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Mapping the transition from arithmetic to algebra: a bibliometric and structural review (2003–2023)

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The shift from arithmetic to algebra entails significant cognitive and pedagogical challenges for both students and teachers. Despite its importance in early mathematics education, few systematic reviews have comprehensively examined the scientific literature on this transition using both quantitative and structural lenses. This study addresses that gap by conducting a bibliometric and structural analysis of 127 academic publications from 2003 to 2023, sourced from the Web of Science and Scopus databases. The review followed PRISMA guidelines and incorporated text mining techniques to harmonize metadata across sources. Graph theory tools were employed to map collaborative networks and thematic clusters, while the Tree of Science model organizes the literature into roots, trunk, and branches to visualize the intellectual development of the field and to identify foundational works, central contributions, and emerging lines of research. The results highlight a growing academic interest in early algebra and algebraic thinking, viewed as essential pedagogical approaches to confront the conceptual hurdles of transitioning from arithmetic. Key thematic areas, leading scholars, and influential research groups were identified, with the United States standing out as the dominant contributor in the field. This review offers relevant insights for researchers, educators, and education policymakers aiming to enhance the teaching and learning of algebra by building on its arithmetic roots and understanding the evolving scholarly landscape.

KEYWORDS

bibliometric analysis, early algebraic thinking, mathematics education, scientific mapping, Tree of Science, arithmetic-algebra transition

1 Introduction

The transition from arithmetic to algebra (TA-A) represents an important step in mathematics education, as it marks the shift from concrete numerical thinking to more abstract and symbolic reasoning. This transition, however, is notoriously difficult for many students, who often face significant challenges when trying to understand more complex algebraic concepts (Kieran, 2007). This process is closely linked to the development of early algebra, which involves introducing algebraic thinking from the earliest levels of schooling through the use of patterns, arithmetic generalizations, the notion of variable, and functional relationships (Kaput, 2008; Carraher and Schliemann, 2007; Radford, 2014). Rather than replacing arithmetic, early algebra builds upon it, supporting students in the gradual appropriation of symbolic representations and relational reasoning. Understanding how research develops in this area is vital for improving educational practices and facilitating this conceptual shift.

Over the last two decades, academic interest in TA-A has grown considerably, reflecting a global concern to optimize teaching and learning methods in this field (Booth et al., 2014). However, despite the growing volume of research, there is still a lack of comprehensive mappings that capture the evolution and current state of scientific production in TA-A. Therefore, the need for an analysis that systematically explores the dynamics of the field, and its intellectual structure becomes evident. To address this need, the present article aims to carry out a systematic and structural mapping of the scientific production related to TA-A, using bibliometric tools and the Tree of Science methodology. The literature has pointed out that, while there have been advances in the teaching of algebra, studies are still needed to better understand the difficulties in the transition and the development of algebraic thinking, as well as how different pedagogical and didactic approaches can contribute to its improvement.

This article performs a bibliometric analysis of the scientific production related to TA-A, covering a period of 20 years (2003–2023), to identify the main trends, intellectual structures, and gaps in the research. The choice of the bibliometric approach is based on its ability to provide an objective and quantifiable view of the development of knowledge in a specific field, allowing for the analysis of publication patterns, collaboration networks, and thematic evolution over time.

To ensure the rigor of the study, the PRISMA methodology (Preferred Reporting Items for Systematic Reviews and Meta-Analyses), widely recognized in systematic literature reviews (Page et al., 2021), was adopted. PRISMA allows for the clear and transparent structuring of the identification, selection, and analysis of relevant documents, ensuring the validity of the findings. In this case, the PRISMA methodology was adapted to the bibliometric context through three stages: document identification, information extraction, and data analysis.

To achieve the objective, a search equation was defined using keywords such as arithmetic, algebra, and transition, expressed as: arithmetic AND algebra AND (transition OR "arithmetic to algebra"). This search was applied to two of the most recognized academic databases—Web of Science (WoS) and Scopus—employing text mining techniques to harmonize metadata formats, which differ between platforms. The selected time frame spanned from 2003 to 2023, covering at least two decades to ensure the relevance and temporal breadth of the included studies. Based on the retrieved data, the evolution of scientific production in the field was mapped, identifying the most prolific countries, journals, and authors, as well as the collaborative networks that structure this area of research.

Additionally, the Tree of Science (ToS) metaphor was used to identify the intellectual structure of the field, organizing the references into three conceptual layers: roots (foundational publications), trunk (key theoretical and methodological developments), and branches (emerging trends and recent applications). This framework not only enables a visualization of how research on the arithmetic—algebra transition has developed over time, but also helps to clarify the articulation of its core contributions. Its application in this study is particularly relevant, as it allows for the identification of theoretical foundations, recognition of formative works, and exploration of new lines of inquiry. By classifying references according to their impact and role in knowledge production, the ToS also supports the detection of gaps in the literature and highlights potential directions for future research.

The ToS algorithm has become established as a widely used methodological tool in systematic reviews across multiple disciplines. Its application has proven effective in identifying research trajectories, intellectual structures, and emerging trends in areas such as digital transport, sustainable agriculture, innovative entrepreneurship, educational museography, artificial intelligence in healthcare, and industrial strategy (Gerrero-Molina et al., 2024; Cano-Vargas and Osorio-Toro, 2024; Ariza-Colpas et al., 2024; Saurith-Moreno et al., 2024; Urina-Triana et al., 2024; Vivares et al., 2022). These diverse implementations demonstrate the methodological versatility of ToS and support its relevance in the present study.

The use of this adapted methodology enabled the identification of the United States, Canada, Indonesia, and Turkey as the countries with the highest scientific output in the topic. In addition, leading journals and authors—along with their collaborative networks—were analyzed, and three thematic trends were identified. One of the key contributions of this work lies in the integration of WoS and Scopus data into a single scientometric analysis. This represents a valuable contribution, as few studies have addressed this integration despite it being recognized as a gap in the literature (Ariza-Colpas et al., 2024; Urina-Triana et al., 2024). Moreover, the findings presented here are novel in that they complement and extend the contributions of previously published systematic reviews (Sibgatullin et al., 2022; Utami and Prabawanto, 2023).

This study seeks to answer the following question: How has scientific production on the transition from arithmetic to algebra evolved within the field of mathematics education, in terms of theoretical approaches, authors, countries, thematic trends, and the development of early algebraic thinking, during the period 2003–2023?

2 Methodology

The methodology employed was structured in three stages, the general aspects of which are summarized in Figure 1, which illustrates the process of identification, refinement, and selection of studies on the TA–A published between 2003 and 2023. The initial search retrieved a total of 153 records (49 from Web of Science and 104 from Scopus). After the removal of 26 duplicates, 127 unique records were retained. These records were then subjected to a preprocessing stage and subsequently organized into an Excel file consisting of 22 sheets for further analysis.

This workflow constitutes an adaptation of the PRISMA methodology (Preferred Reporting Items for Systematic Reviews and Meta-Analyses), a widely accepted framework in academic research for conducting high-quality systematic reviews. Its

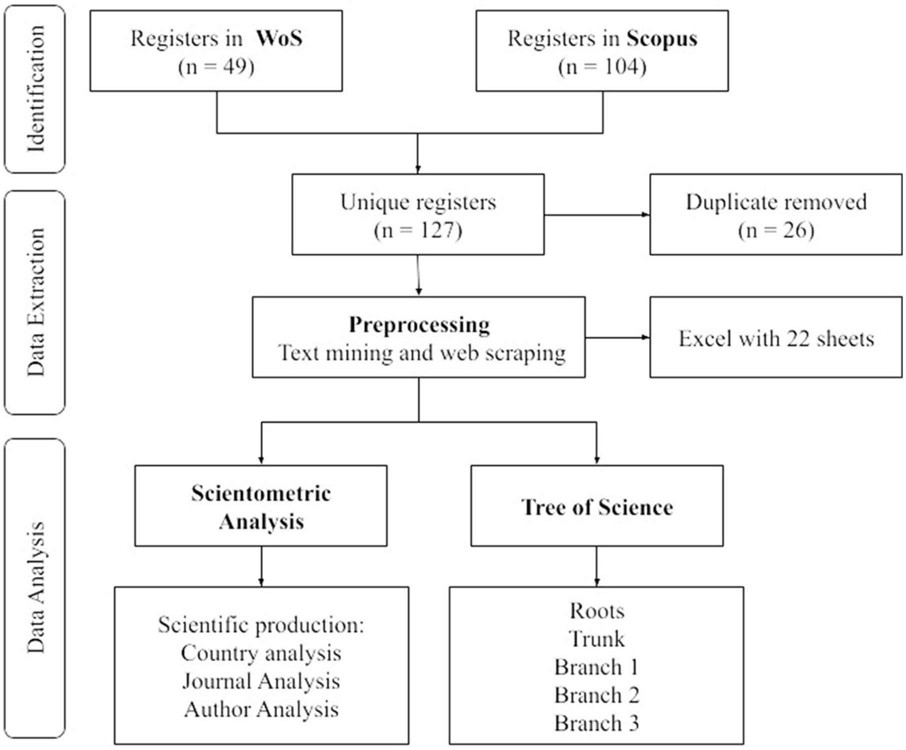


FIGURE 1
Flowchart based on the PRISMA methodology, summarizing the process of identification, preprocessing, and selection of studies on the transition from arithmetic to algebra (TA–A) between 2003 and 2023.

