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Spiral-curricular blended learning for the mathematical methods in physics education—A cross-modular interactive course

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A research based concept for the mathematics education of physics teacher trainees is introduced, which integrates mathematical methods seminars in the experimental physics courses of the first two semesters. This article reports on the implementation and evaluation of a spiral-curricular blended learning approach for the mathematical methods seminars, which is based on design criteria for digital teaching-learning sequences. Evaluations of the course quality show a high level of student satisfaction with the materials, especially with the interactive videos. The interactivity was found to be a central criterion for the quality of the explanatory videos. Surveys on the learning progress for the topics complex numbers and differential equations revealed good to very good test results. Detailed investigations on paired samples showed that knowledge was consolidated, resulting in a trend to a narrower and higher test score distributions with ongoing duration of the course.

KEYWORDS

flipped classroom, mathematical methods, self-regulated learning, interactive explanatory videos, active learning, higher education in physics

1 Introduction

Investigating the hurdles in the introductory phase of studies, with the aim of providing students with appropriate support, is a central element of higher education research. To ensure a successful start to a science study program, students must be supported to find their place in the structures and self-regulating learning environments of university teaching (Woitzik et al., 2023; Tayebi et al., 2021; Kabashi et al., 2022; Echchafi et al., 2021) and to understand the language of mathematics. Recent studies show that there is an enormous need for action and support, especially in helping students to learn applicable mathematics (Woitzik et al., 2023; Lumpe, 2019; Schild, 2021; Slavin, 2008; Kabashi et al., 2022; Kämpf and Stallmach, 2024).

In programs with a very heterogeneous student body attending courses together, it is difficult to provide individual support at this early stage. The physics teacher training program is such a program, where the high level of heterogeneity poses a challenge for the teaching staff at universities. Additionally, governmental regulations for teacher training programs often require, that each teacher trainee studies simultaneously two different sciences which will become his/her teaching subject at schools and the basics of educational science. Since there are many possible combinations with physics, students who join the initial physics courses will be very heterogeneous with respect to their prior physics knowledge, their interests in other fields of science and their own visions as future physics teacher at school. About half of the rookie physics teacher trainees at our university do not choose mathematics as their second teaching subject (Woitzik et al., 2023). These students will not experience further mathematical training in their second subject. The German Physical Society (Deutsche Physikalische Gesellschaft, DPG) conducted a cross-sectional study at 48 German universities to investigate the physics teacher training program and its difficulties (Woitzik et al., 2023). A key finding is that only 50.3% of the universities offer their own mathematics modules for physics teacher training, and only six universities address the individual needs of physics teachers with their own modules from the beginning of the course (Woitzik et al., 2023). This contradicts with the two main challenges of the introductory physics course: support in getting used to university learning (Woitzik et al., 2023; Tayebi et al., 2021; Kabashi et al., 2022; Echchafi et al., 2021) and support in basic mathematical methods (Woitzik et al., 2023; Tayebi et al., 2021; Kabashi et al., 2022).

Closing the gap between mathematics and physics has long been a focus of research based course designs. Several studies have shown that interlinked teaching of mathematics and physics facilitates the transfer of mathematics to physics and vice versa (Dunn and Barbanel, 2000; Yeatts and Hundhausen, 1992; Hundhausen and Yeatts, 1995; Dominguez et al., 2024). A temporally coherent introduction of mathematics to the basic physics lecture enables students to practice formally learned methods and algorithms on relevant application examples. The use of mathematics in physical application tasks promotes motivation and understanding of both mathematics and physics. Standardized notation reduces the cognitive load (Dunn and Barbanel, 2000).

In our previous article, we presented a didactic concept for a two-semester blended learning course to teach mathematical methods (abbreviated as MaMe) integrated in the physics lecture for the physics teacher training program at Leipzig University (Kämpf and Stallmach, 2024). The paper introduced a spiral-curricular concept and the realization of its ideas for the first semester in which we teach classical mechanics and its mathematical methods.

In the present follow-up article, we focus on the three central design criteria of our integrated mathematics course (1) parallelism, (2) spiral-curriculum, (3) interactivity. We present the final content and implementation of the blended learning mathematics course for the 2nd semester, where we teach electrodynamics and its mathematical methods. The development, implementation and revision of the total cross-modular course was carried out for three successive cohorts of physics teacher training students starting in winter terms 22/23; 23/24; 24/25, respectively, using the design-based research approach. The evaluation steps of the entire project period are discussed and important design features of our course are derived. The main research question guiding this study is:

> What is the effect of integrated mathematics-physics teaching on students' perceptions and achievements?

2 Background: Blended learning and flipped classroom

A common finding of educational scientists is that a major difficulty in learning science is the passive role of the student in a traditional lecture (Andrews et al., 2011; Bonwell and Eison, 1991). In recent decades, there has been a great endeavor to develop teaching-learning formats in which students can actively participate and contribute with their individual prior knowledge. There are various proposals and models how to meet the heterogeneous needs of first-year science students. Approaches that offer a high degree of differentiation and support self-regulated learning are blended learning and flipped classroom models. Both approaches are frequently used by teachers and researchers and are accentuated in different ways. It is therefore important to clarify what we understand by blended learning and flipped classroom.

2.1 Blended learning

Two of the most frequently cited definitions of blended learning are those of Graham (2006) and Garrison and Kanuka (2004). Graham (2006) defines Blended learning as: "*Blended learning systems combine face-to-face instruction with computer-mediated instruction.*" Garrison and Kanuka (2004) characterize blended learning as "*... the thoughtful integration of classroom face-to-face learning experiences with online learning experiences. There is considerable intuitive appeal to the concept of integrating the strengths of synchronous (face-to-face) and asynchronous (text-based Internet) learning activities*". Both definitions have in common that they understand blended learning as a mix of online and face-to-face phases. The first definition focuses on teaching process while the second definition describes the learning process. The focus of the second definition lies in the word "thoughtful", emphasizing the importance of the quality of the learning environment.

There are various models for the time distribution between online and face-to-face phases. For Watson (2008), blended learning covers the entire continuum between traditional face-to-face teaching and full online learning. Weigel (2006) defines blended learning as an equal mix of face-to-face and virtual phases. For our newly designed mathematics course we tried to obey the central elements of both definitions. Therefore, our research based development of the mathematics course intended to create a *thoughtful-designed teaching-learning environment with a mixture of online and face-to-face teaching. Both phases are equally important for the learning success of the learners.*

2.2 Flipped classroom

The use of and research on flipped classrooms designs have increased rapidly in recent years. In a systematic review, Lundin et al. (2018) found an increase in publications on flipped or inverted classrooms from four publications in 2010 to 296 publications in 2015. Over 73% of the publications in 2015 are related to higher education. Lage et al. (2000) offers a rudimentary and frequently cited definition. "*Inverting the classroom means that events that have traditionally taken inside the classroom now take place outside the classroom and vice versa*" (Lage et al., 2000). Abeysekera and Dawson (2014) said "*the information-transmission component of a traditional lecture is moved out of class time and replaced by a range of interactive activities designed to entice active learning.*"

The definitions deliberately don't specify how the face-to-face and home learning phases will be organized, as the elements depend on the respective course content and teacher. Most research focuses on interactive learning activities and student-centered learning in the face-to-face phase (Bishop and Verleger, 2013; Abeysekera and Dawson, 2014; Finkenberg, 2018). Video lectures, podcasts, screen casts or texts and an additional pre-class quizzes are usually made available for the self-study phase (Abeysekera and Dawson, 2014; Kämpf and Stallmach, 2024; Bitzenbauer and Hennig, 2023; Lo et al., 2017). DeGrazia et al. (2012) postulated that students who are supposed to watch a video come to the classroom lecture better prepared than those who are supposed to read a text. This finding is in agreement with research by Lin (2021) and Sangermán Jiménez et al. (2021), who showed that students prefer input videos for the self-study phase. As the use of computers for self-study is nowadays quite obvious, some literature characterizes flipped classroom as a special type of blended learning (Graham, 2006; Hrastinski, 2019; Staker and Horn, 2012).

3 Course design of the mathematical methods seminars

Our blended learning mathematics course was designed after analyzing best-practice examples of blended learning mathematics/physics courses (Quinn and Aarão, 2020; Finkenberg, 2018; Fischer, 2014; Bitzenbauer and Hennig, 2023). It was revised with feedback from the testings of our own course in the sense of design-based research. In addition to the general criteria for the design of blended learning courses (Fischer, 2014; Finkenberg, 2018; Kim et al., 2014; Bitzenbauer and Hennig, 2023; Lo et al., 2017), there are three important pillars to our course design.

