recent literature (Pedersen et al., 2021; Caligaris et al., 2019) expands Duval's theory by analyzing how tasks and digital environments mediate semiotic transformations. Moreover, researchers such as Burgos et al. (2021) and Salazar (2018) explore the ontosemiotic complexity and cognitive demands inherent in university-level math problem-solving.

2.3 Motivation theories in mathematics education

Motivation is central to sustaining mathematical engagement and reducing anxiety, especially in abstract or cognitively demanding domains.

Keller (1987) defines four pillars: Attention, Relevance, Confidence, and Satisfaction (ARCS Model of Motivation), often reflected in gamified strategies, game-based learning, problem-based learning, and real-world math applications. Many reviewed studies link improved learning outcomes to instructional designs aligned with these dimensions (Pehlivan and Arabacioglu, 2023; Chapman and Rich, 2018).

The development of autonomy, competence, and relatedness as intrinsic motivators is called Self-Determination Theory (SDT) (Deci and Ryan, 1985). Adaptive gamification environments, flipped classrooms, and collaborative models tend to fulfill these psychological needs, improving students' perseverance and academic self-concept (Rivera and Garden, 2021; Hassan et al., 2021; Bennani and Maalel, 2022).

These ideas help us see how particular teaching decisions, whether using game-like feedback, showing students their progress, or allowing them to choose how they represent their ideas, can boost motivation in university (higher education) classrooms.

2.4 Gamification and educational technology

Gamification (Sobrinho-Duque et al., 2022), defined as the use of game mechanics and game elements in a non-game context, has attracted considerable attention and has been applied across a wide range of fields to motivate and engage individuals in the performance of specific tasks, activities, and the resolution of various problems (Kapp, 2012).

The integration of digital tools in university mathematics teaching has been accelerated by increased access to educational technologies and growing recognition of their motivational affordances (Bouchrika et al., 2021). In this sense, in e-learning education, gamification enriches the math learning experience, as seen in the studies from (Bouchrika et al., 2021; Lubis et al., 2014). Their analysis shows how gamification in math could be translated into smartphone apps to increase the effectiveness of the engagement of math students.

As an educational tool, gamification facilitates learning, encourages motivation, improves student engagement and lesson interactivity, and encourages students to expand their knowledge (Bennani and Maalel, 2022; Behl et al., 2022; Sabri et al., 2022). However, effective use requires alignment with meaningful mathematical reasoning, not merely entertainment (Jiménez-Hernández et al., 2020; Faghihi et al., 2014).

3 Materials and methods

This systematic review identifies didactic strategies in university mathematics education that specifically foster conceptual understanding and engagement motivation. The search strategy was guided by Preferred Reporting Items for Systematic Reviews (PRISMA 2020) criteria (Page et al., 2021) and based on the methodologies proposed by Torres-Carrion et al. (2018) and Kitchenham (2004).

3.1 Search term framework

The conceptual framework used in this work allows us to focus and restrict the subject to didactic strategies for teaching mathematics. The method proposed by (Vicente Torres-Carrion et al., 2018), called “conceptual mindfact” (mentefacto conceptual), helped to organize the scientific thesaurus keywords for the research topic (Figure 1).

3.2 Conducting the review

Once the keywords concerning the research theme are identified, the next step is to organize a semantic search structure that allows us to get the documents for the review analysis. Table 1 presents the semantic search structure (Torres-Carrion et al., 2018), such as entering specific search literature (documents) in scientific databases.

The first level represents the teaching search; the second corresponds to the keyword Mathematics. The third level is relevant for applying the strategy for analyzing scientific documents. The fourth level is the search for global semantic structure.

3.3 PRISMA

3.3.1 Identification and screening

The global semantic structure (see Table 1) used allows us to identify 525 documents through a worldwide search, especially on Scopus and Google Scholar (427 documents) and Web of Science (WoS) (98 documents).

3.3.2 Eligibility and inclusion

Inclusion criteria:

- 1 Studies focused on university/high-level mathematics education, or with extrapolable implications for the higher-education context.
- 2 Studies that implement didactic strategies with measured outcomes in conceptual understanding/comprehension, and/or motivation/engagement.
- 3 Studies concerning didactic strategies (e.g., gamification, semiotic representations, ICT-enhancing teaching, and cooperation).

Exclusion criteria:

- 1 Studies that are not peer-reviewed
- 2 Theoretical studies without any didactic application.
- 3 Studies without a connection to mathematical reasoning, comprehension, or motivation for learning.
- 4 Studies centered on non-university educational levels, unless they offer explicit, extrapolable implications for higher-education contexts.
- 5 Studies related to engineering or psychological strategies

3.3.3 Study selection and reason for exclusion

From 525 records, 160 duplicates were removed; 365 were screened by title/abstracts, and 255 were excluded for document type or out of scope. A total of 110 full texts were assessed, and 80 were