methodological foundations have been broadly endorsed in the literature on review articles (Page et al., 2021; Jabeur, 2024; Ariza-Colpas et al., 2023). PRISMA provides a structured approach designed to ensure reliability and transparency in the identification, selection, and analysis of studies within a review. It consists of a series of sequential steps aimed at minimizing bias in data collection and ensuring that the included studies meet established methodological quality criteria. The phases of this methodology are:

1. **Identification**: Defining search criteria and selecting relevant sources from scientific databases.
2. **Selection**: Applying filters to exclude duplicate, irrelevant studies or those that do not meet the established criteria.
3. **Eligibility**: Reviewing the documents in depth to verify their relevance and methodological quality.
4. **Inclusion**: Compiling the final database of selected studies for analysis.

In this study, the PRISMA methodology was adapted for bibliometric analysis, structuring the review process into three specific stages:

1. Identification of documents through a defined search strategy in WoS and Scopus.
2. Extraction and systematization of relevant information for the scientometric analysis.
3. Structural analysis based on the metaphor of the Tree of Science (ToS).

This approach ensures that the included studies are representative of the existing literature on TA–A and support a robust analysis.

Stage 1 (Identification)

In the identification stage, the search strategy was defined to track relevant studies on the TA-A in high-impact databases. WoS and Scopus were selected due to their recognition in the academic context and their extensive coverage of scientific publications in mathematics education. As a criterion for evaluating the quality of the publications, the impact factor was used, which measures the relevance of a scientific journal based on the average number of citations its articles receive. In this study, the classification of journals in the Scimago Journal Rank (SJR) was used, which groups publications into Q1, Q2, Q3, and Q4, according to their impact. The quartiles divide scientific journals into four categories based on their impact, with Q1 and Q2 representing publications with the highest relevance and influence in the academic community, while Q3 and Q4 group journals with relatively lower impact. In this study, priority was given to the inclusion of articles published in Q1 and Q2 journals, as these ensure a higher level of rigor in the peer-review process and greater visibility in the academic field (Radicchi et al., 2017).

As part of the document identification, the h–index was also considered a bibliometric indicator widely used to measure the

TABLE 1 Search strategy and general results from Web of Science and Scopus.

Criteria	General search settings in Web of Science and Scopus
Range (years)	2003–2024
Search date	July 22, 2024
Document types	Articles, Reviews, Books, Chapters, Conferences
Keywords used	`arithmetic* AND algebra AND (transition OR "arithmetic* to algebra")`
Number of results	
Web of Science	**Scopus**
49	104
	Total (WoS + Scopus)
	127

impact and productivity of a researcher or scientific source. This index is defined as the number *h* of articles that have received at least *h* citations each (Hirsch, 2005).

Table 1 summarizes the main characteristics of the search strategy used to identify the references to be analyzed. The review covers the period from 2003 to 2023 and includes a wide range of documents, such as articles, reviews, books, book chapters, and conference papers, extracted from the two databases. This time frame was chosen because it captures clear phases of initiation and subsequent dynamization in TA–A research. Additionally, it aligns with the thematic consolidation of both Scopus and Web of Science from the early 2000s onward. The span of two decades also enhances the statistical robustness of the study and helps reduce the potential impact of bias. The goal was to map the development and evolution of research on TA–A over the past twenty years.

The process began by identifying relevant keywords related to the topic under study, followed by the formulation of a consolidated search equation to be applied in both databases: [arithmetic AND algebra AND (transition OR "arithmetic to algebra")]. This search returned 49 documents in Web of Science and 104 in Scopus, yielding a total of 127 unique results after duplicate removal.

Stage 2 (data extraction)

This stage involves the refinement and organization of the information obtained in the previous stage. Extracting bibliographic data presents several challenges, including standardizing the different formats used by the two databases. However, combining both sources has been identified as a necessity in other studies (Ariza-Colpas et al., 2024; Urina-Triana et al., 2024). To achieve this, text mining techniques were used to extract bibliographic metadata (authors, year of publication, country, journal or conference, DOI, among others); normalize data formats, as **WoS** and **Scopus** have different structures for presenting references; remove duplicates and correct inconsistencies; analyze keywords and abstracts, identifying thematic trends, and emerging research areas.

Specialized bibliometric analysis tools were used for this process, such as the Bibliometrix package in R, which enabled the cleaning, structuring, and analysis of bibliographic data (Aria and Cuccurullo, 2017; Robledo et al., 2022). This software facilitated the extraction of metadata—including authors, titles, affiliations, abstracts, and citations—the automatic deduplication of records retrieved from both WoS and Scopus, the normalization of author names through automated cleaning functions, and linguistic aggregation to unify idiomatic variants and related terms. Additionally, web scraping techniques were employed on CrossRef to retrieve missing metadata from some records.

The resulting database contains detailed information about the publications, including variables such as authors, year of publication, country of origin, journal or conference where the work was published, number of citations, keywords, and abstracts. It also includes collaboration networks between authors and institutions, which helped identify key studies in TA-A and their subsequent systematic structuring through the ToS metaphor in Stage 3.

Stage 3 (data analysis)

This final stage involved analyzing the data collected in the consolidated database. Bibliometric analyses are relevant for mapping scientific production in a specific field of study and obtaining useful findings for researchers (Khaw et al., 2024). In this article, the analysis was conducted on two fronts. On one side, a mapping of the scientific production in terms of:

- Temporal evolution of publications, identifying the annual growth rate.
- Countries with the highest scientific contribution, highlighting the geographical distribution of knowledge.
- Leading authors and journals, analyzing the impact and influence of the most relevant sources.
- Collaboration networks that are generated in relation to these categories.

On the other hand, the ToS algorithm was used to classify the references in a way that enables a comprehensive analysis of the existing literature in TA–A. The ToS algorithm classifies references into three groups by making an analogy with a tree:

- **Roots**: Foundational studies that establish the theoretical and methodological principles in TA-A. For a study to be considered part of the roots, it must be widely cited, propose theoretical frameworks, methodologies, or approaches for the development of the field, and have a high impact on the structuring of TA-A.
- **Trunk**: Research that consolidates the field by developing key theories, models, and methodological approaches. These studies typically build upon the foundations established by the roots, expanding concepts or validating their applicability, having a significant level of citations, and serving as a bridge between theoretical foundations and emerging applications.

- **Branches**: Recent studies that reflect innovations, applications, and emerging new trends in algebra teaching. For research to belong to this category, it must explore new methodologies, technologies, or didactic approaches in the development of algebraic thinking.

The Tree of Science is a relatively recent methodological approach that has evolved over time through the collaborative contributions of several researchers (e.g., Robledo et al., 2014; Valencia-Hernández et al., 2020; Hurtado-Marín et al., 2021; Eggers et al., 2022). This approach facilitates the identification of key contributions, conceptual structures, and gaps within a given body of literature.

To examine the evolution of scientific production related to the transition from arithmetic to algebra (TA-A), two quantitative indicators were calculated: the compound annual growth rate (CAGR) and the coefficient of variation (CV). The CAGR, which reflects the average annual rate of increase or decrease in scientific output, was computed using the following formula:

$$\mathbf{CAGR} = \left(\frac{V_f}{V_i}\right)^{\frac{1}{n}} - 1,$$

where V_f denotes the number of publications in the final year of the study period, V_i corresponds to the number of publications in the initial year, and n represents the number of years considered.

The variability in annual scientific production was assessed using the coefficient of variation:

$$\mathbf{CV} = \frac{\sigma}{\mu} \times 100,$$

where σ is the standard deviation and μ the arithmetic mean of the annual publication counts. This metric indicates the relative dispersion of scientific output over time.

In addition to production indicators, the scientific collaboration index was employed, defined as the number of articles co-authored by multiple researchers. This indicator is widely recognized in bibliometric studies, as it provides insights into academic collaboration networks, knowledge production capacity, and the consolidation of research communities.

