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A survey on voltage stability indices for power system transmission and distribution systems

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Voltage stability is a critical aspect of power system operation, ensuring the reliable and efficient delivery of electrical energy to consumers. Recently, voltage stability in the power system has received much attention. The primary cause of voltage instability is the lack of real and reactive power generation to cope with the continuous demand increment. Maintaining voltage stability while planning, controlling, and assessing the system's security is a difficult task for power system engineers. From knowledge of past incidents, a lack of reactive power is identified as the primary cause of voltage instability, which may further lead to the total collapse of the system. The importance of voltage stability assessment is essential to maintain the integrity of power systems. It helps prevent voltage instability, which can lead to cascading failures and widespread outages. This paper uses various parameters, including load demand, generator capacity, and system impedance, to develop voltage stability indices (VSIs). The paper mainly focuses on the idea of VSIs that have been developed in order to monitor and control the criticality of the power system. The voltage stability indices for transmission and distributed systems, as well as their subclasses, are thoroughly reviewed in this study. Furthermore, traditional voltage stability methods, as well as various software tools for monitoring the voltage stability problems, are also discussed. In addition, the development of VSIs and its related concepts are clearly described in this paper. This comprehensive survey provides a decent groundwork for future work in this area, and assists professionals in selecting the optimal VSI for various applications. Moreover, it provides a concise overview of methods and the importance of voltage stability assessment in both transmission and distribution systems.

KEYWORDS

voltage instability, voltage stability indices, simulation tools, phasor measurement unit, voltage stability classifications

1 Introduction

The present society needs an uninterruptable and reliable power supply as an essential resource. However, with an incessant growth in power demand, current power systems have been overstressed in delivering real and reactive power. Insufficient reactive power management may lead to voltage collapse and complete blackout of the system. Hence, continuous assessment of the voltage stability of the system should be carried out so as to take preventive measures to avoid voltage collapse. Comprehensive research on voltage stability assessment (VSA) has been performed (Ajjarapu, 1995; Ajjarapu and Lee, 1998;

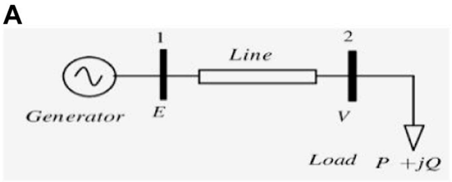
Amroune, 2021; Badru and Taylor, 2000; Boričić et al., 2021). In (Disturbance Analysis Working Group and others, 1979; Li et al., 2023; Liang et al., 2022; Mahseredjian and Alvarado, 1997; MathWorks and Inc, 1993), the cause for the blackouts that occurred in the past decades has been analyzed, and it was identified that reactive power deficiency is the major cause of the blackout. Moreover, sudden variations in atmospheric conditions, poor control over the voltage and frequency of the system, overloading on transmission networks, and transmission congestion were some of the other issues listed as influencing factors for the voltage collapse leading to blackout (Ajjarapu, 2007). Here, reactive power control and voltage limits in generators, the compensation devices that control reactive power and system load characteristics, were enumerated as the major causes for the voltage collapse.

As per IEEE/CIGRE, the definition of voltage stability and the theory on power system stability are well explained in (Kundur et al., 2004). During planning and control of the power system, voltage stability plays a key role in the stable operation of the system. A voltage stability toolbox for load flow and bifurcation analysis has been designed in (Ayasun et al., 2006). Simulation tools and MATLAB are the two software tools discussed here, which are used to analyze the power system stability. Simulation tools can be divided into two types: i) commercial tools such as Power System Simulator for Engineering (PSS/E), CYME Power Engineering, Power World Simulator, Power System Simulator (Simpow), LabVIEW, *etc.*, and ii) customized toolboxes developed for academic and research purposes. Matlab, which is a programming platform widely used to obtain the solution for load flow studies and the VSA of large systems, is described in (Rao and Yuan, 2022; Salama and Vokony, 2022). This analysis was carried out with the help of several toolboxes and packages, such the Electromagnetic Transients Program (Mahseredjian and Alvarado, 1997), Educational Simulation Tool (Vournas et al., 2004), Power System Toolbox (Chow et al., 1992), Sim Power Systems (Sybille, 2004), Power System Simulation Package (Zimmerman, 2005), and Power Analysis Toolbox (Schoder, 2003).

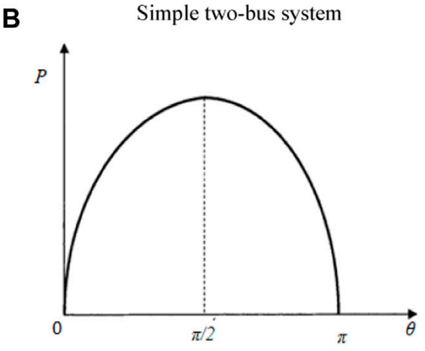
In (Glavic et al., 2012), monitoring and control of system voltage based on supervisory control and data acquisition in the energy management system (EMS) were described. With the development and positioning of phasor measurement units (PMUs), effective online voltage monitoring in power systems is possible. Moreover, the wide-area monitoring system (WAMS) offers a base for online VSA in a large-scale power system (Modarresi et al., 2016; Pereira et al., 2023).

In this paper, different types of methods used in the literature for evaluating VSA have been elaborated. Here, both dynamic and static analyses have been presented clearly. Moreover, voltage stability analysis carried out on both transmission and distribution systems published in the literature has been reviewed. The analyses explored in this paper may help researchers to choose appropriate methods to evaluate VSA for the system considered.

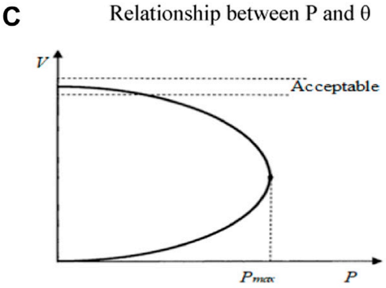
The structural flow of the paper is as follows: in Section 2, the classifications of voltage stability and the cause for voltage instability are described. The traditional voltage stability methods to measure the stability margin from the present operating point to the collapse point are explained in Section 3. The line and bus indices presented in the literature are listed and elaborately explained in Section 4. Furthermore, the interdependence of the indices on the impedance of the system is well described in this section. Moreover, information about the index's critical value and the network type for which the individual indices are most suitable are also presented here. The advantages, disadvantages, and applications of VSIs are explained in Section 5, and the discussion is concluded in Section 6.



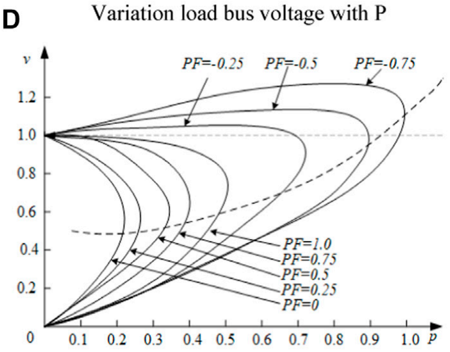
Simple two-bus system



Relationship between P and θ



Variation load bus voltage with P



P vs V curves with respect to power factor

FIGURE 1
(A) Simple two-bus system. **(B)** Relationship between P and θ. **(C)** Variation load bus voltage with P. **(D)** P vs. V curves with respect to power factor.

2 Classification of voltage stability

Voltage stability, otherwise called load stability, is related to the transient stability of the power system. For a given operating

TABLE 1 Categories of voltage stability and its causes.

No.	Type of stability	Causes	Duration of time
1	Large disturbance stability	Loss of generation, faults, and circuit contingencies	Few seconds to minutes
2	Small disturbance stability	Load disturbances, discrete controls at given instant of time, and continuous controls	Minutes
3	Short term stability	Induction motors and HVDC converters	Several seconds
4	Long term stability	Tap changing transformers and generator current limiters	Several minutes

condition, the power system is voltage stable if the voltage at the load side reaches an equilibrium value even after being subjected to a perturbation. However, voltage instability is a dynamic process that involves load and a means for voltage control. The process by which voltage instability leads to a shallow voltage profile is defined as voltage collapse. The voltage instability limit is not directly correlated to the network maximum power transfer limit. However, maximum power transfer is allied to voltage instability if the load is a constant power type. This can be briefly explained with a simple two-bus system, as shown in Figure 1A. The figure depicts a lossless transmission line transferring power from the generator to the load bus. If the generated voltage E and load voltage V are kept constant, at an angle (θ) of 90°, the maximum power transfer occurs in the system. The relation between θ and P is shown in Figure 1B. Instead, if the terminal voltage at the load bus is considered as varying, then the relation between V and P is shown in Figure 1C. From this figure, it can be seen that the load bus voltage reduces with an increase in the load, and reaches a critical value that corresponds to $P_{\max}$. Beyond this point there is no equilibrium. On top of a reduction in the load bus voltage, the critical value reached depends on the power factor of the load, as shown in Figure 1D (Ajjarapu, 2007).

The voltage stability can be evaluated using two different methods of analysis. These are dynamic voltage stability and static voltage stability. In the first method, the dynamic behavior of the system components influences the voltage stability (Taylor, 1994). The second method is used to obtain the maximum loadability limit and the proximity to voltage instability. However, the computation time of the first method is greater than that of the second. Generally, voltage stability is categorized into four classes: large disturbance, small disturbance, short-term, and long-term stability. The causes of voltage stability and the threshold time limit of the perturbation, before which it has to be cleared, are tabulated in Table 1.

3 Traditional voltage stability assessment methods

In recent years, the frequent occurrence of voltage stability problems is receiving special attention from power system engineers. These stability problems normally occur in heavily stressed systems. The analysis of voltage stability can be examined in the given system in terms of the following two aspects (Kundur, 1994).

i) Proximity to voltage instability: this involves how close the system is to voltage instability or the boundary, and it may be measured in terms of physical quantities, such as load levels, active power flow, and reactive power reserve.
ii) Mechanism of voltage instability: this deals with the cause and key factors influencing the instability. It also deals with the voltage-weak area and the various effective measures to enhance the voltage stability. The circumstances and their consequences leading to instability can be trapped by the time-domain simulations. However, as the time constants of the power system components differ on a large scale, the time-domain simulations are very slow and time-consuming. Moreover, this does not readily afford precise information about the degree of stability.

