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Experimental implementation of algebraic identifier for unbalance parameters in a rotor-bearing system

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This article presents experimental results on the algebraic identification of the magnitude and phase angle of unbalance in a rotor-bearing system in a Jeffcott configuration. The algebraic identifier is designed based on a simplified mathematical model of the system and it uses only the measurement of the lateral vibration amplitude of the rotor. The proposed algebraic identifier is first validated by numerical simulation. For experimental implementation, a SpectraQuest Machinery Fault and Rotor Dynamics Simulator is used. The designed identifier is evaluated in two scenarios. In the first, the rotor-bearing system is balanced using the traditional coefficients of influence method, after which a known unbalance is induced and compared with the identified magnitude and phase values. In the second case, the unbalance magnitude and phase values obtained by the algebraic identifier from an unknown original unbalanced configuration are used to balance the rotor-bearing system. The vibration amplitude reduction is quantified to evaluate the identified values. The main contribution of this work is the discussion of practical aspects that cannot be appreciated in simulation, but must be considered in the experimental implementation of the algebraic identification method, as they can limit the performance of the designed identifiers.

KEYWORDS

rotor-bearing system, experimental algebraic identification, unbalance, rotating machinery, vibration

1 Introduction

Unbalance is an inherent phenomenon in rotating machinery and is one of the most common vibration sources in turbomachines. Rotor unbalancing is caused by an asymmetric mass distribution, which can result from many diverse factors: manufacturing imperfections, design tolerances, non-homogenous materials, thermal distortion, eccentricity, geometrical discontinuities, corrosion, or wear (Muszynska, 2005; Novillo, 2022; Subbiah and Littleton, 2018). Unbalance occurs when the center of rotation and the center of gravity of the rotor are not coincident. In this case, unbalance is modeled as a concentrated mass located at a certain distance from the center of rotation (called eccentricity). The product of these parameters (mass and eccentricity) is known as the unbalance magnitude, and the angular location of the unbalance mass relative to a reference system provides the unbalance phase angle. Knowledge of unbalance parameters is necessary to apply balancing methods to attenuate vibration amplitudes during operation of rotating machinery (Li et al., 2021; Schneider, 2023), but also to create more accurate models to analyze and predict the dynamic behavior of rotating systems (Bera et al., 2023; Lalanne and Ferraris, 1998; Tiwari, 2018).

In recent years, unbalance identification has been a very active area of rotordynamics research. Mao et al. (2016) proposed an indirect method for identifying unbalance parameters. The authors considered the eccentricity parameter as known, and the identification of the mass and phase angle is formulated as the unbalance force reconstruction problem. First, the unbalance force is reconstructed from the transient system response, and then the unbalance mass and phase angle are identified from the unbalance force reconstruction. The proposed method is evaluated numerically and experimentally. In the test rig, the identified unbalance parameters were used for balancing the rotor system, achieving a vibration attenuation of about 60%. In a similar way, Shrivastava and Mohanty (2020) used a Kalman filter–based estimation technique to estimate the unbalance force. The proposed technique requires a mathematical model and the displacement/velocity signals at different locations of the shaft which were obtained by numerical integration of experimental acceleration measurements. The estimated force is used to identify the unbalance mass and phase angle at different constant speeds. The authors found that, in some cases when variation in system speed is observed, the estimated phase angle is not accurate. The same authors identified the unbalance parameters using a joint input-state estimation technique. The procedure includes the estimation of the unbalance force and uses the mathematical system model and the displacement measurements at the bearing locations. In the experimental evaluation of the proposed technique, some deviations were observed in the estimated phase angles for certain cases (Shrivastava and Mohanty, 2019). It is important to mention that these methods need to identify the bearing parameters before being applied to estimate the unbalance parameters. Moreover, they consider a rigid rotor model, which may not capture the dynamics of some real-world machines, e.g., high-speed (flexible) rotors, and the obtained results are not used for balancing the rotor system. Machine and deep learning algorithms have been used for rotor unbalance diagnosis in rotating machinery, but there are still some issues to be considered for future analysis, such as the influence of a noisy environment, the speed of convergence of the algorithms, computational cost, volume of data, consistency of data, loss of some input data, and trustworthiness of the input data, etc. (Rajagopalan et al., 2023; Wisal and Oh, 2023). Optimization is another approach utilized to solve the problem of identifying the unbalance parameters. Abbasi et al. (2022) proposed a novel optimization-based method to determine the parameters of a rotating unbalance in a rotor-bearing system. A hybrid algorithm integrating the salp swarm algorithm and Nelder–Mead algorithms for detecting unbalance magnitude and phase angle was developed. The results showed the superiority of the proposed hybrid algorithm in terms of the accuracy of the unbalance parameters and computational efficiency compared to other optimization algorithms in the literature. Recently, Sheng et al. (2024) proposed a method to identify the unbalanced vibration feature, based on the fused cross-correlation fast Fourier transform (FC-CFFT) method. Authors compared this method with FFT, the cross-power, and the sine-approximation methods and they found that FC-CFFT method is more accurate in extracting different unbalanced vibration features of the rotor system. Zhou et al. (2024) investigated the application of the augmented Kalman filter (AKF) for unbalance identification of a practical turbofan engine. The proposed method showed favorable convergence and an accurate estimation for the unbalance magnitude, but results about unbalance phase angle are not reported.