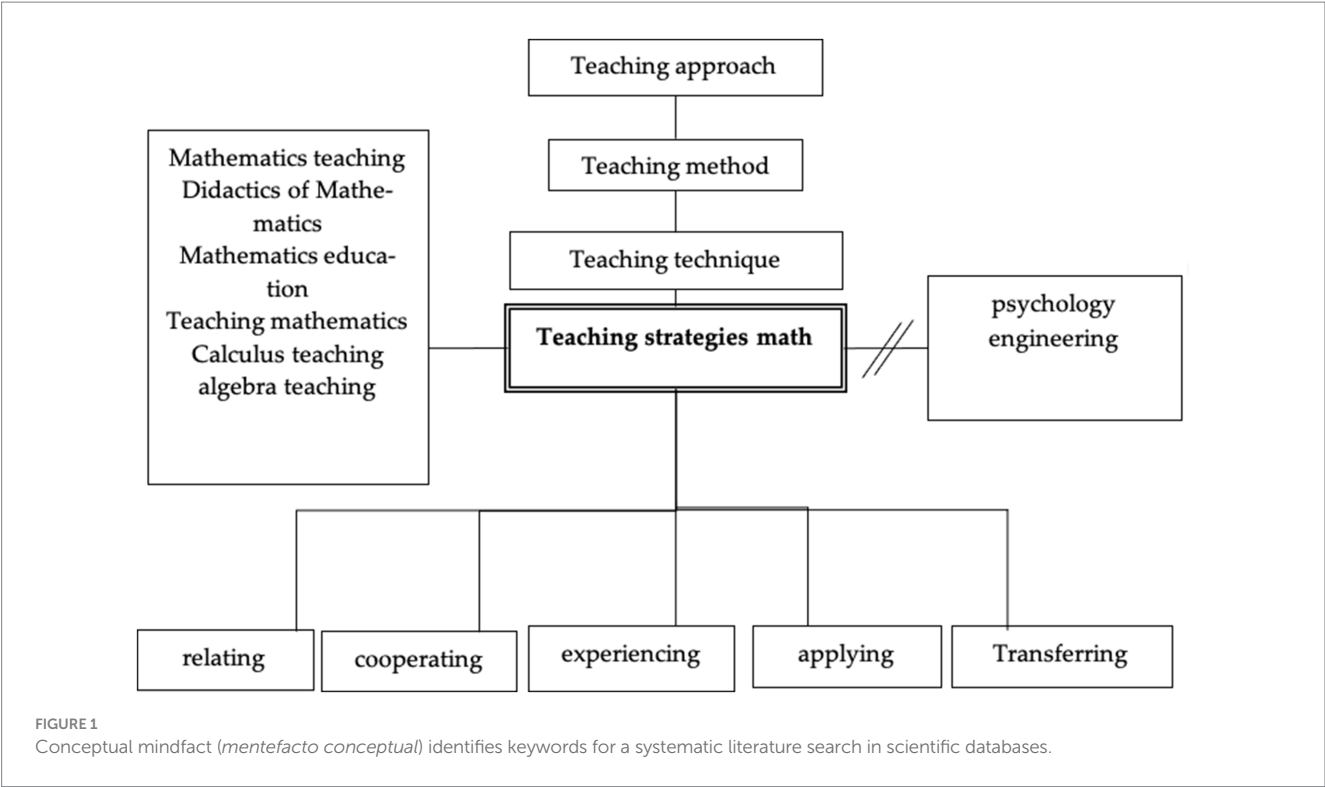


TABLE 1 Keywords used in the search for global semantic structures.

Teaching	((teaching AND (approach AND method AND technique)) AND strateg* AND higher education)
Mathematics	((math*) AND (relating OR cooperating OR experiencing OR applying OR transferring)) OR mathematics AND teaching OR didactics AND of AND mathematics OR didactics AND of AND mathematics OR teaching AND mathematics OR calculus AND teaching OR algebra AND teaching))
Strategy	(Relating AND Cooperating AND Experiencing AND Applying AND Transferring)
Key words for semantic structure search in database	TITLE-ABS-KEY (((teaching AND (approach AND method AND technique)) AND strateg*) AND ((math*) AND (relating OR cooperating OR experiencing OR applying OR transferring)) OR mathematics AND teaching OR didactics AND of AND mathematics OR teaching AND mathematics OR calculus AND teaching OR algebra AND teaching)) AND PUBYEAR > 1999 AND PUBYEAR < 2024 AND higher education or university

The symbol (*) represents a wildcard to help search for a word with multiple spelling variations.

excluded for the following reasons: (R1) no higher education context or non-extrapolable ($n = 43$), (R2) no target outcomes in conceptual/motivation ($n = 16$), (R3) purely theoretical/no didactic application ($n = 12$), (R4) non-math-education focus ($n = 9$). A total of 30 studies were included for synthesis, focused on strategies based on conceptual understanding of math and enhancing the motivation to learn it in university/higher education mathematics contexts (Figure 2).

3.4 Maps in VOSviewer

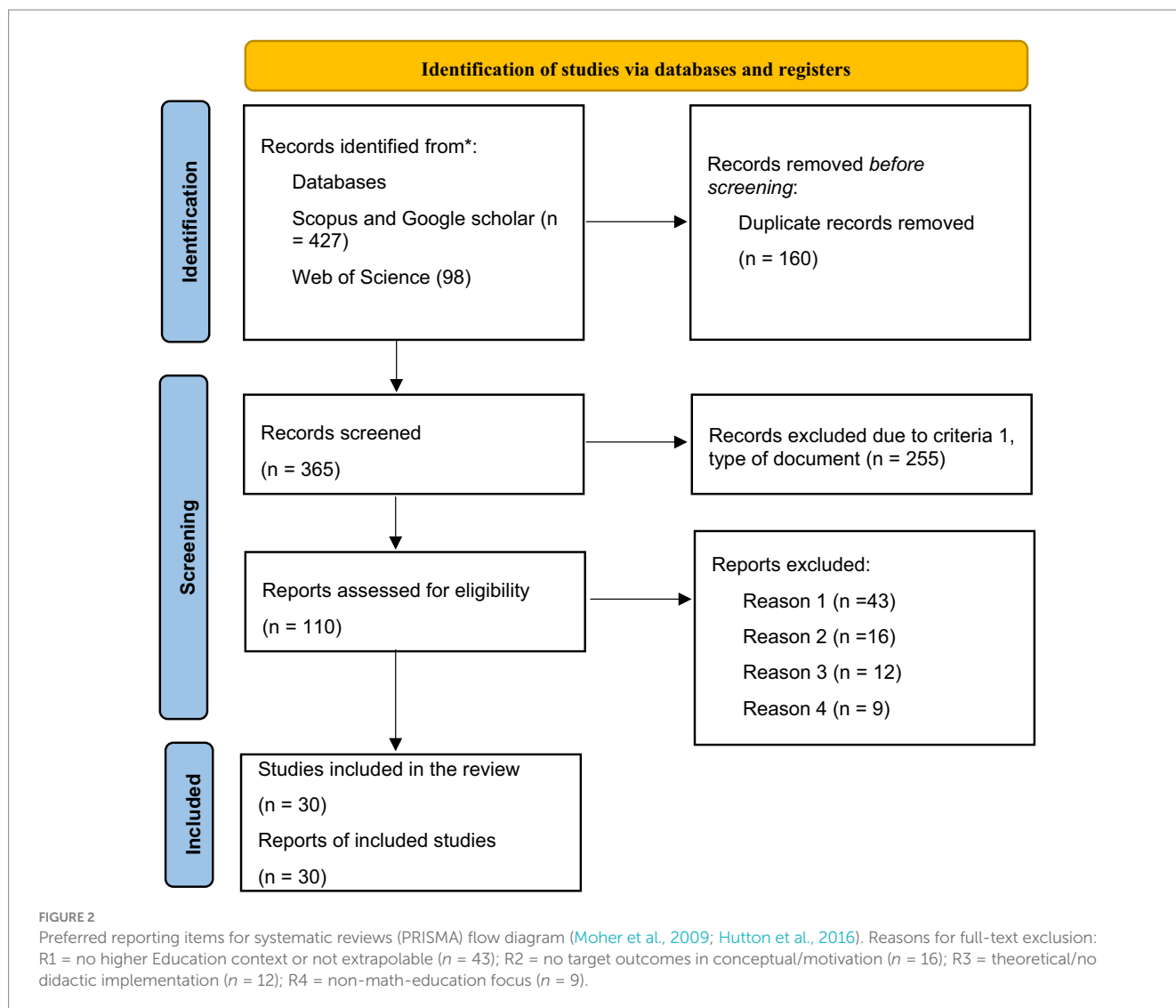
VOS viewer (Van Eck and Waltman, 2010) is a software tool at the Leiden University Center for Science and Technology Studies designed for building and visualizing bibliometric networks. These networks can be built based on citations, bibliographic linkage, co-citation, or co-authorship relationships, including journals, researchers, or individual publications.

Several studies show the application of the VOSviewer in different fields, such as economy (Perianes-Rodriguez et al., 2016; Iliescu, 2021), engineering and computer science (Castillo et al., 2021; Wang et al., 2022), and, of course, also in math education research (Ersozlu and Karakus, 2019; Verma et al., 2021; Hanif Batubara et al., 2022; Veith et al., 2023).

We used VOSviewer software version 1.6.15 for analysis to construct and display bibliometric maps. The data for this objective were obtained from Scopus due to its coverage of a broader range of journals.