As the system dynamics affecting the voltage stability are generally slow, many aspects of the stability problem can be effectively investigated using static methods. The feasibility of the equilibrium point characterized by the specific operating conditions of a power system can be profoundly studied using this method. Alternatively, dynamic analysis is useful for a detailed study of the coordination of control and protection, exact voltage collapse situation, and testing of remedial measures. The way the steady-state equilibrium point is reached is also clearly analyzed by this method.

Traditional voltage stability methods such as the PV and QV curve method, sensitivity analysis, modal analysis, continuation power flow, singular value decomposition, and bifurcation analysis are discussed in the following subsections.

3.1 PV and QV curve method

The most common and conventional PV and QV curves used to evaluate the static voltage stability of the power system have allowed it to operate within the permissible load (Van et al., 2021; Vargas et al., 1999). The critical voltage is derived from Figure 1A and the quadratic voltage equation as given in Eq. 1, (Vournas, 2004; Vournas et al., 2000).

$$v^2 = -(2p\tan\theta - 1) \pm \frac{\sqrt{(2p\tan\theta - 1)^2 - 4p^2 sec^2\,\phi}}{2} \tag{1}$$

Voltage stability is determined by calculating the PV and QV curves at the particular load buses. In practice, these PV and QV curves are used as tools for evaluating the voltage stability at the Bonneville power administration centre. The drawback of this method is that it is more time-consuming and does not provide exact information. However, it can be used to evaluate the stability limit at every bus independently, irrespective of the system characteristics (Kamel et al., 2021).

In summary, the PV-QV curve method remains a fundamental tool in voltage stability assessment, offering valuable insights and aiding in the secure operation of power systems. As power grids continue to evolve, researchers and engineers will continue to refine and adapt this method to meet the demands of modern energy landscapes.

3.2 Sensitivity analysis

Sensitivity analysis can be used as an effective tool to estimate the static stability of the power system (Li and Jiang, 2012; Li and Wang, 2002) and is mainly related to the sensitivity matrix that must be extracted from the non-linear load flow equations given in Eqs. 2, 3.

$$P(\theta, V) = 0 \tag{2}$$

$$Q(\theta, V) = 0 \tag{3}$$

where P represents the system's real power flow and Q denotes the system's reactive power flow. The load flow equations under normal operating conditions can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \left(\because J = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \right) \tag{4}$$

where J represents the Jacobian matrix of the system. An assessment of the sensitivity analysis is used for identifying the week bus, the loadability levels of the bus, and the system stability. However, the sensitivity index alone might not be adequate to evaluate week buses in an interconnected power system (Sinha and Hazarika, 2000). The effectiveness of the power system can become unstable if there is an increase in the sensitivity. The relationships between the node voltage to reactive power, node voltage to active power, load voltage to generator voltage, and the reactive power of the generator to a load of both active and reactive powers are regarded as commonly used sensitivity factors. These play vital roles in the prediction of critical buses in the system.

In summary, sensitivity analysis is a powerful method for voltage state assessment, providing essential insights for both real-time operation and long-term planning. This method provides a nuanced understanding of how variations in different system parameters impact voltage stability. Here are the key takeaways summarizing the significance of sensitivity analysis for voltage state assessment.

1. Granular insights: sensitivity analysis offers granular insights into the sensitivity of voltage-related parameters to changes in system conditions. This level of detail helps power system operators and planners identify critical factors affecting voltage stability.
2. Optimal operation: power system optimization benefits from sensitivity analysis by helping operators find the most efficient operating points while ensuring voltage stability. This is especially valuable in grids with variable renewable energy sources.
3. Planning and expansion: sensitivity analysis supports long-term planning and expansion of power systems. It aids in assessing the impact of new assets, such as generation units or transmission lines, on voltage stability, facilitating informed investment decisions.
4. Integration with advanced tools: modern power system analysis tools incorporate sensitivity analysis to enhance their capabilities. These tools integrate real-time data, advanced algorithms, and visualization techniques, further improving voltage state assessment.

3.3 Modal analysis

The modal analysis approach has been applied to the voltage stability analysis of practical systems. The advantage of this method is that it gives system-wide information related to voltage stability and clearly identifies critical areas in the power system. Moreover, it delivers information about the mechanism of instability. V-Q analysis is the primary step for estimating the modal analysis.

3.3.1 V-Q analysis

Voltage stability is a key factor that is always determined by both active and reactive power in the system. In accordance with this, Eq. 4 describes the relationship between the voltages of the efficient bus and the differences in power injection. Due to the incremental characteristics of V and Q, which are almost identical to the V-Q curve, the measurement of voltage stability is only possible if the real power is unchanged. In this case, in Eq. 4, by equating the value of ΔP with 0, the equation then becomes

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = J \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} \tag{5}$$

By solving this Eq. 5, ΔQ is

$$\Delta Q = J_R \Delta V \ \left(\because J_R = J_{QV} - J_{Q\theta} J_{P\theta}^{-1} J_{PV} \right) \tag{6}$$

where J_R denotes the reduced Jacobian matrix, and from Eq. 6, ΔV is

$$\Delta V = J_R^{-1} \Delta Q \tag{7}$$

With respect to the reactive power injections, the V-Q sensitivity value at the *ith* bus varies (Van Cutsem and Vournas, 1998) so that the positive sensitivity value indicates that the voltage is stable and the negative sensitivity value indicates that the voltage is unstable.

3.3.2 Model sensitivity

In the power system, the physical characteristics of voltage stability can be calculated by the estimation of J_R eigenvalues and eigenvectors (Gao et al., 1992), which can be written as

$$J_R = \xi \Lambda \eta \ \left(\because J_R^{-1} = \xi \Lambda^{-1} \eta \right) \tag{8}$$

where ξ = right eigenvector matrix, η = left eigenvector matrix, and Λ = diagonal eigenvalues matrix. Therefore, the V-Q response at the *ith* mode is given by

$$v_i = (1/\lambda_i) q_i \ (or) = \Delta V = \sum_i (\xi_i \eta_i / \lambda_i) \Delta Q \tag{9}$$

From Eq. 9, the reactive power and modal voltage directions of variations are the same whenever $\lambda_i > 0$, which shows that the system voltage is under stable conditions. However, when $\lambda_i < 0$, the directions of variations in the reactive power and modal voltages will be in opposition, and this will lead the voltage at the *ith* bus to become unstable. Moreover, if $\lambda_i = 0$, modal reactive power changes

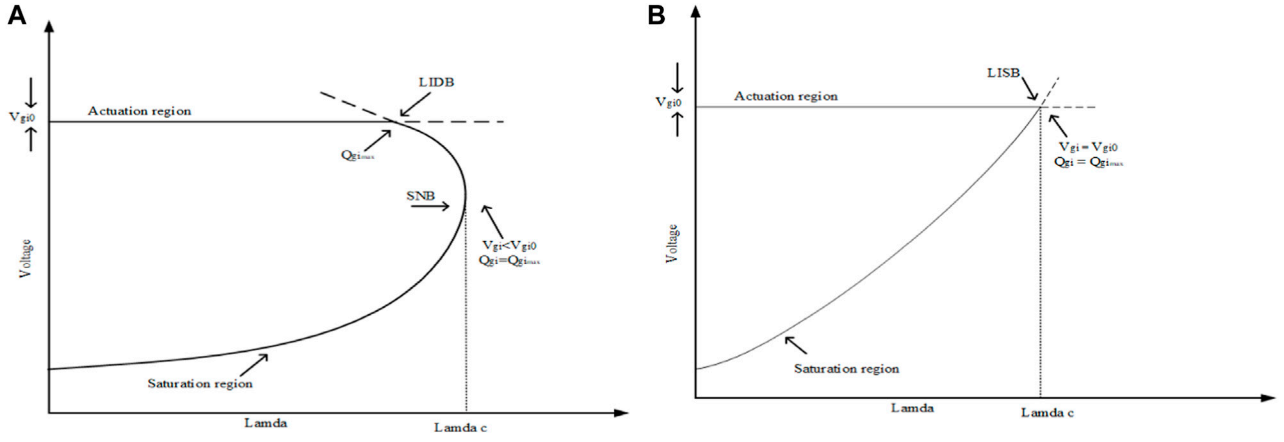


FIGURE 2
(A) LIDB followed by SNB **(B)** LISB.

can cause the modal voltage to change, which will lead to voltage collapse.

3.4 Continuation power flow (CPF) method

Continuation power flow (CPF) (Ajjarapu, 2007) is a numerical method used to analyze the steady-state behavior of power systems under varying operating conditions. It provides a systematic approach to track the behavior of the power system as load levels, generation levels, or control parameters change. The CPF method is particularly useful for studying voltage stability and identifying critical operating points that may lead to voltage collapse.

The CPF method extends the traditional power flow analysis, which solves a set of nonlinear algebraic equations to determine the steady-state conditions of the system. Instead of solving a single power flow equation, CPF solves a series of power flow equations in a sequential manner, gradually varying a continuation parameter. The following provides a comprehensive explanation of the CPF method.

3.4.1 Continuation parameter

The continuation parameter is a scalar parameter that is varied from an initial value to a final value. It represents a quantity of interest in the power system, such as load level, generation level, or control parameter like reactive power limits. The parameter is chosen such that the power system behavior significantly changes as it is varied.

3.4.2 Initialization

The CPF method starts with an initial power flow solution, usually obtained from a base case power flow analysis. This solution serves as the starting point for the continuation process.

3.4.3 Construction of jacobian matrix

The CPF method requires the construction of the Jacobian matrix, which describes the relationship between changes in power injections (P, Q) and voltage magnitudes and angles. The Jacobian matrix is typically sparse, and efficient techniques such as sparse matrix factorization are employed for its solution.