On the other hand, the algebraic identification method has been recently explored in rotordynamics. Mendoza-Larios et al. (2021) used the algebraic identification approach to estimate the stiffness and damping rotordynamic coefficients in a rotor-bearing system under the assumption that the elasticity and dissipative forces are linear. The proposed identifiers are validated only numerically in two scenarios, constant and variable rotational speed of the rotor-bearing system. The simulation results indicate fast convergence in the identification of stiffness and damping parameters, taking less than 0.06 s for both considered operating conditions. Barrerdo et al. (2024) proposed algebraic identifiers for the mass, stiffness and damping parameters of a simplified rotor-bearing system with two degrees of freedom. Authors evaluated numerically the proposed identifiers showing that it is possible to determine the values of the mass, damping and stiffness parameters of the rotor-bearing system in a small-time interval less to 0.28 s. In Beltrán-Carbajal et al. (2013) the algebraic parameter identification methodology is applied to estimate the mass, stiffness, damping, rotor eccentricity, and on-line reconstruction of the unknown centrifugal forces induced by rotor unbalance in order to design an active unbalance controller in a three degrees-of-freedom Jeffcott-like rotor-bearing system. The proposed identifier-controller scheme performance is evaluated by numerical simulations. Arias-Montiel et al. (2014) estimated the unbalance forces in a two-disks rotor-bearing system by using an asymptotic extended-state observer and a reduced-order finite element model. The eccentricity parameter of unbalance in both disks was identified from the estimated unbalance force using the algebraic identification approach. The obtained unbalance forces are used to synthesize an active control law to attenuate the vibration response of the rotor-bearing system. An experimental test rig was used only to validate the finite element model, but the observer and algebraic identifier were validated only in simulation. A methodology for balancing rotor-bearing systems based on an active balancing disk is presented in Mendoza-Larios et al. (2016). The algebraic identification method is used to determine the magnitude of the unbalance and its angular position on the rotor. The proposed methodology is validated numerically. Baltazar-Tadeo et al. (2023) developed and integrated approach for balancing asymmetric rotor-bearing systems by combining algebraic identification, modal balancing, and active balancing disks. The authors concluded that the integration of such elements allows for *in-situ* balancing the asymmetric rotor-bearing system in one single trial run, but the presented results are based solely on numerical simulation. More recently, a novel method for balancing asymmetric rotor-bearing systems, designed to overcome some limitations of previously reported methods, was developed, achieving the simultaneous balance of four vibration modes by a single trial run (Baltazar-Tadeo et al., 2024). The proposed method uses

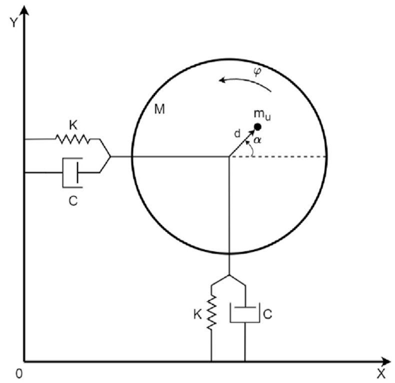

FIGURE 1
Body-free diagram of the Jeffcott rotor.

algebraic identification to calculate a modal masses array for the modal unbalance of the asymmetrical rotor-bearing system. The efficacy of the proposed method was demonstrated numerically.

As can be noted, the algebraic identification method has been widely used for parameters estimation in rotordynamics. This is mainly due to its advantages over other methods: robustness against structured perturbations and noise, high convergence velocity, and it does not require the classical "persistency of excitation condition" (Fliess and Sira-Ramírez, 2003; Sira-Ramírez et al., 2014; Trapero et al., 2008). These properties have been extensively proven by numerical simulation in rotor-bearing systems. Baltazar-Tadeo et al. (2024) showed through numerical simulations that the algebraic identification method is sensitive to noise in vibration signals obtained from rotordynamic systems, and the parameters identified by this approach can be strongly affected by the presence of high levels of noise; therefore, the use of filtering techniques is recommended. Moreover, to the knowledge of the authors, experimental implementation of algebraic identification in real rotordynamic systems has not yet been carried out.

In this work, the experimental implementation of the algebraic identification of magnitude and phase of unbalance in a rotor-bearing system in a Jeffcott configuration is presented. The algebraic identifier is designed based on a two-degree-of-freedom system model and requires only the measurement of the lateral vibration amplitude of the rotor. The proposed algebraic identifier is numerically validated by considering two different operating conditions: constant and variable rotational velocity. After that, a SpectraQuest Machinery Fault and Rotor Dynamics Simulator is used for the experimental implementation of the algebraic identifier. For this case, the identifier is evaluated in two scenarios. Firstly, the rotor-bearing system is balanced using the traditional coefficients of influence method, and then a known unbalance is induced, with the identified balance (magnitude and phase) compared to this. In the second case, the rotor-bearing system starts in an unknown unbalanced condition, and the unbalance magnitude and phase values obtained by the algebraic identifier are used to balance the rotor-bearing system. The vibration amplitude reduction is quantified to evaluate the identified values. Finally, important remarks on practical aspects about experimental implementation of algebraic identifier are provided.

2 Materials and methods

2.1 Algebraic identifier development

Algebraic identification is a model-based method, and the classical Jeffcott rotor model is used to develop the identifier for the unbalance parameters. Different approaches have been proposed for Jeffcott rotor modeling (Friswell et al., 2010; Subbiah and Littleton, 2018; Tiwari, 2018; Vance et al., 2010), but all of them agrees on the idea of a disk with an unbalanced mass at a certain distance (eccentricity) from the geometrical center, as shown in Figure 1, where M is the rotor mass, K and C are the equivalent stiffness and damping of the rotor-bearing system, respectively, m_u is the unbalance mass, d is the eccentricity parameter, α is the unbalance phase angle, and φ denotes the rotation angle of the rotor. In this work, the dynamic model obtained by the Euler-Lagrange formalism described in Beltrán-Carbajal et al. (2014) and given in Equations 1, 2 is employed.

$$M\ddot{x} + C\dot{x} + Kx = m_u d\left(\ddot{\varphi}\sin(\alpha+\varphi) + \dot{\varphi}^2\cos(\alpha+\varphi)\right) \quad (1)$$

$$M\ddot{y} + C\dot{y} + Ky = m_u d\left(\dot{\varphi}^2\sin(\alpha+\varphi) - \ddot{\varphi}\cos(\alpha+\varphi)\right) \quad (2)$$

where x and y are the distances from the origin of the reference frame to the geometrical disk center on the X and Y axes, respectively.

For the development of the algebraic identifier, the time-domain methodology explained by Sira-Ramírez et al. (2014) is followed.