3.5 Clustering

For structure visualization, we one-hot encoded four categorical features per study: didactic strategies, education level (School/Secondary/University), outcome categories (Motivation; Conceptual; Both; Other), and instrument category (e.g.,



questionnaire/test/task-analysis). PCA was applied to reduce dimensionality to 2D, and K-means ($k = 3$) was run to identify profiles.

4 Results and discussion

4.1 Publication evolution

The global semantic structure search (Table 1) found 525 documents of different types (articles, presentations, and reviews) from 2001 to December 1, 2023. Figure 3 shows the number of publications and their evolution in this period.

4.2 Keywords and citations

Regarding the keywords and related publications, Figures 4, 5 present the map of the network of publications about the citations and keywords, respectively. Both of them were

designed in VOSviewer with the database from Scopus. In the map, the density of the yellow color in each keyword indicates the number of repetitions in the total number of scientific documents. The most used keywords are mathematics education, students, and teaching.

4.3 Didactic strategies

Table 2 describes the scientific documents related to our research topic in this project. Figure 6 shows the distribution of the documents analyzed in this review, (a) by level of education and (b) by country.

According to Table 2 and through the analysis of the documents, we can see in Figure 7 that *gamification* (56.7%), *semiotic representations* (36.7%), and *learning theories* (6.7%) are the strategies most applied to improve or research math learning at the university level. These findings reflect that the research trends favoring motivational and interactive approaches in higher education math have evolved over the last two decades.

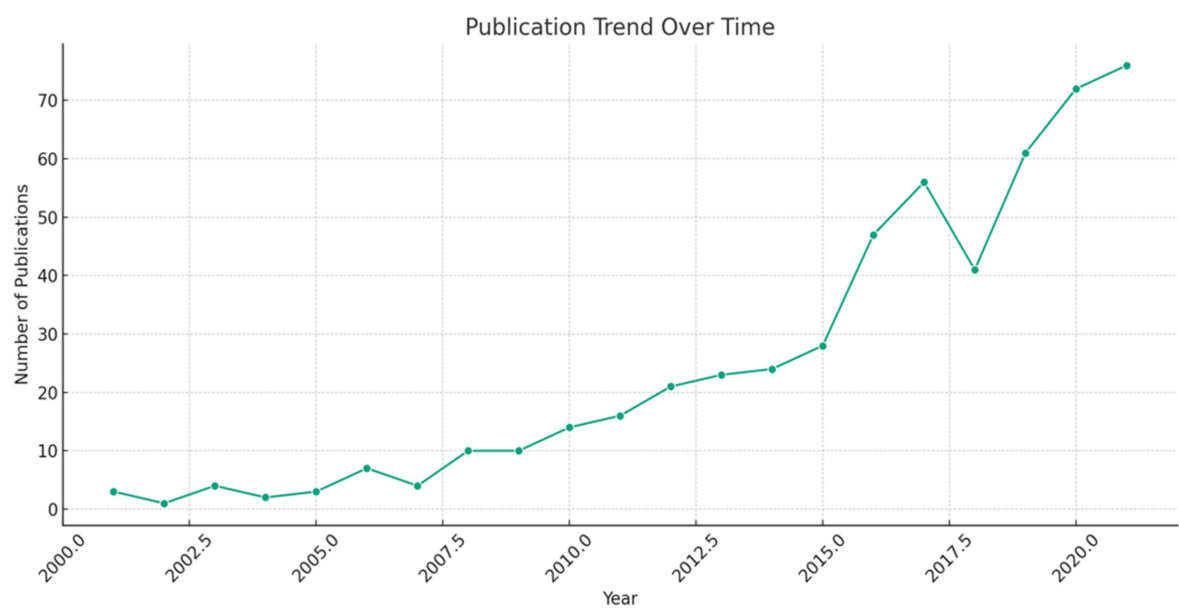


FIGURE 3 Evolution of publications during the last two decades. We can see that there has been an increase in the trend of research in math education since 2015.

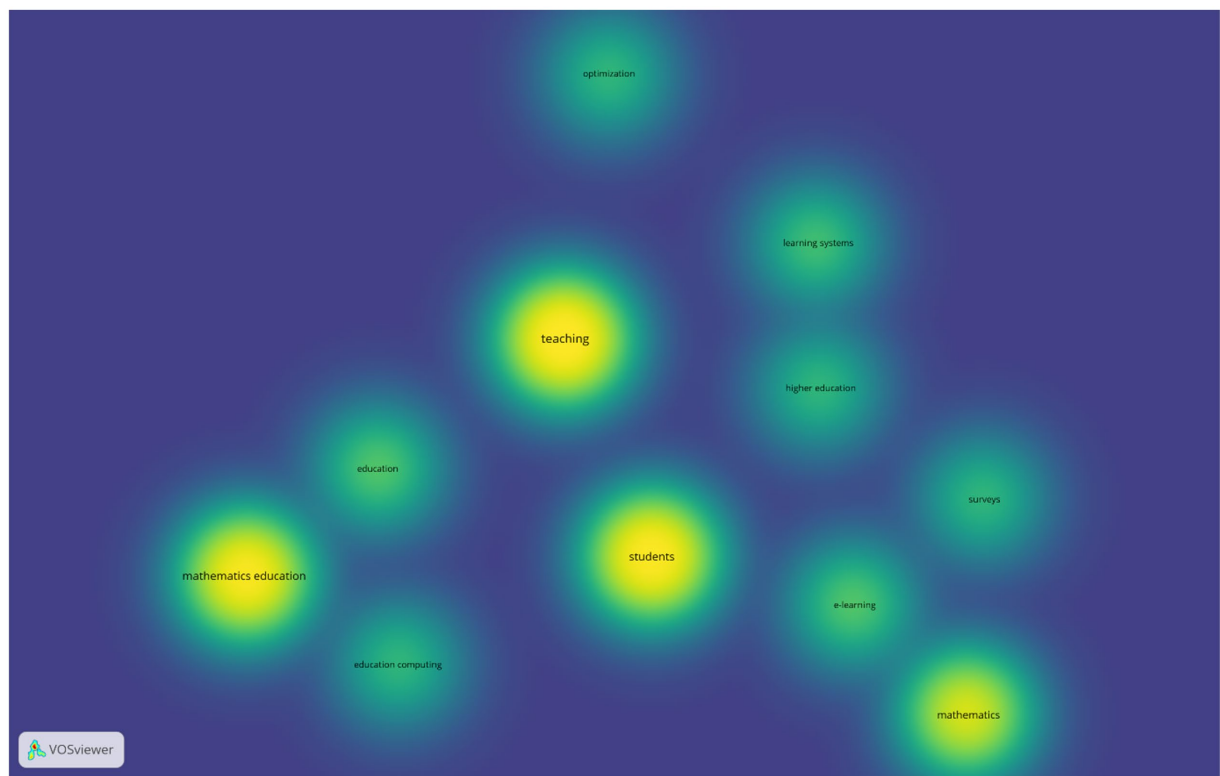
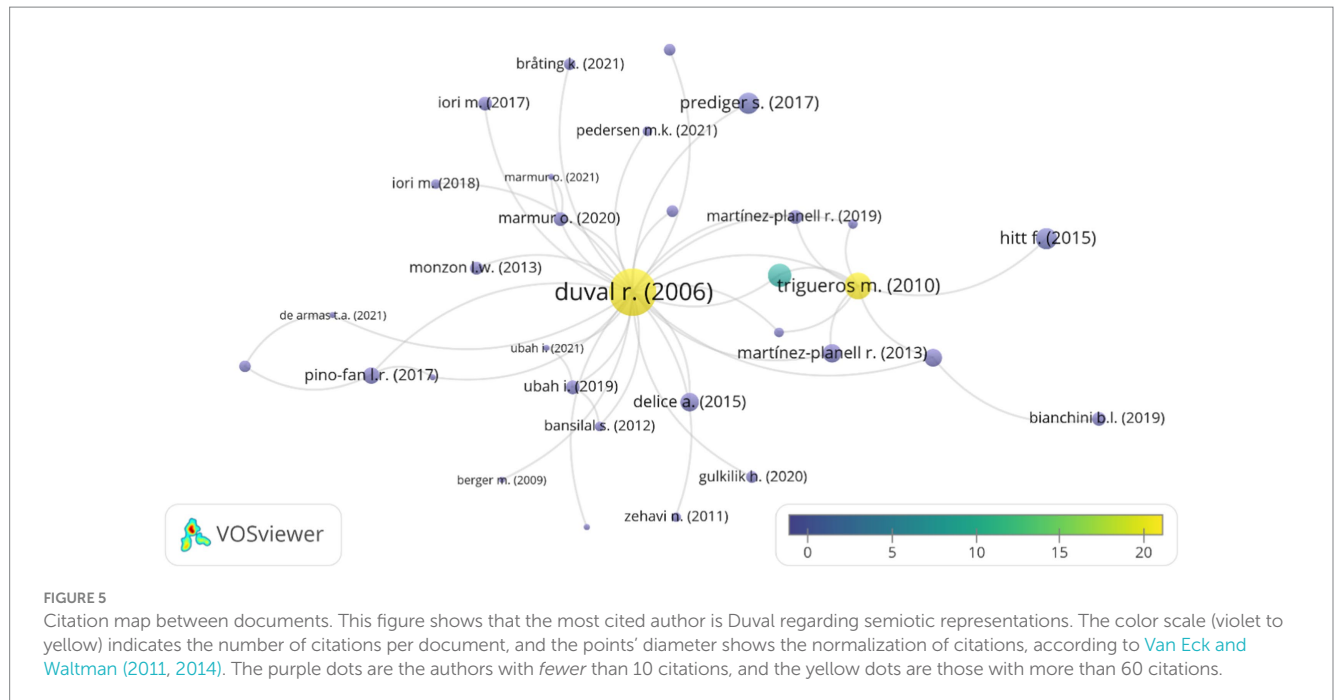


