

Extending resonant inelastic X-ray scattering to the extreme ultraviolet

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In resonant inelastic X-ray scattering (RIXS), core hole resonance modes are used to enhance coupling between photons and low energy electronic degrees of freedom. Resonating with shallow core holes accessed in the extreme ultraviolet (EUV) can provide greatly improved energy resolution at standard resolving power, but has been found to often yield qualitatively different spectra than similar measurements performed with higher energy X-rays. This paper uses experimental data and multiplet-based numerical simulations for the M-edges of Co-, Ni-, and Cu-based Mott insulators to review the properties that distinguish EUV RIXS from more commonly performed higher energy measurements. Key factors such as the origin of the strong EUV elastic line and advantages of EUV spectral functions over soft X-ray RIXS for identifying intrinsic excitation line shapes are discussed.

Keywords: Mott insulator, resonant inelastic X-ray scattering, extreme ultraviolet, collective excitations, X-ray reflectivity

OPEN ACCESS

Edited by:

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Specialty section:

This article was submitted to
Condensed Matter Physics,
a section of the journal
Frontiers in Physics

Received: 15 December 2014

Accepted: 23 April 2015

Published: 18 May 2015

Citation:

Wray LA, Huang S-W, Jarrige I,
Ikeuchi K, Ishii K, Li J, Qiu ZQ,
Hussain Z and Chuang Y-D (2015)
Extending resonant inelastic X-ray
scattering to the extreme ultraviolet.
Front. Phys. 3:32.
doi: 10.3389/fphy.2015.00032

1. Introduction

Resonant inelastic X-ray scattering (RIXS) is a powerful technique for observing the energy states of many-body quantum materials, which include features such as the momentum-resolved Mott gap and emergent particles composed of diverse combinations of charge, orbital, and spin degrees of freedom [1–28]. In RIXS, incident photons are tuned close to an absorption edge to excite high energy core hole states that rapidly decay into low energy many-body excitations, represented by Raman features in scattered light. The core hole resonance states that make RIXS possible are strongly correlated, and undergo complex time evolution that can have a large impact on the appearance of inelastic spectra [1–6, 8, 15, 28–30]. This paper will explore the properties of RIXS performed with core level resonances found in the extreme ultraviolet (EUV), a regime that is infrequently used for resonant photon scattering, and which has very distinctive scattering matrix elements and core hole time evolution.

Extending the RIXS to the EUV incident energy range (roughly $10\text{ eV} < h\nu < 120\text{ eV}$) has several very pronounced consequences. The long wavelength of EUV photons makes it possible to achieve superior energy resolution, opening up significant scientific potential discussed in Section 4. Excitation spectral functions observed in the EUV may also be simpler and easier to attribute, with fewer simultaneously excited collective modes (see Section 5). However, resonant elastic scattering (REXS) is extremely strong in the EUV (the reason for this is explained in Section 3), which can be interesting to measure but is mostly a hindrance as it obscures inelastic features below $E \lesssim 0.1\text{ eV}$.

Inelastic X-ray scattering by contrast has a smaller total cross section in the EUV than in the soft X-ray regime, because of growth in the Auger electron emission rate. Also, EUV photons have insufficient momentum to measure the momentum dispersion of itinerant excitations outside of the Brillouin zone center.

On a more subtle level, EUV RIXS spectra show very different feature intensities than one would find at a corresponding soft X-ray resonance (e.g., comparing the transition metal M- and L- edges). The lifetime of shallow core hole resonance states accessed in the EUV has a complex dependence on incident photon energy [6, 31–33], and can provide the option to access both long and short core hole lifetimes at the same resonance edge. The effective shake-up perturbation caused by shallow core holes also tends to be weaker, and can exclude some classes of excitations that are not directly created by the action of the scattering photon operators (e.g., multi-magnons, phonons, charge transfer modes) [3, 4, 7, 8, 10, 28, 34, 35]. Atomic multiplet (AM) models of RIXS can potentially work better in the EUV, because angular momentum entanglement through multipolar Coulomb interactions is stronger than at the L-edge, and protects multiplet symmetries. These effects will be discussed in Sections 5, 6. Many of our observations about EUV RIXS in this paper are common with earlier studies, and in particular the insightful discussion in Chiuzbaian et al. [16]. However, we have attempted to consider a wider range of effects and provide a broader context with respect to numerics.