First, both sides of Equations 1, 2 are multiplied by t^2:

$$t^2\left(M\ddot{x} + C\dot{x} + Kx = m_u d\left(\ddot{\varphi}\sin(\alpha+\varphi) + \dot{\varphi}^2\cos(\alpha+\varphi)\right)\right) \quad (3)$$

$$t^2\left(M\ddot{y} + C\dot{y} + Ky = m_u d\left(\dot{\varphi}^2\sin(\alpha+\varphi) - \ddot{\varphi}\cos(\alpha+\varphi)\right)\right) \quad (4)$$

After that, the obtained Equations 3, 4 are integrated with respect to t.

$$\iint M\ddot{x}t^2 + \iint C\dot{x}t^2 + \iint Kxt^2 = \iint m_u dt^2\left(\ddot{\varphi}\sin(\alpha+\varphi) + \dot{\varphi}^2\cos(\alpha+\varphi)\right) \quad (5)$$

$$\iint M\ddot{y}t^2 + \iint C\dot{y}t^2 + \iint Kyt^2 = \iint m_u dt^2\left(\dot{\varphi}^2\sin(\alpha+\varphi) - \ddot{\varphi}\cos(\alpha+\varphi)\right) \quad (6)$$

To solve Equations 5, 6, the four terms from each equation are separated and solved individually as it is shown in Equations 7-14.

The integration by parts method is used to integrate the three terms on the left side of Equation 5, resulting in

$$\iint M\ddot{x}t^2 = M\left(t^2x - 4\int xt + 2\iint x\right) \quad (7)$$

$$\iint C\dot{x}t^2 = C\left(\int t^2x - 2\iint xt\right) \quad (8)$$

$$\iint Kxt^2 = K\left(\iint t^2x\right) \quad (9)$$

In an analogous manner, for Equation 6, we have

$$\iint M\ddot{y}t^2 = M\left(t^2y - 4\int yt + 2\iint y\right) \quad (10)$$

$$\iint C\dot{y}t^2 = C\left(\int t^2 y - 2\iint yt\right) \tag{11}$$

$$\iint K y t^2 = K\left(\iint t^2 y\right) \tag{12}$$

For the right-side term in Equation 5, the integration by parts method is applied by considering $u = t^2$, $du = 2t$, $dv = \frac{\mathrm{d}}{\mathrm{d}t}(\dot{\varphi}\sin(\alpha+\varphi))$ and $v = \dot{\varphi}\sin(\alpha+\varphi)$, and using the trigonometric identity $\sin(\alpha+\varphi) = \sin(\alpha)\cos(\varphi) + \cos(\alpha)\sin(\varphi)$, we obtain

$$\begin{aligned}&\int\int m_u d t^2\left(\ddot{\varphi}\sin(\alpha+\varphi) + \dot{\varphi}^2\cos(\alpha+\varphi)\right)\\ &= m_u d\sin(\alpha)\left(\int t^2\dot{\varphi}\cos(\varphi) - 2\int\int t\dot{\varphi}\cos(\varphi)\right) + m_u d\cos(\alpha)\\ &\quad\times\left(\int t^2\dot{\varphi}\sin(\varphi) - 2\int\int t\dot{\varphi}\sin(\varphi)\right)\end{aligned} \tag{13}$$

In a similar way, for the right side on Equation 6, we use $u = t^2$, $du = 2t$, $dv = -\frac{\mathrm{d}}{\mathrm{d}t}(\dot{\varphi}\cos(\alpha+\varphi))$, $v = \dot{\varphi}\sin(\alpha+\varphi)$ and $\cos(\alpha+\varphi) = \cos(\alpha)\cos(\varphi) - \sin(\alpha)\sin(\varphi)$ to obtain

$$\begin{aligned}&\int\int m_u d t^2\left(\dot{\varphi}^2\sin(\alpha+\varphi) - \ddot{\varphi}\cos(\alpha+\varphi)\right)\\ &= m_u d\sin(\alpha)\left(\int t^2\dot{\varphi}\sin(\varphi) - 2\int\int t\dot{\varphi}\sin(\varphi)\right) - m_u d\cos(\alpha)\\ &\quad\times\left(\int t^2\dot{\varphi}\cos(\varphi) - 2\int\int t\dot{\varphi}\cos(\varphi)\right)\end{aligned} \tag{14}$$

Finally, the global solutions for Equations 5, 6 are

$$\begin{aligned}&M\left(t^2x - 4\int xt + 2\int\int x\right) + C\left(\int t^2x - 2\int\int xt\right) + K\left(\int\int t^2x\right)\\ &= m_u d\sin(\alpha)\left(\int t^2\dot{\varphi}\cos(\varphi) - 2\int\int t\dot{\varphi}\cos(\varphi)\right) + m_u d\cos(\alpha)\\ &\quad\times\left(\int t^2\dot{\varphi}\sin(\varphi) - 2\int\int t\dot{\varphi}\sin(\varphi)\right)\end{aligned} \tag{15}$$

$$\begin{aligned}&M\left(t^2y - 4\int yt + 2\int\int y\right) + C\left(\int t^2y - 2\int\int yt\right) + K\left(\int\int t^2y\right)\\ &= m_u d\sin(\alpha)\left(\int t^2\dot{\varphi}\sin(\varphi) - 2\int\int t\dot{\varphi}\sin(\varphi)\right) - m_u d\cos(\alpha)\\ &\quad\times\left(\int t^2\dot{\varphi}\cos(\varphi) - 2\int\int t\dot{\varphi}\cos(\varphi)\right)\end{aligned} \tag{16}$$

Regrouping and renaming terms as

$$mds = m_u d\sin(\alpha) \tag{17}$$

$$mdc = m_u d\cos(\alpha) \tag{18}$$

$$a_1 = \left(\int t^2\dot{\varphi}\cos(\varphi) - 2\iint t\dot{\varphi}\cos(\varphi)\right) \tag{19}$$

$$a_2 = \left(\int t^2\dot{\varphi}\sin(\varphi) - 2\iint t\dot{\varphi}\sin(\varphi)\right) \tag{20}$$

$$b_1 = M\left(t^2x - 4\int xt + 2\iint x\right) + C\left(\int t^2x - 2\iint xt\right) + K\left(\iint t^2x\right) \tag{21}$$

$$b_2 = M\left(t^2y - 4\int yt + 2\iint y\right) + C\left(\int t^2y - 2\iint yt\right) + K\left(\iint t^2y\right) \tag{22}$$

TABLE 1 Numerical parameters for simulation.