FIGURE 4 Network of publications related to keywords in global semantic search. In this figure, it can be seen That the words “teaching,” “student,” and “mathematics education” are the most used.



4.4 Matrix correlation and clustering

A cross-analysis of the didactic strategies and their associated learning outcomes is presented in Figure 8 and Table 3. It is seen that Gamification is associated with motivation principally; semiotic representations with conceptual understanding, and learning theories are related to other outcomes due to their theoretical focus.

4.5 Discussion

This review sets out to identify the didactic strategies that promote conceptual understanding and student motivation in higher/university mathematics teaching. In this sense, the analysis of the 30 studies guides the answer to the research question, showing that there are two families of strategies dominating in their intended outcomes: (i) *gamification* for motivational engagement, and (ii) *semiotic representations* for conceptual learning (see Figures 6a, 7, 8).

4.5.1 Quantitative evidence

Across the analysis of the studies included, gamification is the most prevalent strategy (56.7%), followed by semiotic representations (36.7%), and learning theories (6.7%) (See Figures 6a, 7).

Concerning the learning outcomes and the didactic strategies, it is seen in the strategy-outcomes matrix (see Figure 8; Table 3) that the gamification studies report the motivation/engagement outcomes, while the semiotic representations studies are associated with the conceptual understanding. Only 10% of the studies (3) report outcomes in both domains, underscoring in that way, a narrow intersection between cognitive depth and affective engagement in current practice.

In Figures 9, a clustering analysis using Principal Component Analysis (PCA) + K-means ($k = 3$) is presented, where it is possible to appreciate the formation of three stable clusters through the analysis

of four variables: didactic strategy, education level, outcome category, and instruments.

The three resulting clusters: C0 (yellow squares), C1 (orange circles and pink triangles), and C2 (pink circles) are delineated and labeled at their centroids. The clusters could be interpreted as three distinct profiles of didactic strategies in university mathematics education.

- (a) C0: group studies centered on semiotic conceptual approaches, where the conceptual understanding is prioritized through symbolic, visual, and diagram registers. PCA space indicates low overlap with gamification approaches.
- (b) C1: studies with gamification motivational orientation, where the strategy is predominantly oriented to student motivation and engagement using game mechanics. This cluster also has some integrations with theoretical frameworks, but with less emphasis on semiotic representation.
- (c) C2: gamification general/hybrid orientation, presents gamification strategies with contextualized learning experiences and general applications. The PCA location at the upper left indicates shared variance with C1 in motivational elements but diverges in specific application focus.

The spatial PCA shows a separation between clusters, suggesting that cognitive and motivational approaches are often pursued independently, and it also reflects differences in methodological design and pedagogical intention. In this context, this differentiation highlights the potential to explore hybrid strategies that combine motivation strengths from gamification with the cognitive depth of semiotic representations.

4.5.2 Semiotic representations, depth in conceptual understanding

Studies focusing on semiotic representations highlight the essential role of coordinating multiple representational registers, such

TABLE 2 Documents of the systematic review related to the research topic and their findings.

No.	Author/ reference	Strategies	Mini abstract	Variable	Education level	Participants	Age	Country	Instruments	Outcome category	Cluster	PCA1	PCA2
1	Finck Brandt et al. (2015)	Semiotic representations	This study explores the challenges and opportunities in teaching equations during the transition from high school to higher education, focusing on students' difficulties with finding roots of first or second-degree equations.	Experience rating, Knowledge	University	18	–	Brazil	Questionnaire Interview	Other	0	0.727	–0.53
2	Salazar (2018)	Semiotic representations	It highlights the importance of dynamic representation environments in teaching geometry based on Duval's theory and Peirce's semiotics.	Representation environments Cognitive Impact on teaching and learning processes	University	–	–	Peru	Register analysis specialized software	Both	0	0.7717	0.424
3	Ariza (2009)	Semiotic representations	The interplay between interpretation, meaning, and mathematical objects is examined, emphasizing the role of conceptual construction and visualization in understanding algebraic structures and mathematical events.	Influence: Mathematical Interpretation	University	–	–	Mexico	Text analysis	Other	0	0.727	–0.53
4	Sobrino-Duque et al. (2022)	Gamification	Examines the impact of an automated, card-based gamification strategy on learning Jakob Nielsen's usability rules	Learning	University	55	20–21 years	Spain	Experimental design Survey	Both	1	–0.489	0.45
5	Bennani and Maalel (2022)	Gamification	Adaptive gamification boosts student engagement and learning outcomes through tailored game elements.	Literature Review and Future Challenges"	University	–	–	Tunisia	Literature review Data collection	Other	1	–0.534	–0.51
6	Sabri et al. (2022)	Gamification	Gamification enhances online education, boosting students' motivation and effectiveness through innovative conceptual models.	Motivation academic	University	97	–	Morocco	Games in learning Questionary	Motivation	2	–0.757	0.758