2. Materials and Methods

The Mott insulators CoO, NiO, and SrCuO₂ were chosen as model materials for this study, and provide examples of resonance from strongly correlated $3d^7$, $3d^8$, and $3d^9$ systems, respectively. The principle electronic excitations observed in X-ray absorption spectroscopy (XAS) and RIXS with p-symmetry core holes on Co [7, 16], Ni [6, 8, 14, 15, 17, 18, 36], and Cu [8, 9] are closely identified with excitation matrix elements obtained from AM based calculations for a high-spin 2+ valence atomic state perturbed by the crystal field of surrounding oxygen atoms. The compounds CoO and NiO have a rock salt crystal structure, with an octahedral crystal field on the transition metal site, and strong antiferromagnetic superexchange interactions through σ -bonded oxygen atoms. The Cu atoms of SrCuO₂ are found in a quasi-1D [11, 13, 21, 37] network of copper oxide plaquettes that lack apical oxygens. The ground state includes a single hole in the $3d_{x^2-y^2}$ orbital [36], and the energies of dd crystal field states are known from previous RIXS measurements [8].

Samples of SrCuO₂ and NiO were cleaved single crystals. The CoO sample was a bulk-like well ordered CoO film grown on MgO, and transferred to the RIXS chamber vacuum with minimal exposure to the air. The sample was grown by molecular beam epitaxy (MBE) in an ultrahigh vacuum chamber with the base pressure of 2×10^{-10} torr. Cobalt was evaporated onto a MgO(001) substrate in 2×10^{-6} torr oxygen at room temperature. After evaporation, the sample was annealed to 600°C within a 2×10^{-6} torr oxygen atmosphere for 30 min, yielding a surface roughness under 1nm measured by atomic force microscopy.

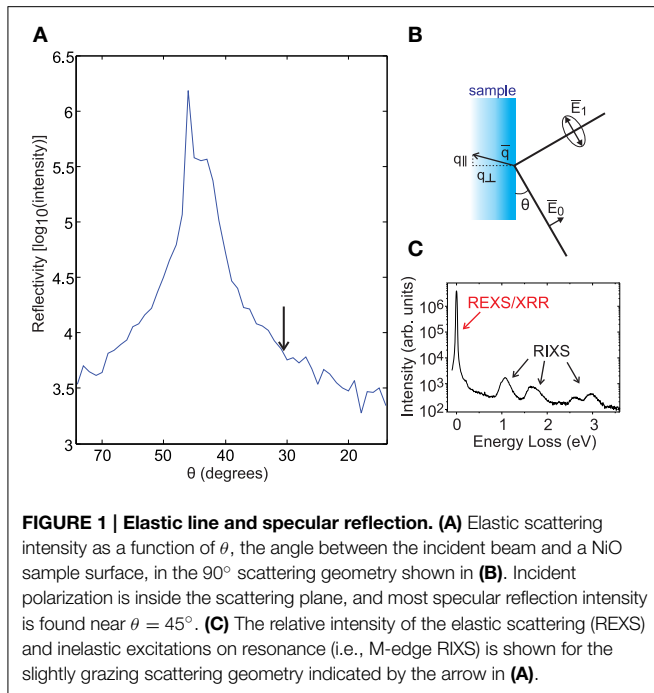
Sample thickness was 45 nm, much greater than the $\lesssim 10$ nm layer over which interface strain and exchange effects may be significant [38], and similar to three times the estimated X-ray penetration depth.

Measurements at the M-edges of Co, Ni, and Cu were performed at the beamline 4.0.3 (MERLIN) RIXS endstation (MERIXS) at the Advanced Light Source (ALS), Lawrence Berkeley National Laboratory. The data were recorded by a VLS based X-ray emission spectrograph equipped with a commercially available CCD detector [39, 40]. Scattering intensity measured at each pixel was normalized to ensure that intensity in the final spectra accurately represents the density of scattered photons per unit energy. Large single crystal samples were measured near room temperature at a pressure of 3×10^{-10} Torr. The resolution-limited full width at half maximum of the elastic line is better than $\delta E \lesssim 35 \pm 2$ meV for all M-edge measurements. Measurements at the copper K-edge were performed with ~ 400 meV resolution using beamline 11-XU at SPring-8. All measurements were performed close to room temperature with in-plane polarization in the [001] scattering plane, with scattered photons measured at 90° to the incident beam trajectory. The photon beam had a grazing (NiO) 25° or (CoO, SrCuO₂) 30° angle of incidence to the cleaved [010] face.