Parameter	Value
Rotor mass M	1.8581 kg
Equivalent damping C	22.0293 $N\cdot s/m$
Equivalent stiffness K	38804.7144 N/m
Unbalance magnitude $m_u d$	1.0752×10^{-4} $kg\cdot m$
Unbalance phase angle α	$\frac{\pi}{6}$ rad
Rotational velocity $\dot{\varphi}$	40π rad/s
Simulation time t_{sim}	1.5 s
Integration step P_{int}	0.5 ms

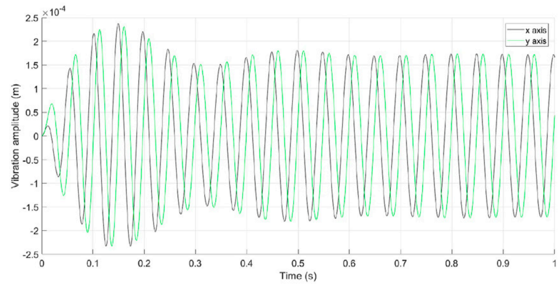

FIGURE 2
Rotor-bearing system response in simulation at constant rotational velocity.

Equations 15, 16 can be rewritten as

$$\begin{bmatrix} a_1 & a_2 \\ a_2 & -a_1 \end{bmatrix}\begin{bmatrix} mds \\ mdc \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \tag{23}$$

The linear Equation 23 can be solved using the Cramer's rule (Strang, 2023) to obtain

$$mds = \frac{b_1a_1 + a_2b_2}{a_1^2 + a_2^2} \tag{24}$$

$$mdc = \frac{a_2b_1 - b_2a_1}{a_1^2 + a_2^2} \tag{25}$$

From the obtained values in Equations 24, 25, magnitude $(m_u d)$ and phase angle α of rotor unbalance can be calculated with

$$m_u d = \sqrt{mds^2 + mdc^2} \tag{26}$$

$$\alpha = \arctan\frac{mds}{mdc} \tag{27}$$

As can be appreciated in Equations 15–27, the algebraic identifier developed to estimate the unbalance parameters is in terms of a_1, a_2, which can be quantified by the known rotational velocity of the rotor-bearing systems, and b_1, b_2, which only depend on the measured lateral vibration system response x and y.

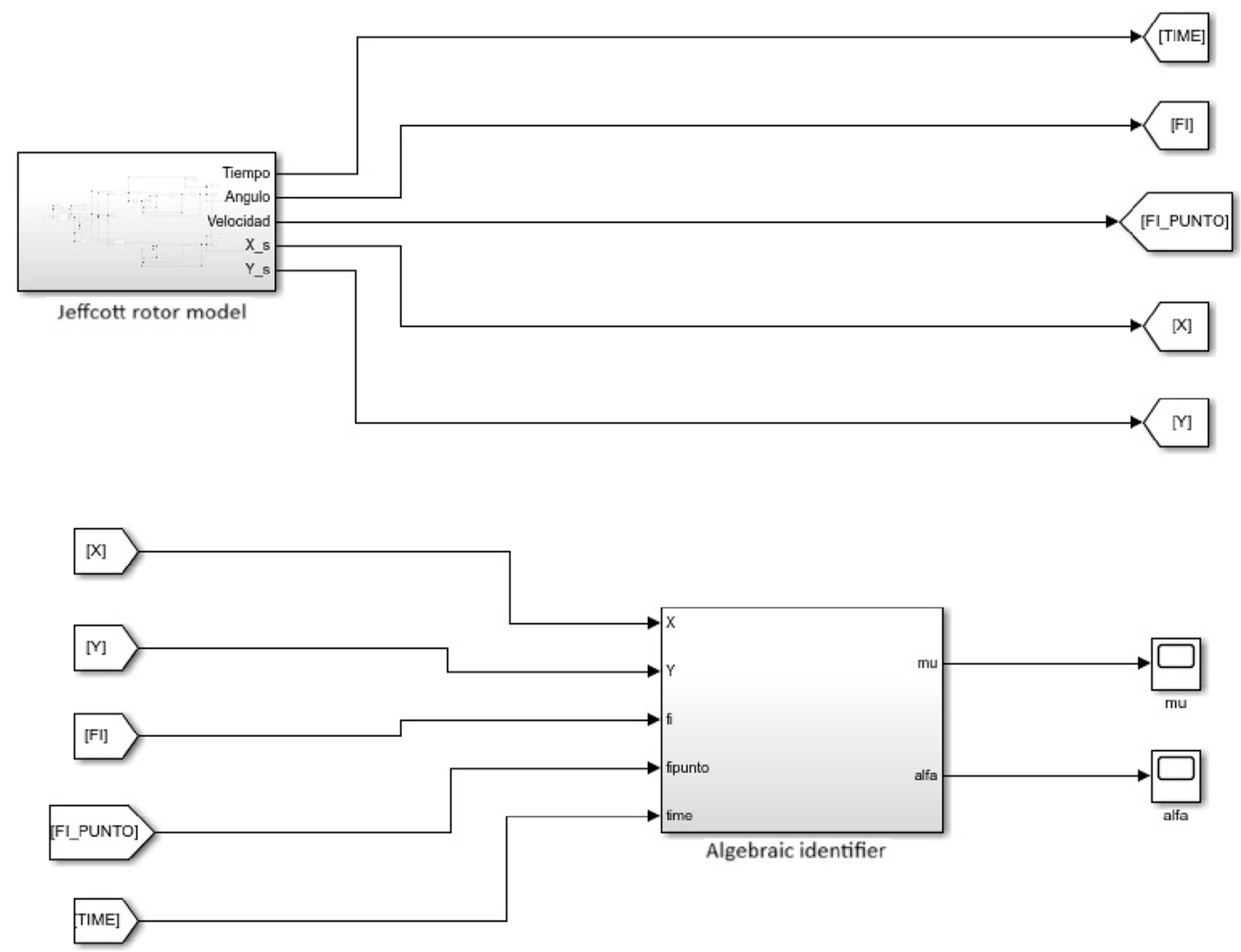

FIGURE 3
Block-diagram for rotor-bearing system model and algebraic identifier.