3.4.4 Tracking critical points

During the continuation process, the CPF method tracks critical points where the system behavior undergoes significant changes. These points can indicate voltage instability or voltage collapse conditions. Critical points are identified by analyzing the Jacobian matrix eigenvalues or other stability indicators.

3.4.5 Analysis of voltage stability

By varying the continuation parameter, the CPF method enables the identification of voltage stability limits, voltage collapse points, and the corresponding load/generation levels or control parameters. The method provides valuable insights into the system's ability to maintain voltage within acceptable limits under different operating conditions.

In (Zhang and Zhang, 2005), an advanced version of the CPF method was implemented in which the theory of PQ decomposition was widely adopted to obtain the solution for the system's power flow. In this connection, the simulation tool for a Lagrange two interpolation technique was developed to predict the process.

Continuation power flow is a powerful tool for voltage stability analysis as it allows for the systematic exploration of the steady-state behavior of power systems. It helps in understanding the impact of load changes, generation changes, and control parameter variations on voltage stability, ultimately aiding in effective system planning, operation, and control.

3.5 Singular value decomposition (SVD)

As seen in Eq. 4, one can transform the Jacobian matrix into a singular matrix if one of the J_R or $J_{P\theta}$ matrices is singular. If $|J_{P\theta}| \neq 0$ (which implies that there is no static angle problem), then it should be noted that under the defined conditions, the Jacobian matrix will become singular. Therefore, it is concluded that in the steady state voltage stability problem, the matrix J_R is a sub-matrix that is in turn

TABLE 2 Maximum power flow–based line indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Power transfer stability index (PTSI)	Nizam et al. (2006)	$PTSI = \frac{2S_r Z(1+Cos(\theta-\phi))}{V_s^2}$	1	Yes	✓	✗
2	L_{sr} index	Albuquerque and Castro (2003)	$L_{sr} = \frac{S_r}{S_{r(max)}} (\because S_{r(max)} = \frac{\frac{V_s^2}{Z}}{4Cos^2(\frac{\theta-\phi}{2})})$	1	Yes	✓	✗
3	Active power performance index (APPI)	Daram et al. (2016)	$APPI = \sum_{k=1}^{L} (\frac{w}{2n}) [\frac{P_k}{P_k{}^{max}} []^{2n}]$	1	Yes	✓	✗
4	VSI	Selim et al. (2020)	$VSI = \frac{4X}{V_1^2} (\frac{P_2^2}{Q_2} + Q_2)$	1	Yes	✗	✓
5	Voltage collapse proximity index (VCPI)	Moghavvemi and Faruque (1998)	$VCPI_1 = \frac{P_r}{P_{r(max)}} (\because P_{r(max)} = \frac{\frac{V_s^2}{Z} Cos\,\phi}{4Cos^2(\frac{\theta-\phi}{2})})$ $VCPI_2 = \frac{Q_r}{Q_{r(max)}} (\because Q_{r(max)} = \frac{\frac{V_s^2}{Z} Sin\,\phi}{4Cos^2(\frac{\theta-\phi}{2})})$ $VCPI_3 = \frac{P_l}{P_{l(max)}} (\because P_{l(max)} = \frac{\frac{V_s^2}{Z} Cos\,\theta}{4Cos^2(\frac{\theta-\phi}{2})})$ $VCPI_4 = \frac{Q_l}{Q_{l(max)}} (\because Q_{l(max)} = \frac{\frac{V_s^2}{Z} Sin\,\theta}{4Cos^2(\frac{\theta-\phi}{2})})$	1	Yes	✓	✗
6	Voltage stability margin (VSM_S)	Deng et al. (2009)	$VSM_s = \frac{S_{cr}-S_l}{S_{cr}}; (\because S_{cr} = \frac{V_s^2}{2Z(1+Cos(\theta-\phi))})$	0	Yes	✗	✓
7	Voltage stability margin index (VSMI)	He et al. (2004)	$VSMI = \frac{\delta_{max}}{\delta_{max}}$	0	Yes	✓	✗
8	LVSI	Ratra et al. (2018)	$LVSI = \frac{2V_r Acos(\beta-\alpha)}{V_s cos(\beta-\alpha)}$	1	No	✓	✗
9	Critical boundary index (CBI)	Furukakoi et al. (2018)	$CBI_{ik} = \sqrt{\Delta P_{ik}^2 + \Delta Q_{ik}^2}$	0	No	✓	✗
10	Line stability index (LSI)	Chakraborty et al. (2016)	$LSI = \frac{\sqrt[2]{Q_L(\frac{1}{X}-\frac{Cos\,\delta}{X})}}{ICos\delta}$	1	Yes	✗	✓

TABLE 3 Maximum power flow–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	P-index	Kamel et al. (2018)	$P - index_j = \frac{-2\frac{P_{Lj}}{V_j}\frac{dV_j}{dP_{Lj}}}{1-2\frac{P_{Lj}}{V_j}\frac{dV_j}{dP_{Lj}}}$	1	No	✓	✗
2	Voltage stability factor (VSF)	Danish et al. (2019)	$VSF_{total} = \sum_{m=1}^{k-1}(2V_{m+1} - V_m)$	0	Yes	✗	✓
3	VSI	Rawat and Vadhera (2020)	$VSI = \lvert V_i\rvert^4 - 4(P_j x_{ij} - Q_j r_{ij})^2 - 4(P_j x_{ij} - Q_j r_{ij})\lvert V_i\rvert^2$	1	Yes	✗	✓
4	VSI	Sadeghi and Foroud (2020)	$VSI = V_1^2 - 4(PR + QX)$	1	Yes	✗	✓
5	VSI	Haque (2015)	$VSI = [2(RP + XQ) - V_s^2]^2 - 4(P^2 + Q^2)(R^2 + X^2)$	1	Yes	✗	✓
6	Voltage margine proximity index (VMPI)	Kataoka et al. (2006)	$VMPI_j = \begin{cases} Cos^{-1}\frac{V^t * V_L}{\lVert V\rVert * \lVert V_L\rVert}(\lVert V\rVert > \lVert V_L\rVert) \\ -Cos^{-1}\frac{V^t * V_L}{\lVert V\rVert * \lVert V_L\rVert}(\lVert V\rVert < \lVert V_L\rVert) \end{cases}$	-	No	✓	✗
7	Sensitivity indices	Simpson-Porco and Bullo (2016)	$\frac{dV}{dQ} Index \Rightarrow I_i = \underset{=}{\Delta} \sum_{J \in L} \frac{Q_j}{V_i}\frac{\delta V_i}{\delta Q_i}, i \in L$	0 = open circuit,	Yes	✓	✗
			$\frac{dV_L}{dQ_G} Index \Rightarrow J_i = \underset{=}{\Delta} \sum_{K \in G} \frac{\delta V_i}{\delta V_k}, i \in L$	∞ = collapse point			
			$\frac{dQ_G}{dQ_L} Index \Rightarrow K_i = \underset{=}{\Delta} \sum_{J \in L} \frac{\delta Q_k}{\delta Q_i}, i \in L$				

TABLE 4 PMU and local measurement–based line indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Recursive least square (RLS)	Milosevic and Begovic (2003)	$x_k = x_{k-1} + G_k(y_k - H_k^T x_{k-1}) G_k = P_{k-1} H_k (\lambda I + H_k^T P_{k-1} H_k)^{-1} P_k = \frac{1}{\lambda}(1 - G_k H_k^T) P_{k-1}$	-	No	✓	✗
2	Voltage stability load bus index (VSLBI)	Milosevic and Begovic (2003)	$VSLBI = \frac{V_r}{\Delta V}$	1	No	✓	✗
3	Voltage instability predictor (VIP)	Julian et al. (2000)	$\Delta S = \frac{(V_k - Z_{Th} I_k)^2}{4 Z_{Th}}$	0	Yes	✓	✗
4	Approximate approach	Li et al. (2010)	$V_{Li} = E_{eq,i} - Z_{eq} I_{Li} Z_{eq} = Z_{LLii}$	-	Yes	✓	✗
5	Simplified voltage stability index (SVSI)	Pérez-Londoño et al. (2014)	$SVSI = \frac{\Delta V_r}{\beta V_r}$	1	Yes	✓	✗
6	Real-time voltage stability assessment (RVSA) indices	Adewole and Tzoneva (2017)	$EGRPR = Q_{gmax}^{c_{gk}}$	0	No	✓	✗
			$RVSA_{Q,A} = (1 - \frac{Q_{gki}}{Q_{g\,max\,i}^c}) \times 100\%$				
			$EGFCR = I_{fdm} = I_{fdmax}^{c_{fdk}}$				
			$RVSA_{Ifd,i} = (1 - \frac{I_{fdki}}{I_{fd\,max\,i}^c}) \times 100\%$				
			$EGSCR = I_{am} = I_{amax}^{c_{ak}}$				
			$RVSAL_{Ia,i} = (1 - \frac{I_{aki}}{I_{a\,max\,i}^c}) \times 100\%$				
7	New line index (NLI)	Vournas et al. (2017)	$NLI = \frac{\Delta P}{\Delta G_P} > 0$	0	Yes	✓	✗
8	$D_v^{ij} = Index$	Ancheng et al. (2016)	$D_v^{ij} = \frac{V_i}{\sqrt{2V_j^2 + 2(P_j' R + Q_j' X)}}$	1	Yes	✓	✗
9	VSI on short circuit capacity (VSI_{SCC})	Huang et al. (2011)	$VSI_{SCC} = \frac{SCC_{min}}{SCC_P \frac{2 S_L X_{th}(1 + Sin\,\phi)}{E_{th}^2}}$	1	Yes	✓	✗