3. Specular Reflection and the Elastic Line

The elastic line observed in EUV RIXS experiments is far stronger than elastic intensity under similar measurement conditions in the soft X-ray regime. For example, M-edge resonant scattering from 3d transition metals is performed in the EUV, and typically yields an elastic line that even under ideal conditions is 3 or more orders of magnitude more intense than inelastic RIXS features (see **Figure 1C**). This contrasts with M-edge atomic multiplet calculations and L-edge soft X-ray data which show that elastic and near-elastic scattering intensity can be within ~ 1 order of magnitude of inelastic features. In this section, we will explain why a strong elastic tail caused by off-specular reflectivity has been unavoidable in EUV X-ray scattering experiments to date. The incident energy dependence of EUV elastic scattering includes a strong resonant component that can be understood from atomic multiplet calculations [6, 7], meaning that the resonant elastic (REXS) signal could potentially be measured on resonance to study electronic correlations on a length scale similar to the incident photon wavelength (e.g., 20–100 nm). On the other hand, it is a significant source of artifacts in RIXS data when the elastic line is 4 or more orders of magnitude stronger than inelastic features, because the RIXS resolution function is convoluted by surface structure in the area of the focused beam spot.

To understand the origin of the EUV elastic line, we have measured how the elastic line intensity varies when approaching the specular reflection condition on a cleaved NiO sample (**Figure 1**). Specular reflection is observed at $\theta = 45^\circ$ for the 90° scattering geometry at the MERLIN beamline (see scattering diagram in **Figure 1B**). The dependence of elastic line intensity on scattering geometry is shown in **Figure 1A**, revealing that elastic intensity decreases exponentially as the scattering



geometry deviates from the specular condition. This trend is common to all sample families we have studied. Similar off-specular and off-Bragg elastic scattering is well-known in the soft X-ray regime, and can provide information about structural and electronic disorder when investigated with techniques such as X-ray photon correlation spectroscopy (XPCS) [41].

However, the tail of specular reflectivity is typically only identified within a few degrees of the specular condition in the soft X-ray regime, and reflection intensity is far weaker than in the EUV [42] due to the roughly $n \sim 1 - c\lambda^2$ dependence of the index of refraction on X-ray wavelength. This leads to an expected increase of about 4 orders of magnitude in the intensity of reflected light at transition metal M-edges relative to L-edges, based on the Fresnel equations ($R_{EUV}/R_X \sim [(n_{EUV} - 1)/(n_X - 1)]^2 \sim (\lambda_{EUV}/\lambda_X)^4 \sim 10^4$). In addition to specular reflection being far stronger in the EUV, the longer wavelengths of EUV photons cause the off-specular elastic tail to cover a much larger solid angle. Surface roughness on a length scale of $\sim 1\mu\text{m}$ is hard to avoid in most samples of interest for RIXS, particularly given the typically $\lesssim 10\text{ nm}$ penetration depth of EUV photons [42]. In-plane scattering with a wavelength of $\lambda > 1\mu\text{m}$ will yield off-specular rays spread over $\delta\theta = 3.2^\circ$ at the nickel M-edge ($h\nu \sim 65\text{ eV}$) in the scattering geometry used for our measurement, qualitatively comparable to the reflectivity feature width in our data, but would only give $\delta\theta = 0.5^\circ$ at the M-edge. If one factors in a 10^4 gain in reflected intensity and a factor of ~ 6 broader rocking curve, it is reasonable to expect off-specular reflectivity tails prominent over several degrees in the soft X-ray regime to cover all solid angles of scattering in the EUV.

The main reason EUV off-specular reflection intensity deviates dramatically from atomic multiplet calculations

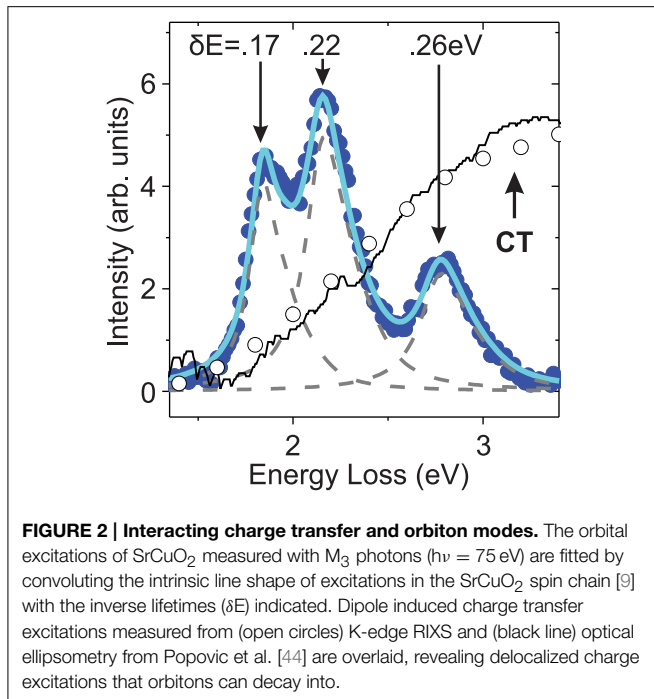
designed for the X-ray regime is that calculations for the X-ray regime make use of the Born approximation, and specular reflection is specifically derived from non-perturbative physics outside of the Born approximation. If we consider even lower photon energies in the optical regime, the observation of off-specular reflection intensity covering all outgoing solid angles is associated with Lambert's cosine law, and has been known of since the eighteenth century [43].