Furthermore, network analysis was utilized to examine patterns of interaction among actors such as countries, authors, and journals. In this framework, each node represents an actor, and links between nodes indicate collaborative relationships. The thickness of the links denotes the strength of the collaboration, while the node degree—defined as the number of connections a node possesses—serves as a measure of centrality and influence. Communities or clusters within the network were identified based on the density of internal connections, revealing the presence of scientific sub-communities.

Risk of bias assessment

In this systematic mapping review with a bibliometric approach, no formal assessment of the risk of bias in individual studies was conducted. This is because the main objective was to analyze the scientific production related to the transition from arithmetic to algebra, focusing on publication patterns, collaboration networks, and thematic evolution, rather than synthesizing specific interventions. Nevertheless, to ensure the quality and relevance of the included sources, rigorous inclusion criteria were established, such as the selection of publications indexed in recognized databases (WoS and Scopus) and the consideration of impact metrics like the H-index of journals. These measures, along with the use of the Tree of Science methodology, help ensure that the studies analyzed originate from reliable sources of high academic quality and provide a comprehensive overview of the field.

3 Results

3.1 Selection and characteristics of the studies

The 127 studies included in this review were selected following the inclusion criteria established under the PRISMA methodology. These studies comprise research articles, theoretical reviews, book chapters, and conference papers, all focused on the transition from arithmetic to algebra. While most contributions center on basic education, relevant studies were also identified in the contexts of teacher training and higher education.

In terms of methodological approach, qualitative studies predominate, followed by theoretical-conceptual investigations and applied instructional proposals. Collectively, these works offer a broad overview of the field's development, ranging from its methodological foundations to its practical applications.

3.2 Bibliometric analysis

3.2.1 Scientific production

The evolution of scientific production over time is presented in the following figures, including a partial result for 2024 due to the cutoff date of the query. In general, Scopus records a higher number of publications than Web of Science for the case of TA-A, except for the year 2014. A relatively increasing trend is observed, with a total growth rate of 7.81% between 2003 and 2023, and a coefficient variation of 0.399 for the number of publications in this same period. Additionally, two periods were assumed, which are referred to as the initiation period and the dynamization period.

Initiation Period (2003–2011): During this period, 40 articles were written, yielding a growth rate of 5.20% with a coefficient of variation of 0.437 (higher variability than the entire observation window). A peak in citations occurred in 2009, driven primarily by two articles: Khng and Lee (2009) titled "Inhibiting interference from prior knowledge: Arithmetic intrusions in algebra word problem solving," and Malisani and Spagnolo (2009) titled "From arithmetical thought to algebraic thought: The role of the 'variable.'" A total of 40 articles were written during this period.

Dynamization Period (2012–2023): During this period, 87 articles were written, showing a growth rate of 9.59% with a coefficient of variation of 0.382 (lower variability than the entire observation window). Citation peaks were observed in 2012,

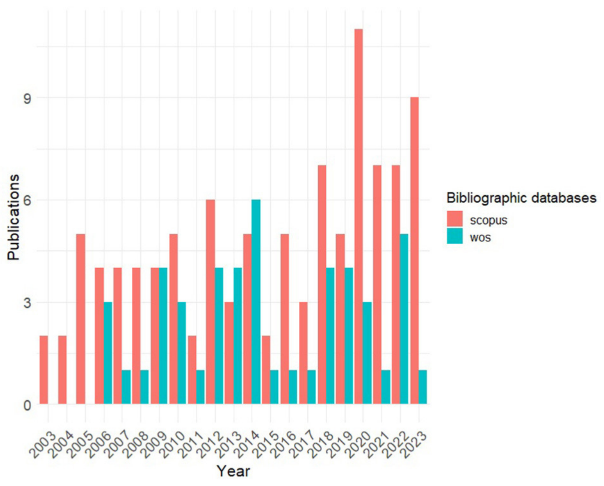


FIGURE 2
Comparison of the number of publications between Scopus and WoS.

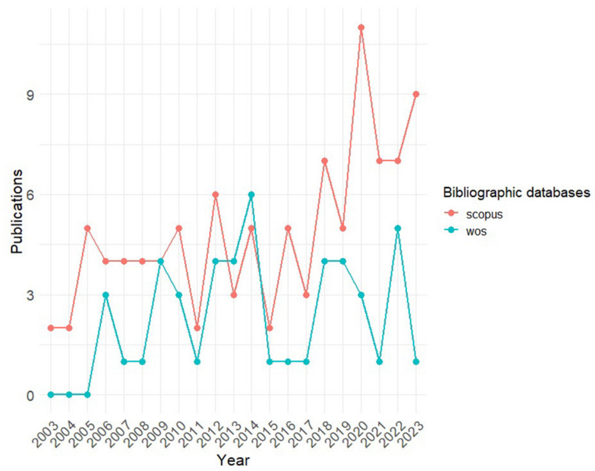


FIGURE 3
Publication trends in Scopus and WoS.

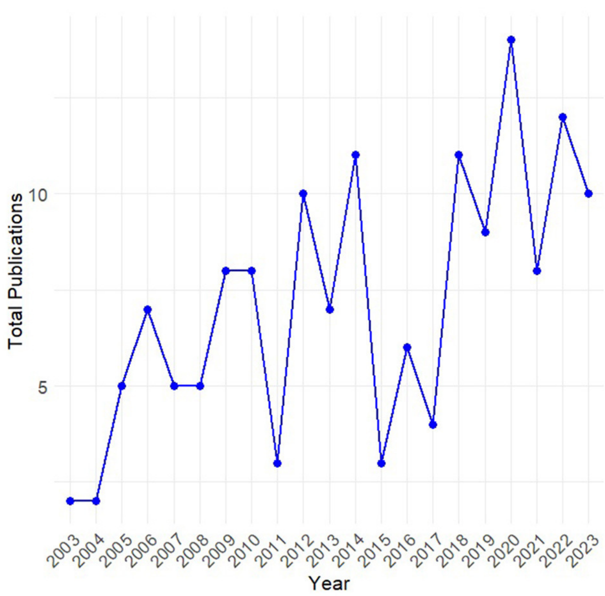


FIGURE 4
Combined publication trend (Scopus + WoS).

2014, and 2019, where three articles were particularly influential: Christou and Vosniadou (2012) conducted several experiments to investigate students' understanding of the real variable concept in algebra; Booth et al. (2014) focused on persistent and harmful errors that students make when solving algebraic problems; and Nortvedt and Siqveland (2019) expanded the discussion to the university level, evaluating the mathematical preparation of students entering calculus and engineering programs.

Figure 2 shows that Scopus maintains a higher number of articles in most years, except for 2013 and 2014, where WoS had a higher number of publications. The progressive increase in both databases suggests growth in scientific production on TA-A.

Scientific production in Scopus has had a more pronounced growth, with peaks in 2012, 2018, and 2020, while WoS shows more pronounced fluctuations and less stable behavior. This trend is evidenced in Figure 3. Similarly, the convergence of trend lines in certain years suggests periods of greater consolidation in the research.

Once the results are consolidated, they are presented in Figure 4, showing the combined publication trend in both databases, which reveals an upward trajectory, with a constant increase in scientific production on TA-A. This behavior could reflect the consolidation of the topic as an area of academic interest.

Figure 5, which shows the cumulative sum of joint publications by year, reveals little stagnation in the accumulation of publications, indicating a sustained increase in academic production over the past two decades, with some positive changes in the slopes of 2012, 2014, and 2017 that suggest an acceleration in the publication of scientific research on TA-A during that period.

3.2.2 Analysis by countries

The top 10 countries with the highest productivity and citation rates in TA-A are shown in Table 2, where scientific production by country is presented along with its impact in terms of citations and publication quality. In the mentioned table, the Production column includes the total number of publications and its percentage relative to the total, while the Citations column shows the total number of citations and the average number of citations per article.

Regarding scientific production, the U.S. leads with the highest number of publications (23 articles), followed by Canada (11), and Indonesia and Turkey (10 each). In terms of citation impact, the U.S. leads with 210 citations and an average of 23.6 citations per publication, followed by Canada (62 citations, 6.97 per publication) and France (59 citations, 6.63 per publication).