1. The parallelism between mathematics and physics
2. The spiral-curriculum between the two semesters of MaMe seminars
3. The interactivity in videos, self-study tasks and in the seminar

In the following subsections the three pillars of the course are described in more detail.

3.1 Parallelism between mathematics and physics

The parallelism of our mathematics course and the experimental physics lectures is established in the module descriptions of the first two courses in experimental physics for the teacher trainee program at Leipzig University (Universität Leipzig, 2018). These descriptions integrate the mathematics education into two 10-credit points experimental physics modules on mechanics in the winter term and on electrodynamics in the summer term with one semester hour per week presence time and three hours per week self study time, i.e. 60 h learning time in sum consisting of 45 h self-study and 15 h attendance during each semester. Since the mathematical training is the responsibility of the physics lecturers, the interlocking and accurate selection of important topics is ensured throughout the whole introductory study phase.

Analogous to Dunn and Barbanel (2000), we use the physics lecture as an organizational discipline, according to which the mathematical methods seminars are arranged in terms of content and time. Ten mathematical topics are introduced in each of the two courses using selected physical applications. Mathematical topics introduced in applications to the mechanics lectures are seminars on digital experiments and data analysis, functions and vectors to describe physical laws, integral calculus in mechanics, multiple integrals, complex numbers, the introduction to differential equations, power series expansions and the introduction to vector analysis. A detailed description of all the mathematical topics and how they are arranged parallel to the mechanics course is given in Table 2 of our previous article, see Kämpf and Stallmach (2024).

Table 1 shows the parallel structure of the electrodynamics lecture and the mathematical methods in terms of content and time. Each mathematics unit is introduced by two interactive videos. The first video introduces new mathematics or a complex physical law. A second video shows a first application of the mathematics/physics law in the specific context of the underlying parallel physics lecture. Further examples are calculated and discussed during the subsequent seminars.

The concepts of the parallelism and of the spiral-curriclarity (see next section) are illustrated in the following on the example of the mathematics needed to understand the Gauss' law of electricity. Since the electrodynamics lectures start with a description of electric fields of point charges and charge distributions, abstract mathematics is quickly required to understand this law

$$\oint_S \vec{E}\,\mathrm{d}\vec{S} = \int_V \frac{1}{\epsilon_0}\left(\vec{\nabla}\cdot\vec{E}\right)\,\mathrm{d}V = \frac{Q_{\text{enclosed}}}{\epsilon_0}. \tag{1}$$

Thus, at the very beginning of the electrodynamics course, one of the two physics lectures during the first week is dedicated to deepen and to consolidate the knowledge of multidimensional functions and the Nabla calculus (see Table 1). This ensures that students have a decent prior knowledge of the individual components of Gauss's divergence theorem and Gauss law of electricity before it is introduced in the physics lecture and the corresponding MaMe videos and presence seminars of the third week (see Table 1; Figure 1).

3.2 The spiral-curriculum between the two semesters of MaMes

Spiral-curricularity between the MaMe seminars across the two semesters means that specific mathematical topics are repeated and successively extended in their complexity. When learners are able to acknowledge and remember concepts or mathematical formalisms applied to solve "simpler" physical problems earlier during the physics courses, we enable them to activate their prior knowledge. This makes it easier to extend their knowledge, to build their own cognitive schemata (Piaget, 2003) and thus to solve more complex physical problems.

The implementation of this cross-modular spiral-curricularity is again illustrated by the stepwise introduction of the mathematics required for the understanding of Gauss' law of electricity.

 10.3389/feduc.2025.1641893

TABLE 1 Subjects of the two MaMe videos to prepare for each of the 10 face-to-face MaMe seminars and the sample application of the new math discussed during respective seminar.

List of the MaMe videos & seminars associated to the lecture topics for the electrodynamic course				
No.	Topics of the electrodynamic lecture	First video: Introduction to mathematics	Second video: Application of mathematics in physics	Application of the video content during the face-to-face seminar
1	Introduction in the electrodynamic course	Functions of several variables	calculus with the Nabla operator (div, grad, rot, Δ)	Symmetry of second derivatives (Schwarz's theorem); Transformation between electric field and electric potential near a point charge
2	Electrical field of point charges	Cartesian and cylindrical coordinates system	Spherical rotation coordinates system	Different applications for volume integrals in cylindrical and spherical rotation coordinates system
3	The electrical field and the potential of point charges—energy conservation	Surface integrals and the flux	Gauss's Divergence Theorem	Derivation of the gravitational field inside and outside a planet using the Gauss Divergence theorem
4	Electrical fields of charge distributions & the electric flux density	–	Gauss's Divergence Theorem to derive electric fields of an uniformly charged sphere	The electric field inside and outside of an uniformly charged cylinder
5	The electrostatic field distributions and its forces in capacitors	Introduction in parameterizing curves	Line integrals to calculate potentials and work	Line integrals to calculate potential inside a capacitor
6	DC networks with ohmic and capacitive resistors & Kirchhoff's rules	Inhomogeneous differential equation	Derivation of the charge curve in a RC circuit	Exercises to derive different charge curves in DC networks
7	The magnetic field of an arbitrary distribution of electric currents & Biot–Savart law	Understanding and application of the Biot–Savart law	Derivation of the magnetic field in the center of a conductor loop	Derivation of the magnetic field in the center of a squared conductor
8	Faraday's law of induction and Lenz's rule	Stokes' theorem to transfer Maxwell's 3rd and 4th equation	Transformation of Farady's law of induction from the differential to integral equation	Derivation of the magnetic field outside an infinite conductor using Maxwell's 4th equation
9	AC networks with complex resistors; phasor diagrams	Derivation of the complex impedances of a resistor, a capacitor and a coil in AC-circuit using complex functions	The RCL series circuit: impedance, voltage and current through its components	The RCL parallel circuit: impedance, voltage and current through its components
10	Open oscillating circuits; Hertzian dipole	–	–	Summary of the course

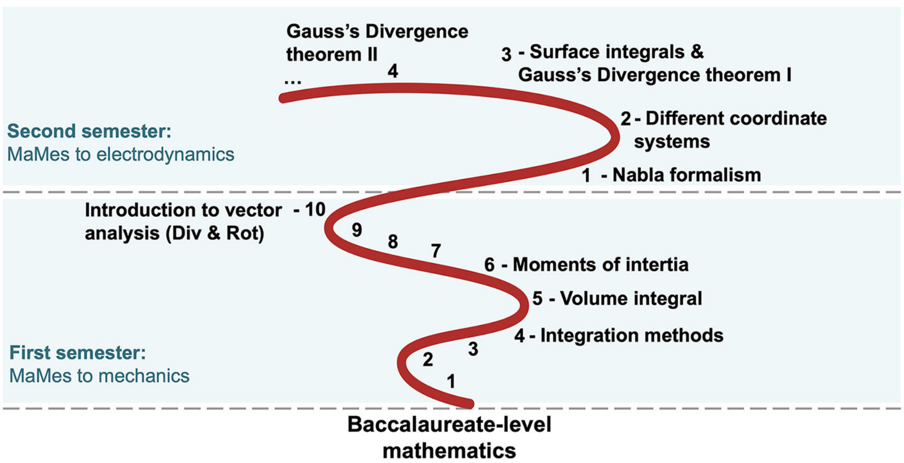

FIGURE 1
Visualization of the spiral-curricular structure of the MaMes using the example of the introduction of Gauss's divergence theorem in the second semester (the numbers in the second semester part refers to the ones given in Table 1).

Figure 1 illustrates from bottom to top the connection between the successive MaMe seminars that eventually allow us to introduce Gauss's Divergence theorem. All MaMe seminars build on the mathematical content of the basic mathematics course at german Baccalaureate-level (detailed curriculum, see State Office for Schools and Education, 2021). Therefore, we are forced to introduce the integration methods substitution and integration by parts during the forth MaMe seminar (no. 01/04) of mechanics course (see Figure 1). During the seminars no. 5 and 6, students learn about volume integrals in order to calculate the center of mass and moments of inertia of a rigid body. The Nabla operator has already been introduced during the last mathematics seminar (no. 01/10) of the mechanics course to analyze conservative force fields by calculating the rotation and the gradient. The second semester builds on this knowledge and links it to new, more complex applications in the context of electrodynamics. As a new application of the Nabla operator for the electrodynamics, the divergence and the Laplace operators are added to describe the electric field and potential of a point charge during the first MaMe seminar (02/01) of the electrodynamics course. With this knowledge, it is only necessary to introduce the flow of a vector field through a surface in the third seminar (02/03) in order to understand Gauss's divergence theorem for calculating the flux (Harden, 1999; Coelho and Moles, 2016).