3 Results

3.1 Numerical validation of the algebraic identifier

In order to prove the algebraic identifier's performance by simulation, the rotor-system dynamics given by Equations 1, 2 were numerically solved using Matlab-Simulink with the numerical parameters presented in Table 1. The obtained lateral vibration system response is shown in Figure 2. As can be observed, the amplitudes on the x and y axes have the same magnitude, but they are 90° out of phase because we are considering a symmetrical rotor. These obtained data, along with the rotational velocity and simulation time, are provided to algebraic identifier, as shown in Figure 3, in which "Algebraic identifier" subsystem contains the programming code for Equations 19–27 and the "Jeffcott rotor model" subsystem the corresponding code for Equations 1, 2.

Results for the identification of unbalance magnitude and phase angle at constant rotational velocity are shown in Figure 4. As can be observed, the identified unbalance parameters rapidly converge to the values considered for simulation in Table 1. It is important to note the high resolution on the vertical scale in both figures, which allows for the precision of the calculated values to be appreciated.

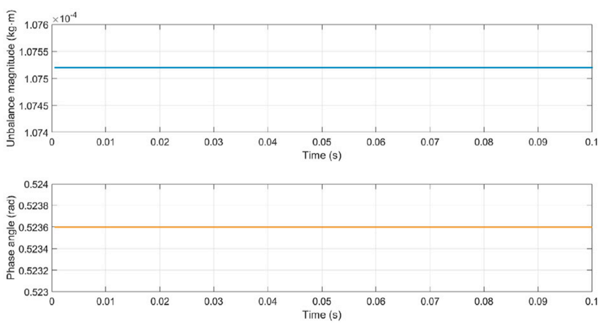

FIGURE 4
Identified unbalance magnitude and phase angle at constant velocity.

Numerical simulations were carried out at variable rotational velocity of the rotor-bearing system to prove the

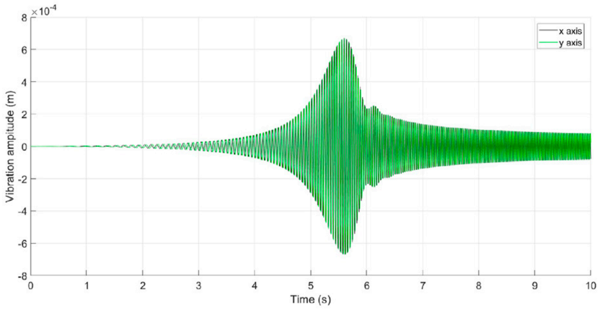

FIGURE 5
Rotor-bearing system response in simulation at variable rotational velocity.

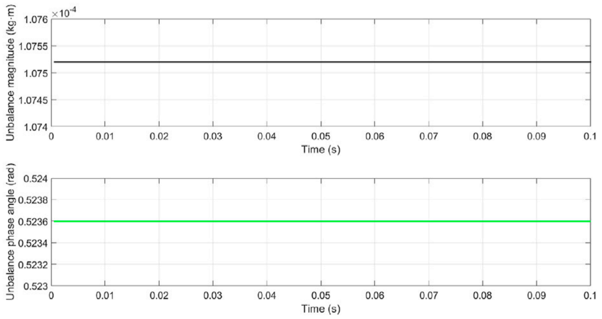

FIGURE 6
Identified unbalance magnitude and phase angle at variable velocity.

algebraic identifier's performance for this operating condition. In this case, a ramp velocity with an angular acceleration of $27 \frac{rad}{s^2}$ was considered. The system response is presented in Figure 5, and the identified values for unbalance parameters are shown in Figure 6. The algebraic identifier's performance is practically the same as for the constant velocity case, the estimated values converge to the real ones in a few milliseconds.

3.2 Test rig description

For experimental implementation of the proposed algebraic identifier, the SpectraQuest Machinery Fault and Rotor Dynamics Simulator shown in Figure 7 was used. Vibration amplitudes were measured by two Eddy current proximity probes (1) by SpectraQuest, located perpendicularly to each other, and the measuring kit includes a signal conditioner (2) that transduces displacement into a voltage signal. A DAQ 6024E data acquisition card by National Instruments (3) was used to acquire the proximity probe signals. The acquired data are processed in Matlab Simulink (4) software to be used in the experimental validation of the algebraic identifier. Finally, an infrared sensor was utilized to generate a pulsing signal to be used as a reference to measure the phase of the proximity probe signals.

3.3 Rotor-bearing system characterization

Mass, damping and stiffness parameters given in Table 1 were experimentally estimated. The mass of the rotor-bearing system M was calculated by Equation 28 (Rao, 2017).

$$M = m_d + 0.5\ m_s \tag{28}$$

where m_d and m_s are the masses of the disk and rotor shaft, respectively. Both elements were directly weighed, yielding $m_d = 1.5\ kg$ and $m_s = 0.7163\ kg$, giving a result of $M = 1.8581\ kg$.

For damping C and stiffness K estimation, the peak-picking method (Ewins, 2000; Inman, 2017) was applied to the experimental frequency response of the rotor-bearing system presented in Figure 8, where amplitude R was calculated by Equation 29.

$$R = \sqrt{x^2 + y^2} \tag{29}$$

and x and y are the vibration amplitudes in steady-state for each operating frequency.