TABLE 5 PMU and local measurement–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Voltage stability risk index (VSRI)	Seethalekshmi et al. (2010)	$VSRI_j = \frac{1}{N}\lfloor\frac{\sum_{i=j-N+1}^{j}(d_i+d_{i-1})\Delta t}{2}\rfloor$	-	No	✓	✗
2	Equivalent node voltage collapse index (ENVCI)	Wang et al. (2009)	$ENVCI = 2(e_k e_n + f_k f_n) - (e_k^2 + f_k^2)$	0	No	✓	✗
3	Linearized motor VSI (LMVSI)	Gu and Wan (2010)	$LMVSI_i = \frac{MVSI_i}{\lvert\frac{d(MVSI_i)}{d\lambda_i}\rvert}$ Where $MVSI = \lvert det(A_i)\rvert$ $\lambda_i = \frac{P_L^{t_i} - P_L^{t_0}}{P_L^{t_0}}$	-	No	✓	✗
4	Simplified L-index	Chen et al. (2017)	$L'_j = \frac{1}{V_j^2}\sqrt{f'^2 + g'^2}$	1	Yes	✓	✗
5	Linear M-index	Matavalam and Ajjarapu (2015)	$M - Index = 1 - \frac{P}{P_{cr}}$	-	Yes	✓	✗

associated with matrix J. Consider matrix A as a quadratic matrix (Lof et al., 1993), the SVD expression is defined in Eq. 10.

$$A = U\sum V^T = \sum\nolimits_{i=1}^{n}\sigma_i u_i v_i^T \tag{10}$$

A new method for estimating the VSM is introduced (Ekwue et al., 1999) based on the closeness of the Jacobian matrix to become singular, which can be called the minimum singular value method. This is a new form of SVD. In connection with this, by correctly evaluating the lad values that can be modified in regular steps, a power flow Jacobian matrix can be calculated. If the Jacobian matrix's singular value is almost equal to 0, the loadability limit of the system is reached. Eq. 11 provides the SVD solution for the linear system given in Eq. 10.

$$x = A^{-1}b = \left(U\sum V^T\right)b = \sum\nolimits_{i=1}^{n}\frac{u_i^T v_i}{\sigma_i}b \tag{11}$$

It is therefore inferred that minor changes in both the value of A and the value of b would result in an enormous change in the value of x, given by a small value of n. Consequently, the small changes in both the active and reactive power injections impact $[\,\Delta\theta \;\; \Delta V\,]^T$. The equation can be written as

$$\begin{bmatrix}\Delta\theta\\ \Delta V\end{bmatrix} = V\sum\nolimits^{-1}U^T\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix} \tag{12}$$

Nevertheless, it is impossible to detect the specific reason for the voltage instability in the system. While this approach can be used to obtain a relative proximity value with respect to the voltage stability limit, it does not provide an exact or linear value for the given power system. This is because, after a given stable operating point that is defined by the system, it experiences a non-linear behavior up to the bifurcation point.

3.6 Bifurcation techniques

The bifurcation technique is a great tool that is widely applied in the study of power system voltage stability. Generally, the power system is expressed by highly nonlinear dynamical system equations, which include system parameters (Wu et al., 2017). If one of the parameters is varied, then the phase portrait of that system may deform slightly. These qualitative changes in the parameters of the dynamical system may lead to system instability. Poorly damped low-frequency oscillations, induction motor dynamics, and the dynamics of voltage collapse near the critical point are the main causes of the change in the dynamical behavior of the system.

The autonomous differential equation of a dynamical system is given in Eq. 13.

$$\dot{x} = F(x,\lambda), x \in R^n, \lambda \in R^k \tag{13}$$

where f represents a differential equation and x denotes the state variables. Load voltage magnitudes or angles, generator angles, and generator angular velocities are the state variables of power system models. λ denotes a vector of the time-invariant scalar parameter. At an equilibrium point (x_0, λ_0), $\dot{x}$ the equation becomes zero, i.e., the steady-state solution of Eq. 13 satisfies Eq.14, representing a set of nonlinear algebraic equations.

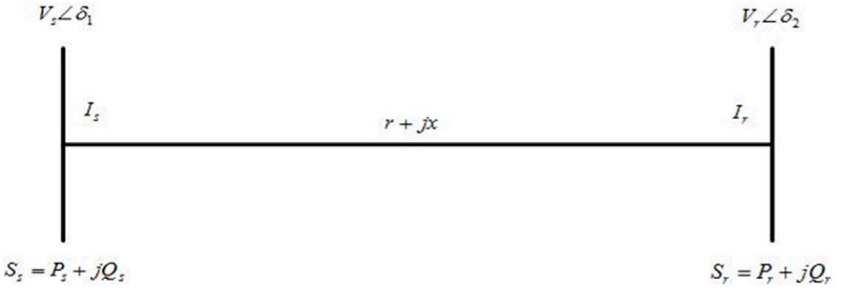


FIGURE 3
Two bus transmission network.

$$F(x_0, \lambda_0) = 0 \tag{14}$$

If the eigenvalues of the partial derivative of Eq.14 with respect to x, $\frac{\partial F}{\partial x}$ (i.e., Jacobian) become non-zero, then according to the implicit function theorem, the equilibria of Eq. 13 can be represented as a smooth function of $x = x(\lambda)$. The function $x(\lambda)$ is known as the branch of equilibria. However, if the eigenvalue of the Jacobian has zero real part occurring at some λ, say λ_c, the system $\dot{x} = F(x_c, \lambda_c)$ becomes structurally unstable, and several branches of $x = x(\lambda)$ can come together at (x_c, λ_c) in R^{n+k}. The parameter set λ_c where the system loses its stability is called a bifurcation set. The point (x_c, λ_c) is called the bifurcation point.

Here, single-parameter bifurcation analysis can be successfully implemented for voltage stability analysis of the real-time system. Multi-parameter bifurcation analysis is too difficult due to the complexity of the electric power system itself. The different types of bifurcation techniques are briefly discussed below.

3.6.1 Saddle node bifurcation (SNB)

The bifurcations are known as a turning point or fold bifurcation and are usually defined by a couple of equilibrium points merging at the bifurcation point and then locally disappearing as the slowly varying parameter λ changes. This bifurcation corresponds to an equilibrium point where the state matrix has a distinct zero eigenvalue, and certain transversal conditions are met (Dobson and Lu, 1993). The direct and continuation methods are two important types of SNB. The direct method is used to find the critical point where the Jacobian is singular by solving the power flow equations of the large power system in one step. The continuation method is described in (Seydel, 2009).

3.6.2 Holf bifurcation method

From Eq. 14, the state of the power system is asymptotically stable if the eigenvalues of the Jacobian have negative real parts at that point. If the real eigenvalue becomes zero or crosses the imaginary axis and moves into the right half-plane, the system can reach a critical point or oscillate with a small amplitude. This phenomenon is explained by the holf bifurcation theory. In this technique, the assumptions to make are (i) $f(x_c, \lambda_c) = 0$, (ii) the Jacobian matrix has a simple pair of purely imaginary eigenvalues, $\mu(\lambda_c) = \pm\ jw$, and (iii) $\frac{d(Re(\mu(\lambda_c)))}{d\lambda} \neq 0$ where λ_c is the parameter at which bifurcation occurs (Hassard et al., 1981; Marsden and McCracken, 2012). The third assumption requires that there is a transversal crossing of the imaginary axis by the pair of complex conjugate eigenvalues. Numerical determination of the holf point involves estimation of the point (x_c, λ_c). Evaluating all the eigenvalues of the Jacobian matrix is a costly way to identify the holf point. However, in the static approach, there are efficient ways to identify the holf point using both the direct and indirect methods (Ajjarapu and Lee, 1992; Rajagopalan et al., 1992; Venkatasubramanian et al., 1995; Seydel, 2009).

3.6.3 Limit-induced bifurcations (LIBs)

Mathematically, the solution points are the limit-induced bifurcations associated with load flow models, where all the eigenvalues of the resulting Jacobian have non-zero real parts, i.e., the Jacobian matrix is non-singular. The LIBs are broadly classified into two types, as follows.

i) *limit-induced dynamic bifurcation (LIDBs)*: this refers to a sudden change in the dynamic behavior of a power system near a voltage stability limit. It occurs when a small change in system parameters or operating conditions causes a qualitative shift in the system's stability and dynamic response. This bifurcation is associated with the onset of voltage instability or voltage collapse.

3.6.3.1 Significance

- Early warning of instability: limit-induced dynamic bifurcation provides an early warning sign of voltage instability and potential voltage collapse in the system. It helps identify critical operating points where small disturbances can lead to significant system disruptions.
- Impact on system dynamics: this type of bifurcation reveals changes in the system's dynamic behavior, such as the appearance of oscillations, unstable voltage modes, or limit cycles. It indicates a shift from stable operation to a regime of unstable or oscillatory behavior.

3.6.3.2 Application

- System planning: understanding limit-induced dynamic bifurcation helps in determining the stability limits of a power system, guiding system planning and expansion decisions to avoid potential voltage collapse scenarios.
- Control strategies: by analyzing the bifurcation behavior, appropriate control strategies can be developed to mitigate