By revisiting the various mathematical topics, we intend to improve understanding (Piaget, 2003; Neumann et al., 2017), and to flexibilise the application of the learned algorithms and approaches (Coelho and Moles, 2016; Harden, 1999) so that they stay available for subsequent physics studies.

3.3 The interactivity in videos, tasks and seminars

Because videos are more appreciated by students than texts, we decided to produce interactive explanatory videos for the self-study phase (DeGrazia et al., 2012). In addition, interactive explanatory videos outperform the traditional videos in learning success and student satisfaction (Bishop and Verleger, 2013; Zhang et al., 2006; Brame, 2016). The conception of our videos and the implementation of interactive elements with the help of the H5P software is described in our previous article, see Kämpf and Stallmach (2024).

Based on our survey data on the quality of the videos published in our previous article (Kämpf and Stallmach, 2024), the explanatory videos on mechanics for the MaMe seminars 01/01–01/10 were revised and the videos for the MaMe seminars 02/01–02/09 on electrodynamics were produced. As part of the development of the videos for the MaMe seminars 01/01–01/10, the last two missing videos for the seminar 01/01 were produced and the existing videos provided with further interactions. The 20 revised videos for the 10 seminars 01/01–01/10 have an average length of 13 min and 25 s. The average number of interactions was increased from 2.5 to 4.25 by adding more interactions, especially in the second videos. After an average of 4 min and 17 s, students are asked to complete a first interactive tasks during watching the videos.

Moreover, for the first nine MaMe seminars 02/01–02/09 on electrodynamics, 17 videos were produced (see Table 1). With an average length of 12 min 50 s and 4.1 interactions per video, they are comparable to the MaMe videos on mechanics. In these videos, the first interaction starts a little earlier, after an average of 3 min and 28 s.

To further support the consolidation of knowledge during the self-study phase, interactive game maps with first practical tasks are invented applying the newly learned contents from the videos of every unit. These voluntary exercises are enriched with motivational gamified elements and formative feedback (Manzano-León et al., 2021; Ariffin et al., 2022). For each lesson, game maps were developed with comprehension questions, simple math problems, and more complex application problems. All digital material including the interactive videos for the two-semester mathematical methods seminars are available on the teaching and learning management system Moodle™ (Moodle Pty Ltd, 2025) of Leipzig University, see Kämpf et al. (2025b).

The attendance phase is also enriched with activating elements. In our experience, the students are very reserved at the beginning of our first mathematical seminars. They hardly dare to ask and answer questions in the beginning of every seminar. Studies show that continuous formative assessment, such as regular quizzes, reduce anxiety and give students more confidence to participate in class (Naseem, 2021; Howell, 2021; Rust, 2002; Dengri et al., 2021). In order to reduce the students inhibition threshold to actively participate, we decided to start each seminar with an introductory quiz before the question and answer time. These quizzes focus on the core content of the previously viewed videos and the basics for the following seminar. The selected topics enable the students to explicitly recognize the important content of the self-study phase and their individual level of understanding. The quizzes are created using the free Microsoft Forms tool and implemented directly into the PowerPoint presentation of the seminar. At the end of each introductory quiz, students are asked to voluntarily assess their own competences in relation to the content of the quiz and the videos.

For the regular self-assessments, a maximum of six competence expectations are formulated. Students are asked to rate their personal level of competence using a four-point Likert scale (uncertain, rather uncertain, rather certain or certain). The lecturer sees the survey results of the knowledge test and the self-assessment immediately and can thus better moderate the following question and answer times. The lecturer is able to address any difficulties that have arisen during the quiz. These quizzes and the following question and answer time cover usually approximately the first 10 min of a face-to-face MaMe seminar time.

The remaining seminar time is used to solve further physical problems with the newly learned mathematical methods. During this phase, the seminar leader moderates the search for a approach and gradually withdraws himself. He is a kind of tutor, providing help and answering questions. It is important that at the end of the seminar the results and the most important calculation steps are noticed and written on the blackboard or presentation so that all students have access to the entire seminar content and solutions. An example of how we designed the didactics task set for a seminar is described for the MaMe seminar 1/02 on functions and vectors (see Kämpf et al., 2025a). Here, the trajectory of a rolling wheel, the trochoide, is investigated according to Bruner's model of modes of representations (Bruner, 1964).

4 Method

To gain a comprehensive research-based insight on our cross modular mathematics course, various short tests were carried out at different times during the intervention, in line with the test approach of Lahme et al. (2024) (see Figure 2). Thus, the most important components were evaluated by the students participating so far in our mathematics course.

As the explanatory videos are the central element of the self-study period, they were analyzed in the first survey (Q1) assessing their explanatory quality according to Kulgemeyer (2020). The timing of the tests is illustrated in Figure 2. The effect of the face-to-face seminars was evaluated for the MaMe seminars dealing with the Nabla operator. The improvement of the students to work with the Nabla calculus was studied by two short self-assessment tests (S1 and S2) before and after the corresponding seminars. These tests were carried out during the tenth seminar of the first semester (S1) and in the first seminar of the second semester (S2) (see Figure 2). The learning progress was assessed with three tests each on complex numbers (K1–K3) and differential equations (D1–D3) (see Figure 2).

So far, three cohorts have participated in our mathematics course. The first cohort C_1 (starting winter term 22/23) pre-tested the materials and tests. Their survey and test results have been published in our previous paper (Kämpf and Stallmach, 2024). While we will refer to these results whenever appropriate, we focus in this article on the data acquired with cohorts C_2 (winter term 23/24) and C_3 (winter term 24/25). These two cohorts learned the mathematical methods with the digital materials and the teaching setups revised after the analysis and evaluation of the surveys and tests of the preceding cohorts. Thus, cohorts C_2 and C_3 received consolidated learning materials as well as revised surveys and tests.

4.1 Survey on the quality of the videos for self-study (Q1)

In line with the design-based research approach, the explanatory power of the videos was tested with all three cohorts (C_1, C_2, and C_3) of the mathematics course. Negative evaluations or suggestions for improvement by students were discussed among the participating teaching staff. Improvements were incorporated by a critical revision of the materials provided to the students or used for the face-to-face seminars of the next cohort. The majority of the revision of the video materials took place after the Q1 survey with cohort C_1, the results of cohort C_1 were used for pre-piloting and published in the previous article, see Kämpf and Stallmach (2024). In the present paper, we show the results of the main study with cohorts C_2 and C_3. Twenty-one students from cohort C_2 and 18 students from cohort C_3 took part in the surveys.

The explanatory power of our videos was tested using Kulgemeyers framework for good explanatory videos (Kulgemeyer, 2020). It defines seven factors for good explanatory videos: (1) structure, (2) adaptation, (3) tools for adaptation, (4) minimal explanation, (5) highlighting relevance, (6) follow-up learning tasks, and (7) new, complex principles. We have adapted the descriptions and characteristics of the first six factors to our videos and their content. The last factor, new complex principles, was not surveyed, because our explanatory videos always deal with topics which are too complex for a self-explanation.

A total of 15 statements were formulated for the six characteristics investigated. The students rate their personal perception of the fulfillment of Kulgemeyer's design criteria on a 4-point Likert scale. Each scale is given a weight, disagree at all $\hat{=}1$, rather disagree $\hat{=}$ 2, rather agree $\hat{=}$ 3, agree at all $\hat{=}$ 4. With these weights, a mean agreement value μ was calculated. The deviation from this mean is expressed by the standard deviation σ (Veith et al., 2022). Moreover, the mean agreement value μ_{total} and standard deviation σ_{total} were calculated for all answers of the different participating cohorts.

At the end of the survey, a free text field was added for strengths of the videos and suggestions for improvement. The written comments were clustered and visualized in a network graph (Veith et al., 2023; Fardian et al., 2024; Chaimani et al., 2013).

4.2 Survey on the benefits of the seminars (S1, S2)

As described in Section 3.3, every MaMe seminar starts with a formative introductory quiz and an associated self-assessment. In order to measure the benefit of the seminar as perceived by the students, the self-assessment questions of the initial quiz were asked again at the end of the seminar. The students assess their respective competences by rating their knowledge on a four-point Likert scale (uncertain $\hat{=}$ 1, rather uncertain $\hat{=}$ 2, rather certain $\hat{=}$ 3, certain $\hat{=}$ 4). As in Section 4.1, each scale is given a weight to calculate the mean μ and the standard deviation σ.