The distribution in terms of quality, measured by publications in different Scimago quartiles, allows for classification of publication quality. In Q1, the U.S. (5 publications), the U.K. (4), and Canada (1) stand out with more articles in this quartile. In Q2,

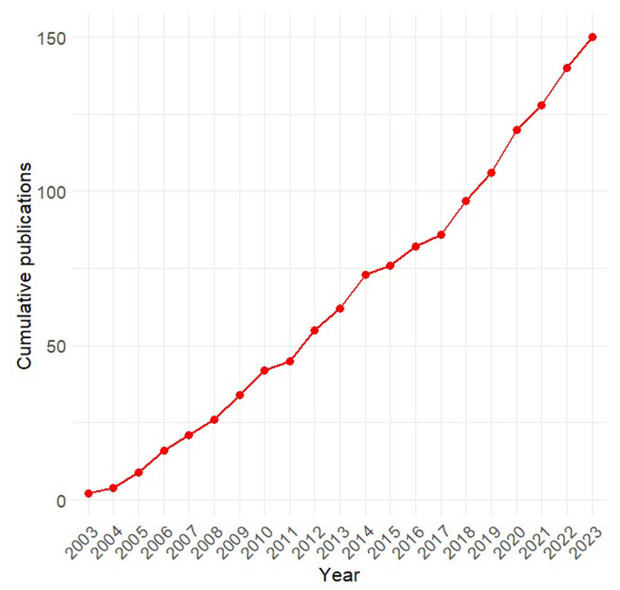


FIGURE 5
Cumulative production.

the U.S. (4), Brazil (2), and Turkey (3) have publications at this level. In Q3 and Q4, Indonesia stands out with 3 publications in each quartile, suggesting less influence in the scientific community compared to other countries.

Figure 6 presents the visual analysis of the impact and scientific collaboration between different countries. Figure 6b illustrates the identification of seven distinct scientific collaboration communities, each with a distinct color. The largest community, led by the USA as the central node with the most connections and collaborations, includes countries such as China, Turkey, Israel, and the United Kingdom, highlighting global collaboration in scientific production. The relationship with Canada is noted, as it appears as the central node of the second most prominent community.

Other smaller groups include countries like Saudi Arabia and Egypt, among others, which seem less connected to the main network.

The upper left graph (Figure 6a) shows how many countries belong to each of the seven identified communities. The smallest communities (6 and 7) each contain only two countries, while the largest community (1) groups six countries, suggesting a more consolidated cooperation network.

The lower left graph (Figure 6c) presents the temporal evolution of new countries (nodes) and the distribution of these collaborations (links). The blue line represents the new nodes, and the red line represents the new links. It shows that until 2020, there was a greater appearance of nodes compared to links between them, with the highest peak of collaborations emerging between 2021 and 2023. One example is the collaboration between Israel and the USA in 2022, as reported by Changala et al. (2022), which highlights how algebra, through advanced techniques, allows solving complex problems in physics, focusing on the transition from arithmetic to more precise mathematical models.

The collaboration between authors from Canada and Mexico is also highlighted through the work of Kieran and Martínez-Hernández (2022), who explored how sixth-grade students in Mexico understand and use the concept of equality in arithmetic and how this can serve as a foundation for algebraic thinking. This study transitions from computational to structural approaches, emphasizing the understanding of equivalence and mathematical properties (such as decomposition and reflexivity) in arithmetic. This preparation helps students work with algebraic equations and fosters the development of mathematical language and justification in arithmetic, aiding students in expressing and understanding more complex algebraic concepts.

3.2.3 Analysis of journals

Table 3 presents a comparison of the 10 most representative academic journals in terms of publications in the field of study, Impact Factor, h–index, and quartile ranking. Educational Studies in Mathematics stands out with the highest impact factor (1.48) and an h–index of 83, ranked in the top quartile (Q1), indicating its high influence and prestige in the educational field. On the other hand, Lecture Notes in Computer Science have a notably high h–index (470), reflecting extensive citation, though with a moderate impact factor (0.61) and ranked in Q2. Journals like School Science and Mathematics and ZDM–International Journal on Mathematics Education highlight their relevance, both ranked in Q1.

Figure 7 represents the citation structure between scientific journals, analyzing how publications in different journals are connected through citations. Figure 7b shows three large journal communities dominate in terms of their networks and cross–citations. The first community has its central node in the Journal of the Acoustical Society of America, reflecting a more specialized but active focus on acoustic research. In this journal, Nethercote et al. (2023) show how the analytical techniques and matrix equations used in their article reflect advanced algebraic concepts. The second community, with its node in the journal Advances in Nonlinear Variational Inequalities, is where Ali et al. (2023) published an article on the abacus, showing how basic arithmetic operations can be formalized and understood through advanced mathematical principles, such as number theory and algebra. Finally, the third community, with its node in the journal Mathematics, features Lengyelfalusy and Gonda (2023), who explore the decomposition and reformulation of problems to facilitate algebraic resolution.

In the upper–left graph (Figure 7a), the number of journals in each citation community is shown, indicating that the smaller communities (6 and 7) include only a few journals, while the largest community (1) has around 18 journals. As the number of communities decreases, the number of journals in each one increases, which may suggest a consolidation of knowledge in certain groups.

Figure 7c shows how the citation network has grown over time, with the blue line representing new journals (nodes) and the red line representing new connections (links). Peaks in the incorporation of new journals and the establishment of new connections can be observed, such as during the period from 2009 to 2012, suggesting a key moment for the expansion of the citation network.

TABLE 2 Scientific production and its impact by country.

Country	Production		Citations		Q1	Q2	Q3	Q4
	Count	Percentage	Count	Average				
USA	23	18.7	210	23.6	5	4	1	0
Canada	11	8.94	62	6.97	1	2	1	0
Indonesia	10	8.13	13	1.46	1	0	3	3
Turkey	10	8.13	51	5.73	1	0	3	0
France	5	4.07	59	6.63	0	1	1	0
United Kingdom	5	4.07	46	5.17	4	0	0	0
Brazil	4	3.25	9	1.01	0	2	1	0
China	4	3.25	33	3.71	2	0	0	0
Colombia	4	3.25	0	0	0	0	0	0
Australia	3	2.44	55	6.18	1	1	0	1

3.2.4 Author collaboration network

Table 4 presents the top 10 researchers based on their academic production and impact, measured through the h–index from Scopus. Akkan Y, with four articles and an h–index of 5, leads the list, followed by Baki A in sixth place, with two articles and an h–index of 13. Both are from the University of Trabzon in Turkey and stand out for the number of publications. Herman T, from Indonesia, ranks second with an h–index of 12 despite having fewer articles, suggesting publications with greater impact. Notable researchers from Colombia include Cruz J and Estévez M from the Universidad Distrital Francisco José de Caldas, who stand out in Figure 8 as part of the third most productive author network. With the participation of these Colombian authors, Romero Cruz et al. (2014) stand out with their article "Processes of unitization and normalization in the construction of a TA-A object: multiplication as a change of unit", an approach that helps students understand algebraic concepts through basic arithmetic operations.

Figure 8 presents the collaboration networks among authors contributing to the topic. Figure 8b shows that the most consolidated network stems from Eberle, particularly through one of his collaborations with Rüede et al. (2019). This study addresses the mathematical competencies required for university preparation in Switzerland, identifying two key areas: procedural skills in arithmetic and elementary algebra, and conceptual skills in interpreting graphs and formulas. The second most connected network involves Gürbüz and Akkan (2008), who examined the TA–A process in a sample of students from grades 5 to 8 in Turkey. Figure 8a shows the number of authors in each community. For example, in the smaller communities (9 and 10), it is evident that there are few authors, resulting in low interconnectivity. However, the largest community (1) includes over 100 authors, suggesting a denser and more stable cooperation network. Figure 8c graph shows the evolution of the collaboration network over time. In this graph, the lines represent two aspects: the blue line represents new authors (nodes) entering the collaboration network, and the red line (links) represents new collaborations established between authors. The most significant growth peaks occur in 2005 and 2017, where the incorporation of new authors and collaborations increases significantly.