The damping ratio ζ can be estimated by

$$\zeta = \frac{\omega_2 - \omega_1}{2 \cdot \omega_r} \tag{30}$$

where ω_1 and ω_2 are the frequencies corresponding to an amplitude $\frac{A_{max}}{\sqrt{2}}$, A_{max} is the maximum amplitude of the

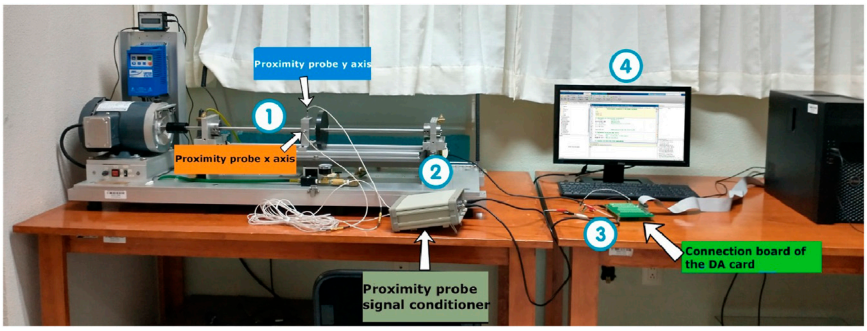

FIGURE 7
Rotor-bearing test rig.

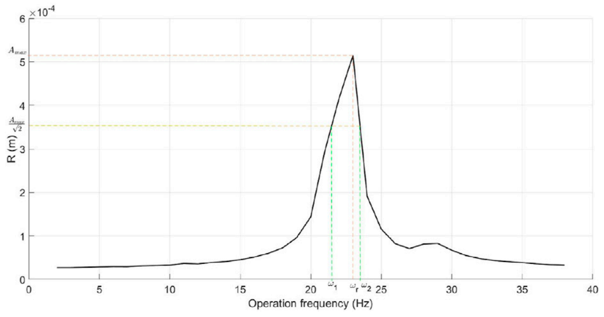

FIGURE 8
Experimental frequency response of the rotor-bearing system.

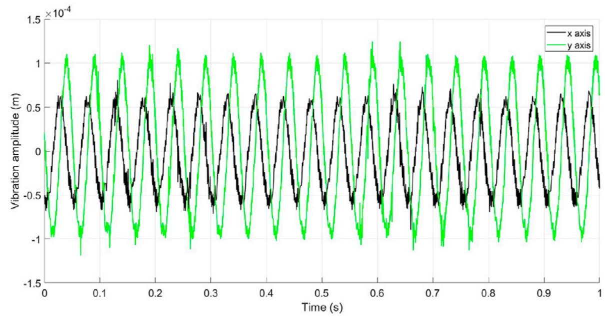

FIGURE 9
Experimental unfiltered response of the rotor-bearing system at 20 Hz.

frequency response, and ω_r is the frequency corresponding to the maximum amplitude A_{max}.

From Figure 8, we can obtain the following values: $A_{max} = 5.1502 \times^{-4} m$, $\omega_r = 144.5132 \frac{rad}{s} = 23\ Hz$, $\frac{A_{max}}{\sqrt{2}} = 3.6418 \times^{-4} m$, $\omega_1 = 21.5787\, Hz$, and $\omega_2 = 23.4659\, Hz$. Substituting them in Equation 30, we obtain the damping ratio as

$$\zeta = \frac{23.4659\ Hz - 21.5787\ Hz}{2 \cdot 23\ Hz} = 0.04102$$

The equivalent stiffness coefficient can be obtained from the natural frequency definition (Rao, 2017) given in Equation 31.

$$K = M \cdot w_r^2 \tag{31}$$

and substituting the mass and natural frequency values:

$$K = 1.8581\ kg \cdot (144.5132\ rad/s)^2 = 38804.7144 \frac{N}{m}$$

Finally, the equivalent damping coefficient is obtained from the critical damping (C_r) definition (Rao, 2017) as follows

$$C_r = 2 \cdot \sqrt{K \cdot M} = 2 \cdot \sqrt{38804.7144 \frac{N}{m} \cdot 1.8581\ kg} = 537.0401 \frac{N \cdot s}{m}$$

$$C = C_r \cdot \zeta = 537.0401 \frac{N \cdot s}{m} \cdot 0.04102 = 22.0293 \frac{N \cdot s}{m}$$

Once the rotor-bearing system has been characterized, the proposed algebraic identifier is experimentally proven for two cases. In the first, the rotor-bearing system is balanced, and then a known unbalance is induced. In the second scenario, the algebraic identifier is used to estimate the unbalance parameters of an unknown unbalanced condition, and the identified parameters are used to balance the rotor-bearing system.

3.4 Identification of induced unbalance parameters

The rotor-bearing system was balanced using the well-known influence coefficients method (Bently and Hatch, 2002; Lees, 2016). It is important to mention that this method requires a reference signal to measure the phase angle, which was obtained from the infrared sensor mentioned in the previous section. This reference signal is also used to measure the unbalance phase angle estimated by the algebraic identifier.

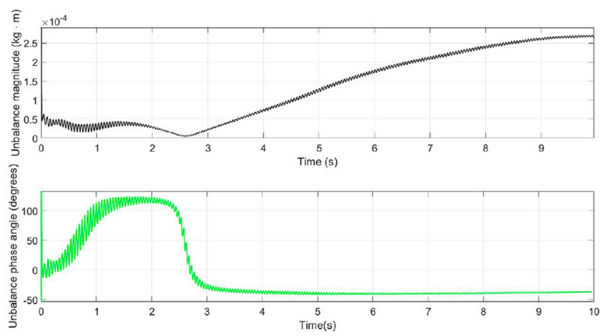

FIGURE 10
Identified unbalance magnitude and phase angle with unfiltered signals.

From this balanced condition, a heavy spot was induced. For the first case, an unbalance mass $m_u = 1.2868 \times 10^{-3} kg$ at a radial distance $d = 57 \times 10^{-3} m$ from the geometrical center of the disk was applied. These parameters result in an unbalance magnitude $m_u d = 7.3347 \times 10^{-3} kg \cdot m$, and the mass location corresponds to a phase angle of 0° from the reference signal provided by the infrared sensor. In Figure 9, the experimental response of the rotor-bearing system operating at a constant rotational velocity of 20 Hz is shown. These data were introduced into the proposed algebraic identifier, and the results for the estimated unbalance magnitude and phase angle are depicted in Figure 10.