The self assessments for the subsequent seminars 01/10 and 02/01, in which the Nabla operator is introduced for application in mechanics and electrodynamics, respectively, are analyzed in detail (see Figure 2). Their results are visualized in a divergent bar chart, with uncertain assessments to the left of zero and certain assessments to the right of zero (Robbins and Heiberge, 2011). The self-assessment was only carried out by cohort C_3. In the first seminar 01/10, $N = 13$ and $N = 14$ respectively participated in the self-assessment. In the second seminar 02/01 almost the whole cohort ($N = 33$) participated.

To compare the self-assessments before and after one single seminar, a Wilcoxon test is performed with the paired samples. A Man–Whitney U-test was used to examine the differences in self-assessment of the partially paired results between the end of the first seminar 01/10 and the beginning of the second seminar 02/01 (Guo and Yuan, 2017).

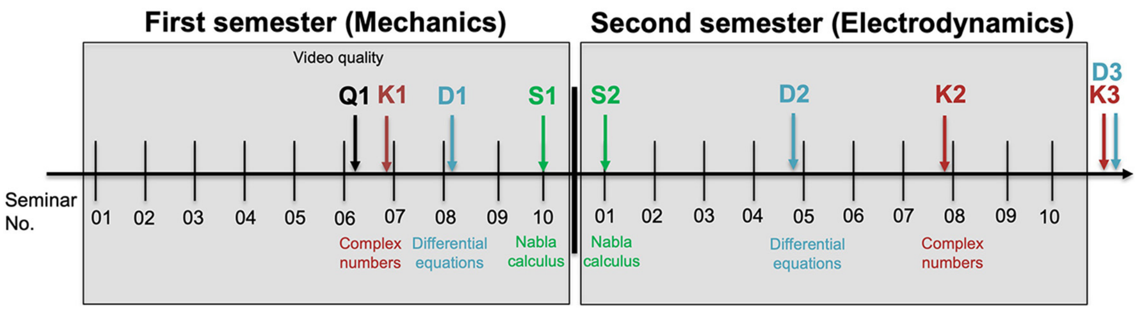

FIGURE 2
Timetable of the various tests and quality checks in the two semesters of the mathematical methods course.

4.3 Knowledge tests on complex numbers (K1–K3) and differential equations (D1–D3)

4.3.1 Knowledge tests on complex numbers (K1–K3)

The knowledge tests on complex numbers contain five items with a total of 11 sub-tasks, which can be solved in a total of 10 min. The tasks include items on arithmetic operations, the complex plane, the complex conjugate and the representation of oscillations as a complex function. Three similar tests were done at three different times K1–K3 (see Figure 2). The first test K1 was taken immediately after the introduction to complex numbers (seminar 01/07), the second test K2 6 months later, before the topic was revisited in electrodynamics (seminar 02/08). The third test K3 was taken at least 1 year after the introduction in the third semester.

Cohorts C_1 ($N_1 = 24, N_2 = 10$ and $N_3 = 18$) and C_2 ($N_1 = 24$, $N_2 = 23$ and $N_3 = 30$) participated in the tests as pilot group and main study group respectively. The test results are presented graphically in a 25% whisker box plot. In order to take into account all test results from small cohorts, all subjects of the respective tests are examined as unpaired samples using a Man–Whitney U-test (Guo and Yuan, 2017).

4.3.2 Knowledge tests on differential equations (D1–D3)

Similarly, three tests on differential equations were carried out after the introduction (D1 after seminar 01/08), before the reintroduction after 6 months (D2 before seminar 02/05) and after 1 year (D3). The tests on differential equations contain five tasks with 14 items on the characterization of differential equations, the application of the method of separation of variables, the exponential ansatz to solve a linear differential equation with constant coefficients and the specific solution of a differential equation.

The test was carried out with the students of cohort C_2. The number of participating students was $N_1 = 16$ for the voluntary online test D1 during winter term 2023/24. It increased to a more representative participation of $N_2 = 28$ and $N_3 = 26$ for tests D2 (summer term 2025) and D3 (winter term 2024/25), respectively. The changes are analyzed for all three tests using the Man–Whitney U-test and for the 10 paired samples of D1–D3 using the Wilcoxon test.

5 Results

5.1 Survey on the quality of the videos for self-study (Q1)

The results of the students rating of the quality of our interactive learning videos provided for the self-study phase of the course are displayed in Figure 3. It compares the answers of the students of the two cohorts C_2 and C_3 investigating the six criteria of good explanatory videos (Kulgemeyer, 2020) written on the right column of the figure. 80%–90% of the students of both cohorts agree that our videos fulfill the six criteria. On average we received 85% positive responses for all questions. Focusing our videos on the design criteria of good explanatory videos (Kulgemeyer, 2020) has resulted in a very highly rating of their explanatory power ($3.24 \leq \mu \leq 3.39$). The results indicate that the vast majority of students found the material very helpful for their self-study phase. Thus, the videos appear to the students as comprehensive and suitable for an initial learning of the mathematical methods.

Many of our students mentioned in the free text answer of this survey that our explanatory videos are comprehensible and provided many other helpful comments for improvements. Twenty-nine of the 39 students who participated in the survey Q1 made a total of 76 positive comments on the videos. Obviously, students' motivation to provide written feedback was very high. In order to better understand the written comments and their relationships, they were grouped into different characteristics. Each feature that appeared at least twice was further investigated. This resulted in 11 positive characteristics and nine suggestions for improvement. The characteristics are described in Table 2 with anchor examples and the number of mentions. We grouped these answers and investigated their interdependencies by means of the circular network graph, shown in Figure 4. The most frequent positive comments relate to the comprehensibility ($N = 16$) and the interactivity ($N = 17$). They are presented in the center part of Figure 4 and are highlighted in green as all other positive comments. The yellow-orange highlighted comments (nodes) are mostly hints and additional whishes of the students to improve the video material. The graph has 20 nodes and 142 edges.

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FIGURE 3
Assessment of the quality criteria of good explanatory videos according to Kulgemeyer (2020) from cohorts C_2 ($N = 21$) and C_3 ($N = 18$).

The diameter of the nodes scales with the number of mentions (Chaimani et al., 2013). Their edges (connections) illuminate links between elements mentioned together in a comment. The weight and darkness of the edges scale with the frequency of common mentions (Chaimani et al., 2013).

High connected nodes act as bridges and conduits between large but otherwise isolated regions of a network (Aldrich, 2014). As the two nodes "Interactivity" and "Comprehensibility" were the most frequently mentioned and were mentioned together with most of the other features ("Comprehensibility": 72 edges; "Interactivity": 66 edges), they can be considered as bridges in Figure 4 to explain the hidden structure in students' assessments. The bridges "Interactivity" and "Comprehensibility" contain several diffuse (soft-wired) connections in light gray and seven hard-wired connections in black. The strongest connection between two features is between "Interactivity" and "Comprehensibility" themselves with a weight of nine. Both nodes have further strong connections to the positive features "Flexibility," "Structure," "Optics," "Showing examples," and "Animations." They thus combine the characteristics of good explanatory videos (Kulgemeyer, 2020).

The connections between the individual suggestions for improvement are soft-wired. Strong connections can be seen to the two bridges "Comprehensibility" and "Interactivity" in the center of the diagram. The connections between "Comprehensibility" and the two orange nodes "Tasks too easy" and "More tasks" are hard-wired with a weight of six each. The "Comprehensibility" node thus represents the bridge between the good explanatory quality of the videos and the request for more and/or more difficult application tasks. This can only be requested once the contents of the videos have been understood.

The basic idea of this study was to determine the explanatory power of the videos. The network analysis of the free text responses shows that this is the central positive aspect of the feedback. A frequently mentioned suggestion for improvement is the "Difficulty gradient between the videos." This was especially mentioned in cohort C_2 (see Table 2). In the subsequent revision of the videos, more interactions and more detailed explanations were added to the series of the second videos, which were perceived as more difficult (see Section 3.3). This reduced the number of mentions of the difficulty gradient from six in C_2 to one in C_3, so this weakness of the videos has been remedied.

