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Sustainable ferry leasing strategies: the option contract perspective

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Ferry demand fluctuates unpredictably across different seasons and holidays, posing significant scheduling challenges for operators and resulting in high operating costs and increased carbon emissions. To adapt to market demand variations, ferry operators often supplement their own fleets with leased vessels. Therefore, this paper explores sustainable leasing strategies between ferry leasing companies and operators under uncertain demand conditions, aiming to maximize ferry utilization efficiency. First, this paper develops leasing models under four contract types: wholesale pricing, unilateral options (call and put), and bidirectional options (a classic game-theoretic approach for optimizing decisions under demand fluctuations). Subsequently, it determines the optimal number of leased ferries for each strategy. Then, this paper conducts a comparative analysis of the four contracts, supplemented by sensitivity analysis. Finally, it examines the scenario where an operator purchases ferries instead of leasing them. A case study of a high-speed passenger ferry company in Zhuhai demonstrates that option contracts can mitigate demand uncertainty, thereby improving fleet utilization. The bidirectional option proves more flexible than the unilateral option. However, leasing is not always preferable to purchasing. The findings provide sustainable insights for ferry operators in designing leasing strategies, ultimately reducing operating costs and carbon emissions.

KEYWORDS
ferry leasing, sustainability, uncertain demand, unilateral option, bidirectional option

1 Introduction

Environmental and climate problems caused by carbon emissions are becoming increasingly serious. To this end, many countries and regions around the world have taken some measures to reduce carbon emissions in transportation (Taghvaee et al., 2017; Mohamad Taghvaee et al., 2023). The current carbon emissions mitigation measures, primarily concentrate on land, more or less neglecting the effects of ferry transportation, which is a substantial contributor to carbon emissions in the transportation sector (Taghvaee et al., 2019). In recent years, with the continuous development of the marine economy and coastal tourism, the role of ferry transportation has gradually increased (Tsoi and Loo, 2021). Indeed, ferry transport services are essential for the coastal tourism economy with strong and resilient public demand. In other words, ferry transport is an important (or only) way for island tourists and residents to travel or freight (Chu et al., 2020). However, while ferry transportation provides convenient travel services, it also brings a lot of carbon emissions (Anwar et al., 2020; Wu et al., 2022). Although the current development of the ferry operation system is relatively complete, it is still challenging to reduce the impact of uncertainty in the ferry passenger market when planning ferry capacity. Improper capacity planning will reduce ferry utilization, resulting in higher operating costs and carbon emissions, contradicting strategies for developing sustainable transport. Therefore, this paper aims to seek sustainable ferry leasing strategies between ferry leasing companies and ferry operators under uncertain market demand, so as to maximize the utilization of ferries.

As we all know, the ferry passenger transport market is affected by seasons, weather, holidays and other factors, so the market demand faced by ferry operators is uncertain (Chu et al., 2020). Furthermore, due to the high purchase and maintenance costs of ferries, many operators usually choose to fully (or partially) lease ferries to operate. However, the uncertainty of market demand has brought great difficulties to operators' ferry leasing. Ferry leasing is a long and complicated procedure. Generally, the leasing contract is signed by both parties ex-ante, and the ferry cannot be renewed or returned ex-post. If the operator leases more ferries than the market demand, some ferries will be idle, resulting in a waste of funds. On the contrary, if the operator leases fewer ferries than the market demand, part of the passengers cannot be served, resulting in an inevitable loss. Therefore, whether the number of leased ferries is too much or too little will cause a profit loss for ferry operators. The uncertainty of market demand is one of the most fundamental reasons operators lease too many/too few ferries (Cheng et al., 2021). Therefore, the main problem faced by ferry operators is how to design ferry leasing contracts to reduce (or eliminate) the loss of interest caused by uncertain demand.

The literature on ferry transportation is very rich, and current research mainly focuses on optimization strategies of ferry routes (Lai and Lo, 2004; Wang and Lo, 2008; An and Lo, 2014; Karapetyan and Punnen, 2015; Ng and Lo, 2016; Škurić et al., 2021), container leasing strategies (Liu et al., 2013; Luo et al., 2021), competition and cooperation strategies in the ferry service supply chain (Liu and Wang, 2019; Choi et al., 2020; Xu et al., 2021; Xu et al., 2024b), and the impact of carbon emissions on ferry transportation (Psaraftis and Kontovas, 2010; Wu et al., 2024; Xiao et al., 2024; Xu et al., 2024c), ferry capacity allocation strategy (Xing et al., 2023; Yi et al., 2023; Xu et al., 2024a) and many other topics.

Game theory and optimization methods are the two main methods for studying ferry transportation related issues, and this article adopts game analysis methods. Research on the container leasing strategies, Liu et al. (2013) tackles the container planning problem from the carrier's perspective in a two-echelon container shipping service chain (CSSC), which includes one carrier and one upstream rental company. A flexible contract with options is introduced into the one-period container planning mechanism. Luo et al. (2021) considered a container transportation service chain (CTSC) consisting of container leasing companies, carriers, freight forwarders, ports, and shippers. They introduced a one-period two-level option contract into the empty container ordering problem from the perspective of freight forwarders. Research on the competition and cooperation strategies in the ferry service supply chain, Liu and Wang (2019) studied the motivations for horizontal alliances and the value of vertical cooperation between two competing carriers in a one-to-two shipping service competition model. They found that alliances can reduce carrier service competition, lower port service prices, and weaken the port's monopoly advantage. Choi et al. (2020) used the mean-risk formulation to analyze the impact of risk attitude and demand fluctuations on the service pricing game between two Container-shipping-lines (CSLs) and found that when container shipping companies are able to accept greater risks, the equilibrium price will rise, while slight risk seeking helps maximize the expected profits of both CSLs. Xu et al. (2021) studied the equilibriums of platform encroachment and price matching in a sea-cargo supply chain with a liner company and two asymmetric forwarders. Furthermore, Xu et al. (2024b) introduced independent freight forwarders and subsidiary freight forwarders into the shipping supply chain and explored the interaction between carrier channel encroachment and freight forwarder ordering timing. Research on the impact of carbon emissions on ferry transportation, Psaraftis and Kontovas (2010) studied the impact of various maritime emission reduction policies on maritime logistics. Wu et al. (2024) developed a game model to analyze the motivations of two competing carriers to adopt port-of-call strategies and the effects of The European Union Emissions Trading System (EU ETS). They also conducted an empirical study based on European routes and explored potential ways to improve the effects of the EU ETS. Xu et al. (2024c) introduced the data of 19 coastal countries in the European Union (EU) to build the weighting matrices and spatial Durbin model to reflect the spatial-temporal characteristics and driving factors. The observation results showed that the carbon emissions from shipping trade in the EU coastal countries have a positive spatial correlation and spatial clustering. Xiao et al. (2024) discussed the topic of carbon neutralization in sustainable port and shipping, emphasizing that efforts of the port and shipping industry to achieve carbon peak and carbon neutrality through high-quality advancement, corporate social responsibility, and the integration of green and low-carbon development principles. Research on the ferry capacity allocation strategy, Xing et al. (2023) established a game-theoretic model to explore the strategic procurement of logistics services in a shipping supply chain, where a freight forwarder canvasses orders and provides one-stop service to shippers. Yi et al. (2023) constructed a maritime supply chain with one port and two carriers, and analyzed how risk-aversion behavior and contract unobservability impact the pricing and contract preferences. Xu et al. (2024a) considered a freight market with differentiated capital consisting of a shipping company and two freight forwarding companies, and explored the equilibrium of capacity allocation caused by sudden external event.

A review of the literature shows that although there are many related studies on ferry operation strategies, few scholars have paid attention to the research on ferry leasing strategies, which is an essential but currently ignored issue. Compared with container leasing, a significant feature of ferry leasing is that the ship scheduling procedure is complex and requires prior planning, and leased ferries generally cannot be sublet (Cheng et al., 2021).

As an effective tool to deal with supply chain risks (such as demand uncertainty, price volatility, unreliable supply, etc.), options contracts have been widely used in many fields. For example, the telecommunications industry (Wang et al., 2017; Wang et al., 2018), the supply chain of fresh agricultural products (Zhao et al., 2013; Chen et al., 2017; Yang et al., 2017), emergency material reserves (Liang et al., 2012; Rabbani et al., 2015; Wang et al., 2015), the ferry container industry (Liu et al., 2013; Luo et al., 2021), etc. Although there are many pieces of literature on options contracts, few scholars have applied option contracts to the field of public transportation. With the increasing role of ferries, the phenomenon of ferry leasing is more prevalent in the ferry market, and there is currently less research on ferry leasing (Cheng et al., 2021). Therefore, this paper will take the ferry as the research object, and study several strategies of ferry leasing in the ferry market, such as wholesale price contract, call option contract, put option contract and two-way option contract, so as to obtain the best leasing strategy of ferry operators. Moreover, the paper considers the case of purchasing a ferry operation in order to find the basis for a ferry operator to choose a ferry purchasing (or leasing) strategy. Finally, we validated our derived model using a real case study of Zhuhai High Speed Ferry Company. The options contracts considered in this paper are similar to those studied in (Nosoohi and Nookabadi, 2016), but there are some important differences. First, their research is focused on logistics service supply chains, whereas this paper focuses on ferry planning problems for ferry operators. Due to the complex scheduling procedures for ferries, excess ferries have no residual value and cannot be returned to the rental Company. Secondly, the seasonality of the ferry market is obvious, and the demand is uncertain. Demand forecasting is critical to ferries scheduling, i.e., operators need to also consider the risk of ferries being out of stock or idling. Aggressive operators will see a peak season in the passenger market, while conservative operators will see a low season in the passenger market, and options contracts can be used to address these issues, thereby reducing the risk of an imbalance in ferry leases. Aggressive ferry operators can buy call options contracts to reduce the risk of ferry shortages, while conservative ferry operators can buy put options to reduce the risk of ferry idling. Furthermore, they only study call and bidirectional options contracts that respond to a surge in demand. In addition to call and bidirectional options contracts, this paper also examines put option contracts that respond to a sudden drop in demand.

In particular, this paper mainly addresses the following questions: (1) Compared with wholesale price contracts, can option contracts reduce the impact of market demand uncertainty, thereby enhancing the profitability and efficiency of ferry operators? (2) What is the difference between call, put, and bidirectional option contracts? (3) Is it more beneficial for a ferry operator to purchase a ferry than to lease a ferry?

This paper summarizes our contributions as follows:

First, this paper develop a ferry leasing model for ferry operators, examining the impact of demand uncertainty on leasing ferry strategies. Second, this paper develop different options contracts to reduce the impact of demand uncertainty, obtain equilibrium strategies for each option contract, and elucidate the effects of different parameters on options contracts in numerical experiments. Finally, this paper discuss the circumstances of purchasing a ferry operation and arrive at the basis for a ferry operator to choose to purchase or lease a ferry. The research results of this paper will provide decision-making basis for ferry operators to formulate the best ferry leasing strategy, thereby improving the utilization rate of ferries and reducing carbon emissions. This will promote the sustainable development of ferry transportation and increase the benefits of multiple participants in the ferry transportation system, such as ferry operators, passengers, and government departments.