TABLE 6 Voltage equation of transmission line–based line indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	LSZ	Jalboub et al. (2010)	$LSZ = \frac{2\vert Z\vert\vert S_r\vert}{\vert V_s\vert^2-2\vert Z\vert(P_r \cos\theta+Q_r \sin\theta)}$	1	Yes	✓	✗
3	Improved LSZ	Yari and Khoshkhoo (2019)	$ILSZ = \frac{2\vert Z_{th}+Z_l\vert\vert S_l\vert}{\vert V_{th}\vert^2-2\vert Z_{th}+Z_l\vert(P_l \cos\theta+Q_l \sin\theta)}$	1	Yes	✓	✗
4	Maximum loadability index (MLI)	Venkatesh et al. (2004)	$MLI = \frac{V_i^2[-(r_{ij}.P_{ij}+x_{ij}.Q_{ij})+\sqrt{(r_{ij}^2+x_{ij}^2)(P_{ij}^2+Q_{ij}^2)}]}{2.(x_{ij}.P_{ij}-r_{ij}.Q_{ij})^2}$	1	Yes	✗	✓
5	Fast VSI	Musirin and Abdul Rahman (2002)	$FVSI = \frac{4Z^2Q_r}{V_s^2X}$	1	Yes	✓	✓
6	Novel line stability index (NLSI)	Yazdanpanah-Goharrizi and Asghari (2007)	$NLSI = \frac{P_rR+Q_rX}{0.25V_s^2}$	1	Yes	✓	✗
7	Stability index (SI)	Eminoglu and Hocaoglu (2007)	$SI = 2V_s^2V_r^2 - V_r^4 - 2V_r^2(P_rR + Q_rX) - Z^2(P_r^2 + Q_r^2)$	0	Yes	✗	✓
8	Line collapse proximity index (LCPI)	Tiwari et al. (2012)	$\mathrm{LCPI} = \frac{4\vert A\vert Cos(\alpha) \times [P_r\vert B\vert Cos(\beta)+Q_r\vert C\vert sin(\beta)]}{[V_s Cos(\delta)]^2}$	1	Yes	✓	✗
9	Revamp voltage stability index (RVSI)	Rath et al. (2017)	$RVSI = \frac{4Z_{ij}^2Q_rX_{ij}}{V_s^2(R\sin\delta+X\cos\delta)^2}$	1	Yes	✓	✗
10	New line stability index ($NLSI_1$)	Samuel et al. (2017)	$NLSI_1 = \frac{4Q_r}{\vert V_s\vert^2}[\frac{(\vert Z\vert)^2}{X}\sigma - \frac{X}{\sin^2(\theta-\delta)}(\sigma - 1)] \leq 1$	1	Yes	✓	✗
11	Voltage reactive power index (VQI_{Line})	Althowibi and Mustafa (2010)	$VQI_{line} = \frac{4Q_r}{\vert B\vert V_s^2}$	1	Yes	✓	✗
12	L	Sahari et al. (2003)	$L = \frac{4[V_sV_r-V_r^2]}{V_s^2}$	1	No	✓	✗
13	Voltage stability load index (VSLI)	Abdul Rahman and Jasmon, (1995), Makasa and Venayagamoorthy, (2011)	$VSLI = \frac{4[V_sV_r Cos(\delta)-V_r^2 Cos(\delta)]}{V_s^2}$	1	No	✓	✓
14	Voltage stability indicator (VSI_B)	Chattopadhyay et al. (2014)	$VSI_B = \frac{4Q_r(R+X)^2}{X(V_s^2+8RQ_r)}$	1	Yes	✗	✓
15	Line stability index (L_{ij})	Subramani et al. (2009)	$L_{ij} = \frac{4Z^2Q_rX}{V_s^2(R\sin\delta-X\cos\delta)^2}$	1	Yes	✓	✗
16	Line stability index (L_{mn})	Moghavvemi and Omar (1998)	$L_{mn} = \frac{4XQ_r}{(V_s \sin(\theta-\delta))^2}$	1	Yes	✓	✓
17	Line stability factor (LQP)	MOhamed et al. (1989)	$LQP = 4(\frac{X}{V_s^2})(\frac{P_s^2X}{V_s^2} + Q_r)$	1	Yes	✓	✓
18	Stability index (L_P)	Moghavvemi and Faruque (2001)	$L_P = \frac{4RP_r}{V_s Cos(\theta-\delta)^2}$	1	Yes	✗	✓
19	New VSI	Kanimozhi and Selvi (2013)	$NVSI = \frac{2X\sqrt{P_r^2+Q_r^2}}{2Q_rX-V_s^2}$	1	Yes	✓	✗

TABLE 7 Voltage equation of transmission line–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Voltage stability index (L_d)	Kaya et al. (2013)	$L_d = \frac{4\sqrt{(P_j^2+Q_j^2)(R_{ij}^2+X_{ij}^2)}}{V_i^2}$	1	Yes	✗	✓
2	Critical voltage (V_{cr})	Kaya et al. (2013)	$V_{cr} = \frac{E}{2\cos\theta}$	Critical voltage value	No	✓	✗
3	L-index	Kessel et al. (1986)	$L_j = \lvert 1 + \frac{V_{0j}}{V_j} \rvert = \frac{S_j^*}{Y_{jj}V_j^2}$	1	Yes	✓	✗
4	VCPI_{Bus}	Balamourougan et al. (2004)	$VCPI = \left\lvert 1 - \frac{\sum_{\substack{m=1 \\ m \neq i}}^{N} V'_m}{V_i} \right\rvert \left(\because V'_m = \frac{Y_{im}}{\sum_{\substack{j=1 \\ j \neq i}}^{N} Y_{ij}{}^* V_m} \right)$	1	Yes	✓	✗
5	Improved VSI (IVSI)	Yang et al. (2012)	$IVSI = \frac{-4\sum_{j=0}^{n}(G_{ij}-B_{ij})(P_i+Q_i)}{[\sum_{j=1}^{n}\lvert V_j \rvert [G_{ij}(\cos\delta_{ij}+\sin\delta_{ij})-B_{ij}(\cos\delta_{ij}-\sin\delta_{ij})]]^2}$	1	Yes	✓	✗
6	Voltage deviation index (VDI)	Yang et al. (2012)	$VDI_T = \sum_{j=1}^{N}\lvert 1 - V_j \rvert$	-	-	✓	✗

TABLE 8 Maximum power transfer theorem–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Impedance stability index (ISI)	Smon et al. (2006)	$ISI = \frac{Z_L - Z_{th}}{Z_L} = 1 - \frac{\lvert I_r \Delta V_r \rvert}{\lvert V_r \Delta I_r \rvert}; \left(\because Z_L = \frac{V_r}{I_r}; Z_{th} = \frac{\Delta V_r}{\Delta I_r}\right)$	0	No	✓	✗
2	S difference criterion (SDC)	Smon et al. (2006)	$SDC = \left\lvert 1 + \frac{\Delta V_r I_r^*}{V_r \Delta I_r^*} \right\rvert$	0	No	✓	✗
3	Z_L/Z_S ratio	Wiszniewski (2007)	$\frac{Z_L}{Z_S} = \frac{M+1}{[(M\cos\beta)^2 - M^2 + 1]^{0.5} - M\cos\beta} \left(\because M = \frac{(S_b - S_a)(Y_b + Y_a)}{(S_b + S_a)(Y_b - Y_a)}\right)$	1	No	✓	✗
4	Voltage stability index (VSI_{Bus})	Haque (2008)	$VSI = \left[1 + \left(\frac{I_i}{V_i}\right)\left(\frac{\Delta V_i}{\Delta I_i}\right)\right]^{\alpha}$	0	No	✓	✗

voltage instability, enhance system damping, and improve the system's dynamic performance.

ii) *Limit-induced static bifurcations (LISBs)*: static bifurcation refers to the qualitative change in the steady-state behavior of a power system near a voltage stability limit. It occurs when there is a change in the number or type of steady-state solutions as system parameters or operating conditions are varied. Static bifurcations are associated with the appearance or disappearance of multiple equilibrium points or voltage collapse points (Avalos et al., 2009). In the case of LIDBs, the equilibrium points continue to exist after being reached as the bifurcation parameter λ varies, as shown in Figure 2A. On the other hand, LISBs are approximately identical to SNBs, in that these correspond to the points at which two solutions converge and vanish as the bifurcation parameter λ changes, as shown in Figure 2B. LISBs are often correlated with a maximum loadability margin.

3.7 Limitations and challenges of traditional voltage stavility methods

Voltage stability analysis is an important aspect of power system analysis and operation. Several methods are employed to assess voltage stability, such as the PV and QV curve method, sensitivity method, modal analysis, continuation power flow analysis, singular value decomposition (SVD), and bifurcation analysis. While these methods provide valuable insights, they also have their limitations and challenges. The following is an overview of the limitations and challenges associated with each method.

3.7.1 PV and QV curve method

- Limited accuracy: the PV and QV curve method provides a simplified representation of the system, assuming a constant power factor or voltage magnitude. This simplification may lead to inaccuracies in representing the actual system behavior.
- Lack of dynamic information: this method does not capture dynamic phenomena and transient effects that can impact voltage stability.
- Limited applicability: it may not be suitable for analyzing complex systems with significant variations in load and generation conditions.

3.7.2 Sensitivity method

- Complexity: sensitivity analysis involves calculating derivatives and requires accurate system models, making it computationally intensive and complex.
- Model uncertainties: the sensitivity method is sensitive to model uncertainties, such as load models and parameter variations, which can affect the accuracy of the results.
- Limited scope: it may not provide a comprehensive assessment of voltage stability under large disturbances or system-wide events.

3.7.3 Modal analysis

- Linear approximation: modal analysis assumes linearity, which may not capture the non-linear behavior of power systems accurately.