5.2 Survey on the benefits of the seminars (S1, S2)

Figure 5 shows the C_3 students' self-assessment of the competences needed in relation to Nabla calculus in the two MaMe seminars 01/10 (time t_1, mechanics) and 02/01 (time t_2, electrodynamics). The divergent bar chart compares the pre- and post-seminar assessments for both seminars. The pre-test results are displayed above the post-test results. The first two items on partial derivative and cross product are prior knowledge that has already been dealt with in school and several previous physics as well as mathematics seminars. Both topics are important

TABLE 2 Description of the positive characteristics and suggestions for improvement of the free text responses to the explanatory videos.

Category	Anchor example	N_{C_2}	N_{C_3}	N_{tot}
Positive comments				
Interactivity	– "Interactive content promotes fun and motivation to understand the topic" – "... I really like the direct application, as it consolidates the newly learned knowledge. ..."	11	6	17
Comprehension	– "Always goal-orientated and comprehensible, always enables a relatively easy and understandable introduction to the respective topic" – "Complex content explained simply, didactically good teaching"	8	8	16
Flexibility	– "Can watch them as often as you like and at any time + can pause to edit content at your own pace" – "The videos can be re-watched at any time (ideal for learning)"	5	3	8
Structure	– "Clear structure—the topic is worked through step by step from beginning to end" – "The videos are optimally structured"	4	4	8
Optics	– "They are visually appealing without too many frills" – "Standardized design"	3	4	7
Showing examples	– "Positive: theory with the following application example" – "Address the physical content from the perspective of the underlying mathematical structures and explain them clearly and concisely"	2	5	7
Animations	– 3D animations; The fades are especially good to show which expression has replaced which term	1	6	7
Video length	– "Are neither too short nor too long"	4	2	6
Audio quality	– "Appealing voice/acoustically appealing" – "She explains with a loud, clear voice"	0	3	3
Vocabulary	– "Both speakers use an understandable vocabulary, technical terms that do not correspond to our previous knowledge are clarified"	0	2	2
Summary	– "Positive: summaries at the end"	0	2	2
Suggestions for improvement				
More tasks	– "... More tasks with different levels of difficulty and different approaches would be brilliant (to be able to apply it safely to complex tasks) ..."	3	4	7
Difficult gradient between videos	– "... The video that follows the theory is more complex and not as easy to understand as the previous one. ... "	6	1	7
Tasks too easy	– "... The only weakness I sometimes see is that the tasks are too easy and therefore tend to bother me, but then you would also have the option of continuing the video without answering. .."	3	3	6
Videos too fast	– "... Sometimes a lot of information input in a very short time (but can be slowed down using the stop function). ..."	5	1	6
Audio quality	– "... Sometimes the audio quality is poor. ... "	3	2	5
More physical application	– " ... The references to physics could be made clearer. ..."	1	3	4
Tasks too difficult	– " ... The tasks that are set during the video can only be solved by guessing if you don't have an approach (or you can skip the task). It would be cool to have help if needed. ..."	1	1	2
Video too long	– "... Length of the videos ..."	1	1	2
Many technical terms	– ".. Sometimes a lot of technical terms are thrown around . ..."	1	1	2

Each characteristic is associated with anchor examples (freely translated from German) and the number of mentions of cohort C_2 and C_3.

foundations for the Nabla calculus. At the beginning of the first seminar, more than 62% of the students are confident in their knowledge, only one is uncertain. As the cross product and partial derivative were further practiced during the seminars, the students felt on average more confident in using them after the seminar than before. Please note that we believe that this positive result is caused by our concept of the spiral-curricularity as discussed in Section 3.2.

The operations "grad" and "rot" were introduced in the videos of the previous self-study period of the MaMe seminar 01/10. It is therefore not surprising that only 16% of the students say that they can perform these operations at least rather certainly before the first in depth-seminar. The average self-assessment is $\mu = 2.08$, just within the range of "rather uncertain." During the seminar, these operations are practiced on the physical example of a leaf spring. After the first seminar, the students estimate their skills on average at $\mu = 2.86$ and $\mu = 3.0$, within the range of "rather certain." The increase in competence was tested with a Wilcoxon test. With a respective significance level of $p = 0.003$, the competence assessments for both rotation and gradient improved significantly by the teaching during the face-to-face seminars.

Finally, the ability to apply the "rot" and "grad" calculus to study force fields for conservativity (rotation) and to transform the

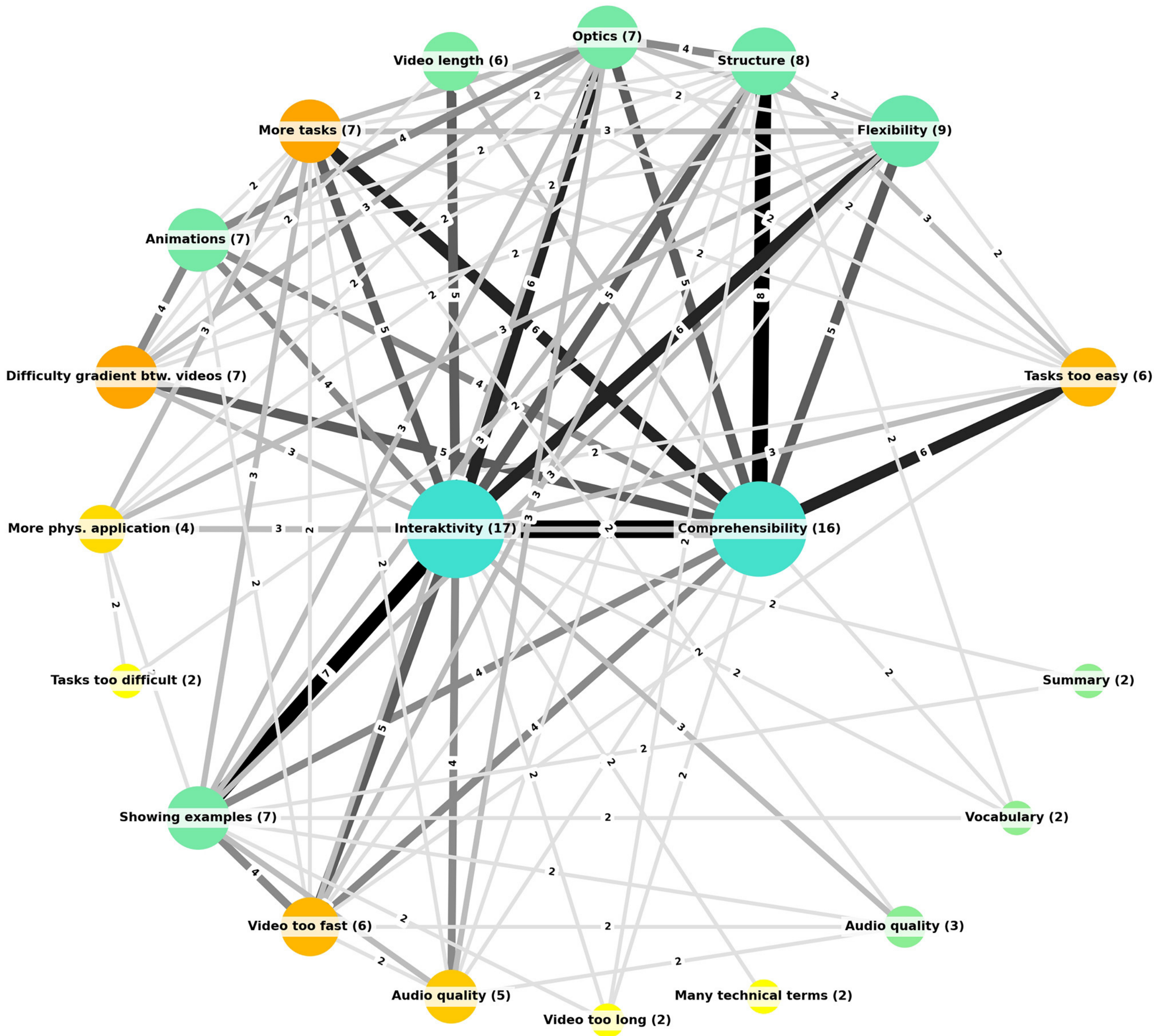

FIGURE 4
Network graph for the free text responses to the survey regarding the quality of the interactive videos for cohort C_2 and C_3. The green-turquoise nodes represent 76 positive comments, the 33 yellow-orange nodes illustrate suggestions for improvement. The diameter of the nodes and the thickness of the edges scales with the frequency.

potential energy to the associated force (gradient) was assessed. Before the first seminar, the students felt more confident in checking the force field for conservativity (38% positive, $\mu = 2.23$) than in the necessary tool to perform the required calculation of the rotation (16% positive, $\mu = 2.08$). After the first seminar, 79% of the students stated that they were able to check a force field for conservatism with a high degree of confidence ($\mu = 3.14$). It is remarkable that none of the students is really uncertain. The Wilcoxon test shows a significant increase ($p = 0.001$) in positive self-assessment. There is also a significant increase ($p < 0.001$) in the self-assessment responses on the competences for transforming energy to the associated force field between before ($\mu = 1.77$) and after the first seminar ($\mu = 3.00$).