As can be appreciated, the unbalance parameter values do not converge to any value. This is mainly because the influence of noise in the acquired data from the proximity probes. Some digital filtering techniques were applied to the raw signals; however, the identifier performance could not be improved because digital filters induce delays in the filtered signals. This characteristic affects the phase angle of the acquired signals and, as a consequence, the identifier's performance. Results of the identified values with filtered signals are included as Supplementary Material. Experimental filtered signals used as input to the algebraic identifier are presented in Supplementary

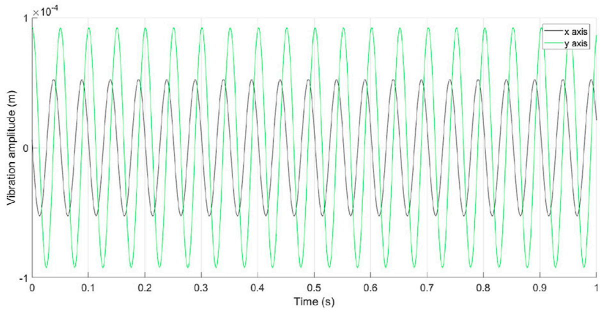

FIGURE 11
Artificially reconstructed signals for response of the rotor-bearing system at 20 *Hz*.

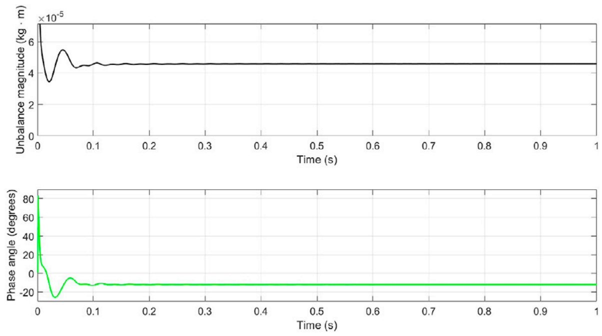

FIGURE 12
Identified unbalance magnitude and phase angle with reconstructed signals.

Figures S1, S2, and the obtained results for the identified values of the unbalance parameters are shown in Supplementary Figures S3, S4.

As a solution for this problem, we propose a signal reconstruction procedure to obtain the system response free of noise and without any delay that could affect the identifier's performance. Firstly, the raw signals are filtered. Then, amplitude, frequency and phase angle of the signals are obtained by analyzing the first two cycles, since according to Baltazar-Tadeo et al. (2023), the unbalance parameters can be obtained with the algebraic approach using just a small fraction of the system response. The reconstructed signals of the rotor-bearing system response are presented in Figure 11. These data were used as input for the algebraic identifier, and the corresponding identified values for unbalance magnitude and phase angle are shown in Figure 12.

The identified unbalance parameter values converge rapidly (in $0.1s$) to a constant value. The unbalance mass can be calculated from the identified unbalance magnitude in Figure 12 as

$$m_b = \frac{4.59 \times^{-5} kg \cdot m}{57 \times 10^{-3} m} = 8.0526 \times 10^{-4} kg$$

This value is compared with the induced unbalance mass ($1.2868 \times 10^{-3} kg$), resulting in a percentage difference of 37.4%. The identified value for the unbalance phase angle is 12° and in this case it is not possible to calculate a percentage difference because the induced angle value is 0°.

The obtained differences may be due to various factors, such as the residual unbalance after the rotor balancing by the influence coefficient method (the rotor cannot be perfectly balanced), the consideration of a simplified symmetric mathematical model when the obtained experimental system response is asymmetric, a possible shaft runout, and other unmodeled dynamics. Moreover, it was not possible to obtain balancing masses with the exact calculated values, and only approximated values were used.

More experimental tests were carried out with the same unbalance mass at different angular locations, following the procedure for the system response signals reconstruction. The obtained results for the identified unbalance mass and phase angle are summarized in Table 2. These values were used for rotor balancing by locating a similar mass 180° from the identified value for the unbalance phase angle. In Table 3 the balancing mass and angular location as well as the vibration attenuation percentages for each experimental test, are presented. Graphical results for vibration amplitude attenuation in the case of the first row in Table 3 are shown in Figure 13 for x and y-axes. It is important to mention that all the signals in Figure 13 were digitally filtered to improve visualization and comparison, as well as to facilitate the quantification of vibration attenuation.

3.5 Identification of original unbalance parameters

For the second scenario, all the trim, balancing and unbalance masses were removed from the disk of the rotor-bearing system. From this original unbalanced condition, the vibration system response was obtained, and following the reconstruction signal procedure described in the previous section, the unbalance parameters were estimated by the proposed algebraic identifier.

TABLE 2 Identified unbalance parameters by experimental tests.

Induced mass	Induced phase angle	Identified mass	Identified phase angle
$1.2868 \times 10^{-3} kg$	120°	$0.69 \times 10^{-3} kg$	121.6°
$1.2868 \times 10^{-3} kg$	180°	$0.68 \times 10^{-3} kg$	−178.4°
$1.2868 \times 10^{-3} kg$	240°	$0.82 \times 10^{-3} kg$	−118.3°
$1.2868 \times 10^{-3} kg$	270°	$0.85 \times 10^{-3} kg$	−89.33°

TABLE 3 Vibration attenuation for rotor balancing by the identified unbalance parameters.