TABLE 9 Jacobian matrix–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Impedance ratio indicator (IRI)	Chebbo et al. (1992)	$IRI = \frac{Z_{ii}}{Z_i}$	1	Yes	✓	✗
2	Minimum eigenvalue and eight eigenvector method (RE)	Gao et al. (1992)	$\Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q$	All eigenvalues should be positive	Yes	✓	✗
3	Minimum singular value	Lof et al. (1993)	$\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = V \sum^{-1} U^T \begin{bmatrix} \Delta F \\ \Delta G \end{bmatrix}$	-	Yes	✓	✗
4	Second-order index	Berizzi et al. (1998)	$i = \frac{1}{i_0} \frac{\sigma_{max}}{d\sigma total_{max}}$	i should be greater than 0	No	✓	✗
5	Integral steady-state margin (ISSM)	Hatziargyriou et al. (1994)	$ISSM = (\frac{J_c}{J_0})$	Index value between 0 and 1	-	✓	✗
6	Voltage stability index approximation (VSIA)	Aolaritei et al. (2018)	$VSIA = \frac{In(det_{approx})}{n}$	-	Yes	✗	✓
7	Diagonal element dependant index (DEDI)	Sinha and Hazarika (2000)	$I_{qi} = \frac{\frac{\partial Q_i}{\partial V_i}}{-B_{ii}}; I_{pi} = \frac{\frac{\partial P_i}{\partial \delta_i}}{-B_{ii}}; I_i = \frac{\frac{\partial P_i}{\partial \delta_i}}{\sum_{\substack{j=1 \\ j \neq 1}}^{N} B_{ii} V_j}$	-	Yes	✓	✗

TABLE 10 P-V curve–based bus indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Approximate collapse power index (ACPI)	Tran (2009)	$ACPI = \frac{P_{ia}}{P_{C,i}}; ACPI = \frac{1}{1+d\lambda_{C,i}}$	Varies between 0 and 1	No	✓	✗
2	Test function (TF)	Seydel (1994)	$t = e_l^t J(x,\lambda)v$	-	Yes	✓	✗
3	Tangent vector indices (TVI)	De Souza et al. (1997)	$TVI_i = \lvert\frac{dV_i}{d\lambda}\rvert^{-1}$	0	No	✓	✗
4	Disturbance detection index (DDI)	Aghdam and Khoshkhoo (2020)	$DDI^m = \lvert\frac{\bar{k}_1^m - \bar{k}_1^{m-1}\rvert}{\bar{k}_1^{m-1}} \times 100$	DDI >0.5 × 10^{nr}	No	✗	✓
5	V/V_0	Kessel et al. (1986)	V/V_0	The ratio at which weak and effective sensitivity is identified at every node.	Yes	✓	✗
6	Voltage collapse index (VCI)	Raja et al. (2013)	$VCI_i = [1 + (\frac{I_i \Delta V_i}{V_i \Delta I_i})]^{\alpha}$	0; the index value depends on alpha	Yes	✓	✗
7	P_{Lmgn}	Nagao et al. (1997)	$P_{Lmgn} = \frac{P0_{max}}{P_{max\,n}}$	Index value between 0 and 1	No	✓	✓
8	Sensitivity index	Konar et al. (2015)	$Index = \left\{ \max_{i=1\cdots n} [diag(J_{Ri}^{-1})\lvert t2 - diag(J_{Ri}^{-1})\rvert_{t1}] \times 10^3 \right\}$	-	No	✓	✗
9	Novel VSA algorithm	Aghdam and Khoshkhoo (2020)	$NVSAAI = \frac{p}{p_{max}}$	$0 \leq NVSAAI \leq 1$	No	✗	✓
10	SBI	Vanfretti and Sevilla (2015)	$SBI = \begin{bmatrix} sb_{11} & sb_{12} & sb_{13} & sb_{14} & sb_{15} & sb_{16} \\ sb_{21} & sb_{22} & sb_{23} & sb_{24} & sb_{25} & sb_{26} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ sb_{n1_{n2}} & sb_{n3} & sb_{n4} & sb_{n5} & sb_{n5} & sb_{n6} \\ sb_{m1} & sb_{m2} & sb_{m3} & sb_{m4} & sb_{m5} & sb_{m6} \end{bmatrix}$	-	Yes	✓	✗
	ABI		$\frac{ABI=[\Delta P_{a11,} \Delta P_{b12,} \Delta P_{c13,} \Delta P_{a14,} \Delta P_{b15,} \Delta P_{c16}]}{GBI=[\Delta P,\Delta V]}$				
	GBI						
11	Network sensitivity approach (SG)	Althowibi and Mustafa (2012)	$SG_p = \frac{P_{gt}}{P_{dt}}; SG_q = \frac{P_{gt}}{Q_{dt}}$	Sharp rise to ∞ values	No	✓	✗
12	Voltage instability proximity index (VIPI)	Tamura et al. (1988)	$VIPI = Cos^{-1} (\frac{y_s^T y(a)}{\lVert y_s \rVert \lVert y(a) \rVert})$	By using the fictitious and exact solution, the critical point has been estimated.	Yes	✓	✗

TABLE 11 Line indices.

No.	Index name	Author	Formulae	Critical value	Is impedance dependent on the index?	Applicable system	
						TS	DS
1	Voltage collapse proximity index ($VCPI_A$)	Wang et al. (2005)	$VCPI_A = V_r \cos(\delta) - 0.5V_s$	0	No	✓	✗
2	Integrated transmission line transfer index (ITLTI)	Chuang et al. (2016)	$S_{R-Index} = \frac{Sin(\theta_R^{'}+\delta') Sin\,\delta'}{Cos(\frac{\theta_R^{'}}{2})}$	1	No	✓	✗
3	Composite severity index (CSI)	Mishra et al. (2016)	$CSI = w_1 \times \frac{w_{mn}}{2a} (\frac{P_{mn}}{P_{mn}{}^{max}} ()_2^2 \frac{4xQ_n}{[V_m \sin(\theta-\delta)]^2})$	1	Yes	✓	✗
4	Voltage sag severity index (VSSI)	Dong et al. (2015)	$VSSI = D + S$ $D = \begin{cases} 0, \ldots if \ldots T_1 < 10s \\ 10 \ldots\ldots Otherwise \end{cases}$ $S = \int_0^\infty \Delta V(t)dt = \int_{t_0}^{t_0+T_2} (0.9 - V(t))dt$	-	No	✓	✗
5	VSQI	Katsanevakis et al. (2019)	$VSQI = \sqrt{(x_\varepsilon^{vs} - x_\varepsilon^{vq})^2 + (y_\varepsilon^{vs} - y_\varepsilon^{vq})^2}$	-	No	✗	✓
6	I_P and I_Q	Fang et al. (2015)	$I_P = \frac{\Delta l}{\Delta C_i} = \frac{n_{Pi}}{n_P^T P_l + n_Q^T Q_l}; I_Q = \frac{\Delta l}{\Delta C_i} = \frac{n_{Qi}}{n_P^T P_l + n_Q^T Q_l}$	-	No	✓	✗
7	Reactive power loss index (RPLI)	Moger and Dhadbanjan (2015)	$RPLI_i = Qloss_{i,0}^n + \sum Qloss_{i,k}^n \times NCOSI_k$	-	Yes	✓	✗
8	$\frac{dS_l}{dY_l}$ index (DSY)	Parniani and Vanouni (2010)	$DSY = \frac{\Delta S_i}{V_i^2 \Delta Y_i}$	0	Yes	✓	✗
9	M_i^0, M_i^j	De Moura and Prada (2005)	$M_i^0 = S_{m_i}^0 - S_i^0 M_i^j = S_{m_i}^0 - S_i^j$	-	No	✓	✗
10	Influence index	De Moura and Prada (2005)	$II_i^j = (signof\,\beta_i^0) \times (\frac{M_i^j}{M_i^0} - 1)$	-	Yes	✓	✗
11	Short term VSI	Zhao (2017)	$SVSI_r = \int_{T_{Flt}}^{T_{SVSI_r}} \lvert v(t) - V_s \rvert dt$ $SVSI_0 = \int_{t_{Clear}}^{t_{End}} \lvert v_{TVSI_0}(t) - v(t) \rvert dt$ $SVSI_S = (v(0) - V_s) \times (t_{stable} - t_{Clear})$	-	No	✓	✗

TABLE 12 Merits and demerits of VSIs.

VSI name & ref. No.	Merits	Demerits	Test systems considered
P-index (Kamel et al., 2018)	Compared to the L-index, this gives a better indication of proximity to voltage collapse.	Some assumptions are required to calculate the index, e.g., generation and load must be considered to vary in the same proportion.	Load shedding and dynamical VSA are some of the applications of this index. IEEE-14, 57, and 118 buses are the test systems considered.
CBI (Furukakoi et al., 2018)	1. The accuracy of this index is very high compared to conventional VSIs. 2. Requires fewer parameters (ΔP, ΔQ) to calculate VSI.	The computational time of this index is high compared to conventional VSIs.	IEEE 14 and 57 bus systems.
Improved LSZ (Yari and Khoshkhoo, 2019)	1. This index has the ability to detect the stability status of the system even when P and Q are flowing in the opposite direction. 2. Shows better performance compared to other line indices and has a low computational burden.	Many parameters were required to compute this index.	IEEE-39 and 118 bus system.
NLI (Vournas et al., 2017)	This index has the ability to obtain the stability status of large power systems.	The procedure for calculating this index requires more time.	IEEE Nordic test system.
Dij (Ancheng et al., 2016)	Has the ability to assess the online stability of the system.	1. This index is influenced by time delays, dead bands, and voltage regulators. 2. The computation of this index is high due to its nonlinear nature.	The bus system and the Chinese regional power grid were the test system considered.
CSI (Mishra et al., 2016)	This index is a hybrid of PI and Lmn indices. Has the ability to identify the critical lines at post-contingency conditions.	The calculation time of this index is high.	IEEE-30 and Indian utility 62 bus system.
RPLI (Moger and Dhadbanjan, 2015)	1. This index can be used to sense the weak load bus under peak load conditions under severe contingency conditions.	1. The validity of this index has only been confirmed with traditional stability methods such as CPF, modal analysis, and Q-V sensitivity.	This index can be utilized in EMS centers to identify vulnerabilities in real time.
	2. It is very helpful in the placement of reactive power devices for additional voltage support. 3. The identification of weak buses in the system using this index is completely non-iterative, thus, requires minimal computational effort and as compared with other traditional methods.	2. No other recent stability indices were considered in this validation. Therefore, the performance of this index in terms of identification of stability status, computational effort, and computational time in comparison with recent techniques remains unknown.	The Southern India 72 bus practical system is the test system considered.
LCPI (Tiwari et al., 2012)	1. The significant advantage of this index is that it can identify the stability status of the system more precisely when the system is heavily loaded. 2. Another advantage of this index is that it is derived by considering both the exact model of the transmission line and the effects of the system's real and reactive power flow.	The main drawback of this index is that it cannot identify the weak areas and lines under low load conditions.	IEEE 30 and 118 bus system.
Z_L/Z_S (Wiszniewski, 2007)	This method eliminates the main drawback of using voltage level as an indicator of voltage stability.	The main drawback of this index is that it depends on the load power factor.	Applicable for load shedding in the power system.
VMPI (Kataoka et al., 2006)	Very sensitive in identifying the low-voltage buses in the system.	This index can be applied only for low voltage limits and not for high voltage limits.	IEEJ west 30 machines and 115 bus system.
VCPI bus (Balamourougan et al., 2004)	This index is most appropriate for predicting the voltage collapse of real-time systems.	In the case of a large system, the computational time required by this index is too high.	This index can be applied in central load dispatch centers to predict the power system voltage collapse online; it is a very helpful tool to initiate corrective actions. IEEE-118 bus system is the test system considered.
FVSI (Musirin and Abdul Rahman, 2002)	This index can be used to identify the critical lines and also to predict the occurrence of voltage collapse.	The calculation time of this index is high compared to LQP and Lmn.	IEEE-30 bus system.