(Continued)

TABLE 2 (Continued)

No.	Author/ reference	Strategies	Mini abstract	Variable	Education level	Participants	Age	Country	Instruments	Outcome category	Cluster	PCA1	PCA2
7	Rincon-Flores et al. (2023)	Gamification	his research aimed primarily to assess the change in attitude toward mathematics in high school students through a gamified methodology involving a reward system managed through a web platform called Gamit!	E-learning technology	Secondary University	454	–	Mexico	Gamification ICT Performance	Both	1	–0.489	0.45
8	Chapman and Rich (2018)	Gamification	The impact of educational gamification on student motivation and learning outcomes, finding that gamified courses are more motivating than traditional ones, with elements like progress tracking and feedback being particularly effective.	Gamified interface	University	–	–	United States	Pre/post-tests, questionnaire	Other	1	–0.534	–0.51
9	Rivera and Garden (2021)	Gamification	Use of gamification in higher education to increase student motivation and engagement.	Student motivation	University	–	–	United Kingdom	Implementation of gamification Theory of Gamified Learning de Landers	Motivation	2	–0.757	0.758
10	Uzun and Arslan (2009)	Semiotic representations	Examines the semiotic representation skills of future primary school teachers in mathematics, focusing on how students use and transform different representations for the same concept.	Perception	University	28	–	Turkey	Studies activities Evaluation	Other	0	0.727	–0.53
11	Caligaris et al. (2019)	Semiotic representations	It examines the communication competence of first-year students in mathematics, focusing on natural, graphic, and symbolic registers.	Representations	University	–	–	Argentina	Data collection Questionary	Other	0	0.727	–0.53
12	Ledesma (2011)	Semiotic representations	Examines how engineering students use representation registers in solving Calculus problems, exploring challenges, simulations, and teaching strategies.	Understanding	University	–	–	México	Use simulations Questionary	Conceptual Understanding	0	1.1351	0.569

(Continued)

TABLE 2 (Continued)

No.	Author/ reference	Strategies	Mini abstract	Variable	Education level	Participants	Age	Country	Instruments	Outcome category	Cluster	PCA1	PCA2
13	Burgos et al. (2021)	Semiotic representations	Analyzes the onto semiotic complexity of the definite integral in calculus instruction.	Conceptual understanding in calculus	University	–	–	Spain	Investigative methodological tools	Conceptual Understanding	0	1.1351	0.569
14	Bouchrika et al. (2021)	Gamification	Reveals positive impact on student engagement and motivation through gamified question platform, fostering interaction and adoption of e-learning technologies.	Motivation	University	899	18–26	Argelia	e-Learning technology Questionary	Motivation	2	–0.757	0.758
15	Hassan et al. (2021)	Gamification	Challenges in engaging students with diverse learning styles in e-learning and proposes adaptive gamification to enhance motivation and reduce dropout rates.	Interactions	University	200	22–28	Pakistan	Adaptative gamification Questionary	Other	1	–0.534	–0.51
16	Faghihi et al. (2017)	Gamification	Gamification is used to make learning algebra, especially the quadratic formula, fun and effective, through entertainment software that reduces stress and improves understanding.	Software interactivity	University	15	–	United States	Gamification techniques Software Flunky Math Mayhem	Other	1	–0.534	–0.51
17	Faghihi et al. (2014)	Gamification	Impact of gamification in college algebra education, show success in improving student performance and math concept retention through interactive gaming elements.	Interactive gaming	University	30	–	United States	Tutoring Software Math Dungeon	Other	1	–0.534	–0.51
18	Hafzah et al. (2019)	Gamification	The development of a gamification linear algebra application using storytelling to engage students in learning mathematics.	Player Flow Concept	University	30	–	Malasia	Three-Stage Thinking Model Questionary	Other	1	–0.534	–0.51
19	Jiménez-Hernández et al. (2020)	Gamification	Introduces MiniBool, a web-based tool designed to support the learning of Boolean algebra in a blended learning setting, showing positive effects on student motivation and academic performance.	Student performance	University	54	–	Mexico	Face to face learning Software minibool	Other	1	–0.534	–0.51

(Continued)

TABLE 2 (Continued)

No.	Author/ reference	Strategies	Mini abstract	Variable	Education level	Participants	Age	Country	Instruments	Outcome category	Cluster	PCA1	PCA2
20	Lubis et al. (2014)	Gamification	Improves user engagement in learning applications, such as Math Workout Series on smartphones, through game design elements and mechanics.	User interaction	University	-	-	Indonesia	Game activities Evaluation	Other	1	-0.534	-0.51
21	Saleem et al. (2022)	Gamification	Examines the use of gamification in online education, focusing on its benefits and challenges.	Impact on educational processes	University	120	-	Turkey	Quantitative and qualitative methods Questionary and interview	Other	1	-0.534	-0.51
22	Pedersen et al. (2021)	Digital tasks, semiotics	Explores how digital tasks foster use of semiotic registers in university math.	Conceptual understanding	University	-	-	USA/Norway	Task analysis	Conceptual Understanding	0	1.1351	0.569
23	Pehlivan and Arabacioglu (2023)	Flipped + Gamification	Quasi-experimental study in higher ed. Increased motivation and math performance.	Motivation, achievement	University	-	-	Turkey	Pre/post-tests, questionnaire	Motivation	2	-0.757	0.758
24	Duval (2006)	Semiotic representations	Proposes the theoretical framework of semiotic representation registers to support mathematical understanding.	Cognitive structure, register coordination	Secondary	-	-	France	Literature analysis	Other	0	0.727	-0.53
25	Zabala-Vargas et al. (2022)	Game-Based Learning (ARCS Model)	Uses game-based learning strategies framed within the ARCS model to promote deep learning in engineering math.	Engagement, deep learning	University	-	-	Latin America	ARCS model evaluation	Motivation	2	-0.757	0.758
26	Maarif et al. (2018)	Learning theories	The use of Cabri II software as a tool for learning and improving geometry skills in virtual classes, advantages and disadvantages are explored.	Geometry skills, Academic performance, Student participation	University	32	19–22	Indonesia	Usage and data collection through Cabri II software Test	Other	1	0.1187	-0.73
27	Chinna and Sunkesula (2023)	Learning theories	Evaluate the Jigsaw Cooperative Learning in enhancing basic numeracy among sixth graders, addressing the decline in literacy and numeracy skills.	Foundational numeracy skills	Secondary school/ extrapolable	60	12	India	Jigsaw Method of Cooperative Learning. Oral and written tests	Other	1	0.1187	-0.73
28	Temple and Doerr (2012)	Semiotic representations	Developing fluency in the mathematical register through conversation in a tenth-grade classroom	Conceptual understanding	Secondary school/ extrapolable	24		Rumania		Conceptual Understanding	0	1.1351	0.569