This paper is organized as follows. Section 2 describes the research problem and explains the associated assumptions. In Section 3, we solve the equilibrium strategies of ferry operators under four scenarios: wholesale price contracts, call option contracts, put option contracts, and bidirectional option contracts. Section 4 conducts numerical experiments and sensitivity analyses and summarizes the corresponding managerial implications. Finally, Section 5 concludes the full text and proposes future research directions. We give proof of all results in the Appendix.

2 Problem description

Consider a single-cycle and two-stage ferry leasing supply chain consisting of a ferry leasing Company (Company A) and a ferry operator (Company B). Company B leases ferries from Company A to maintain its daily passenger transportation. Since the passenger ferry market is greatly affected by seasons, weather, and holidays, the market demand faced by Company B is uncertain (Chu et al., 2020). Company B can only judge the weak and peak seasons of the ferry passenger transport market based on previous experience and cannot estimate the change in market demand ex-ante. According to the market research results, the wholesale price contract is a ferry leasing strategy commonly adopted by Company B. Company B wholesales ferries from Company A before the market demand changes. Due to the complicated scheduling procedure of the ferry, the fatal disadvantage of adopting the wholesale price contract is that Company B cannot renew the lease or return the ferries after the market demand changes, resulting in the phenomenon that the ferry is out of stock or idle (Cheng et al., 2021). To overcome the shortcoming of wholesale price contracts, Company B can use option contracts to lease ferries.

Like most literature on options contracts (Nosoohi and Nookabadi, 2016; Liu et al., 2021), the ferry leasing problem considering option contracts also consists of two stages. In the first stage, Company B leases a certain number of ferries from Company A to meet the potential market demand; this stage is ex-ante ferry leasing. Due to the uncertainty of demand, Company B and Company A signed the option contract for ferry leasing in the second stage. Following a change in demand, Company B decides to execute an option contract, an ex-post ferry leasing. Note that in this paper, we assume that the ferries required by Company B are all leased from Company A, i.e., we do not consider the ferries owned by Company B. As mentioned earlier, due to the complicated schedule of ferries, Company B will not have the opportunity to replenish or refund the ferry once the demand changes. Therefore, Company B needs to decide before the demand changes, which is a significant feature of products with a long supply cycle, short sales season, and uncertain demand (Nosoohi and Nookabadi, 2016). In reality, operators can purchase ferries to operate. Therefore, we also discussed the situation in which Company B purchased the ferry operation.

2.1 Notation and assumptions

The variables and parameters used in this paper are shown in Table 1.

Without loss of generality, this paper makes the following reasonable assumptions:

Assumption 1: Company A and Company B are risk-neutral decision-makers, and their decisions are entirely rational (Liu et al., 2021).

Assumption 2: This paper considers only one type of ferry. There are different types of ferries in practice. For the problem of multiple types of ferries, the optimal order quantity of each type of ferries can be obtained through the model in this paper (Liu et al., 2013).

Assumption 3: Each ferry's operating profit (or cost) is the same. Although the operating profit (or cost) of a ferry is related to factors such as its passenger capacity, route, etc., this paper examines the optimal number of ferries to lease, assuming for simplicity that the operating profit (or cost) of each ferry is the same (Liu et al., 2021).

Assumption 4: The number of ferries owned by Company B is zero. Company B prefers to lease ferries because purchasing ferries requires higher procurement and maintenance costs. Therefore, assume that the number of ferries owned by Company B is zero. Even if Company B owns a small number of ferries, it can be subtracted from the optimal quantity of the leased ferries (Liu et al., 2013).

Assumption 5: All prices in this paper are exogenous variables, so they are all constants. According to Arani et al. (2016) and Yuan et al. (Yuan et al., 2020), the price in this paper satisfies the following relationship: 1) $w > w_1 > 0$ is the premise to incentivize Company B to buy the option contract, 2) $p - c > w_1 + p_c > w$ is to guarantee the profit of Company A and Company B, 3) $w > p_p - w_1 > 0$ is the premise to prevent Company B from executing put option arbitrage.

Although some of the assumptions listed above are conducive to the establishment and derivation of the model in this paper, they are not in line with our real life. Because the real environment cannot be so ideal, this is a limitation of the research in this paper and also a common problem existing in many related studies in the field of game theory (Liu et al., 2013).

TABLE 1 Notation used in this paper.

Notation	Explanation
D	Market demand, D is continuous and derivable
$f(\cdot)$	The probability density function of D
$F(\cdot)$	The cumulative distribution function of D, $F(\cdot)$ is non-negative, reversible, strictly increasing, and $F(0) = 0$
μ	The expected value of D
p	Per ferry's revenue
w	Per ferry's wholesale prices
w_1	Per ferry's option prices
a	Ferry order quantity in the wholesale contract
b_1	Ferry initial order quantity in the call option contract
b_2	Ferry option order quantity in the call option contract
c_1	Ferry initial order quantity in the put option contract
c_2	Ferry option order quantity in the put option contract
d_1	Ferry initial order quantity in the bidirectional option contract
d_2^c	Ferry call option order quantity in the bidirectional option contract
d_2^p	Ferry put option order quantity in the bidirectional option contract
e	Ferry purchase quantity
p_c	Strike price of the call option
p_p	Strike price of the put option
g	Ferry out of stock cost
c_l	Ferry idle cost
c	Unit operating cost of leasing a ferry
c_s	Unit operating cost of purchasing a ferry
π_0	Company B's profit in the wholesale price contract
π_c	Company B's profit in the call option contract
π_p	Company B's profit in the put option contract
π_b	Company B's profit in the bidirectional option contract
π_s	Company B's profit when purchasing ferries operation

2.2 Decision processes

The decision processes of the four ferry leasing contracts (i.e., wholesale price contracts, call option contracts, put option contracts and bidirectional option contracts) are as follows:

2.2.1 Case 1: the wholesale price contract

Company B leases ferries of quantity a at the wholesale price w, which is determined by Company A. When the market demand changes, if the demand increases, Company B will have insufficient distribution capacity and bear certain out-of-stock cost g. And if demand decreases, the ferries leased by Company B will be partially idle.

2.2.2 Case 2: the call option contract

Stage 1: Before demand increases

1.1. Company B leases ferries of quantity b_1 at the wholesale price w.
1.2. Company B signs a call option contract with Company A, the quantity of options is b_2, and the option price is w_1, where the option price w_1 is determined by Company A.

Stage 2: After demand increases

2.1. Based on the change in demand, Company B decides whether to exercise the call option at the price p_c.

1. If the increased demand is less than the initial lease quantity of Company B, there will be some idle ferries, and Company B will not exercise the call option. Note that idle ferries have no surplus value.
2. If the increased demand is greater than Company B's initial lease quantity, Company B exercises the call option at the price p_c.
3. If the increased demand is greater than the sum of Company B's initial lease quantity and options order quantity, some tourists cannot be served, and Company B will bear the out-of-stock cost g.

2.2. End leasing.

2.2.3 Case 3: the put option contract

Stage 1: Before demand decreases

1.1. Company B leases ferries of quantity c_1 at the wholesale price w.
1.2. Company B signs a put option contract with Company A, the quantity of options is c_2, and the option price is w_1.

Stage 2: After demand decreases

2.1. Based on the change in demand, Company B decides whether to exercise the put option at the price p_b.

1. If the reduced demand is less than the difference between firm B's initial order quantity and the options order quantity, firm B exercises the put option at the price p_b. In addition, there are some idle ferries, and the idle ferries have no surplus value.
2. If the reduced demand is greater than the difference between firm B's initial order quantity and the options order quantity, firm B exercises the put option at price p_b.
3. If the reduced demand is greater than Company B's initial order quantity, Company B will not exercise the put option. And some tourists cannot be served, Company B will bear the out-of-stock cost g.

2.2. End leasing.

2.2.4 Case 4: the bidirectional option contract

Stage 1: Before demand changes

1.1. Company B leases ferries of quantity d_1 at the wholesale price w.
1.2. Company B signs a bidirectional option contract with company A, the quantity of options is (d_2^c, d_2^p), and the option price is w_1.

Stage 1: After demand changes

2.1. Based on the change in demand, Company B decides whether to exercise the bidirectional option.

1. If the changed demand is less than the difference between firm B's initial order quantity and the put options order quantity, firm B exercises the put option of the bidirectional option at price p_b. In addition, there are some idle ferries, and the idle ferries have no surplus value.
2. If the changed demand is greater than the difference between Company B's initial order quantity and the put options order quantity, Company B exercises the put option of the bidirectional option at price p_b.
3. If the changed demand is greater than Company B's initial order quantity, Company B exercises the call option of the bidirectional option at price p_c.
4. If the changed demand is greater than the sum of Company B's initial order quantity and the call option order quantity, some tourists will be lost, and Company B will bear the out-of-stock cost g.

2.2. End leasing.

The decision processes of ferry leasing are shown in Figure 1.

3 Models and equilibrium solutions

In this section, we first develop the ferry leasing model of the wholesale price contract and take it as the benchmark model. Then, we also developed different types of option (including call option, put option and bidirectional option) contract to analyze the optimal ferry leasing strategy of Company B. Finally, we study the situation of Company B purchasing the ferry operation. The optimal leasing and purchasing strategies for ferries derived in this section will provide ferry operators with the best decision-making choices, which was overlooked in previous literature.

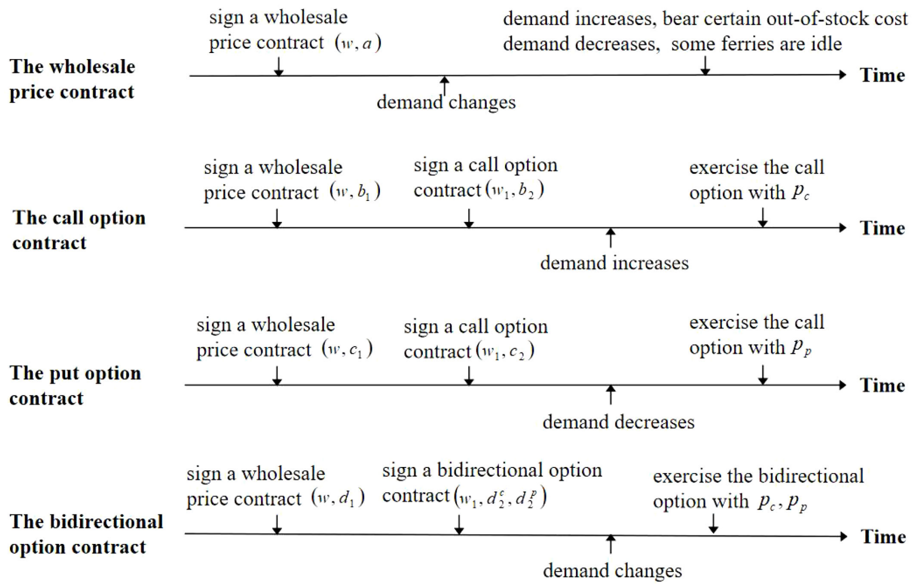

FIGURE 1
The decision processes of ferry leasing.