(Continued on following page)

TABLE 12 (*Continued*) Merits and demerits of VSIs.

VSI name & ref. No.	Merits	Demerits	Test systems considered
TPSI (Gubina and Strmčnik, 1995)	1. Requires only voltage phasor information and can be used to determine the proximity of voltage collapse directly from the phasor measurements. 2. Computational time is very low.	Many assumptions have to be considered when this index is applied to meshed systems, which reduces the accuracy of the prediction.	This index can be applied to radial distribution systems. The New England 39 bus system and IEEE 118 system are the test systems considered.

- Modal truncation: the accuracy of modal analysis depends on the number of modes considered. Truncating the modal analysis may lead to the exclusion of important system dynamics.
- Assumptions: modal analysis often assumes decoupled modes, which may not hold true for all system conditions.

3.7.4 Continuation power flow analysis

- Computational requirements: continuation power flow analysis requires solving a series of power flow equations iteratively. It can be computationally demanding, particularly for large-scale systems.
- Convergence issues: the method may encounter convergence problems when dealing with ill-conditioned systems or in the presence of voltage collapse phenomena.
- Representation of dynamic effects: continuation power flow analysis typically focuses on steady-state conditions and may not capture transient or dynamic effects adequately.

3.7.5 Singular value decomposition (SVD)

- Limited applicability: SVD is primarily used for analyzing linearized models and may not fully capture the non-linear dynamics associated with voltage stability.
- Data requirements: SVD relies on system response data, and obtaining accurate and sufficient data for analysis can be challenging.
- Interpretability: interpreting the results of SVD and relating them to voltage stability issues can be complex and requires expertise.

3.7.6 Bifurcation analysis

- Computational complexity: bifurcation analysis involves exploring system behavior near critical points, which can be computationally intensive, particularly for large-scale systems.
- Non-linear system representation: bifurcation analysis assumes non-linear models, which may require accurate modelling of the system dynamics and associated parameters.
- Sensitivity to initial conditions: the results of bifurcation analysis can be sensitive to the initial conditions and small perturbations, making it necessary to carefully select the initial operating point.

It is worth noting that each method has its advantages and can provide valuable insights into voltage stability. However, it is essential to consider these limitations and challenges when applying these methods to ensure accurate and reliable results. Additionally, combining multiple methods and considering a holistic approach to voltage stability analysis can help mitigate some of these limitations.

4 VSIs

In the study of power systems, voltage instability is a significant and challenging problem for electrical scientists and engineers. Numerous approaches and practices implemented on IEEE test systems are discussed in the following sections to assess voltage collapse prediction and voltage instability (Zambroni de Souza et al., 2011). Some of the conventional approaches were conferred in Section 3. Several VSIs are discussed to judge the dynamic problems for planning and controlling power systems. VSIs are categorized into two types.

(i) Line VSIs: “The line VSIs are used to identify the capability of the TS to deliver power to the load area.” The value of these indices determined for each branch is used to find the gap between the current operating point and the maximum power transfer capability of the line.
(ii) Bus VSIs: “The bus indices have the ability to detect the margin between the present operating point of the buses and maximum loading of load buses."

In (Reis and Maciel Barbosa, 2009; Karbalaei et al., 2010; Zhao J and Yang, 2011; Lim et al., 2012; Ismail et al., 2014; Baleboina et al., 2017; Baleboina and Mageshvaran, 2018; Yuva Kishore and Guru Mohan, 2018; Yari and Khoshkhoo, 2019; Nageswa Rao et al., 2021), a comprehensive assessment of several line stability indices (LSIs) was carried out. Moreover, a modified LSI has been proposed to precisely measure the stability status of system networks at different conditions. The performance of nearly 17 indices was analyzed for the following conditions.

(i) Different ratios of R/X,
(ii) Different directions of load increment and load power factors, and
(iii) Different sending end bus voltages.

From the analysis, it is observed that indices such as LQP, LSZ, VSMI, VCPI (1), VCPI (3), MLI, and L were assessing the stability of the system properly in spite of the different direction of load increment and R/X ratios. Moreover, LQP, NVSI, VCPI (1), VCPI (3), Lsr, VSI, VSMI, L, LSZ, and MLI were the indices that have the ability to assess the stability during both low power factors (PF) and typical PFs (i.e., PF = 0.85 or higher) conditions. However, except for LSZ, the remaining indices did not perform well when P and Q flow in different directions. Furthermore, even the LSZ index is able to measure the stability of a simple two-bus system only and fails to analyze the stability stiffness of a real, large power system. In order to overcome this drawback, an improved LSZ has been proposed (Yari and Khoshkhoo, 2019).

4.1 Limitations of voltage stability indices and potential improvements

1. Simplified system representation: voltage stability indices often rely on simplified system models and assumptions, which may not capture all aspects of system behavior accurately. To address this limitation, improved models and algorithms can be developed that consider more detailed system representations, accounting for factors such as non-linear loads, voltage-dependent models, and more accurate representations of distributed energy resources.

2. Uncertain input data: voltage stability indices depend on accurate input data, including load models, system parameters, and real-time measurements. However, uncertainties in data can impact the accuracy of the indices. Advanced data estimation and assimilation techniques can be employed to enhance the quality and accuracy of input data, thus improving the reliability of voltage stability indices.

3. Lack of dynamic information: voltage stability indices primarily focus on steady-state conditions and may not capture transient or dynamic effects that can impact voltage stability. Incorporating dynamic models and considering dynamic stability analysis methods can provide a more comprehensive assessment of voltage stability, addressing the limitations of static indices.

4. Scalability: voltage stability indices should be applicable to large-scale systems with complex network topologies and various operational scenarios. Efficient computational algorithms and parallel computing techniques can be utilized to ensure scalability and computational efficiency of the indices.

5. Integration of control actions: voltage stability indices can be improved by incorporating control actions and their impact on voltage stability. This includes the integration of online monitoring systems, automated control schemes, and advanced control strategies to maintain voltage stability in real-time.

In this section, the VSIs were listed based on different concepts such as maximum power flow, PMU and local measurements, voltage equation, *etc.* Here, information about the critical value of the index, formulae, whether the index is impedance dependent or not, and the applicability of stability indices to the transmission system (TS) and distribution system (DS) (Nguyen et al., 2021) is clearly shown in all the tables.

4.2 Indices based on maximum power flow equations

A single line of an interconnected network and the parameters of a line are considered, as shown in Figure 1A (Balamourougan et al., 2004). The model of power flow analysis and the apparent power expressed at the sending end bus and receiving end bus can be given by Eqs. 15, 16.

$$S_s = \frac{|V_s|^2}{Z}\angle\theta - \frac{|V_S||V_r|}{Z}\angle(\theta+\delta_1-\delta_2) \tag{15}$$

$$S_r = \frac{|V_s||V_r|}{Z}\angle(\theta-\delta_1+\delta_2) - \frac{|V_r|^2}{Z}\angle\theta \tag{16}$$

From Eq. 15 and Eq. 16, separating the true and reactive power we obtain

$$P_r = \frac{V_sV_r}{Z}cos(\theta-\delta_1+\delta_2) - \frac{V_r^2}{Z}cos\,\theta \tag{17}$$

$$Q_r = \frac{V_sV_r}{Z}sin(\theta-\delta_1+\delta_2) - \frac{V_r^2}{Z}sin\,\theta \tag{18}$$

By using Eq. 15 to Eq. 18, the maximum power transfer through a line and the VSIs can be derived and are listed in Table 2 for the line and Table 3 for the bus.

4.3 Indices based on PMU & local measurements

Advanced technology in smart devices such as PMU was introduced in the power system to monitor the power injections at all the buses and also to track the parameters of the system (i.e., current, voltage, *etc.*). Generally, PMUs are located on the generator side as well as on the load side. This smart device can measure the voltage and current phasor information, in discrete time intervals, of the system where it is installed. PMU measurements can be used to estimate the proximity of voltage collapse in the system. The VSI has been derived from load flow studies with the aid of data attained from PMUs, as given in Tables 4, 5. It provides an exact estimation of the strength/weakness of the system connected to the estimated capacity based on local measurements.

4.4 Indices based on the voltage equation of a transmission line

This section presents the indices based on the voltage equation of the transmission line (Ajjarapu, 2007). The bus voltage of the transmission system shown in Figure 3 is given in Eq. 19.

$$V_s = \sqrt{\left(V_r + \left(\frac{P_r r + Q_r x}{V_r}\right)^2 + \left(\frac{P_r x - Q_r r}{V_r}\right)^2\right)} \tag{19}$$

where V_s and V_r are the sending and receiving end voltages, respectively, P_r and Q_r are active and reactive power flowing from the sending end to the receiving end, respectively, and r and x are the equivalent resistance and reactance, respectively. Using the above equation, the various VSIs for lines and buses, along with their critical value and information about their dependency on the impedance, are elaborated in Tables 6, 7.