(Continued)

TABLE 2 (Continued)

No.	Author/ reference	Strategies	Mini abstract	Variable	Education level	Participants	Age	Country	Instruments	Outcome category	Cluster	PCA1	PCA2
29	Mildenhall and Sherriff (2018)	Semiotic representations	Using multimodal learning experiences can be effective in teaching mathematics. Using a social semiotic lens within a participationist framework, this paper reports on a professional learning collaboration with a primary school teacher designed to explore the use of metaphors and modalities in mathematics instruction.	Understanding, Participation	School/ extrapolable	6		Australia	Metaphors and Semiotic representations	Conceptual Understanding	0	1.1351	0.569
30	Lee et al. (2023)	Gamification		Engagement Learning Outcomes	University	218	18–24	South Korea	Digital twin technology	Motivation	2	–0.757	0.758

as symbolic, graphical, verbal, and algebraic, to facilitate and promote a deeper mathematical reasoning.

Several studies (Hitt, 1998; Ledesma, 2011; Burgos et al., 2021) mention the importance of semiotic representations as a key to understanding and addressing the challenges of acquiring the mathematical concepts of calculus and precalculus. Similarly, other authors say representations are crucial for students' and expert mathematicians' mathematical activity (Morgan, 2006; Iori, 2018). The different representations foster deep understanding and conceptual learning by reinforcing students' ideas and skills (Ainsworth et al., 1997; Even, 1998; Winslow, 2003). Studies such as those proposed by Brock et al. (2020) raise the bar on rigor and encourage students to solve problems creatively while gaining valuable data on their growth as thinkers and mathematicians (Hancock and Karakok, 2021).

In that way, it emphasizes the importance of research in the semiotic representations, which, according to Duval (2006, 2017), produce a deep comprehension of learning abstract mathematical objects. For instance, when students use semiotic representations to explore and solve real-world problems collaboratively, they engage more deeply with the material, applying theoretical knowledge in practical contexts (Moyer-Packenham et al., 2022). Integrating semiotic representations with other instructional strategies, such as problem-based learning or collaborative projects, can enhance their effectiveness.

4.5.3 Gamification enhances engagement and motivation

Gamification strategies, including Game-Based Learning (GBL), have shown strong potential to increase learner engagement, reduce mathematics anxiety, and create meaningful learning experiences when tasks are situated in authentic contexts (Lubis et al., 2014).

Moreover, nowadays, gamification can be enhanced through artificial intelligence tools. In the literature, Alneyadi and Wardat (2023) show how ChatGPT could provide students with a positive influence in the learning of magnetism concepts, so in that way, the use of AI models could improve educational outcomes (Lubis et al., 2014). Their study shows how gamification in math could be translated into smartphone and AI apps to increase the effectiveness of the engagement of math students. According to the explored literature, in recent years, there has been a substantial increase in research focused on gamification, which could be influenced by the educational disruptions of the COVID-19 pandemic (Bouchrika et al., 2021).

Gamification as an active learning strategy is combined with other active methodologies such as Problem-Based Learning (PBL), Flipped Classroom, Project-based learning, and real context situations; e.g., several studies demonstrated better performance and understanding of the students about the math concepts (Hassan et al., 2021; Pehlivan and Arabacioglu, 2023) and also emphasized the importance of gamification, combining with flipped classrooms, due to its allowing students to engage in more active and motivating learning activities (Husain et al., 2023). The Flipped classroom provides the possibility that the students could develop their ideas and acquire skills that directly have implications for the progress of significant learning (Lo and Hew, 2020; Lo et al., 2021), and is also recognized as a powerful strategy for teaching and learning math in university courses.

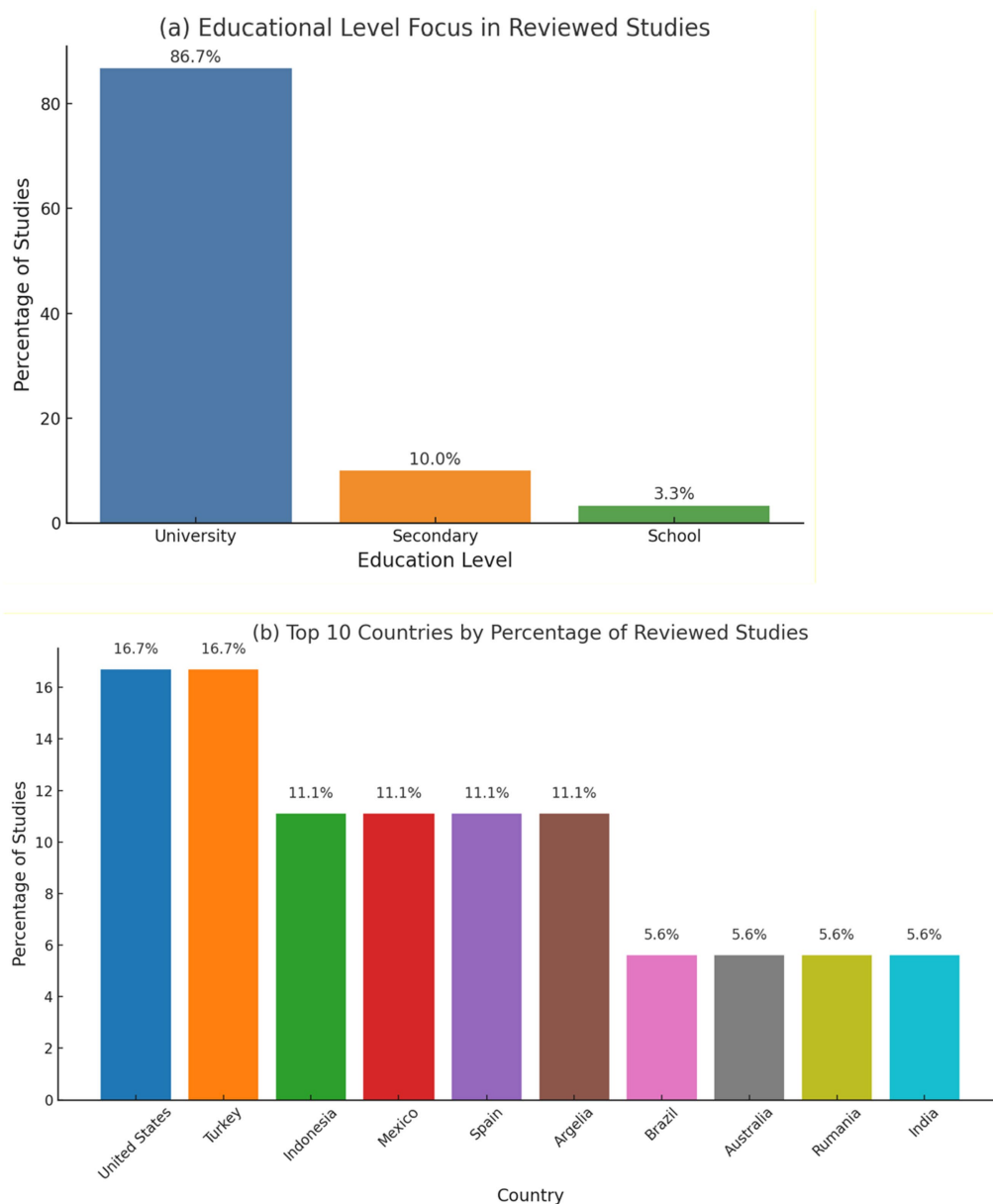


FIGURE 6

The distribution of the documents analyzed in this review is (a) by level of education, and (b) by country.