3.1 The wholesale price contract

Since the wholesale price contract is the typical contract used in the supply chain and the only contract currently employed in the ferry lease market, it is used as a benchmark for comparing option contracts. In practice, a wholesale price contract is a particular case of an option contract in which Company B does not have the flexibility to adjust its initial ferry order quantity. In the wholesale price contract, Company B leases ferries of quantity a at the wholesale price w from Company A. At this point, referring to (Liu et al., 2013) and (Luo et al., 2021), the profit function π_0 of Company B can be deduced as

$$\pi_0 = p\min(a, D) - wa - c\min(a, D) - g[D - a]^+ \tag{1}$$

In Equation 1, the first term on the right side is the revenue generated by the operation of the ferry, the second term is the rental cost of the ferry, the third term is the operating cost of the ferry, and the fourth term is the out-of-stock cost of the ferry. As mentioned above, due to the complicated schedule of ferries, Company B will not have the opportunity to replenish ferries once the demand changes and the idle ferries cannot be sublet, so Company B has a shortage cost.

According to Equation 1, we can get the expected profit of Company B $E[\pi_0]$

$$E[\pi_0] = (p + g - c)\left(a - \int_0^a F(x)dx\right) - wa - g\mu \tag{2}$$

By solving the first derivative of $E[\pi_0]$ (i.e. Equation 2) with respect to a, we can get

$$\frac{dE[\pi_0]}{da} = (p + g - c)(1 - F(a)) - w \tag{3}$$

Then, by solving the equation $\frac{dE[\pi_0]}{da} = 0$, we can get the optimal number of ferries leased by Company B in the wholesale price contract.

Proposition 1: In the wholesale price contract, the Company B's optimal quantity of ferries leased is $a^* = F^{-1}(1 - \frac{w}{p+g-c})$.

Proof. For readability, all the proofs are included in the Appendix.

According to Proposition 1, we can know that in the wholesale price contract, the number of ferry rentals of Company B is affected by the wholesale price. More precisely, the larger the wholesale price w, the smaller the number a of ferry rentals by Company B. Uncertainty in the ferry market demand will cause changes in wholesale prices and ferry numbers. Therefore, reasonable wholesale prices are crucial to the number of ferry rentals.

3.2 The call option contract

Suppose Company B is optimistic about future market demand changes (peak season approaching). In that case, it will buy call options contracts from Company A, and each call option gives Company B the right to lease a ferry to Company A at the agreed option strike price during the sales season. Company B's option exercise quantity cannot exceed its purchase quantity, and Company A guarantees the supply (i.e., Company A's supply cannot be less than Company B's total order quantity). In the wholesale price contract, Company B alone bears the risks of uncertain ferry demand, price fluctuations and unreliable supply. In contrast, the call option contract provides flexibility, price stability and supply reliability for Company B to obtain the ferry.

In the call option contract, Company B firstly formulates the initial ferry lease quantity b_1, then the call option order quantity b_2, and finally decides whether to exercise the call option at price p_c based on the change of demand. At this point, referring to (Liu et al., 2013) and (Luo et al., 2021), the profit function π_c of Company B can be deduced as

$$\pi_c = p\min(b_1+b_2,D) - wb_1 - w_1b_2 - p_c\min[(D-b_1)^+, b_2] - c\min(b_1+b_2,D) - g[D-b_1-b_2]^+ \quad (4)$$

In Equation 4, the first item on the right is the income generated by the operation of the ferry, the second item is the fixed order cost, the third item is the order cost of the call option, the fourth item is the exercise cost of the call option, the fifth item is the operating cost of the ferry, and the sixth item is the shortage cost of the ferry.

According to Equation 4, we can get the expected profit of Company B $E[\pi_c]$

$$E[\pi_c] = (p+g-c-p_c)\left(b_1+b_2-\int_0^{b_1+b_2}F(x)dx\right) + p_c\left(b_1-\int_0^{b_1}F(x)dx\right) - wb_1 - w_1b_2 - g\mu \quad (5)$$

By solving the first derivatives of $E[\pi_c]$ (i.e. Equation 5) with respect to b_1 and b_2 respectively, and setting them equal to zero, we can obtain

$$\frac{\partial E[\pi_c]}{\partial b_1} = (p_c+c-p-g)F(b_1+b_2) - p_cF(b_1) - c + g + p - w = 0 \quad (6)$$

$$\frac{\partial E[\pi_c]}{\partial b_2} = (p_c+c-p-g)(F(b_1+b_2)-1) - w_1 = 0 \quad (7)$$

Then, we can derive the optimal ferry leasing strategy for Company B under the call option contract by solving the above two equations.

Proposition 2: In the call option contract, the Company B's optimal initial leasing ferries quantity is $b_1^* = F^{-1}(1-\frac{w-w_1}{p_c})$ and the optimal total leasing ferries quantity is $b_1^*+b_2^* = F^{-1}(1-\frac{w_1}{p+g-p_c-c})$.

Proposition 3: In the call option contract, we have the following conclusion

1. The Company B's optimal initial leasing ferries quantity b_1^* is negatively related to the wholesale price w and positively related to the option price w_1.
2. The Company B's optimal total leasing ferries quantity $b_1^*+b_2^*$ is negatively related to the option price w_1.

Propositions 2 and 3 indicate that in the call option contract, the Company B's leasing ferries quantity is affected by the wholesale price w and the option price w_1, where both w and w_1 are determined by Company A. More precisely, the larger the wholesale price w is, the Company B will reduce the initial order quantity of ferries, but increase the option order quantity. Conversely, the larger the option price w_1 is, the Company B will increase the initial order quantity of ferries, but reduce the option order quantity. Note that increasing w_1 will reduce the total number of ferries leased by Company B. The above conclusion can be interpreted as follows: for Company B, the increase in wholesale price will reduce its willingness to use the wholesale price contract, and the increase in option order price will reduce its willingness to use the option contract.

3.3 The put option contract

Suppose Company B is pessimistic about future market demand changes (low season approaching). In that case, it will buy put options contracts from Company A, and each put option gives Company B the right to return a ferry to Company A at the agreed option strike price during the sales season.

Similarly, In the put option contract, Company B firstly formulates the initial ferry lease quantity c_1, then the put option order quantity c_2, and finally decides whether to exercise the put option at price p_p based on the change of demand. At this point, referring to (Liu et al., 2013) and (Luo et al., 2021), the profit function π_p of Company B can be deduced as

$$\pi_p = p\min(c_1,D) + p_p[\min(c_1-D,c_2)]^+ - wc_1 - w_1c_2 - c\min(c_1,D) - g[D-c_1]^+ \quad (8)$$

In Equation 8, the first item on the right is the income generated by the operation of the ferry, the second item is the income generated when the put option is exercised, the third item is the fixed order cost, the fourth item is the order cost of the put option, the fifth item is the operating cost of the ferry, and the sixth item is the out-of-stock cost of the ferry.

According to Equation 8, we can get the expected profit of Company B $E[\pi_p]$

$$E[\pi_p] = (p+g-c-p_p)\left(c_1-\int_0^{c_1}F(x)dx\right) + p_p\left(c_1-\int_0^{c_1-c_2}F(x)dx\right) - wc_1 - w_1c_2 - g\mu \quad (9)$$

By solving the first derivatives of $E[\pi_p]$ (i.e. Equation 9) with respect to c_1 and c_2 respectively, and setting them equal to zero, we can obtain

$$\frac{\partial E[\pi_p]}{\partial c_1} = (p+g-c-p_p)(1-F(c_1)) + p_p(1-F(c_1-c_2)) - w = 0 \quad (10)$$

$$\frac{\partial E[\pi_p]}{\partial c_2} = p_pF(c_1-c_2) - w_1 = 0 \quad (11)$$

Then, we can derive the optimal ferry leasing strategy for Company B under the put option contract by solving the above two equations.

Proposition 4: In the put option contract, the Company B's optimal initial leasing ferries quantity is $c_1^* = F^{-1}(1-\frac{w+w_1-p_p}{p+g-c-p_p})$ and the optimal option leasing ferries quantity is $c_2^* = F^{-1}(1-\frac{w+w_1-p_p}{p+g-c-p_p}) - F^{-1}(\frac{w_1}{p_p})$.

Proposition 5: In the put option contract, we have the following conclusion

1. The Company B's optimal initial leasing ferries quantity b_1^* is negatively related to the wholesale price w and the option price w_1.
2. The Company B's optimal option leasing ferries quantity c_2^* is positively related to the option price w_1.

Similarly, Proposition 4 and Proposition 5 show that in the put option contract, the Company B's initial leasing ferries quantity is affected by the wholesale price w and the option price w_1. More precisely, the larger the wholesale price w (or the option price w_1) is, the Company B will reduce the initial order quantity of ferries. Moreover, the larger the w_1, the greater the quantity of ferry operations $(c_1^* - c_2^*)$. This is because with w_1 increases, both the initial order quantity c_1^* and the option order quantity c_2^* decrease, while c_1^* decreases more than c_2^*, resulting in $c_1^* - c_2^*$ increase.

3.4 The bidirectional option contract

Suppose Company B is uncertain about future changes in market demand. In that case, it will purchase bidirectional option contracts from Company A, and each bidirectional option gives Company B the right to lease or return a ferry to Company A at the agreed option strike price during the selling season.

Similarly, In the bidirectional option contract, Company B firstly formulates the initial ferry lease quantity d_1, then the option order quantity (d_2^c, d_2^p), and finally decides whether to exercise the call (or put) option at price p_c(or p_p) based on the change of demand. referring to (Liu et al., 2013) and (Luo et al., 2021), the profit function

π_b of Company B can be deduced as

$$\pi_b = p\min(d_1, D) + (p - p_c)\min([D - d_1]^+, d_2^c) + p_p[\min(d_1 - D, d_2^p)]^+ - wd_1 - w_1(d_2^c + d_2^p) - c[\min(d_1, D) + \min([D - d_1]^+, d_2^c)] - g[D - d_1 - d_2^c]^+ \quad (12)$$

In Equation 12, the first term on the right side is the revenue generated by the fixed subscription ferry operation, the second term is the revenue of the ferry operation when the bidirectional option is exercised as a call option, and the third term is the revenue when the bidirectional option is exercised as a put option, the fourth item is the fixed subscription cost, the fifth item is the option subscription cost, the sixth item is the operating cost of the ferry, and the seventh item is the out-of-stock cost of the ferry.