4.5 Indices based on maximum power transfer theorem

In this section, a detailed list of indices derived from the well-known maximum power transfer theorem (MPTT) is given. In AC systems, the MPTT states that the maximum power will be delivered from source to load when the load impedance becomes equal to the complex conjugate of the source impedance. These indices were derived by applying the condition of MPTT on the power flow equations given in Eq. 15 to Eq.18. Table 8 provides a clear picture of these line and bus indices, along with their critical value and information about their dependency on the impedance. Moreover, Eq. 20 (Gong et al., 2006) and Eq. 21 (Gubina and Strmčnik, 1995) are developed for transmission systems to check the status of critical lines presented in the system.

$$VSI_A = Min\left(\frac{P_{margin}}{P_{max}\frac{Q_{margin}}{Q_{max}\frac{S_{margin}}{S_{max}}}}\right) \tag{20}$$

$$TPSI = 0.5V_i - v_d \tag{21}$$

4.6 Indices based on the Jacobian matrix

This section deals with static voltage stability indices that are based on the Jacobian matrix of the power flow equations. These indices provide a clear view of the proximity to voltage instability and detail on the critical buses and areas. These VSIs are listed in Table 9. The critical value of these indices and the dependency on impedance are also mentioned in the table.

4.7 Indices based on the P-V curve

Using Eq. 1, the PV and QV curves can be drawn considering the constant power factor and constant real power, respectively (Ajjarapu, 2007). Moreover, the critical voltage and maximum power derived from the same equation are given in Eqs. 22, 23, respectively.

$$v_{crit} = \frac{1}{\sqrt{2}^{*}\sqrt{1+sin\,\phi}} \tag{22}$$

$$p\frac{cos\,\phi}{2(1+sin\,\phi)}_{max} \tag{23}$$

At the unity power factor (UPF), the maximum permissible power can be obtained by substituting $\phi = 0$ into Eq. 22 $p\frac{1}{2}_{max}$. Similarly, the value of the critical voltage at UPF obtained from Eq. 21 is $v_{crit} = \frac{1}{\sqrt{2}} = 0.707$. Various line and bus indices were derived based on the concept of the P-V curve and the maximum permissible power and critical voltage. Table 10 summarises the VSIs generated from the P-V curve. The rows 1–10 represent the bus indices and 11 and 12 represent line indices developed by the P-V curve.

4.8 Miscellaneous indices

In this section, the line and bus indices based on ABCD parameters of the transmission line, the combination of two indices, reactive power loss, transmission paths, *etc.*, other than those listed and described in the earlier sections, are discussed. The different types of miscellaneous line VSIs are listed in Table 11. The significant features of these indices are described in the following.

- The index $VCPI_A$ tracks information about the weakest transmission paths and the reactive power reserve of the network (Wang et al., 2005).
- The integrated transmission line transfer index (ITLTI) is computed with the help of the power factor of the receiving end load and the power angle between sending and receiving ends. This index provides reliable information on the critical margin of voltage instability of the power system (Chuang et al., 2016).
- The CSI index is a combination of PI and Lmn indices, and provides information about the real power and voltage stability of the system (Mishra et al., 2016).
- VSSI can be used to evaluate the short-term VS. of the load bus. This index is obtained from the fault-induced voltage sag (Dong et al., 2015).
- VSQI is derived with the help of a voltage stability curve to examine the voltage stability and to check the quality of the system voltage (Katsanevakis et al., 2019).

The different miscellaneous bus indices are given in Table 11. Some important features of these indices are listed in the following.

- The index Ip and Iq is obtained from the bifurcation technique. It is used to find the critical nodes in the power system during abnormal operating and contingency conditions (Fang et al., 2015).
- RPLI is derived from the reactive power support and loss allocation algorithm using the Ybus method under both normal and severe contingency conditions (Moger and Dhadbanjan, 2015).
- The DSY index is a derivative of the apparent load power with respect to its admittance. As the apparent load power and admittance can be evaluated via the measurement of current and voltage, this index is easily determined via the locally measured scalar values of *V* and *I*. This index is used to find the loadability limit of the system (Parniani and Vanouni, 2010).
- The influence index, which is derived from the SV curve, is used to find the critical margin of the power system under contingency conditions (De Moura and Prada, 2005).
- Short-term VSI is computed with the help of three components: SVSIr, SVSIo, and SVSIs. These three components reflect transient voltage restoration, transient voltage oscillation, and the ability to reach a steady state, respectively (Zhao, 2017).

5 Merits and demerits of line and bus VSIs

In this section, the merits, demerits, and applications of various line and bus VSIs are described in Table 12. Reactive power planning, load shedding, power system planning and control, online stability monitoring, *etc.*, are some of the different power system problems to which these indices can be applied in order to identify critical areas, buses, and lines. This may help power system engineers to operate the power system within safe limits. For further clarifications, readers may refer to (Furukakoi et al., 2018), in which a comparison based on the computation time taken by the indices was neatly described. Moreover (Yari and Khoshkhoo, 2019), provided a performance-based comparison of various VSIs.

6 Applications of VSIs

Voltage stability indices play a crucial role in assessing and managing voltage stability in real-world power systems. These indices provide quantitative measures of system stability and are used in various applications to ensure secure and reliable operation. The following are some common applications of voltage stability indices in real-world scenarios.

6.1 Voltage stability assessment

Voltage stability indices are employed to assess the voltage stability of power systems under different operating conditions. They provide a quantitative measure of the system's proximity to voltage collapse or instability. By analyzing these indices, system operators and planners can identify critical operating points, voltage stability limits, and potential voltage collapse scenarios.

6.2 System planning and expansion

Voltage stability indices are used in system planning and expansion studies to ensure the long-term reliability and security of power systems. These indices help determine the maximum load or generation levels that the system can support without voltage stability issues. By incorporating voltage stability considerations into planning studies, system operators can make informed decisions regarding system reinforcements, network upgrades, and capacity expansion to mitigate voltage stability concerns.

6.3 Contingency analysis

Voltage stability indices are utilized in contingency analysis to evaluate the impact of contingencies, such as generator or transmission line outages, on voltage stability. By calculating these indices for different contingency scenarios, operators can identify critical contingencies that may lead to voltage instability and take appropriate preventive or corrective measures, such as load shedding or re-dispatching, to maintain system stability.

6.4 Online monitoring and control

Voltage stability indices are used for real-time monitoring and control of power systems. By continuously computing these indices based on real-time measurements, operators can detect and identify deteriorating voltage stability conditions in the system. This allows for timely implementation of control actions, such as reactive power control, voltage regulation, and load shedding, to maintain voltage stability within acceptable limits.

6.5 Remedial action schemes

Voltage stability indices are utilized in the design and operation of remedial action schemes (RAS). RAS are automatic control systems that respond to voltage instability conditions by taking predefined corrective actions. These indices serve as triggers for activating specific RAS, such as fast-acting voltage control devices, capacitor banks, or generator tripping, to prevent voltage collapse or mitigate voltage stability issues.

6.6 Dynamic security assessment

Voltage stability indices are integrated into dynamic security assessment tools to evaluate the system's dynamic response under various disturbances or contingencies. These tools utilize real-time measurements and dynamic models to assess the system's transient stability and voltage stability. Voltage stability indices provide valuable information to operators for making critical decisions during emergency situations or system restoration processes.

The voltage stability indices have diverse applications in real-world power systems, ranging from voltage stability assessment and system planning to online monitoring, control, and dynamic security assessment. These indices enable operators and planners to make informed decisions, implement preventive and corrective measures, and ensure the reliable and secure operation of power systems in the face of voltage stability challenges.

7 Conclusion

To analyze the evolution over time of some of the proposed different voltage stability methods and to identify voltage collapse, a detailed survey of these methods was carried out in this paper. An introduction, a brief literature survey of the different voltage stability methods, and indices proposed by various researchers were presented. The classification of indices based on the disturbances that occurred in the power system, traditional methods such as PV-QV curve, sensitivty, model analysis, CPF methods, and Bifurcation tecniques for voltage stability, and their limitations and applications as well as conclusions were described in the next sections.

A clear description of the various line and bus indices derived based on different voltage stability constraints such as PMU and local measurements, the Jacobian matrix, PV curve, and concepts such as the maximum power flow equations of the transmission line, voltage equation of the transmission line, and maximum power transfer theorem were presented in the subsequent sections. Moreover, the merits, demerits, and applications of the various VSIs were briefed in the last section. Finally, this paper gives a deep insight into most of the voltage stability methods and indices in the literature. This can assist researchers in the future in selecting the optimal VSI for various applications.

8 Future scope

The future scope of this article is as follows.

1. Integration of artificial intelligence (AI) and machine learning: the incorporation of AI and machine learning algorithms into voltage stability assessment will enable systems to learn from historical data and real-time measurements. This will enhance the accuracy of voltage stability predictions, especially in dynamic and complex grids with high renewable energy penetration.
2. Cyber-physical security: future research should focus on developing voltage stability assessment methods that account for cyber-physical security risks. This includes protecting systems from cyberattacks that can compromise voltage stability.
3. Advanced sensors and IoT: the deployment of advanced sensors and Internet of Things (IoT) devices in the grid will provide a wealth of real-time data. Future voltage stability assessments should harness this data to improve monitoring and control strategies.
4. Grid modernization: with the ongoing modernization of power grids, future voltage stability assessment methods must adapt to accommodate the increased complexity introduced by smart grids, microgrids, and distributed energy resources.
5. Multi-objective optimization: researchers can explore multi-objective optimization techniques to simultaneously optimize voltage stability, economic operation, and environmental impact, considering factors such as renewable energy generation and grid resiliency.
6. Energy storage integration: as energy storage systems become more prevalent, future assessments should incorporate the dynamic capabilities of energy storage devices to enhance voltage stability and grid reliability.
7. Real-time adaptive control: develop real-time adaptive control strategies that can dynamically respond to changing grid conditions, such as fluctuations in renewable generation, to maintain voltage stability.
8. Decentralized control: investigate decentralized control strategies that allow distributed energy resources, such as solar panels and batteries, to participate actively in voltage stability management.
9. Standardization and regulatory frameworks: future efforts should include the development of standardized methods for voltage stability assessment, facilitating consistent practices across utilities and regulatory bodies.
10. Cross-disciplinary collaboration: encourage collaboration between power system engineers, data scientists, cybersecurity experts, and policymakers to develop holistic approaches to voltage stability assessment, considering both technical and regulatory aspects.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

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