3.3 Thematic trends and intellectual structure

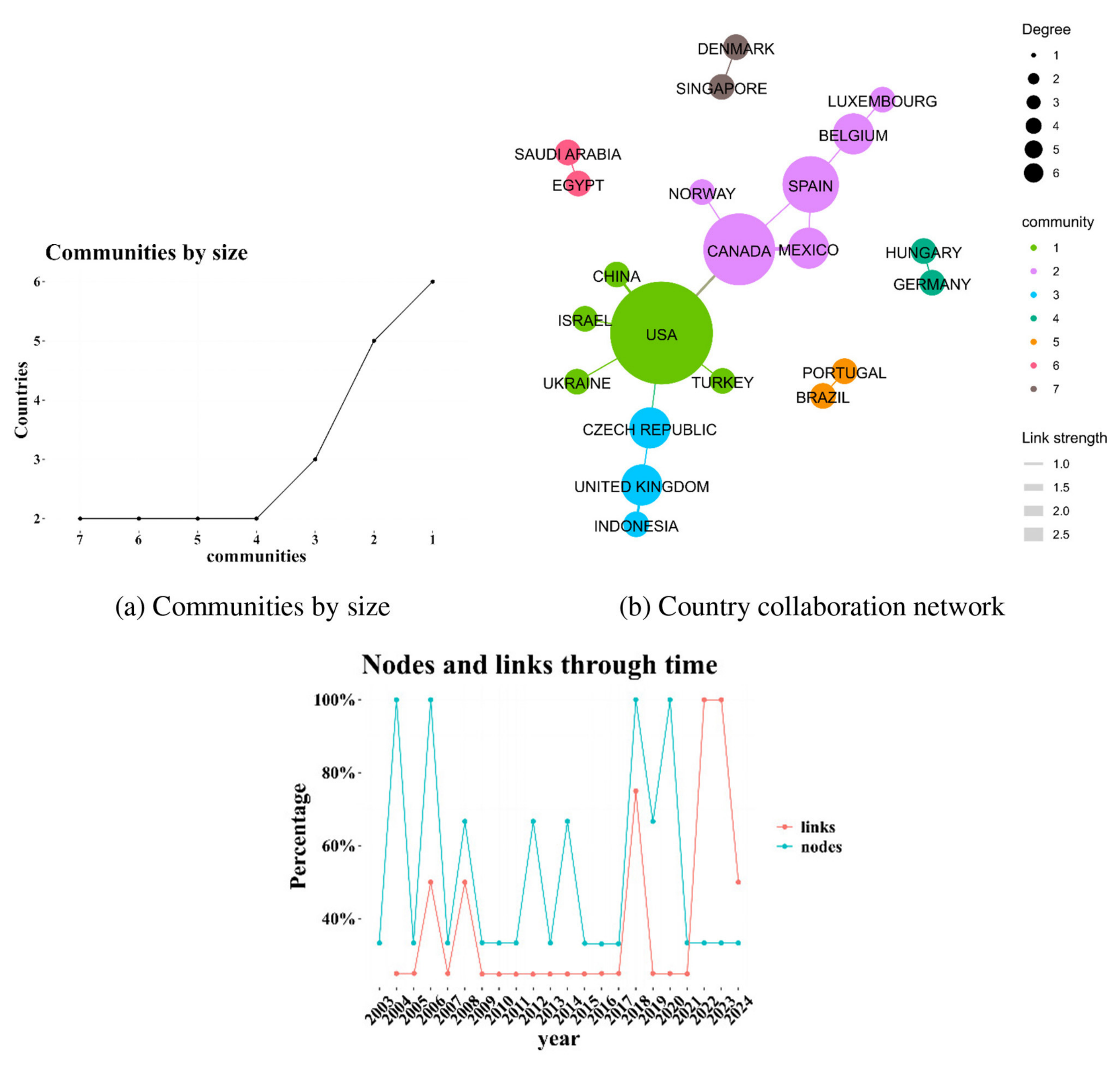
Using the Tree of Science (ToS) metaphor, the intellectual structure of research on TA-A was analyzed by classifying publications into three categories—roots, trunk, and branches—as detailed in the methodology section. This analysis made it possible to identify how research in the field has evolved from its theoretical foundations to its current practical applications within primary, elementary, and secondary education contexts.

Figure 9 shows a representation of the structure of the Tree of Science (ToS), which results from categorizing research into roots, trunks, and branches.

3.3.1 Roots

It was found that the understanding of generalized arithmetic or algebra in high school students has been a topic of interest since the pioneering studies of Küchemann (1980), who investigated how young people handle algebraic ideas and TA-A (algebraic thinking), revealing a significant gap between these two approaches. A year later, Küchemann (1981) delved into the difficulties and learning strategies that students face when tackling algebra, emphasizing the importance of teaching that promotes a solid conceptual understanding of algebraic operations and relationships.

Continuing this line of research, Lesley R (1984) explored the strategies and common mistakes students make when learning algebra, showing that many errors are not simple mistakes but rather the result of arithmetic thinking strategies that are not appropriately applied in the algebraic context. This study further underscores the need to understand these strategies



(a) Communities by size (b) Country collaboration network

(c) Nodes and links through time

FIGURE 6
Scientific collaboration by country. **(a)** Size of each community. **(b)** Country-level collaboration network. **(c)** Temporal evolution of active nodes and links.

to design more effective teaching methods. In this regard, Usiskin (1999) expanded the discussion by introducing different conceptions of algebra in the school context, such as generalized arithmetic, problem-solving procedures, studying relationships between quantities, and algebra as the study of structures and the use of variables.

Filloy and Rojano (1989) focused on the process of solving equations, highlighting TA–A. Their study revealed a "breaking point" where students must abandon familiar arithmetic strategies and adopt a more algebraic approach, which requires a profound conceptual shift. Complementing this, Kieran (1992) consolidated much of this research into a comprehensive framework, exploring how algebraic thinking develops from an early age and proposing new directions to improve algebra teaching.

Building upon these early studies, the field of school algebra has evolved toward more structured conceptual models, among which two widely recognized frameworks stand out. On the one hand, Kieran (2004) proposed that algebraic thinking can be expressed through three types of activity:

TABLE 3 Number of publications and impact of the top journals.

Journals	Count in WoS	Count in scopus	Impact factor	h–index	Quartile
Lecture Notes In Computer Science	0	8	0.61	470	Q2
Journal Of Physics: Conference Series	0	5	0.18	99	
Educational Studies In Mathematics	3	5	1.48	83	Q1
Revista Cientifica	3	0	0.3		
School Science And Mathematics	2	2	0.42	54	Q1
ZDM- International Journal On Mathematics Education	0	3	1.1	66	Q1
AIP Conference Proceedings	0	2	0.15	83	
Bollettino Di Storia Delle Scienze Matematiche	1	0	0.1	9	Q4
Canadian Journal Of Science, Mathematics And Technology Education	0	2	0.43	32	Q2
Egitim Ve Bilim	0	2	0.24	28	Q3

- Generational activity, related to the creation and expression of generalizations;
- Transformational activity, associated with the manipulation of expressions and equations; and
- Meta-level activity, involving reflection on the meaning of algebraic procedures.

This model has been adopted in various empirical studies and has served as the foundation for the design and analysis of classroom tasks aimed at developing algebraic thinking.

On the other hand, the framework proposed by Kaput (2008) defines algebraic thinking as the capacity to formulate and express generalizations within increasingly formal symbolic systems, and to operate on those symbols in a structured way. This perspective is organized into three fundamental content strands:

1. Algebra as the study of structures and systems that emerge from arithmetic;
2. Algebra as the analysis of functions, relations, and change; and
3. Algebra as a set of modeling languages.

This conceptual framework has been widely used as an analytical lens in recent research, particularly focusing on the first two strands, while the third is often integrated transversally in problem-solving contexts (Kieran et al., 2016). Building on this model, Blanton et al. (2018) reorganized the strands proposed by Kaput (2008) into three key areas for the development of early algebra curricula:

1. Generalized arithmetic;
2. Equivalence, expressions, equations, and inequalities; and
3. Functional thinking.

The first two areas are closely related to the study of structures that arise from arithmetic, whereas the third connects functional understanding with the modeling of real-world phenomena and situations.

In 1994, Herscovics (1994) introduced the idea of a "cognitive gap" between arithmetic and algebra, suggesting that this conceptual leap is more complex than previously assumed and highlighting the need for a specific pedagogical approach to address these difficulties. That same year, Sfard and Linchevski (1994) explored the advantages and disadvantages of reification in algebra learning, arguing that while it facilitates the understanding of complex concepts, it can lead to misinterpretations if the conceptualization process is not properly guided.