According to Equation 12, we can get the expected profit of Company B $E[\pi_b]$

$$E[\pi_b] = (p + g - c)\left(d_1 + d_2^c - \int_0^{d_1+d_2^c} F(x)dx\right) - p_c\left(d_2^c - \int_{d_1}^{d_1+d_2^c} F(x)dx\right) + p_b\int_{d_1-d_2^p}^{d_1} F(x)dx - wd_1 - w_1(d_2^c + d_2^p) - g\mu \quad (13)$$

By solving the first derivatives of $E[\pi_b]$ (i.e. Equation 13) with respect to d_1, d_2^c, and d_2^p respectively, and setting them equal to zero, we can obtain

$$\frac{\partial E[\pi_b]}{\partial d_1} = (p + g - c)(1 - F(d_1 + d_2^c)) + p_b(F(d_1) - F(d_1 - d_2^p)) - p_c(F(d_1) - F(d_1 + d_2^c)) - w = 0 \quad (14)$$

$$\frac{\partial E[\pi_b]}{\partial d_2^c} = (p + g - c)(1 - F(d_1 + d_2^c)) - p_c(1 - F(d_1 + d_2^c)) - w_1 = 0 \quad (15)$$

$$\frac{\partial E[\pi_b]}{\partial d_2^p} = p_pF(d_1 - d_2^p) - w_1 = 0 \quad (16)$$

Then, we can derive the optimal ferry leasing strategy for company B under the bidirectional option contract by solving the above three equations.

Proposition 6: In the bidirectional option contract, the Company B's optimal initial leasing ferries quantity is $d_1^* = F^{-1}(1 - \frac{w-p_p}{p_c-p_p})$, the optimal call option leasing ferries quantity is $d_2^c* = F^{-1}(1 - \frac{w_1}{p+g-c-p_p}) - F^{-1}(1 - \frac{w-p_p}{p_c-p_p})$, and the optimal put option leasing ferries quantity is $d_2^p* = F^{-1}(1 - \frac{w-p_p}{p_c-p_p}) - F^{-1}(\frac{w_1}{p_p})$.

Proposition 7: In the bidirectional option contract, we have the following conclusion

1. The Company B's optimal initial leasing ferries quantity d_1^* is negatively related to the wholesale price w.
2. The Company B's optimal total leasing ferries quantity under the call option (i.e., $d_1^* + d_2^c*$) is negatively related to the option order price w_1.
3. The Company B's optimal total leasing ferries quantity under the put option (i.e., $d_1^* - d_2^p*$) is positively related to the option order price w_1.

Similarly, Proposition 6 and Proposition 7 show that in the bidirectional option contract, the Company B's leasing ferries quantity is affected by the wholesale price w and the option price w_1. In particular, in the bidirectional option contract, when the option order price increases, Company B will increase the order quantity of put options but decrease the order quantity of call options. In fact, Proposition 7 is a generalization of Proposition 3 and Proposition 5.

According to the above results, in the bidirectional option contract, we can get $d_1^* - d_2^p* \le d_1^* \le d_1^* + d_2^c*$, Company B thus can decide its option ordering strategy based on changes in market demand. If Company B predicts that the market will be a peak season in the future, it will not buy the put option in the bidirectional option contract. Similarly, if Company B predicts that the market will be low season in the future, it will not buy the call option in the bidirectional option contract. If Company B is uncertain about future market changes, it will buy both call and put options. The bidirectional option increases the flexibility of the online Company B's profitability. We summarize this insight in the following result.

Proposition 8: Company B can choose options according to the following rules

1. When $d_2^c{}^* = d_2^p{}^* = 0$, choose the unilateral option (call or put).
2. When $d_1^* - d_2^p{}^* < d_1^* < d_1^* + d_2^c{}^*$, choose the bidirectional option.
3. When $d_1^* - d_2^p{}^* = d_1^* = d_1^* + d_2^c{}^*$ and $d_2^c{}^* = d_2^p{}^* \neq 0$, the option does not exist.

In fact, Proposition 8 is a theoretically derived proposition without mathematical derivation, which aims to illustrate the flexibility of bidirectional options. In particular, in Case (2), we have $d_1^* = F^{-1}(1 - \frac{w - p_p}{p_c - p_p})$, $d_2^c{}^* = F^{-1}(1 - \frac{w_1}{p+g-c-p_p}) - d_1^*$, and $d_2^p{}^* = d_1^* - F^{-1}(\frac{w_1}{p_p})$. When $d_1^* - d_2^p{}^* = d_1^*$, we have $d_2^p{}^* = 0$, which means that the order quantity of the put option in the bidirectional option contract is zero, and the bidirectional option can be regarded as the call option. Similarly, when $d_1^* = d_1^* + d_2^c{}^*$, we have $d_2^c{}^* = 0$, which means that the order quantity of the call option in the bidirectional option contract is zero, and the bidirectional option can be regarded as the put option. It follows that the need for the bidirectional contract only arises when $d_1^* - d_2^p{}^* < d_1^* < d_1^* + d_2^c{}^*$.

Proposition 8 shows that when Company B is uncertain about future market demand changes, the bidirectional option is its optimal choice. Company B has the flexibility to exercise the bidirectional option when demand increases or decreases. In particular, Case (1) is a scenario where Company B uses the unilateral option (call or put); Case (2) is a scenario where Company B uses the bidirectional option; Case (3) is a scenario where Company B abandons the options.

Moreover, Proposition 8 gives the conditions under which the bidirectional option is applicable. Only when $\frac{w_1}{p_p} < 1 - \frac{w - p_p}{p_c - p_p} < 1 - \frac{w_1}{p+g-c-p_p}$, Company B will choose the bidirectional option; otherwise, the unilateral option. In the ferry leasing market, if ferry operators can accurately predict the changes in market demand (that is, reasonable contract parameters can be found), their optimal decision is to buy bidirectional options to reduce the risk of idle or shortage of leased ferries, which shows that bidirectional options are usually more flexible than unilateral options.

3.5 Purchasing ferries operation

To provide better ferry services, Company B may purchase ferry operations from a sustainable perspective. Notice, purchasing a ferry requires much capital, and the ferry requires routine maintenance. We use c_s to represent the unit cost of company B operating its ferry (since the service life of the ferry is known, its purchase capital can be included in its operating costs). Obviously, we can find $c_s > c$. Moreover, in this case, even if the ferry is idle, company B needs to pay a certain capital to maintain the ferry, so we denote the idle cost of the ferry by c_l (if company B leases the ferry, the idle cost is paid by company A). At this point, referring to (Liu et al., 2013), the profit function π_s of Company B can be deduced as

$$\pi_s = p\min(e, D) - c_s \min(e, D) - g[D - e]^+ - c_l[e - D]^+ \tag{17}$$

In Equation 17, the first term on the right side is the revenue generated by the ferry, the second term is the operating cost of the ferry, the third term is the out-of-stock cost of the ferry, and the fourth term is the idle cost of the ferry.

According to Equation 17, we can get the expected profit of Company B

$E[\pi_s]$

$$E[\pi_s] = (p + g - c_s)\left(e - \int_0^e F(x)dx\right) - c_l \int_0^e F(x)dx - g\mu \tag{18}$$

By solving the first derivatives of $E[\pi_s]$ (i.e. Equation 18) with respect to e_1, we can obtain

$$\frac{dE[\pi_s]}{de} = (p + g - c_s)(1 - F(e)) - c_l F(e) \tag{19}$$

Then, we can derive the optimal ferry leasing strategy for Company B under purchase ferry operation by solving the $\frac{dE[\pi_s]}{de} = 0$.

Proposition 9: In the scenario of purchasing ferries operation, the Company B's optimal purchase ferries quantity is $e^* = F^{-1}(1 - \frac{c_l}{p+g-c_s+c_l})$. Moreover, Company B can choose the strategy of purchasing or leasing ferries according to the following criteria:

1. If $c_s = p + g + c_l - \frac{(p+g-c)c_l}{w}$, purchasing and leasing ferries yield the same profit outcome.
2. If $c_s > p + g + c_l - \frac{(p+g-c)c_l}{w}$, leasing ferries is more profitable.
3. If $c_s < p + g + c_l - \frac{(p+g-c)c_l}{w}$, purchasing ferries is more profitable.

By comparing the Company's optimal quantity of ferries leased in Proposition 1 and the optimal purchase ferries quantity in Proposition 9, this paper can draw the basis for Company B to choose the strategy of leasing (or purchasing) ferries. Proposition 9 shows that leasing a ferry is not always the optimal strategy, and that company B needs to choose to purchase (or lease) ferries based on the relationship between leasing operating cost c and purchasing ferry operating cost c_s.

4 Numerical results

In this section, numerical analysis is used to verify the previous propositions and analyze the corresponding management significance. First, we set the corresponding parameters reasonably by citing a practical case. Next, the impact of option subscription price on different ferry leasing strategies is analyzed. Then, the sensitivity of some parameters was analyzed. Finally, the management implications of this paper are refined.

4.1 A practical case

Company B is a high-speed passenger ferry Co., Ltd. in Zhuhai, which mainly engages in marine transportation, waterfront tourism and island development. According to the official website (http://www.zh-gs.com/) of Company B, there are 130 daily flights between

FIGURE 2
Company B ferry operating route map.

Hong Kong, Macau, Shenzhen and Wanshan Islands in Zhuhai, with an annual passenger volume of over 5.4 million. The specific route map is shown in Figure 2.

Although the water transportation system in the jurisdiction managed by Company B is relatively complete, as mentioned above, ferry operations are greatly affected by factors such as seasons, weather, and holidays, so the market demand faced by Company B is random, and there are obvious weak, peak season. Taking Xiangzhou Port as an example, it is mainly responsible for passenger transport between Xiangzhou Port, Wanshan Island, Dongao Island, Wailingding Island, Kuishan Island and Dangan Island. In 2016, 7, 10 and 16 ferries were required to maintain normal operation of Xiangzhou Port during the low season, peak season and National Day respectively.

Therefore, referring to Nosoohi and Nookabadi (2016), combined with the survey results of company B, this paper assumes that the ferry demand D obeys a normal distribution with mean $\mu = 30$ and variance $\sigma = 10$. The specific values of other parameters are shown in Table 2.

4.2 Comparison of different leasing contracts

In order to illustrate the validity of options contracts and the differences between different options contracts, this section analyzes and compares the initial order quantity, option order quantity, total order quantity and profit of ferries under different leasing contracts. The specific results are shown in Figures 3–6. Note that this section examines the effect of a change in option order price from 200 to 1000.

Figure 3 shows the effect of option order price on the initial order quantity of ferries in the four leasing contracts. From Figure 3, we can see that the relationship between the initial order quantity of ferries in the leasing contracts is $c_1 > a > d_1 > b_1$. This is because for Company B, if he buys the put option contract, he can only return the ferry but not renew the ferry. In order to cope with the potential market demand, his initial order quantity is the largest at this time. If Company B buys the call option contract, its initial order quantity is lower than the wholesale price contract because it has the right to renew the ferry without refunding the ferry. If Company B buys the bidirectional option contract, its initial order quantity includes put options for insufficient demand and put options for excess demand, so the initial order quantity of bidirectional options is larger than that of call options, and because the bidirectional option is flexible, as a result, the initial order quantity is lower than the wholesale price contract. In summary, we can get $c_1 > a > d_1 > b_1$.