Even the lecture free semester break between the two MaMe seminars 01/10 and 02/1 has no effect on students' self-assessment on the topics of partial derivatives and cross product, which shows the test at time t_2 (see Figure 5). On average, both are applied "rather certain" to "certain." There is a clear difference in self-assessment in the lower four categories in Figure 5. Weakly significant differences can be seen in the self-assessments of both the calculation of the rotation ($U = 314.5$, $z = 2.390$, $p = 0.017$) and of the gradient ($U = 329.5$, $z = 2.291$, $p = 0.0221$). The self-assessments in the application of these operations to "Check a force field for conservatism" ($U = 383$, $z = 3.535$, $p = 0.0002$) and also to "Transform from potential energy to the associated force" ($U = 387$, $z = 3.629$, $p = 0.0001$) shows highly significant

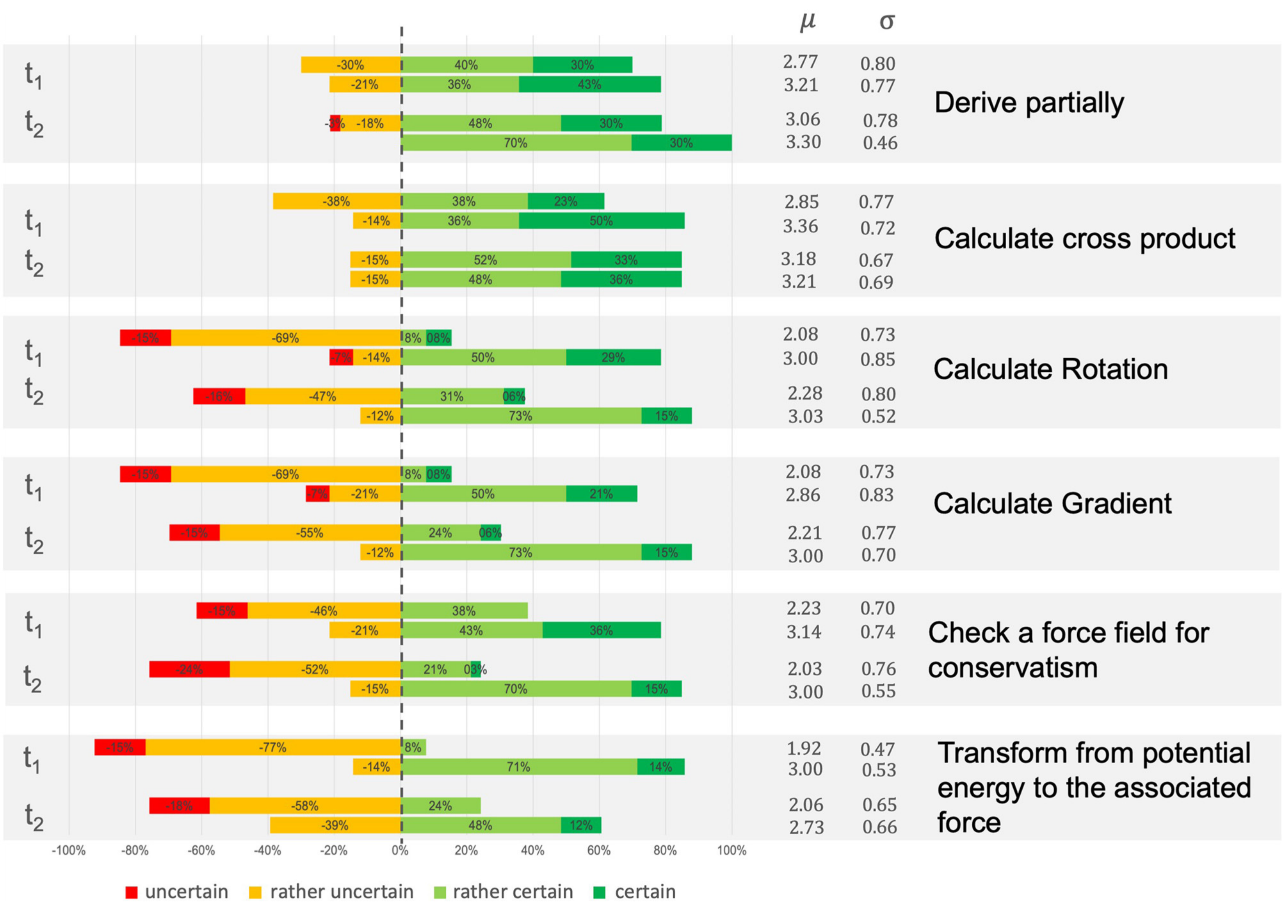

FIGURE 5
Self-assessment of students' skills in working with the Nabla operator. At time t_1 the Nabla operator was introduced in MaMe seminar 01/10 and at time t_2 in MaMe seminar 02/01 with electrostatic examples. Number of participants: $t_1: N_{\text{before}} = 13;\ N_{\text{after}} = 14;\ t_2: N_{\text{before}} = N_{\text{after}} = 33$.

differences to the still fresh knowledge after the first seminar (see Figure 5). Obviously, the results of the self-assessment on the new topics rotation, gradient and their applications decrease during the lecture-free period. This is not unexpected, for these rather complex and demanding mathematical skills.

The second seminar on Nabla calculus has a very similar influence on self-assessment as the first one. After the second repetition during this seminar students feel more confident in calculation of the rotation ($\mu = 2.28$ to $\mu = 3.03$) and the gradient ($\mu = 2.21$ to $\mu = 3.00$) as well as in using rotation to check the conservativity of a vector field ($\mu = 2.02$ to $\mu = 3.00$). None of the total 33 participants in these surveys, covering almost the entire cohort C_2, is still uncertain after the second seminar. Similar, but not quite as large effects are seen when the gradient is used to transform between potential energy and the associated force field ($\mu = 2.06$ to $\mu = 2.73$).

The Sankey diagram shown in Figure 6 summarizes all 95 paired self-assessments of the four new topics rotation, gradient and their application before and after the seminar 02/01 in electrodynamics. The self-assessments before the seminar are shown on the left side. Across all items, 52 assessments (55%) were "rather uncertain" and 14 were "uncertain" before MaMe seminar 02/01. Only about a third of the respondents rated their use of gradient and rotation as "rather certain" to "certain." The self-assessments after the seminar 02/01 are shown on the right. Only 20 students' self-assessments (21%) are still in the category "rather uncertain." 51 students' self-assessments (54%) are "rather certain" and 24 (25%) are even "certain" after the seminar. There are no students who still feel "uncertain" on any point after the seminar. The majority of the students who were unsure at the beginning were "rather certain" after the seminar, 4 were even "certain." Figure 6 shows that not only the means of the self-assessments improve, but with one exception all students rate themselves as equally good or better in all categories after the seminar. The biggest improvement in self-assessment are made by students who rated themselves as "uncertain" or "rather uncertain" at the beginning.

These assessments show that the students rate their learning time in the seminar as beneficial. Between seminars, students become less confident applying new mathematics. It must be emphasized that no direct conclusion can be drawn from the self-assessment to the actual performance, but these assessments provide a strong indicator of the value of the activating face-to-face seminars and the spiral-curricular repetition of themes.