Balancing mass	Balancing angle	Attenuation percentage in x-axis	Attenuation percentage in y-axis
$0.76 \times 10^{-3} kg$	-60°	67%	69%
$0.76 \times 10^{-3} kg$	0°	60%	65%
$0.80 \times 10^{-3} kg$	60°	54%	60%
$0.80 \times 10^{-3} kg$	90°	55%	61%

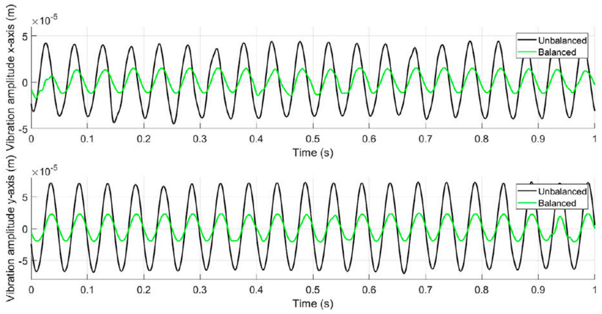

FIGURE 13
Vibration attenuation in x and y-axes (mass balance at 60°).

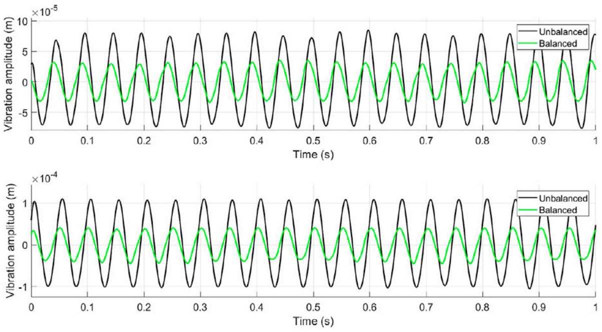

FIGURE 14
Vibration attenuation in x and y-axes for the original unbalanced condition.

The resulting balancing parameters were: a balancing mass of $1.14 \times 10^{-3} kg$ located at 30°. In Figure 14, the obtained vibration attenuation for a balancing mass of $1.2868 \times 10^{-3} kg$ at 30° is presented. The calculated attenuation percentage is 63% for x-axis and 62% for y-axis.

4 Conclusion and outlook

In this article, an identifier for the unbalance parameters of a Jeffcott-like rotor-bearing system based on the algebraic method was developed. A simplified mathematical model for the rotor-bearing system and Matlab Simulink software were employed to design and numerically prove the proposed identifier, finding a rapid convergence of the estimated parameters to the correct values. In experimental testing, however, the designed identifier was negatively affected by the inevitable noise in the measured signals needed for the identification procedure, finding that in presence of noise, the identified unbalance parameters did not converge to any constant value. Digital filtering techniques were not effective because the induced delay in the filtered signals affected the estimated values, mainly the unbalance phase angle. Taking advantage of the velocity convergence of the algebraic identification method, a reconstruction signal procedure was proposed in order to decrease the noise influence on the identifier's performance without phase delay effects. By applying the reconstruction signal procedure, the identifier performance was improved, achieving a rapid convergence of the estimated unbalance parameters. This procedure has resulted a viable and practical approach for off-line rotor balancing using algebraic identifiers, but its applicability in cases of on-line balancing must be addressed both numerically and experimentally to verify the scope and limitations.

The proposed algebraic identifier was evaluated by using the estimated unbalance parameters for rotor balancing. Experimental results showed vibration amplitude attenuations around 60% for all tests carried out. These vibration attenuation percentages are comparable with those previously reported in literature. Mao et al. (2016) achieved a reduction between 58% and 65% in vibration amplitude but the proposed method must reconstruct the unbalance force before the unbalance parameters could be estimated. Shrivastava and Mohanty (2019) reported vibration amplitudes attenuation of around 50% using a joint input-state estimation technique. This approach also requires the calculation of the unbalance force previously to the unbalance parameters estimation.

The obtained results could be improved by using a more complex mathematical model of the rotor-bearing system that considers dynamical effects ignored by the simplified model used here. Moreover, the importance of properly measuring the phase of signals was noted in the experimental tests. It is necessary to have the same reference for both numerical data obtained by mathematical model simulation and experimental signals. This is achieved by placing the infrared sensor in a correct location, in this case, in the same angular position than proximity probe used to measure the vibration amplitudes in y-axis. Since algebraic identification is a model-based identification method, as a future work we propose the consideration of finite element (FE) models which describes more precisely the dynamics of the real system. FE models can include various characteristics ignored by the Jeffcott model such as more inertial disks along the shaft, changes in the shaft diameter, gyroscopic effects, anisotropy in bearings and shaft properties. These models have already been used in the development of unbalance parameters identifiers using the algebraic identification method (Arias-Montiel et al., 2014; Mendoza-Larios et al., 2016; Baltazar-Tadeo et al., 2023; Baltazar-Tadeo et al., 2024) but the reported results are only in numerical simulations. Besides, some additional problems have emerged by the use of this kind of model, mainly the need to measure a great amount of vibration amplitudes along the shaft. Some solutions for this problem has been reported as the use of state observers (Arias-Montiel et al., 2014) or pseudo-modal models (Baltazar-Tadeo et al., 2024). However, these proposals have been evaluated only numerically. Therefore further experimental work is required to overcome these difficulties and achieve the implementation of algebraic identification in more complex systems. Theoretically, the algebraic identification method is robust against the noise in the acquired signals (Fliess and Sira-Ramírez, 2003; Sira-Ramírez et al., 2014; Trapero et al., 2008), but in our experience, noise is an important factor which negatively affect the estimated parameters value. Thus, we believe that more experimental evidence is needed about the influence of noise in the measured signals on the performance of identifiers based on algebraic method, in order to verify its robustness in comparison with other methods in real systems applications.

Data availability statement

The datasets presented in this study can be found in online repositories. The names of the repository/repositories and accession number(s) can be found below: Google drive https://drive.google.com/drive/folders/1UXa3e1aIR2iBXHUSbaRpMKoVKb0bAJRN.

Author contributions

JQ-B: Investigation, Software, Validation, Writing–review and editing. MA-M: Conceptualization, Formal Analysis, Methodology, Project administration, Resources, Writing–original draft, Writing–review and editing. JM-L: Conceptualization, Formal Analysis, Methodology, Resources, Writing–review and editing. LV-S: Formal Analysis, Resources, Writing–review and editing.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

The author(s) declare that no Generative AI was used in the creation of this manuscript.

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Supplementary material

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fmech.2025.1553759/full#supplementary-material

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