Moreover, Figure 3 also shows that with the increase of option order price w_1, the initial order quantity of call option increases gradually. This is because the cost of using options increases, so

TABLE 2 Parameter setting.

Parameters	p	w	w_1	p_c	p_p	c	g	c_l
Value	20000	5000	600	6000	3000	2000	10000	500

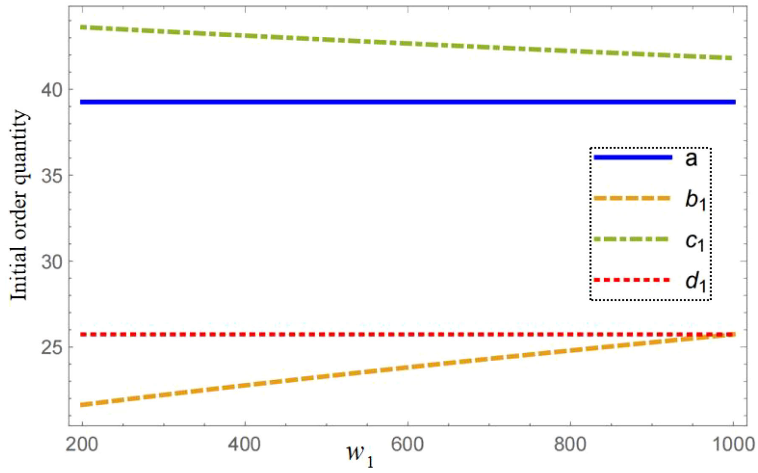

FIGURE 3
The effect of option order price on the initial order quantity of ferries.

Company B must reduce the used quantity of options but increase the initial order quantity of ferry. In the put option contract, the initial order quantity of the ferry decreases with the increase of w_1, because the higher option order price makes the ferry operator lose the incentive obtained from the higher initial order quantity. In the bidirectional option contract, w_1 has no effect on the initial order quantity, because of the high flexibility of the bidirectional option.

Figure 4 shows the effect of option order price on the option order quantity of ferries in the four leasing contracts. From Figure 4, we can find that the relationship between the option order quantity of ferries in three option contracts is $d_2^c + d_2^p > b_2 > c_2$. In the bidirectional option contract, the order quantity of option includes call option and put option, which must be larger than that of the unilateral option (call or put) contract. In the call option contract, the initial order quantity is smaller than that of the put option contract. In order to meet the change of market demand as much as possible, the order quantity of the option should be larger than that of the put option contract. In addition, as the option order price w_1 increases, the Company B's option order quantity (b_2, c_2, and $d_2^c+ \ d_2^p$) will all decrease, because the increase in option price reduces company B's willingness to buy options.

Figure 5 shows the effect of option order price on the total order quantity of ferries in the four leasing contracts. From Figure 5, we can see that the call option has the largest total order quantity, followed by the bidirectional option and the put option. In the call option contract, Company B only has the right to continue to purchase without returning the ferries. In order to meet the changes of market demand, it often orders a large quantity of ferries, which will make part of the ferries idle. In the put option contract, Company B only has the right to refund but not to continue to purchase the ferry, so it may be out of stock, so its total order quantity is small. In the bidirectional option contract, Company B

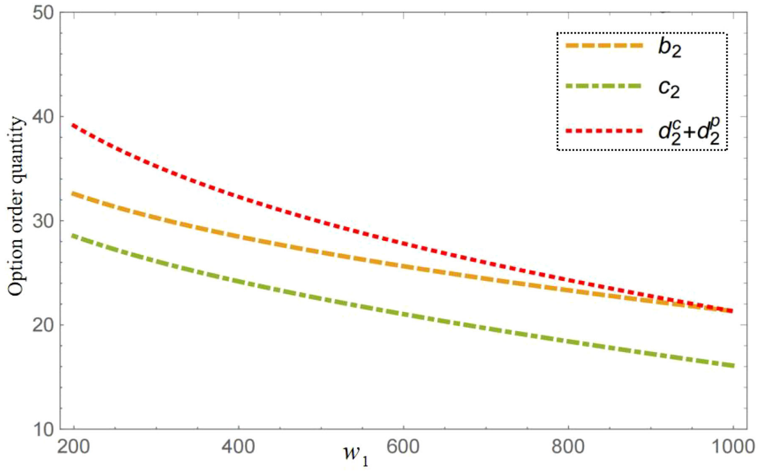

FIGURE 4
The effect of option order price on the option order quantity of ferries.

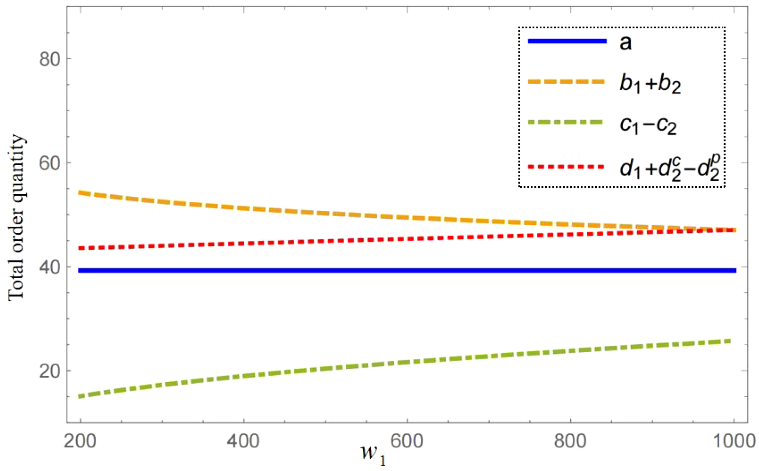

FIGURE 5
The effect of option order price on the total order quantity of ferries.

has both the right to return and the right to continue purchasing ferries, which can reduce (or eliminate) the risk of ferries shortage and excess, so that the total number of ferries is between the call option and the put option.

Figure 6 shows the effect of option order price on Company B's expected profit in the four leasing contracts. From Figure 6, we can find that the relationship between the Company's profit in four leasing contracts is $\pi_b > \pi_c > \pi_p > \pi_0$. In other word, Company B obtains the most profit from the bidirectional option contract and the least profit from the wholesale price contract. Moreover, when Company B is uncertain about the change of market demand, it is better to adopt call option contract than put option contract, because put option contract usually requires a large initial order quantity, which increases the risk of idle ferry, thus increasing Company's loss. The analysis in this section makes a good explanation of propositions 1 to 8. In addition, it can also be concluded that the adoption of the option contract is an effective method to deal with the change in ferry market demand because it can help operators alleviate the problem of ferry shortage or idle under normal requests. Both the quantity of option orders and the expected earnings of ferry operators will decrease with the increase of option prices because the increase of option prices makes operators less willing to use option contracts. In addition, bidirectional options are favored by ferry operators because of their flexibility.

4.3 Sensitivity analysis

In the following, Company B's optimal orders and profit under the wholesale price and three option contracts are investigated based on different parameter settings. Along this line, each time the

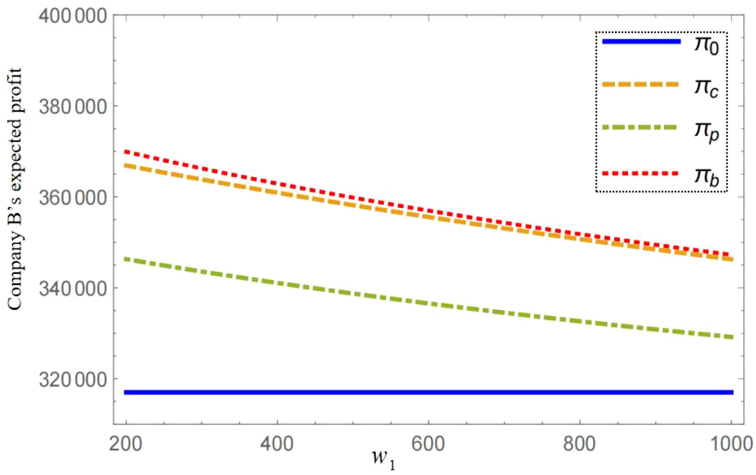

FIGURE 6
The effect of option order price on Company B's expected profit.

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value for one or more of the parameters is changed, the others remain unchanged as described in Section 4.1.

Table 3 gives Company's optimal orders and profit under the four scenarios of wholesale contract, put, call and bidirectional options. In this Table, different values are considered for (w, w_1, p_c, p_p). Note that we used '-' as not applicable, bold for the optimal strategy. Since we have analyzed the impact of w_1 in the previous section, we will not describe it here.

Effects of w: as expected, in the four contracts, the initial ferry leasing quantity and expected revenue of Company B will decrease with the increase of the wholesale price w. In the call option contract, the option order quantity increases with the increase of w, because more call options will compensate for the lower initial lease quantity to balance the expected operating income of Company B. In the put option contract, the option order quantity decreases with the increase of w, which is also because the initial lease quantity decreases. In order to balance the income of company B, the option quantity must be reduced. Similarly, calls must increase and puts must decrease in the bidirectional option contract. In addition, we also find that company B has the highest expected profit when it adopts the bidirectional option contract.

Effects of p_c: In the call and bidirectional option contracts, it can be seen that the expected profit of company B decreases as p_c increases because the cost of exercising the option keeps increasing. In the call option contract, the quantity of initial leases increases as p_c increases, but the quantity of call leases decreases as p_c increases, because Company B will choose more initial order quantity to cope with the higher option exercise costs. Similarly, the initial order quantity in the bidirectional option contract will increase and the call option order quantity will decrease. Note that in the bidirectional option contract, put option order quantity will increase, which is a response to the risk of uncertain market demand.

Effects of p_p: In the put option and bidirectional option contracts, it can be found that the expected profit and initial lease quantity of Company B increase as p_p increases, because the risk of company B over leasing is reduced, and company B will get more profit from Company A under the exercise of the put option. In the bidirectional option contract, the leasing quantity of call options will undoubtedly decrease as the initial order quantity increases.

Table 4 gives Company B's optimal orders and profit for different values of p, c, and g.

Effects of p: as expected, Company's expected profit increases as p increases in all four cases. In the wholesale price and put option contracts, the initial lease quantity increases as p increases, while the initial lease quantity of the call and bidirectional options is independent of p. In the call and put option contracts, the option

TABLE 3 The effect of different parameters on Company B's optimal orders and profit in the four contracts.