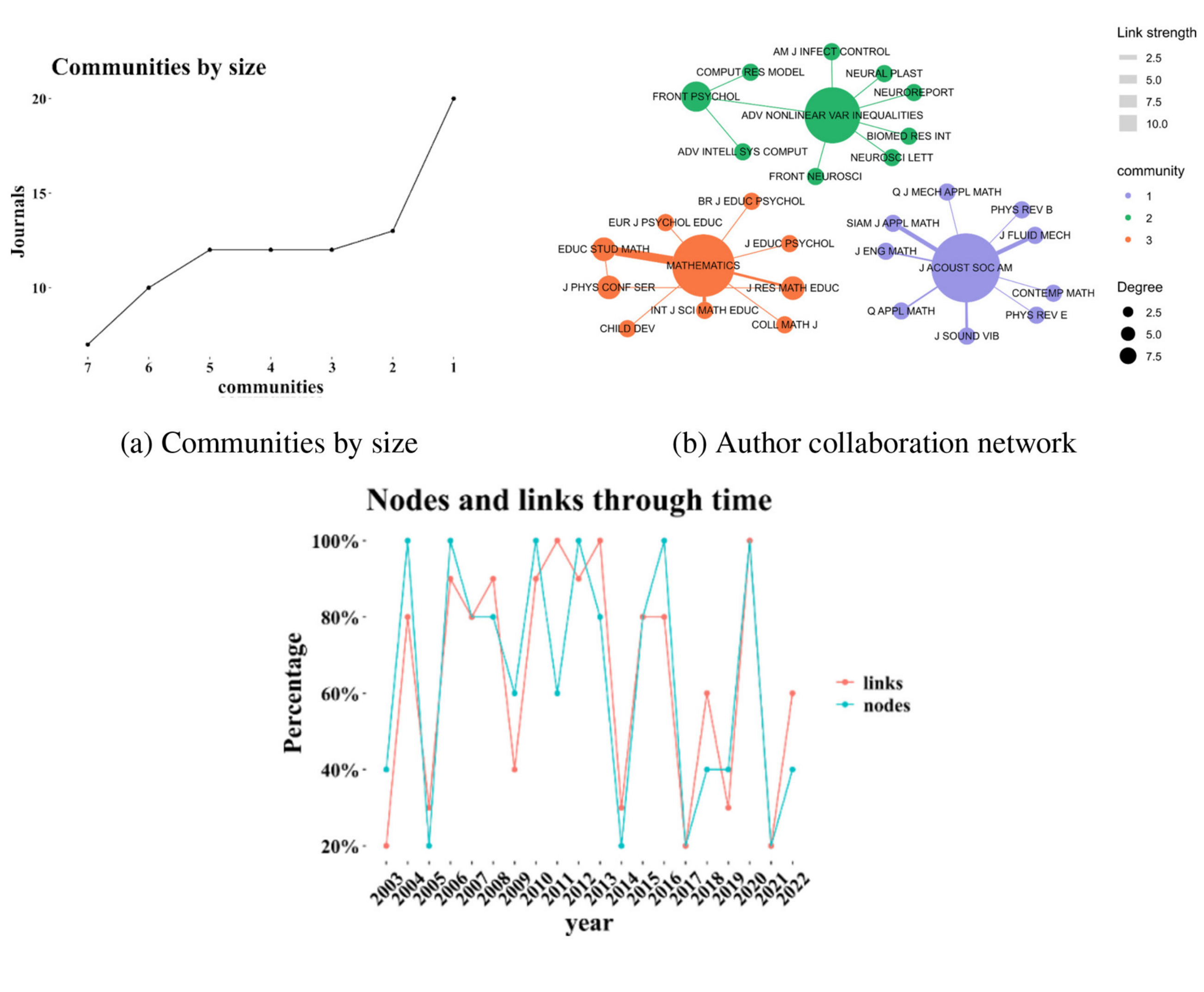
Mason (1996) continued this line of research, addressing the importance of teaching algebra as a tool to express mathematical generalities, emphasizing the need to develop in students a deep and flexible understanding of algebraic concepts. Stacey and MacGregor (1999) investigated the algebraic method of problem-solving, highlighting that true understanding of algebra as a generalized process requires teaching that integrates problem-solving into algebraic learning.

Subsequently, Kieran (2004) explored algebraic thinking in early grades, emphasizing the importance of introducing basic algebraic concepts at an early age. Later, Gutiérrez and Boero (2006) expanded this analysis by addressing the teaching and learning of algebra from a multidimensional approach, integrating cognitive, semiotic, and didactic perspectives.

Finally, Carraher et al. (2008) investigated the interrelationship between arithmetic and algebra in early mathematics education, proposing that integrating algebraic thinking from the earliest stages can facilitate a smoother transition into formal algebra, better preparing students to face mathematical challenges in higher grades.

3.3.2 Trunk

According to the analogy used, the trunk represents the contributions that address structural aspects of algebraic thinking. In this regard, Küchemann (1978) proposed a hierarchy in students' understanding of the algebraic variable—ranging from basic arithmetic interpretations to functional and structural uses—which has been widely employed to analyze how students progress in their use and understanding of variables. This perspective provides a foundation for structuring the analysis of the following studies. For example, Malisani and Spagnolo (2009) analyzed



(a) Communities by size

(b) Author collaboration network

(c) Nodes and links through time

FIGURE 7
Citation networks of the top journals. **(a)** Community sizes based on the number of authors. **(b)** Co-authorship network showing components, node degree, and link strength. Node size is proportional to the number of connections, and colors represent different components within the network. **(c)** Evolution of active nodes and links over time.

TABLE 4 Production by Author.

Researcher	Total articles	Scopus h–index	Affiliation	City - Country
Akkan Y	4	5	Trabzon University	Trabzon - Turkey
Herman T	3	12	Universitas Pendidikan Indonesia	Bandung - Indonesia
Wahyuni R	3	2	Universitas Islam Riau	Pekanbaru - Indonesia
Ali S	2	3	King Faisal University	Al-Ahsa - Saudi Arabia
André M	2	2	Laurentian University	Sudbury - Canada
Baki A	2	13	Trabzon University	Trabzon - Turkey
Boucard J	2	2	Nantes Université	Nantes - France
Cruz J	2		Universidad Distrital Francisco José de Caldas	Bogotá - Colombia
Eberle F	2	5	Universität Zürich	Zurich - Switzerland
Estévez M	2		Universidad Distrital Francisco José de Caldas	Bogotá - Colombia

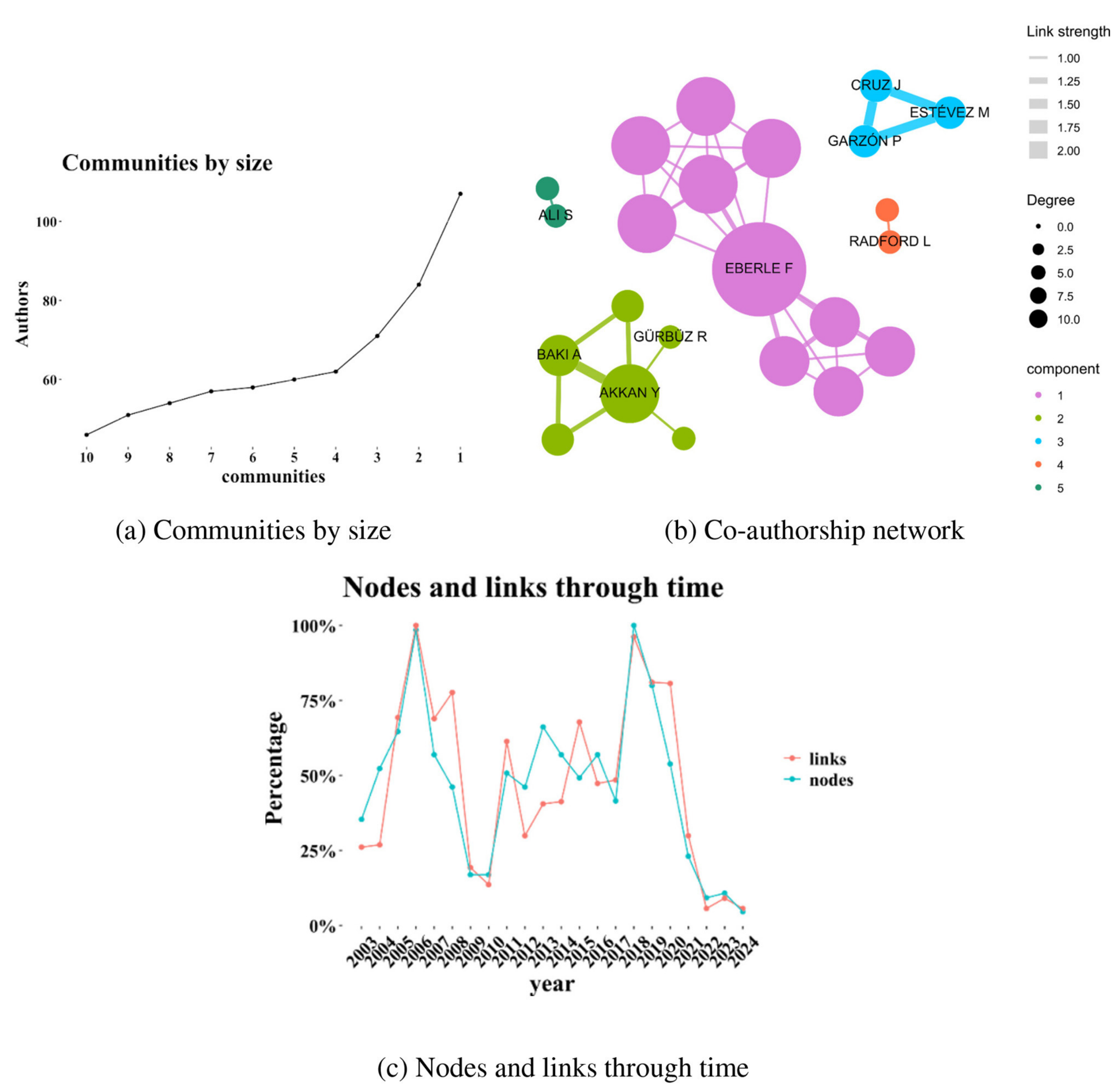


FIGURE 8
Collaboration network of the most productive authors. **(a)** Community size based on the number of authors. **(b)** Co-authorship network highlighting collaborative components, link strength, and node degree. Node size is proportional to the number of connections, and colors represent distinct components identified in the collaboration network. **(c)** Temporal distribution of active nodes and links.

the crucial role of the "variable" in the TA–A transition. Their study revealed that the variable, a central concept in algebra, poses significant challenges for students accustomed to arithmetic reasoning. Therefore, the authors emphasize the importance of explicitly addressing the conceptual difficulties associated with introducing variables in order to facilitate a smoother transition from arithmetic to algebra.