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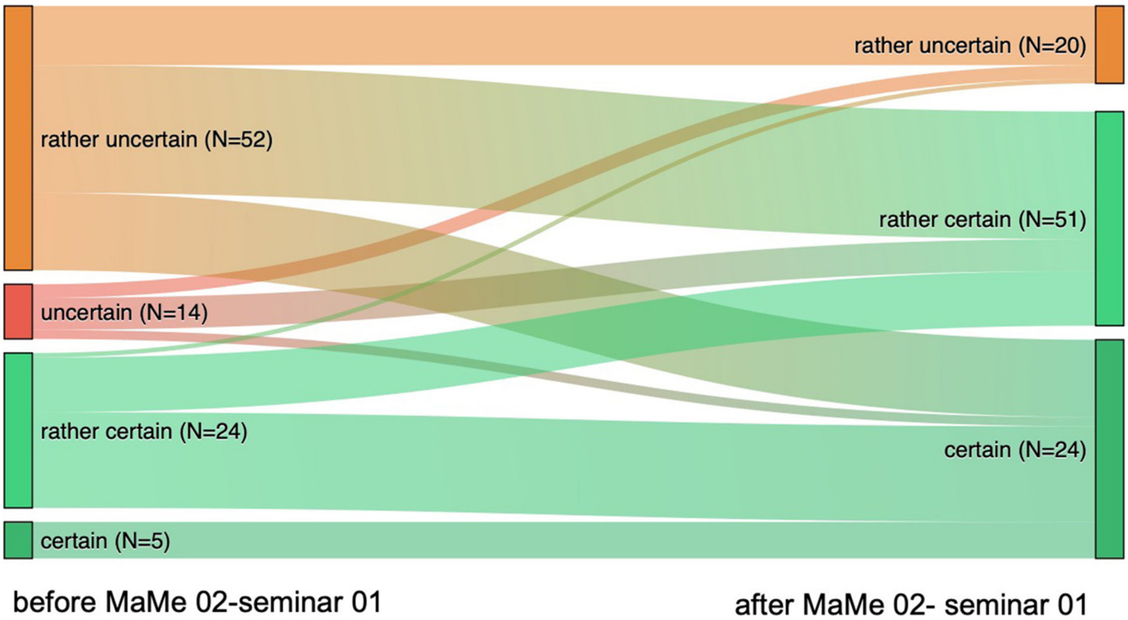

FIGURE 6
Sankey diagram showing the development of all paired self-assessments on the topics of rotation, gradient and their application before and after the MaMe seminar 02/01 (electrodynamics). With one exception, all students remain equally confident or feel more confident after the seminar.

5.3 Knowledge tests on complex numbers (K1–K3) and differential equations (D1–D3)

5.3.1 Knowledge tests on complex numbers (K1–K3)

Figure 7 shows the results of three knowledge tests on the calculus with complex numbers for cohorts C_1 and C_2. The mean value μ of the test results is shown as a square and the median as a line.

The first test K1 is written immediately after the introduction of complex numbers to describe oscillations. This is the first time that students use complex numbers. On average they immediately achieve satisfactory results ($\mu_{C_1} = 70.1\%$; $\mu_{C_2} = 63.3\%$). In the following mechanics course, students deepen their knowledge of complex numbers by applying their calculus to solve the differential equation of vibration. In the second semester on electrodynamics, students again need complex numbers to describe complex impedances. The corresponding mathematical methods seminar takes place 6 months after the first introduction to complex numbers. The test K2 is given to the students before this seminar. Despite the longer period between the two seminars, the students retain their knowledge and the average results are comparable ($\mu_{C_1} = 77.3\%$; $\mu_{C_2} = 73.9\%$). The satisfactory to good test results suggest that the knowledge of complex numbers is still applicable and can be used as a solid basis for the forthcoming revision of the description of impedances.

The final test K3 was administered at least 1 year after the initial introduction. In this long-term knowledge test, cohort C_1 achieved good results, in fact eight of the 18 participants achieved very good results. Cohort C_2 achieved good to very good results as well. Eleven of the 30 participating students scoring 100% in the test. The test results for both groups tend to narrow and to shift to better scores compared to the previous tests K1 and K2 (see Figure 7). The initially very heterogeneous knowledge in the tests on complex numbers with scores of 10% − 100% in K1 is homogenized and improved significantly by the time of test K3 to scores of 60% − 100%.

As the group size fluctuated during the study periods, Mann–Whitney U-tests were performed to analyze changes in the results of one cohort at each study period and a Wilcoxon-test to analyze only the paired (see Figure 7). The consistently non-significant results of Cohort C_1 in both cases suggest that students retained their knowledge during the entire study period. The significant differences in the results of Cohort C_2 on the long-term knowledge test suggest that knowledge has been consolidated and deepened.

5.3.2 Knowledge tests on differential equations (D1–D3)

Figure 8a shows the distribution of all students' answers in the three tests D1-D3 on differential equations given at three instants of time during the cross-modular MaMe seminars. The first test D1 was performed immediately after the MaMe seminar 01/08 (mechanics) on the exponential approach to solve the vibration differential equation. The 16 participants achieved on average good results ($\mu_{D1} = 81.7\%$). After 6 months, a similar test D2 was given just before the advanced MaMe seminar 02/06 (electrodynamics) on differential equations for solving discharging processes in electrical circuits. The knowledge retained in this test is more heterogeneous and on average only sufficient to satisfactory ($\mu_{D2} = 59.9\%$). The larger number of participants of cohort C_2 in test D2 exhibits more heterogeneous tests results

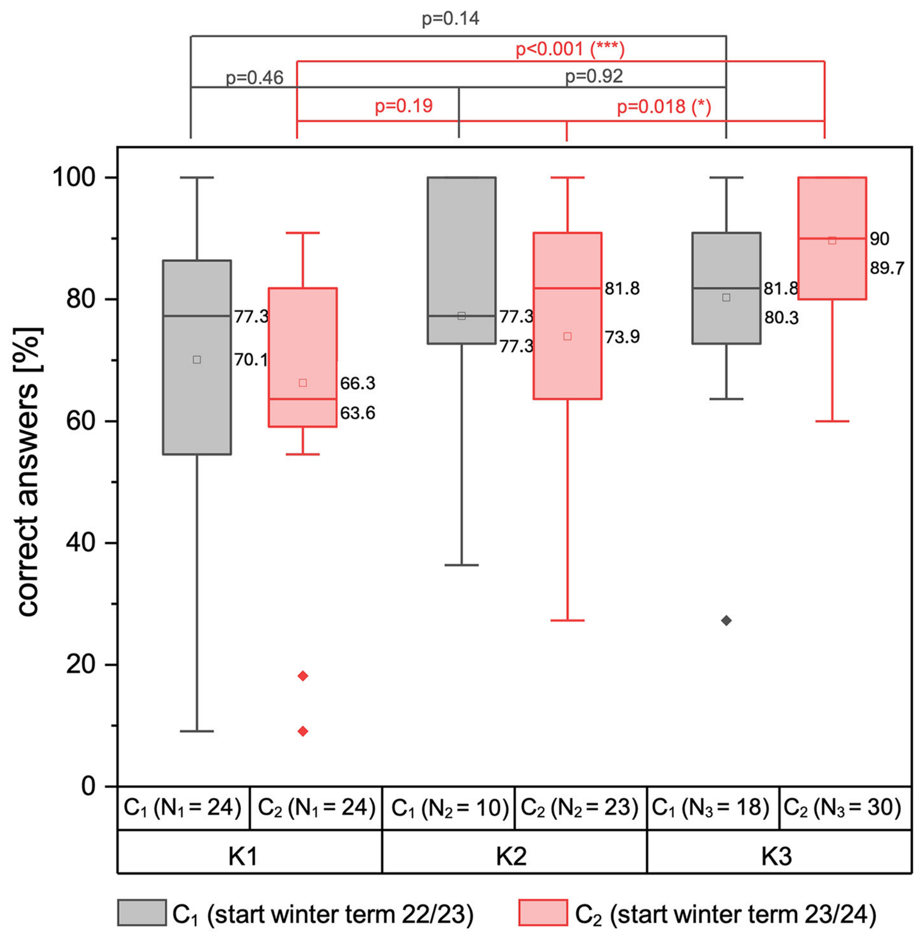

FIGURE 7
Box plots of the results of the knowledge tests on complex numbers K1-K3 of cohorts C_1 and C_2. Over the duration of the course, the knowledge of complex numbers increases and the students achieve homogeneous good to very good results in the long-term knowledge test K3. The stars indicate the significance level.

in D2 than in D1 with less participants of the same cohort. When comparing the paired samples of both tests, the mean value only drops from $\mu_{D1'} = 83.6\%$ to $\mu_{D2'} = 78.6\%$, which is not significant (see Figure 8b). The test was repeated after 1 year in order to assess long-term knowledge without further MaMe seminars. All 26 participants in test D3 again showed a similarly heterogeneous distribution of answers as in test D2. The mean of the test scores did not increase significantly to $\mu_{D3} = 68.7\%$.

Regarding the results of the 10 paired samples of D2 and D3, their results improve to good to very good ($\mu_{D3'} = 90\%$). The effect of the overall improvement and homogenization of the results between D1 and D3 in the paired sample is consistent with the results of the tests on complex numbers (see Section 5.3.1). The results confirm that these 10 students, who took part in all the tests, have achieved our learning objectives in relation to differential equations. They are able to characterize differential equations, solve simple differential equations by separation of variables and the exponential approach, and determine the specific solution of a differential equation by initial values.