Parameters	Value	Wholesale price		Call option			Put option			Bidirectional option			
		a	π_0	b_1	b_2	π_c	c_1	c_2	π_p	d_1	d_2^c	d_2^p	π_b
w	4200	40.36	348822.55	27.47	21.76	377098.04	44.61	23.03	371406.97	32.53	16.69	10.95	**380289.46**
	4400	40.06	340778.00	26.59	22.63	371691.70	44.05	22.47	362541.59	30.84	18.39	9.25	**373953.27**
	4600	39.77	332796.87	25.69	23.53	366462.60	43.53	21.95	353783.96	29.16	20.06	7.58	**367953.27**
	4800	39.49	324871.47	24.76	24.47	361417.03	43.05	21.46	345126.62	27.47	21.76	5.88	**362289.46**
	5000	39.21	317002.24	23.77	25.45	356563.43	42.59	21.01	336563.30	25.69	23.53	4.11	**356971.74**
w_1	200	–	–	21.58	32.04	367914.59	43.53	28.54	346303.62	25.69	27.93	10.70	**369925.11**
	400	–	–	22.72	28.21	361918.67	43.05	24.15	341067.27	25.69	25.24	6.80	**362918.02**
	600	–	–	23.77	25.45	356563.43	42.59	21.01	336563.30	25.69	23.53	4.11	**356971.74**
	800	–	–	24.76	23.19	351705.08	42.16	18.39	332629.92	25.69	22.25	1.92	**351801.06**
	1000	–	–	25.69	21.21	347268.62	41.75	16.06	329188.89	25.69	21.21	0	**347268.62**
p_c	6000	–	–	23.77	25.45	356563.43	–	–	–	25.69	23.53	4.11	**356971.74**
	6200	–	–	24.48	24.71	355065.64	–	–	–	26.81	22.37	5.23	**355766.44**
	6400	–	–	25.11	24.03	353663.17	–	–	–	27.77	21.38	6.19	**354690.96**
	6600	–	–	25.69	23.41	352344.54	–	–	–	28.60	20.50	7.02	**353720.83**
	6800	–	–	26.23	22.84	351100.36	–	–	–	29.34	19.72	7.76	**352838.09**
p_p	3000	–	–	–	–		42.59	21.01	336563.30	25.69	23.53	4.11	**356971.74**
	3200	–	–	–	–	–	43.00	21.87	339002.98	26.34	22.89	4.76	**357210.15**
	3400	–	–	–	–	–	43.44	22.73	341535.91	27.07	22.16	6.36	**357532.07**
	3600	–	–	–	–	–	43.92	23.59	344166.53	27.90	21.33	7.57	**357919.00**
	3800	–	–	–	–	–	44.44	24.47	346900.91	28.86	20.37	8.89	**358392.95**

Bold values represent the highest profit value among several contracts.

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TABLE 4 The effect of price and cost on Company B's optimal orders and profit in the four contracts.

Parameters	Value	Wholesale price		Call option			Put option			Bidirectional option			
		a	π_0	b_1	b_2	π_c	c_1	c_2	π_p	d_1	d_2^c	d_2^p	π_b
p	15000	37.81	172452.34	–	24.31	207146.33	41.26	19.68	189380.02	–	22.39	–	**207554.63**
	17500	38.56	244555.81	–	24.93	281832.83	41.98	20.39	262875.68	–	23.00	–	**282241.14**
	20000	39.21	317002.24	–	25.45	356563.43	42.59	21.01	336563.30	–	23.53	–	**356971.74**
	22500	39.78	389722.78	–	25.92	431327.90	43.13	21.55	410401.61	–	23.99	–	**431736.21**
	25000	40.30	462667.11	–	26.32	506119.16	43.62	22.03	484361.18	–	24.41	–	**506527.47**
c	1500	39.33	331526.24	–	25.55	371513.88	42.70	21.12	351319.99	–	23.63	–	**371922.19**
	1750	39.27	324262.90	–	25.50	364038.49	42.65	21.06	343940.91	–	23.58	–	**364446.80**
	2000	39.21	317002.24	–	25.45	356563.43	42.59	21.01	336563.30	–	23.53	–	**356971.74**
	2250	39.15	309744.29	–	25.40	349088.71	42.53	20.95	329187.18	–	23.48	–	**349497.02**
	2500	39.08	302489.13	–	25.35	341614.33	42.47	20.89	321812.60	–	23.43	–	**342022.64**
g	5000	37.81	322452.34	–	24.31	357146.33	41.26	19.68	339380.02	–	22.39	–	**357554.63**
	7500	38.56	319555.81	–	24.93	356832.83	41.98	20.39	337875.68	–	23.00	–	**357241.14**
	10000	39.21	317002.24	–	25.45	356563.43	42.59	21.01	336563.30	–	23.53	–	**356971.74**
	12500	39.78	314722.78	–	25.92	356327.90	43.13	21.55	335401.61	–	23.99	–	**356736.21**
	15000	40.23	312667.12	–	26.33	356119.16	43.62	22.03	334361.18	–	24.41	–	**356527.47**

Bold values represent the highest profit value among several contracts.

order quantity increases with the increase of p. In the bidirectional option contract, the call option order quantity increases as p increases, while the put option order quantity is independent of p. This is because the increase of p will inevitably lead to the increase of the total order quantity. In particular, the bidirectional option at this time can be regarded as a call option.

Effects of c: contrary to the influence of p, the expected profit of Company B decreases with the increase of c in all four cases. In the wholesale price and put option contracts, the initial lease quantity decreases as c increases, while the initial lease quantity of the call and bidirectional options is independent of c. In the call and put option contracts, option order quantity decreases as c increases. In the bidirectional option contract, call option order quantity decreases as c increases, while put option order quantity is independent of c. This is because the increase of c will inevitably lead to a decrease in the total order quantity.

Effects of g: the expected profit of Company B decreases with the increase of g in all four cases. It is worth noting that the profit in the wholesale price contract fell the fastest, followed by the call (or put) option contract, and the bidirectional option contract was the slowest, indicating that the option contract can cope well with the shortage situation in the ferry market, and the bidirectional option contract is optimal. In the wholesale price and put option contracts, the initial lease quantity increases as g increases, while the initial lease quantity of the call and bidirectional options is independent of g. In the call and put option contracts, the option order quantity increases with the increase of g. In the bidirectional option contract, the call option order quantity increases as g increases, while the put option order quantity is independent of g. This is because with the increase of g, Company B will increase the total order quantity in order to better deal with the risk of stock shortage.

Table 5 shows the impact of demand variance on Company B's optimal orders and profit. From Table 5, it can be seen that Company's expected profit decreases as the demand variance σ increases in all four cases. In the wholesale price and put option contracts, the initial lease quantity increases as σ increases. The initial lease quantity in the call and bidirectional option contracts decreases as σ increases. In each of the three options contracts, the order quantity of the call and put options increases to balance the total order quantity. This is because the greater σ, the greater the uncertainty in the ferry market and the higher the risk to Company B. Therefore, the greater the volatility of demand, the greater the incentive for Company B to purchase the option contract. The bold values provided in Tables 3–5 represent the highest profit value among several contracts.

In addition to studying the influence of various parameters and variables on Company B's optimal orders and profit in the four contracts, this section also conducts digital experiments on the two situations of purchasing ferry and leasing ferry, and the specific results are shown in Figure 7.

As can be seen from Figure 7, when the purchase cost of ferry $c_s < 27700$, Company B will choose to purchase ferry, otherwise it will choose to lease ferry. Moreover, with the increase of purchase cost, the purchase quantity of company B will gradually decrease, which further verifies and expands proposition 9. Therefore, proposition 9 can be used as the basis for Company B to choose to purchase or lease a ferry.

TABLE 5 The effect of variance on Company B's optimal orders and profit in the four contracts.

Parameters	Value	Wholesale price		Call option			Put option			Bidirectional option			
		a	π_0	b_1	b_2	π_c	c_1	c_2	π_p	d_1	d_2^c	d_2^p	π_b
σ	1	30.92	382689.52	29.38	2.55	386645.64	31.26	2.10	384645.63	29.57	2.35	0.41	**386686.47**
	5	34.60	353447.62	26.89	12.73	373228.21	36.30	10.50	363228.15	27.85	11.77	2.05	**373432.37**
	10	39.21	317002.24	23.77	25.45	356563.43	42.59	21.01	336563.30	25.69	23.53	4.11	**356971.74**
	15	43.81	283908.94	20.66	38.18	343250.74	48.89	31.51	313250.54	23.54	35.30	6.16	**343863.20**
	20	48.42	260202.27	17.54	50.91	339324.66	55.18	42.01	299324.39	21.39	47.06	8.22	**340141.28**

Bold values represent the highest profit value among several contracts.

4.4 Managerial insights

Section 4.2 and 4.3 summarize the influence of different parameter settings on ferry operators' order quantity and profit under the four scenarios of wholesale contract, put, call and bidirectional options. The experimental results show that the increase of wholesale price w, option order price w_1, call option strike price p_c, lease ferry operation cost c and out-of-stock cost g will reduce ferry operators' profit. On the contrary, the increase of the put option strike price p_b and the operating income p of each ferry will increase the operators' profit. Therefore, for ferry operators (Company B), it is necessary to increase the capacity of each ferry, optimize routes and reduce operating costs. For ferry leasing Companies (Company A), ferries' wholesale price and option price should be reasonably set.

According to the above analysis, for ferry operators, all option contracts are more profitable than wholesale price contracts, because option contracts are more helpful to company B to reduce its interest losses caused by demand uncertainty. In options contracts, the bidirectional option contract is always more profitable than the unilateral option (call or put), because the bidirectional option is more flexible than the unilateral option (call or put). The more volatile the market demand, the more incentive operators to purchase options. Moreover, it is not always advantageous for operators to choose the strategy of leasing ferries. When the operating cost of purchasing ferries is less than a certain threshold value, the strategy of purchasing ferries is more profitable.

In conclusion, the adoption of options contracts by ferry operators will increase ferry utilization, thereby reducing ferry operating costs and carbon emissions, which is consistent with the global goal of developing sustainable transportation.

5 Conclusions

While existing research on ferry operations has predominantly focused on route optimization and container leasing strategies, the critical domain of ferry leasing strategies has remained underexplored. This study addresses this gap by developing a comprehensive analytical framework to evaluate optimal ferry leasing quantities and expected profits across four contractual arrangements: wholesale pricing, call options, put options, and bidirectional options. Through rigorous modeling and sensitivity analysis of key operational parameters, we establish a robust decision-making framework that compares leasing and purchasing scenarios, offering actionable insights for ferry operators. Our findings reveal that option contracts consistently outperform wholesale pricing in terms of profitability, with

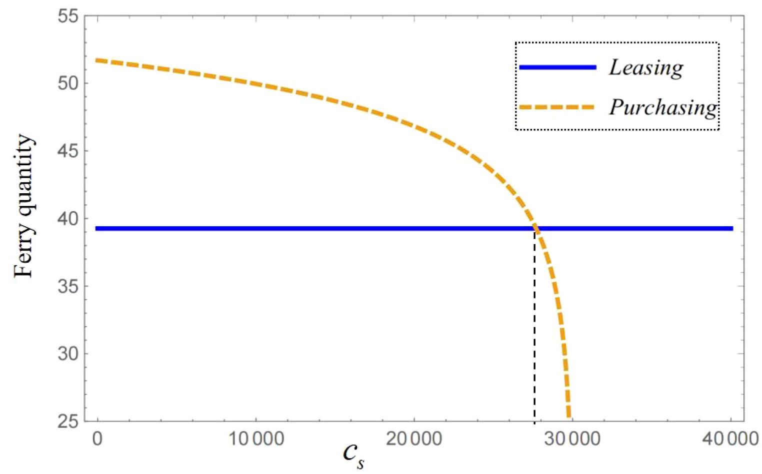

FIGURE 7
Purchasing VS leasing.

bidirectional options demonstrating superior flexibility, particularly in volatile market conditions where they serve as an effective risk mitigation tool. Notably, the analysis challenges the conventional assumption that leasing is universally advantageous; instead, it identifies scenarios where purchasing ferries becomes the economically preferable option—specifically when acquisition costs fall below a critical threshold. These insights carry significant implications for sustainable ferry transportation development. By optimizing resource allocation through tailored leasing strategies, operators can enhance operational efficiency, reduce carbon emissions, and improve service reliability for passengers. Furthermore, our results provide valuable guidance for policymakers seeking to foster a more resilient and sustainable ferry transportation ecosystem.