Two years later, in 2011, Cai et al. (2011) addressed the development of algebraic thinking in early grades by comparing educational practices in China and Singapore. This study highlighted how early and well-structured introductions to algebra can foster more effective TA-A. The following year, Christou and Vosniadou (2012) investigated how students assign numbers to literal symbols, exploring key aspects of TA-A. Their work revealed misconceptions that students may have when facing the literal aspect of algebra, highlighting the need for pedagogical approaches that clarify these concepts from early stages.

In the same year, Banerjee and Subramaniam (2012) analyzed the evolution of a didactic approach to teaching early algebra, showing how progressively adapting teaching strategies can improve students' understanding of fundamental algebraic

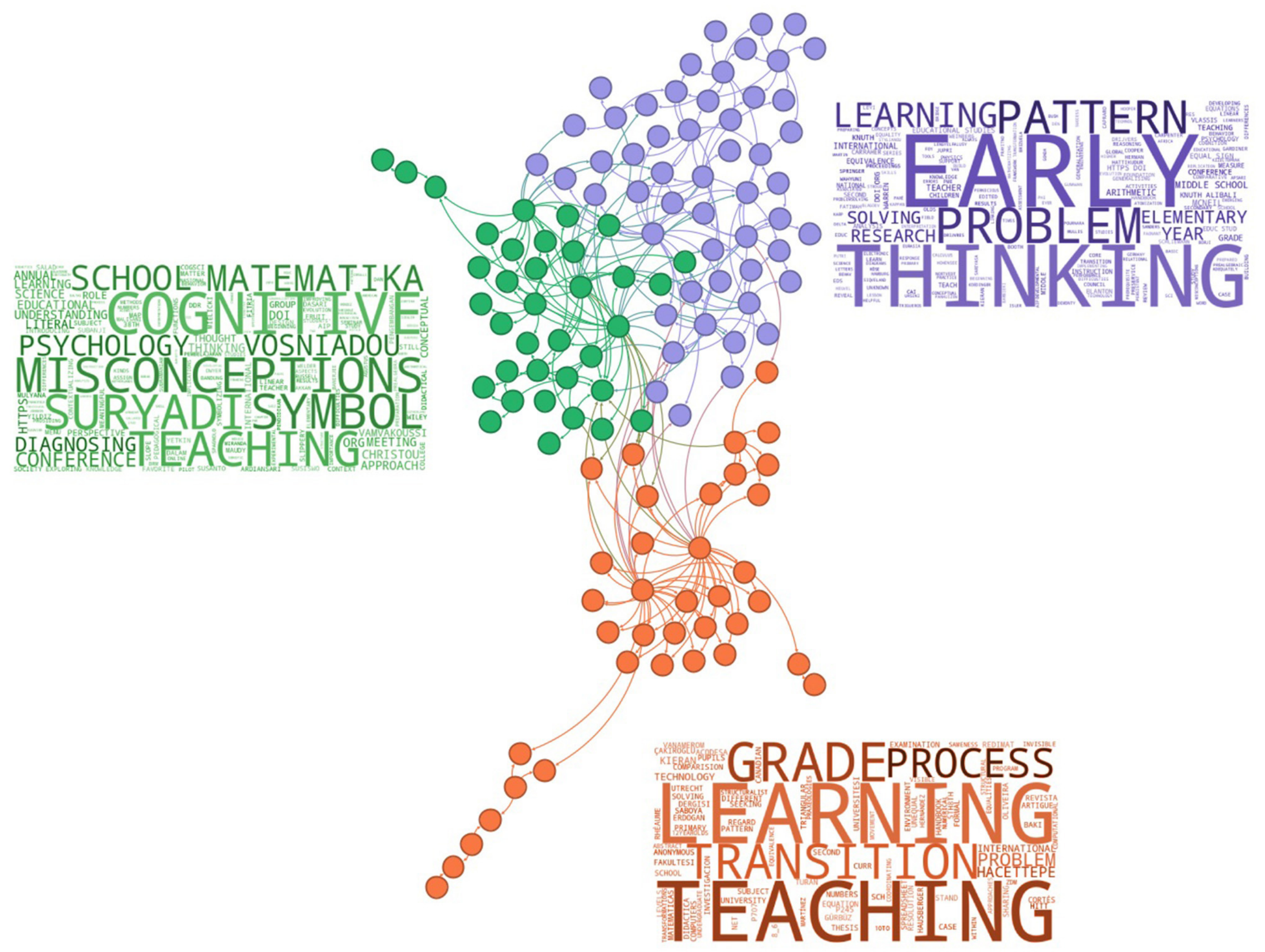


FIGURE 9
Structure of the Tree of Science (ToS) in the TA-A research field: studies are categorized into roots (green), trunk (orange), and branches (purple). The accompanying word clouds summarize the main conceptual focus of each group.

concepts, thereby facilitating their transition from an arithmetic approach. This adaptation is crucial to building a solid foundation in algebra that develops from the early grades.

Finally, Kieran and Martínez-Hernández (2022) explored how students aged 10 to 12 coordinate "invisible and visible equality" within transformations of equivalence in numerical equalities. This study highlights students' movement from computational approaches to structural approaches in algebra, representing a significant evolution in their mathematical thinking. This research emphasizes the importance of guiding students in understanding the underlying structure in algebraic expressions, a fundamental skill for their success in advanced mathematics.

3.3.3 Branch 1: development of algebraic thinking in early education

The first trend identified is related to the teaching of algebraic thinking and problem-solving in the early school years, focusing on aspects such as the development of algebraic thinking skills from an early age and the recognition and use of patterns to solve problems. For example, Hohensee and Borji (2024) conducted a conceptual replication study focused on preparing future primary education teachers to teach early algebra. This study emphasizes the importance of teacher training to ensure that students understand fundamental algebraic concepts from the early grades, facilitating a smoother transition from arithmetic.

In 2023, Lengyelfalusy and Gonda (2023) explored the relationship between transformation and atomization of problems in solving algebraic problems, highlighting how students decompose and reformulate problems to make them more manageable in an algebraic context. This research complements the work of Hitt et al. (2023), who adopted a sociocultural approach to studying the construction of arithmetic-algebraic generalization skills in schools in Quebec and Mexico, emphasizing the influence of cultural context and social interaction in learning algebra.

In 2022, Wahyuni et al. (2022) investigated how early secondary school students interpret algebraic letters, finding that difficulties persist when trying to understand them as variables rather than specific unknowns. This research is closely related to the earlier work of Wahyuni et al. (2020), which examined how students use arithmetic thinking when solving mathematical problems, revealing that while they apply arithmetic concepts, they often struggle to extend these to the algebraic context.

That same year, Apsari et al. (2020) explored how students perform TA-A during pre-algebra lessons, highlighting the common difficulties faced during this transition. Finally, Pournara and Sanders (2020) conducted a response pattern analysis that revealed how students perform in arithmetic equivalence and algebraic equations.

3.3.4 Branch 2: teaching and learning processes in TA-A

The second branch identified a trend toward the teaching and learning processes in TA-A, with several significant contributions. Studies such as those by Ennassiri et al. (2023) analyzed the institutional relationship of modeling activities in a Moroccan high school textbook, revealing how the pedagogical content is structured to facilitate the learning of algebraic concepts through modeling activities. This study highlights the importance of the selection and organization of teaching materials in algebra education, emphasizing how modeling activities can serve as a bridge between arithmetic understanding and algebraic abstraction.

In the same vein, Kieran and Martínez-Hernández (2022) examined how students between the ages of 10 and 12 coordinate "invisible and visible equality" in transformations of equivalence in numerical equalities. This research emphasizes the students' movement from a computational approach to a more structural one, which could signify a successful transition from arithmetic to algebraic thinking. Complementing this, Polotskaia et al. (2022) found that students' ability to recognize and use algebraic relationships is a key indicator of their progress in transitioning to algebraic thinking.