6 Discussion

Obeying the design criteria for the explanatory quality of a video (Kulgemeyer, 2020; Fyfield et al., 2022) resulted in a very positive evaluation of our interactive learning videos (see Section 5.1). The "comprehensibility" of the videos was emphasized in the free text responses. Obviously, this factor acts as a bridge between the clustered items of the free text responses, which according to Kulgemeyer (2020) also make up a video with high explanatory power. Like Kulgemeyer (2020) and Fyfield et al. (2022), we can confirm that the five factors structure, optics, animations, examples and length are core elements of good explanatory videos. Additionally, we suggest to add the factor "interactivity" to the catalog of good explanatory videos (Kulgemeyer, 2020) since it complements the "follow-up learning tasks" with embedded initial exercises to promote understanding of the students. Our results are consistent with those of other studies, which show that interactive elements such as quizzes embedded in videos motivate students to engage more deeply with physics (Ketsman et al., 2018; Kosmaca and Siiman, 2022) and also more general STEM (Preradovic et al., 2020; Dampil, 2024) contents. There are currently still different

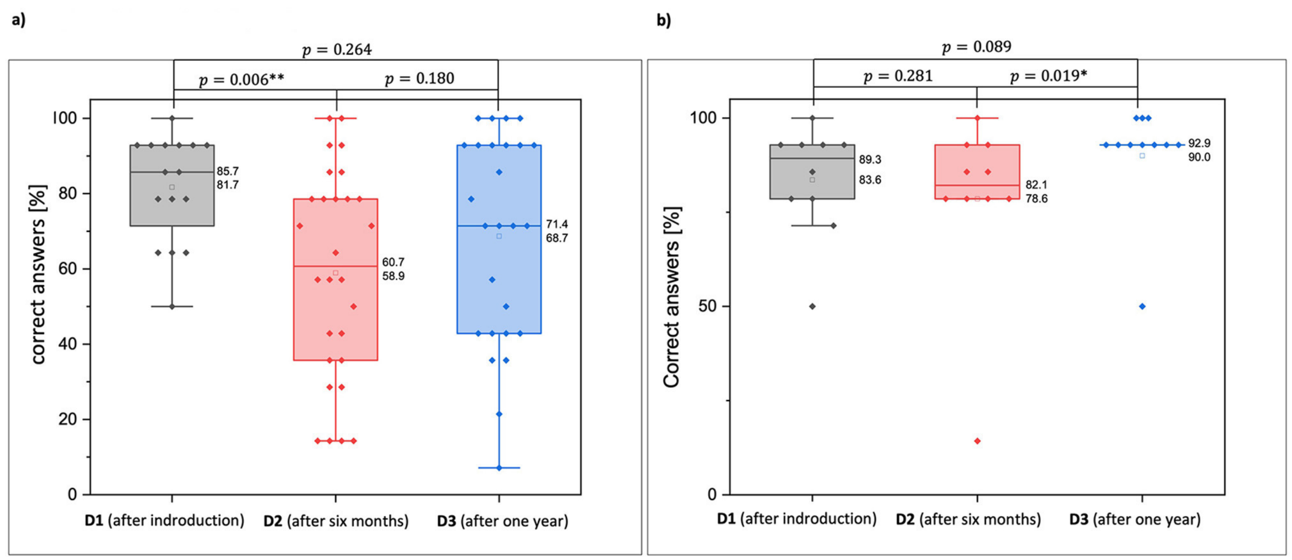

FIGURE 8
Box plots of the results of the knowledge tests on differential equations D1-D3 of cohort C_2. **(A)** Shows the results of all participating students ($N_1 = 16$, $N_2 = 28$ and $N_3 = 26$), **(B)** shows the result of the 10 paired samples. The stars indicate the significance level.

opinions about the effectiveness of the interactive features in videos, which classify them as having either the same (Ketsman et al., 2018) or a better (Ploetzner, 2022; Preradovic et al., 2020; Kestin and Miller, 2022; Ibrahim and Abu Hmaid, 2017; Dampil, 2024) effect compared to normal videos.

Our self-assessment surveys before and after the Nabla calculus seminars show that activating seminars have a positive effect on students' confidence in dealing with the newly learned topics (see Section 5.2). Both surveys demonstrate the importance of revisiting important topics in line with the spiral-curriculum. Students tend to be more confident with topics that have been covered repeatedly in the explanatory interactive videos and the face-to-face seminars. Self-assessment in dealing with new mathematics decreases significantly during the practice-free period of the semester break. Similar loss of knowledge after a lecture-free period has been found by Weggemans et al. (2017), Alshamrani et al. (2021), and Osborne and Shaw (2020). Thus, our study confirms the findings of Tirol (2014), Coelho and Moles (2016), Neumann et al. (2017), Harden (1999), Weggemans et al. (2017), and Alshamrani et al. (2021) that the opportunity to consolidate previously learned knowledge by revisiting topics is a very prominent advantage of a spiral-curriculum approach.

The fact that the knowledge acquired in the spiral-curriculum is becoming increasingly consolidated (Coelho and Moles, 2016; Neumann et al., 2017; Harden, 1999; Catindig and Scheiter, 2018) is also evident in our knowledge tests on complex numbers (see Figure 7). After successfully completing the mathematical methods seminars, students are equipped with good to very good average knowledge of calculating with complex numbers, the complex plane and the complex representation of periodic functions.

These findings are not obviously confirmed in the three tests on students competences in differential equations (see Figure 8). However, the paired test samples of students, who participated in all seminars where the tests were written, achieved consistently good to very good results. In our opinion, these students are the highly motivated physics teacher training students of the cohort who actively participate in almost all mathematics and physics classes. Our data confirm the findings of Gladys and Dastoor (2024), Sharma et al. (2005), Harisson (2008), Credé et al. (2010), Nieuwoudt (2020), Devadoss and Foltz (1996), and Lukkarinen et al. (2016), who postulate a strong correlation between active attendance time and test points. It is therefore important to actively participate in the course and to actively follow both the self-study videos and the classroom seminars in order to achieve a high level of learning success.

However, the whole student body must also be considered. Although it seems to us that we meet the needs of the highly active students group, it remains a fraction of students who do not achieve sufficient or satisfactory results after the interventions. It is therefore important to continue to support these students. Therefore, we decided to make all materials available so that this group as well as all students can refresh and deepen their knowledge.

7 Summary

With our present and preceding (Kämpf and Stallmach, 2024) paper we report on the research-based development and successful implementation of a cross-modular blended learning mathematical methods course for the first two semesters (mechanics and electrodynamics) of physics teacher training courses. We agree, that blended learning and flipped classroom approaches are ways of responding as distinct as possible to the heterogeneous content and structural needs of the learning group during the first semesters (Finkenberg, 2018; O'Flaherty and Phillips, 2015). Our course is based on comparable best-practice examples (Quinn and Aarão, 2020; Finkenberg, 2018; Bitzenbauer and Hennig, 2023) and has been continuously improved over the 3 years from 2022 to 2025 in line with a design-based research approach.

The design principles of our integrated mathematical methods course rests on the three pillars parallelism, spiral-curricularity, and interactivity:

- By integrating the mathematics course into the physics course, it is possible to align the mathematics training both temporally and thematically with the underlying physics lecture. We can confirm the positive effects of teaching mathematics and physics in parallel as already been found by Dunn and Barbanel (2000), Yeatts and Hundhausen (1992), and Dominguez et al. (2024).
- The spiral-curricular structure is important for mathematics education. By revisiting the topic, knowledge will be consolidated and deepened (Coelho and Moles, 2016; Neumann et al., 2017; Harden, 1999).
- The interactivity of the provided digital material is the best way to transfer knowledge in the self-study phase (DeGrazia et al., 2012; Lin, 2021; Sangermán Jiménez et al., 2021). We see a further improvement in the explanatory videos by adding interactive elements such as tasks and additional information. Their use is rated very positively by the students. We suggest to add the "interactivity" to the criteria of good explanatory videos.

We believe that our concept for teaching mathematical methods in college or university courses may be adapted for many science and engineering study programs which do not explicitly reserve extended study time for a specific mathematics education.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Author contributions

LK: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Validation, Visualization, Writing – original draft. JB: Conceptualization, Investigation, Methodology, Validation, Writing – review & editing. FS: Conceptualization, Data curation, Formal analysis, Methodology, Project administration, Resources, Supervision, Writing – review & editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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