To address seasonal demand fluctuations, we propose the following strategic recommendations for ferry operators: 1) Peak Seasons: Adopt wholesale price contracts for base capacity and supplement with call options to flexibly expand fleet size in response to surging demand. 2) Off-Seasons: Combine wholesale pricing with put options to mitigate the risks of overcapacity while maintaining service readiness. 3) Uncertain Demand: Leverage bidirectional options to dynamically adjust fleet size, balancing call and put options to align with unpredictable market conditions. This study not only advances theoretical understanding but also equips industry stakeholders with practical tools to navigate demand uncertainty, ultimately contributing to the long-term sustainability and efficiency of ferry transportation systems.

We highlight a few potential directions for future work. First, this paper only studies the ordering strategies of ferry operators, ignoring the options pricing strategies and supply chain coordination of ferry leasing companies. Options pricing (Grabbe, 1983; Hua et al., 2019) and supply chain coordination (Hasani et al., 2013; Luo et al., 2015) based on options contracts are critical for improving supply chain performance. Second, this paper only considers one ferry operator. If there are multiple ferry operators, the ferry options transactions between operators cannot be ignored. Third, for simplicity, this paper assumes that the bidirectional option and the unilateral option have the same order price. In reality, the bidirectional option may have a higher order price because it is more flexible (Patra and Jha, 2022), so the issue of different order prices for different options is also worth studying. Finally, the ferry information (e.g., quality, cost, etc.) is usually the private information of the leasing company, and this information will affect the operation and decision of the ferry operator. Therefore, the ferry lease contract under asymmetric information is worth studying (Hu et al., 2014; Chen et al., 2018). As the model introduced in this article is presently limited to analyzing scenarios involving a single ferry operator, it possesses inherent constraints. However, we are actively refining the model's structure and underlying assumptions to broaden its applicability to large-scale issues encompassing multiple ferry operators. Our upcoming work will make more contributions to improving operational efficiency, reducing operational costs, and enhancing passenger satisfaction for ferry operators, thereby promoting the efficient and sustainable development of the ferry operation market.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material. Further inquiries can be directed to the corresponding author.

Author contributions

HC: Software, Writing – original draft, Methodology. HH: Data curation, Writing – original draft, Formal Analysis. SZ: Validation, Conceptualization, Writing – review & editing. LZ: Formal Analysis, Writing – original draft, Software. LX: Visualization, Supervision, Conceptualization, Writing – review & editing. CW: Funding acquisition, Writing – review & editing, Project administration.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

The author(s) declare that no Generative AI was used in the creation of this manuscript.

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References

An, K., and Lo, H. K. (2014). Ferry service network design with stochastic demand under user equilibrium flows. *Transportation Res. Part B: Methodological* 66, 70–89. doi: 10.1016/j.trb.2013.10.008

Anwar, S., Zia, M. Y. I., Rashid, M., Rubens, G. Z. D., and Enevoldsen, P. (2020). Towards ferry electrification in the maritime sector. *Energies* 13, 6506. doi: 10.3390/en13246506

Arani, H. V., Rabbani, M., and Rafiei, H. (2016). A revenue-sharing option contract toward coordination of supply chains. *Int. J. Production Economics* 178, 42–56. doi: 10.1016/j.ijpe.2016.05.001

Chen, X., Li, B., and An, S. (2018). Option contract design for supply chains under asymmetric cost information. *Kybernetes* 48, 835–860. doi: 10.1108/K-12-2017-0495

Chen, X., Wan, N., and Wang, X. (2017). Flexibility and coordination in a supply chain with bidirectional option contracts and service requirement. *Int. J. Production Economics* 193, 183–192. doi: 10.1016/j.ijpe.2017.07.013

Cheng, H., Xu, S. X., Huang, G. Q., Shao, S., and Xu, G. (2021). Optimal pricing for ferry services with a new entrant: a game-theoretic perspective. *Transportmetrica A: Transport Sci.* 18 (3), 1626–1655. doi: 10.1080/23249935.2021.1956635

Choi, T. M., Chung, S. H., and Zhuo, X. (2020). Pricing with risk sensitive competing container shipping lines: Will risk seeking do more good than harm? *Transportation Res. Part B: Methodological* 133, 210–229. doi: 10.1016/j.trb.2020.01.003

Chu, X., Shao, S., Xu, S. X., and Kang, K. (2020). Data-driven ferry network design with candidate service arcs: the case of Zhuhai Islands in China. *Maritime Policy Manage.* 47, 598–614. doi: 10.1080/03088839.2020.1747650

Grabbe, J. O. (1983). The pricing of call and put options on foreign exchange. *J. Int. Money Finance* 2, 239–253. doi: 10.1016/S0261-5606(83)80002-3

Hasani, R., Jafarzadeh, H., and Khoshalhan, F. (2013). A new method for supply chain coordination with credit option contract and customers' backordered demand. *Uncertain Supply Chain Manage.* 1, 207–218. doi: 10.5267/j.uscm.2013.09.002

Hu, F., Lim, C. C., and Lu, Z. (2014). Optimal production and procurement decisions in a supply chain with an option contract and partial backordering under uncertainties. *Appl. Mathematics Comput.* 232, 1225–1234. doi: 10.1016/j.amc.2014.01.149

Hua, S., Liu, J., Cheng, T. E., and Zhai, X. (2019). Financing and ordering strategies for a supply chain under the option contract. *Int. J. production economics* 208, 100–121. doi: 10.1016/j.ijpe.2018.10.008

Karapetyan, D., and Punnen, A. P. (2015). "An integer programming model for the ferry scheduling problem," in *Case Studies in Operations Research* (Springer, New York, NY), 517–537.

Lai, M. F., and Lo, H. K. (2004). Ferry service network design: optimal fleet size, routing, and scheduling. *Transportation Res. Part A: Policy Pract.* 38 (4), 305–328. doi: 10.1016/j.tra.2003.08.003

Liang, L., Wang, X., and Gao, J. (2012). An option contract pricing model of relief material supply chain. *Omega* 40, 594–600. doi: 10.1016/j.omega.2011.11.004

Liu, C., Jiang, Z., Liu, L., and Geng, N. (2013). Solutions for flexible container leasing contracts with options under capacity and order constraints. *Int. J. Production Economics* 141, 403–413. doi: 10.1016/j.ijpe.2012.09.005

Liu, J., and Wang, J. (2019). Carrier alliance incentive analysis and coordination in a maritime transport chain based on service competition. *Transportation Res. Part E: Logistics Transportation Rev.* 128, 333–355. doi: 10.1016/j.tre.2019.06.009

Liu, S., Hua, G., Cheng, T. C. E., and Dong, J. (2021). Unmanned vehicle distribution capacity sharing with demand surge under option contracts. *Transportation Res. Part E: Logistics Transportation Rev.* 149, 102320. doi: 10.1016/j.tre.2021.102320

Luo, M., Li, G., Wan, C. J., Qu, R., and Ji, P. (2015). Supply chain coordination with dual procurement sources via real-option contract. *Comput. Ind. Eng.* 80, 274–283. doi: 10.1016/j.cie.2014.12.019

Luo, T., Chang, D., and Xu, Z. (2021). Forwarder's empty container ordering and coordination considering option trading in the container transportation service chain. *Comput. Ind. Eng.* 156, 107251. doi: 10.1016/j.cie.2021.107251

Mohamad Taghvaee, V., Assari Arani, A., Nodehi, M., Khodaparast Shirazi, J., Agheli, L., Neshat Ghojogh, H. M., et al. (2023). Sustainable development goals: transportation, health and public policy. *Rev. Economics Political Sci.* 8, 134–161. doi: 10.1108/REPS-12-2019-0168

Ng, M., and Lo, H. K. (2016). Robust models for transportation service network design. *Transportation Res. Part B: Methodological* 94, 378–386. doi: 10.1016/j.trb.2016.10.001

Nosoohi, I., and Nookabadi, A. S. (2016). Outsource planning through option contracts with demand and cost uncertainty. *Eur. J. Operational Res.* 250, 131–142. doi: 10.1016/j.ejor.2015.10.030

Patra, T. D. P., and Jha, J. K. (2022). Bidirectional option contract for prepositioning of relief supplies under demand uncertainty. *Comput. Ind. Eng.* 163, 107861. doi: 10.1016/j.cie.2021.107861

Psaraftis, H. N., and Kontovas, C. A. (2010). Balancing the economic and environmental performance of maritime transportation. *Transportation Res. Part D: Transport Environ.* 15, 458–462. doi: 10.1016/j.trd.2010.05.001

Rabbani, M., Arani, H. V., and Rafiei, H. (2015). Option contract application in emergency supply chains. *Int. J. Serv. Operations Manage.* 20, 385–397. doi: 10.1504/IJSOM.2015.068523

Škurić, M., Maraš, V., Davidović, T., and Radonjić, A. (2021). Optimal allocating and sizing of passenger ferry fleet in maritime transport. *Res. Transportation Economics* 90, 100868. doi: 10.1016/j.retrec.2020.100868

Taghvaee, V. M., Agheli, L., Assari Arani, A., Nodehi, M., and Shirazi, J. K. (2019). Environmental pollution and economic growth elasticities of maritime and air transportations in Iran. *Mar. Econ Manag* 2, 114–123. doi: 10.1108/MAEM-09-2019-0008

Taghvaee, S. M., Omaraee, B., and Mohamad Taghvaee, V. (2017). Maritime transportation, environmental pollution, and economic growth in Iran: Using dynamic log linear model and granger causality approach. *Iranian Economic Rev.* 21, 185–210. doi: 10.22059/ier.2017.62100

Tsoi, K. H., and Loo, B. P. (2021). Cutting the loss: International benchmarking of a sustainable ferry business model. *Transportation Res. Part A: Policy Pract.* 145, 167–188. doi: 10.1016/j.tra.2021.01.007

Wang, C., Chen, J., and Chen, X. (2017). Pricing and order decisions with option contracts in the presence of customer returns. *Int. J. Production Economics* 193, 422–436. doi: 10.1016/j.ijpe.2017.08.011