Hausberger (2018) expanded this discussion by proposing structuralist praxeologies as a research program in the teaching and learning of abstract algebra. His approach focuses on how mathematical structures can be effectively taught, promoting a deep and abstract understanding of algebra from the early educational levels. Likewise, Cortés et al. (2014) recognize that using numbers as context can facilitate TA-A. Their research emphasizes the importance of using technological tools and interactive teaching methods to help students internalize algebraic concepts based on arithmetic foundations.

In general, these investigations highlight the significance of pedagogical practices in the transition from arithmetic thinking to handling more complex algebraic structures.

3.3.5 Branch 3: cognitive processes, errors, and algebraic symbols: challenges and pedagogical solutions

In this branch, a trend was identified to improve algebra teaching by understanding students' misconceptions and the cognitive processes involved in learning algebraic symbols, as well as using psychological and diagnostic approaches to identify and correct conceptual difficulties. For example, Fitria et al. (2023) presented a study focused on the use of cognitive maps to diagnose and explore students' misconceptions about algebra, finding that these maps are effective tools for identifying errors or misunderstandings and contributing to a better understanding of algebraic concepts by students.

In the same year, Ardiansari et al. (2023) examined the continued popularity of the pedagogical approach known as "Fruit Salad Algebra," used to introduce algebraic concepts in schools. This research explored the effectiveness of this method and found that although it is engaging and playful, it may not be sufficient to foster a deep understanding of algebra. In a previous study, Yildiz and Yetkin Ozdemir (2021) addressed teachers' content knowledge as a determining factor for TA-A, emphasizing that teacher mastery of content is essential to guide students through this transition, enabling them not only to manipulate algebraic symbols but also to understand the meaning and logic behind these manipulations. Therefore, ongoing teacher training is required.

Finally, Maudy et al. (2017) explored the contextualization of symbols in algebra learning, investigating how students contextualize algebraic symbols in their learning process, emphasizing the importance of connecting algebraic symbols to real-life situations to make learning more meaningful and applicable to their daily lives.

3.4 Limitations

As part of this systematic review with a bibliometric focus on TA-A, several limitations must be considered when interpreting the results. First, the search was limited to two major databases—WoS and Scopus—which may have excluded relevant publications indexed in other sources.

Additionally, the Journal of the Acoustical Society of America emerged as one of the nodes in the co-citation network. Although this journal is not directly related to the field of mathematics education or the study of early algebra, its presence may be attributed to the broad use of the term algebra in the search strategy, which was not limited exclusively to expressions such as early algebra or algebraic thinking. Upon reviewing the associated content, it was confirmed that the retrieved articles were correctly indexed with the keywords defined in the search protocol, although they were not necessarily aligned with the specific focus of this review. In this regard, the appearance of this node is understood as a collateral effect of the inclusive criteria adopted, which aimed to ensure comprehensive coverage of the field. While this does not compromise the validity of the findings, it does highlight the importance of employing more refined search strategies in future studies to delineate the scope of analysis with greater precision.

Regarding the metrics used, such as the number of publications, citations, and impact factors, although these indicators offer a quantitative view of the scientific output, they do not necessarily capture the depth or quality of the studies analyzed. To mitigate this limitation, the Tree of Science (ToS) algorithm was employed, allowing for an analysis of the intellectual structure of the field and providing a more comprehensive perspective on thematic developments and scholarly connections.

4 Discussion

This section addresses some complementary issues to the results presented, divided into two lines of discussion.

4.1 Information on the most cited articles

The impact of scientific publications is often linked to the number of citations they receive, suggesting their level of influence in the field. However, this quantitative criterion does not always guarantee that the most cited research is necessarily the most innovative or transformative. In this study, five contributions with a high number of citations were identified: Khng and Lee (2009); Malisani and Spagnolo (2009); Christou and Vosniadou (2012); Booth et al. (2014); Nortvedt and Siqveland (2019).

An analysis of these studies reveals that, although they come from different approaches and contexts, there is a consensus on the need for an approach that also addresses the cognitive interferences arising from previous arithmetic strategies. Khng and Lee (2009) emphasizes the role of cognitive inhibition in TA-A, while Malisani and Spagnolo (2009) argue that the structuring of mathematical thinking is not just a change of tools but a conceptual transformation that requires specific metacognitive processes.

Additionally, these contributions illustrate that TA-A not only involves a change in mathematical content but a profound transformation in the way reasoning is structured. Overcoming natural biases, correcting persistent errors, and reinforcing mathematical foundations are essential aspects to ensure successful learning and adequate preparation for higher educational levels.

Regarding the number of publications, it is important to question whether their impact is genuinely attributable to the quality and originality of their contributions or if, instead, it responds to external factors such as academic visibility or the volume of publications in certain countries. In this sense, the predominance of the U.S. in terms of production and citations could skew the perception of the real impact of these studies, as this country maintains a high level of publications in multiple areas of knowledge, not only in TA-A. This raises uncertainty about whether the recognition of these studies is due to their intrinsic value or if it is the result of academic inertia, where certain authors and universities receive greater exposure due to their global presence in scientific literature.

4.2 Evolution of trends in TA-A research: a comparative analysis and future projection

The bibliometric analysis conducted in this study reveals progressive growth in scientific production on TA-A, with a notable acceleration during the period of dynamization. This increase in publications may suggest a growing interest in the field.

Three emerging thematic trends stand out in the intellectual structure of the field, indicating a transition in the way algebra teaching is conceived. These trends reveal a shift toward more integrative pedagogical models, combining the cognitive development of students with teaching strategies aimed at the practical application of algebraic concepts.

When comparing the results of this study with previous research, there is an observable evolution in the focus areas and methodologies. The foundational studies that form the roots of the field, such as those by Küchemann (1980, 1981); Lesley R (1984); Usiskin (1999), focused on identifying common strategies and errors students face when encountering algebra, particularly in understanding and using the concept of a variable. While these studies laid the foundation for the field, their perspective was mostly descriptive and centered on specific difficulties in learning algebra.

In contrast, recent research has expanded the perspective, integrating more complex technological and cognitive approaches. For example, works such as those by Hohensee and Borji (2024) have addressed the preparation of future teachers to improve algebra teaching, while Lengyelfalusy and Gonda (2023) have explored the atomization of algebraic problems as a teaching strategy. This shift reflects an interest in more structured and applicable educational models in real teaching contexts, marking a distinction from the early investigations. Other works that complement this line of discussion were reviewed in the results section.

The trends identified in this study not only provide an updated view of the current state of research but also constitute key lines of work for future investigations.

5 Conclusions

The transition from arithmetic to algebra represents a crucial milestone in mathematical education, as it involves moving from a concrete numerical approach to a more abstract and symbolic reasoning. This transition presents significant challenges for many students, and further research is required to provide evidence on how to address this transition in different contexts.

The use of bibliometric and scientometric tools in this study was essential to identify trends and patterns in TA-A research. Through the analysis of scientific publications in WoS and Scopus, a database was consolidated, allowing for an exploration of the evolution of the field and its intellectual structure.

Regarding the stated objectives, the study achieved a systematic approach to scientific production in TA-A, employing scientometric analysis that facilitated the identification of theoretical, methodological, and emerging influences within the area. Additionally, the metaphor of the Tree of Science (ToS) allowed for a visualization of how knowledge in the field is structured, distinguishing foundational studies, theoretical consolidations, and trends.

This type of analysis highlights the importance of applying advanced data analysis techniques in bibliometric studies. Without these methodologies, the exploration of scientific literature becomes a complex, demanding process prone to bias, which would affect the quality and reliability of the findings.

The scientometric analysis identified two key aspects that facilitate the transition from arithmetic to algebraic thinking: the importance of introducing variables and pedagogical practices combined with the use of technological tools. When applied in the early education of students, these aspects help enhance the understanding of algebraic concepts based on arithmetic foundations.

From the intellectual structure analyzed through ToS, it was found that the roots of TA-A research date back to the 1980s, and it remains a relevant area of research today. Three trends were identified: (1) Development of algebraic thinking in early education, (2) Teaching and learning processes in the transition to algebraic thinking, and (3) Cognitive processes, errors, and algebraic symbols: challenges and pedagogical solutions.

Author contributions

VG: Conceptualization, Formal analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing. DB: Conceptualization, Funding acquisition, Resources, Supervision, Validation, Writing – review & editing. CU-L: Data curation, Formal analysis, Methodology, Software, Validation, Visualization, Writing – review & editing.

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Conflict of interest

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