In math problems, contextualization means that the students can identify the variables in a specific real problem and, after, could propose different ways of solving the situation using the mathematical concepts and relations between them. Some successful examples reported in the university context and secondary education with extrapolable results are mentioned in math contextualized studies in different fields like health, engineering, and biology (Jiménez and Castillo, 2017; Chapman and Rich, 2018; Jiménez, 2018; Jiménez-Gaona et al., 2019; Rivera and Garden, 2021; Sobrino-Duque et al., 2022; Amable Vivanco-Galvan et al., 2018).

In sum, gamification can increase intrinsic motivation and engagement to learn math (Buckley and Doyle, 2014; Chapman and Rich, 2018; Bouchrika et al., 2021), contribute to improving performance and

consolidating concepts, e.g., in subjects such as algebra and calculus (Faghihi et al., 2014; Faghihi et al., 2017; Hafzah et al., 2019; Jiménez-Hernández et al., 2020); and represents a powerful tool for teachers at all levels of the education system (Rivera and Garden, 2021).

4.5.4 Challenges, future proposals, and limitations

Concerning the challenges, studies, such as Meij et al. (2022), mention the gap between the theories of teaching, the reality in education, and the formation of educators. This point suggests the potential existence of a discrepancy between theoretical educational paradigms and their practical implementation in teacher education (Uzun and Arslan, 2009; Pedersen et al., 2021).

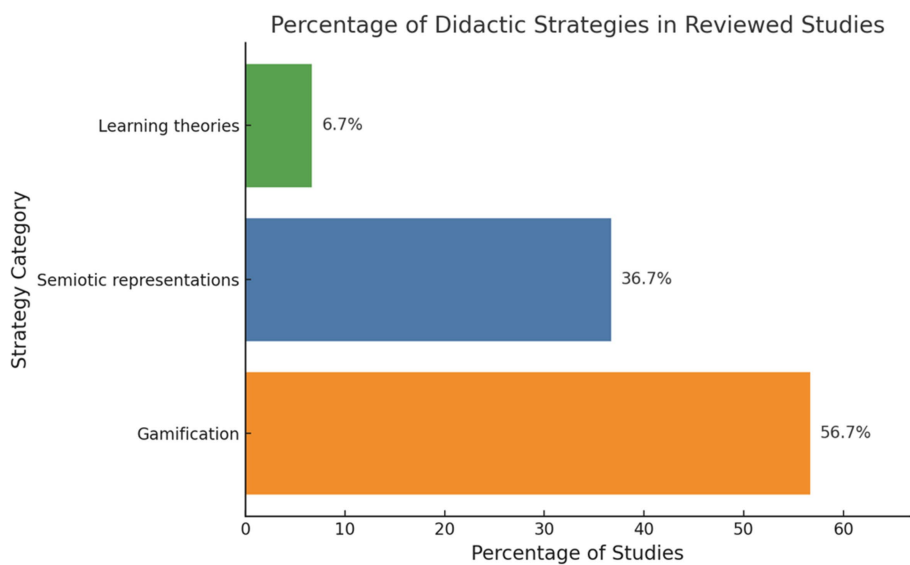


FIGURE 7 Percentage distribution of didactic strategies identified in this project of university-level mathematics education studies. Gamification is the most prevalent strategy (56.7%), followed by semiotic representations (36.7%) and learning theories (6.7%).

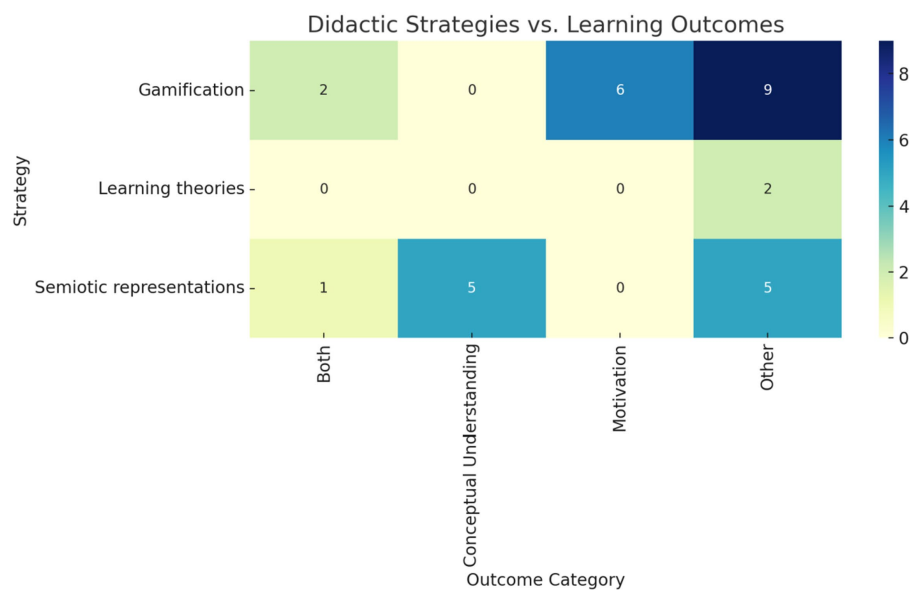


FIGURE 8 Matrix correlation of didactic strategies and learning outcomes identified in the review studies. It is seen that gamification is strongly associated with motivational outcomes; semiotic representations are linked to conceptual understanding. A limited number of studies simultaneously address (3) both cognitive and affective domains, highlighting a gap in integrative pedagogical approaches.

Implementing semiotic representations in teaching practices could be challenging, especially for teachers without a pedagogical formation and training in semiotic strategies. As [Iori \(2018\)](#) highlights, the success of semiotic representations in teaching mathematics depends on the ability of the teachers to select the appropriate representations that align with the learning objectives and levels of understanding of the students.

Similarly, embedding real-world contextualized mathematical tasks within gamified environments could be challenging due to the demand for careful design when the aim is to achieve

deep conceptual learning while maintaining the students' motivation.

In this sense, one of the most significant gaps identified is the lack of studies integrating semiotic representations and gamification within a unified pedagogical design (see [Figures 8, 9](#)). Under this consideration, our result suggests exploring hybrid approaches that combine cognitive depth of semiotics with the motivational dynamics of gamification, creating richer learning experiences that support comprehension and sustained engagement. This aligns with the observed evolution in the literature from representation-focused

TABLE 3 Studies by strategy and outcomes concerning the matrix correlation of didactic strategies and learning outcomes identified in the review studies.