Wang, X., Li, F., Liang, L., Huang, Z., and Ashley, A. (2015). Pre-purchasing with option contract and coordination in a relief supply chain. *Int. J. Production Economics* 167, 170–176. doi: 10.1016/j.ijpe.2015.05.031

Wang, D. Z., and Lo, H. K. (2008). Multi-fleet ferry service network design with passenger preferences for differential services. *Transportation Res. Part B: Methodological* 42, 798–822. doi: 10.1016/j.trb.2008.01.008

Wang, H., Olsen, T. L., and Liu, G. (2018). Service capacity competition with peak arrivals and delay sensitive customers. *Omega* 77, 80–95. doi: 10.1016/j.omega.2017.06.001

Wu, M., Li, K. X., Xiao, Y., and Yuen, K. F. (2022). Carbon Emission Trading Scheme in the shipping sector: Drivers, challenges, and impacts. *Mar. Policy* 138, 104989. doi: 10.1016/j.marpol.2022.104989

Wu, J., Liu, J., and Li, N. (2024). The evasion strategy options for competitive ocean carriers under the EU ETS. *Transportation Res. Part E: Logistics Transportation Rev.* 183, 103439. doi: 10.1016/j.tre.2024.103439

Xiao, G., Xu, L., Shu, Y., and Liu, G. (2024). Carbon neutralization in sustainable port and shipping. *Sustainability (2071-1050)* 16. doi: 10.3390/su16114765

Xing, W., Yan, L., and Zhou, S. (2023). Strategic logistics service procurement in shipping supply chains. *Ocean Coast. Manage.* 242, 106714. doi: 10.1016/j.ocecoaman.2023.106714

Xu, L., Luo, Y., Chen, J., and Zhou, S. (2024a). Capacity prioritization allocation and credit financing option in shipping freight forwarding market. *Comput. Ind. Eng.* 189, 109987. doi: 10.1016/j.cie.2024.109987

Xu, L., Shi, J., and Chen, J. (2021). Platform encroachment with price matching: introducing a self-constructing online platform into the sea-cargo market. *Comput. Ind. Eng.* 156, 107266. doi: 10.1016/j.cie.2021.107266

Xu, L., Wei, Y., Chen, J., and Luo, Y. (2024b). Carrier channel encroachment and forwarder order sequence: an equilibrium analysis of shipping service supply chain. *Int. J. Shipping Transport Logistics* 19, 82–97. doi: 10.1504/IJSTL.2024.142370

Xu, L., Yang, Z., Chen, J., Zou, Z., and Wang, Y. (2024c). Spatial-temporal evolution characteristics and spillover effects of carbon emissions from shipping trade in EU coastal countries. *Ocean Coast. Manage.* 250, 107029. doi: 10.1016/j.ocecoaman.2024.107029

Yang, L., Tang, R., and Chen, K. (2017). Call, put and bidirectional option contracts in agricultural supply chains with sales effort. *Appl. Math. Model.* 47, 1–16. doi: 10.1016/j.apm.2017.03.002

Yi, T., Meiping, W., and Shaorui, Z. (2023). Pricing and contract preference in maritime supply chains with downstream competition impact of risk-aversion and contract unobservability. *Ocean Coast. Manage.* 242, 106691. doi: 10.1016/j.ocecoaman.2023.106691

Yuan, X., Bi, G., Zhang, B., and Yu, Y. (2020). Option contract strategies with risk-aversion and emergency purchase. *Int. Trans. Operational Res.* 27, 3079–3103. doi: 10.1111/itor.12519

Zhao, Y., Ma, L., Xie, G., and Cheng, T. E. (2013). Coordination of supply chains with bidirectional option contracts. *Eur. J. Operational Res.* 229, 375–381. doi: 10.1016/j.ejor.2013.03.020

Appendix

Proof of Proposition 1.

According to Equation 3, we can obtain $\frac{d^2E[\pi_0]}{da^2} = -(p+g-c)f(a) < 0$. Therefore, π_0 is a strictly concave function in a. We find the unique a^* by solving $\frac{dE[\pi_0]}{da} = 0$.

Thus, Proposition 1 is proved. □

Proof of Proposition 2.

According to Equations 6, 7, we can obtain $\frac{\partial^2 E[\pi_c]}{\partial b_1^2} = -p_c f(b_1) - (p+g-p_c-c)f(b_1+b_2) < 0$, $\frac{\partial^2 E[\pi_c]}{\partial b_1 \partial b_2} = -(p+g-p_c-c)f(b_1+b_2) < 0$, $\frac{\partial^2 E[\pi_c]}{\partial b_2 \partial b_1} = -(p+g-p_c-c)f(b_1+b_2) < 0$, and $\frac{\partial^2 E[\pi_c]}{\partial b_2^2} = -(p+g-p_c-c)f(b_1+b_2) < 0$.

Furthermore, we can obtain the Hessian matrix of $E[\pi_c]$ as

$$H_1 = \begin{bmatrix} \frac{\partial^2 E[\pi_c]}{\partial b_1^2} & \frac{\partial^2 E[\pi_c]}{\partial b_1 \partial b_2} \\ \frac{\partial^2 E[\pi_c]}{\partial b_2 \partial b_1} & \frac{\partial^2 E[\pi_c]}{\partial b_2^2} \end{bmatrix}$$ Since H_1's first-order conditions

$D_1 = \frac{\partial^2 E[\pi_c]}{\partial b_1^2} < 0$, and second-order conditions $D_2 = (p+g-p_c-c)p_c f(b_1)f(b_1+b_2) > 0$. Thus, the Hessian matrix of $E[\pi_c]$ is negative definite, i.e., $E[\pi_c]$ is jointly concave in b_1 and b_2. We find the unique b_1^* and $b_2^*$by solving $\frac{\partial E[\pi_c]}{\partial b_1} = 0$ and $\frac{\partial E[\pi_c]}{\partial b_2} = 0$, respectively.

Therefore, Proposition 2 is proved. □

Proof of Proposition 4.

According to Equations 10, 11), we can obtain $\frac{\partial^2 E[\pi_p]}{\partial c_1^2} = -(p+g-p_p-c)f(c_1) - p_p f(c_1-c_2) < 0$, $\frac{\partial^2 E[\pi_p]}{\partial c_1 \partial c_2} = p_p f(c_1-c_2) > 0$, $\frac{\partial^2 E[\pi_p]}{\partial c_2 \partial c_1} = p_p f(c_1-c_2) > 0$, and $\frac{\partial^2 E[\pi_p]}{\partial c_2^2} = -p_p f(c_1-c_2) < 0$.

Furthermore, we can obtain the Hessian matrix of $E[\pi_p]$ as

$$H_2 = \begin{bmatrix} \frac{\partial^2 E[\pi_p]}{\partial c_1^2} & \frac{\partial^2 E[\pi_p]}{\partial c_1 \partial c_2} \\ \frac{\partial^2 E[\pi_p]}{\partial c_2 \partial c_1} & \frac{\partial^2 E[\pi_p]}{\partial c_2^2} \end{bmatrix}$$ Since H_2's first-order conditions

$D_1 = \frac{\partial^2 E[\pi_p]}{\partial c_1^2} < 0$, and second-order conditions $D_2 = (p+g-p_p-c)p_p f(c_1)f(c_1-c_2) > 0$. Thus, the Hessian matrix of $E[\pi_p]$ is negative definite, i.e., $E[\pi_p]$ is jointly concave in c_1 and c_2. We find the unique c_1^* and $c_1^*$by solving $\frac{\partial E[\pi_p]}{\partial c_1} = 0$ and $\frac{\partial E[\pi_p]}{\partial c_2} = 0$, respectively.

Thus, Proposition 4 is proved. □

Proof of Proposition 6.

According to Equations 14, 15, 16, we can obtain $\frac{\partial^2 E[\pi_b]}{\partial d_1^2} = -(p+g-p_c-c)f(d_1+d_2^c) - (p_c-p_p)f(d_1) - p_p f(d_1-d_2^p) < 0$,

$\frac{\partial^2 E[\pi_b]}{\partial d_1 \partial d_2^c} = -(p+g-p_c-c)f(d_1+d_2^c) < 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_1 \partial d_2^p} = p_p f(d_1-d_2^p) > 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_2^c \partial d_1} = -(p+g-p_c-c)f(d_1+d_2^c) < 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_2^c 2} = -(p+g-p_c-c)f(d_1+d_2^c) < 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_2^c \partial d_2^p} = 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_2^p \partial d_1} = p_p f(d_1-d_2^p) > 0$, $\frac{\partial^2 E[\pi_b]}{\partial d_2^p \partial d_2^c} = 0$, and $\frac{\partial^2 E[\pi_b]}{\partial d_2^p 2} = -p_p f(d_1-d_2^p) < 0$.

Furthermore, we can obtain the Hessian matrix of $E[\pi_b]$ as

$$H_3 = \begin{bmatrix} \frac{\partial^2 E[\pi_b]}{\partial d_1^2} & \frac{\partial^2 E[\pi_b]}{\partial d_1 \partial d_2^c} & \frac{\partial^2 E[\pi_b]}{\partial d_1 \partial d_2^p} \\ \frac{\partial^2 E[\pi_b]}{\partial d_2^c \partial d_1} & \frac{\partial^2 E[\pi_b]}{\partial d_2^c 2} & \frac{\partial^2 E[\pi_b]}{\partial d_2^c \partial d_2^p} \\ \frac{\partial^2 E[\pi_b]}{\partial d_2^p \partial d_1} & \frac{\partial^2 E[\pi_b]}{\partial d_2^p \partial d_2^c} & \frac{\partial^2 E[\pi_b]}{\partial d_2^p 2} \end{bmatrix}$$ Since H_3's first-order conditions

$D_1 = \frac{\partial^2 E[\pi_b]}{\partial d_1^2} < 0$, second-order conditions $D_2 = (p+g-p_c-c)f(d_1+d_2^c)((p_c-p_p)f(d_1) + p_p f(d_1-d_2^p)) > 0$, and third-order conditions $D_3 = -p_p(p_c-p_p)(p+g-p_c-c)f(d_1)f(d_1+d_2^c)f(d_1-d_2^p) < 0$.

Thus, the Hessian matrix of $E[\pi_b]$ is negative definite, i.e., $E[\pi_b]$ is jointly concave in d_1, d_2^c, and d_2^p. We find the unique d_1^*, d_2^c*, and d_2^p* by solving $\frac{\partial E[\pi_b]}{\partial d_1} = 0$, $\frac{\partial E[\pi_b]}{\partial d_2^c} = 0$, and $\frac{\partial E[\pi_b]}{\partial d_2^p} = 0$, respectively.

Thus, Proposition 6 is proved. □

Proof of Proposition 9.

According to Equation 19, we can obtain $\frac{d^2E[\pi_s]}{de^2} = -(p+g-c_s+c_l)f(e) < 0$. Therefore, π_s is a strictly concave function in e. We find the unique e^* by solving $\frac{dE[\pi_s]}{de} = 0$.

Thus, Proposition 9 is proved. □