Author/reference	Strategies	Outcome Category
Sobrinho-Duque et al. (2022)	Gamification	Both
Rincon-Flores et al. (2023)	Gamification	Both
Sabri et al. (2022)	Gamification	Motivation
Rivera and Garden (2021)	Gamification	Motivation
Bouchrika et al. (2021)	Gamification	Motivation
Pehlivan and Arabacioglu (2023)	Gamification	Motivation
Zabala-Vargas et al. (2022)	Gamification	Motivation
Lee et al. (2023)	Gamification	Motivation
Bennani and Maalel (2022)	Gamification	Other
Chapman and Rich (2018)	Gamification	Other
Hassan et al. (2021)	Gamification	Other
Faghihi et al. (2017)	Gamification	Other
Faghihi et al. (2014)	Gamification	Other
Hafzah et al. (2019)	Gamification	Other
Jiménez-Hernández et al. (2020)	Gamification	Other
Lubis et al. (2014)	Gamification	Other
Saleem et al. (2022)	Gamification	Other
Maarif et al. (2018)	Learning theories	Other
Chinna and Sunkesula (2023)	Learning theories	Other
Salazar (2018)	Semiotic representations	Both
Ledesma (2011)	Semiotic representations	Conceptual Understanding
Burgos et al. (2021)	Semiotic representations	Conceptual Understanding
Pedersen et al. (2021)	Semiotic representations	Conceptual Understanding
Temple and Doerr (2012)	Semiotic representations	Conceptual Understanding
Mildenhall and Sherriff (2018)	Semiotic representations	Conceptual Understanding
Finck Brandt et al. (2015)	Semiotic representations	Other
Ariza (2009)	Semiotic representations	Other
Uzun and Arslan (2009)	Semiotic representations	Other
Caligaris et al. (2019)	Semiotic representations	Other
Duval (2006)	Semiotic representations	Other

research to motivation-focused approaches and now toward hybrid frameworks.

Finally, building on these insights, rather than claiming established synergistic effects, we frame integration as a research agenda: embed semiotic rigor (semiotic register conversions) inside motivationally sound gamified progressions; pair conceptual and motivational outcomes with validated instruments; and use active controls to isolate mechanisms. This combination could encourage students' teaching and preparation for solving complex problems in authentic, real-world scenarios in higher education and at all levels.

4.6 Limitations

One of the principal limitations of this project is that the studies were explored only for a unique level of education; thus, future

research should focus on diverse student populations and studies that combine experimental and mixed methods across diverse educational contexts and learning outcomes could enrich the generalizability of findings. Also, a limitation of this study is that the evidence is limited to a set of countries, and it does not consider the classroom climates and teacher roles, constraining generalizability across levels and regions.

5 Conclusion

The literature review of this work has identified two primary didactic strategies that have garnered significant research attention and demonstrated effectiveness in teaching mathematics: the use of semiotic representations and gamification.

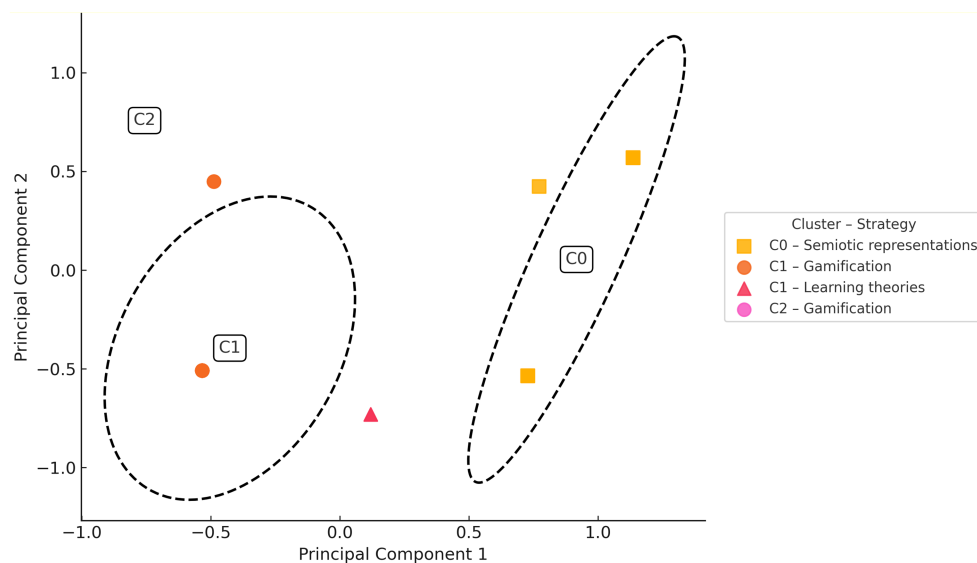


FIGURE 9

Cluster profiles ($k = 3$) of didactic strategies in university mathematics education, categorized as semiotic-conceptual (C0), gamification-motivational (C1), and gamification-general (C2). Principal Component Analysis (PCA) was applied to variables related to strategies, instruments, and educational levels, resulting in three distinct clusters. Ellipses indicate the approximate dispersion and grouping of studies within each cluster.

The theoretical basis for using semiotic representations in university-level mathematics education is grounded in Duval's theory, which aims to foster a deep cognitive understanding of mathematical concepts by focusing on symbolizing and interpreting mathematical ideas.

Conversely, gamification is an active teaching-learning strategy that presents mathematics as an engaging and approachable subject. By incorporating game-like elements into the learning process, gamification helps to reduce students' anxiety toward mathematics and promotes a more positive learning experience. However, it is essential to mention that if we only apply gamification, we risk losing the rigor of the math.

In this sense, a proposal that combines semiotic representations with gamification can create a synergistic effect that optimizes engagement and comprehension in mathematics education. Semiotic representations facilitate deep cognitive processing, while gamification makes learning more enjoyable and accessible. This integrated approach can enhance students' ability to visualize and conceptualize mathematical ideas, strengthening their analytical skills and understanding of complex concepts.

Additionally, it is crucial to create environments that encourage discussion and collaborative knowledge construction, recognizing the teacher's role as a facilitator and guide in the learning process. By strategically implementing semiotic representations and gamification across all educational levels, from higher education to lower levels, educators can offer a compelling approach to enhance both the learning and teaching experiences in mathematics. This comprehensive strategy makes learning more enjoyable and significantly improves students' analytical capabilities and conceptual understanding.

Author contributions

DC: Conceptualization, Formal analysis, Methodology, Visualization, Writing – original draft, Writing – review & editing. JC: Formal analysis, Visualization, Writing – original draft, Writing – review & editing. CC: Conceptualization, Data curation, Investigation, Methodology, Visualization, Writing – original draft. YJ: Conceptualization, Methodology, Visualization, Writing – review & editing. MR-Á: Conceptualization, Formal analysis, Supervision, Validation, Visualization, Writing – review & editing. VL: Formal analysis, Supervision, Validation, Visualization, Writing – review & editing.

Funding

The author(s) declare that no financial support was received for the research and/or publication of this article.

Acknowledgments

The authors acknowledge Universidad Técnica Particular de Loja for supporting this project. DC also acknowledges the support from Universitat Politècnica de València through the Assistance Call Doctoral Student Mobility.

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