4. A New Regime of Energy Resolution

A primary incentive to perform RIXS in the EUV regime is that with the same resolving power and qualitatively similar optics, one can achieve energy resolution that is as much as an order of magnitude better than RIXS spectrometers in the soft X-ray regime [39, 40]. At the transition metal M-edge, this has so far enabled studies with energy resolution similar to $\delta E \sim 20$ to 30 meV , which is similar to the energy scale of room temperature thermal fluctuations. A principle success of high resolution studies over the last few years has been more detailed observations of how thermal energy changes the appearance of atomic multiplet excitations such as transition metal dd modes. Temperature dependence in dd modes has been linked to thermal lattice expansion and the thermal population of low energy degrees of freedom [7, 45]. The thermal population of low energy electronic degrees of freedom can introduce an intensity tail on the energy gain side (i.e., the leading edge) of excitations in a pattern resembling anti-Stokes scattering. This tail is termed “pseudo-anti-Stokes” emission in Wray et al. [7].

Entering the $\delta E \sim 20$ to 30 meV resolution regime also enables the observation of a broader range of many-body effects, such as the interaction between atomic multiplet excitations and other many-body degrees of freedom [7]. Measurements in **Figure 2** show that the energy resolution range accessed at the M-edge is sufficient to resolve the decay dynamics of orbital excitations in the cuprate Mott insulator SrCuO_2 . The copper oxide lattice of SrCuO_2 has a similar electron configuration to undoped 2D cuprates [36] but electrons are constrained to propagate primarily within one-dimensional chains of corner sharing CuO plaquettes [11, 21, 37]. The momentum dispersion of similar orbital excitations has been studied at the soft X-ray L-edge [20, 21], however energy resolution at that energy did not facilitate a detailed study of excitation line shapes.

In SrCuO_2 , charge transfer excitations and atomic multiplet orbital excitations overlap in energy close to the Brillouin zone center. This can be seen from overlaying the M-edge spectrum of SrCuO_2 orbitons with measurements of $Q = 0$ charge transfer modes by copper K-edge RIXS and optical ellipsometry in **Figure 2**. Energetic overlap with the charge transfer continuum implies that the orbitons observed in this measurement are actually semi-stable “resonance modes.” Degeneracy with the continuum allows resonance modes to delocalize into separately propagating electrons, holes, and spinons, without first absorbing energy from an outside source. If delocalization happened instantaneously, the M-edge RIXS spectrum of SrCuO_2 would largely be shaped by the spectral contour of delocalized



states seen in K-edge RIXS (circles in **Figure 2**). Instead, the experimental results show an M-edge spectrum that strongly resembles expectations from an AM model.

Orbiton line widths fitted in **Figure 2** provide a rough estimate of the time scale of charge transfer delocalization from a local orbiton resonance. The leading edge of the lowest energy ($3d_{xy}$) excitation does not overlap with a large density of charge transfer excitations, and has a sharper line shape than any other feature in the spectrum, roughly matching a $\delta = 0.15 \pm 0.02$ eV Lorentzian. As the energies of the orbitons increase beyond the charge transfer gap energy, a broadening trend is observed. The highest energy excitation ($3d_{3z^2-r^2}$) has the greatest quantum overlap with charge transfer excitations, and the fitted inverse lifetime of $\delta = 0.26$ eV suggest that it can decay into delocalized states on a time scale of $\hbar/0.26 \pm 0.02$ eV = 2.5 ± 0.2 fs.

The line shapes resolved from SrCuO₂ illustrate the need for adequate resolution to observe how simple atomic excitations decompose into more complex and delocalized many-body states, and are broad enough to also be well resolved at recently developed soft X-ray RIXS spectrometers that achieve ~ 100 meV resolution [46–49]. Our earlier EUV studies have shown that orbital excitations inside the insulating gap of CoO and NiO have significantly sharper line shapes that are sensitive to thermal fluctuations [7], with a $\delta E \sim 70$ meV leading edge contour identified at low temperature for the 1 eV excitation in CoO. Resolving these widths and the larger feature widths in the SrCuO₂ spectrum illustrates the femtosecond time scale on which local multiplet excitations can persist and interact meaningfully with the surrounding many-body environment. Several upcoming soft X-ray spectrometers are anticipated to reveal momentum dispersion in the critically important “thermal” resolution regime ($\delta E \sim k_B T$ near or below room

temperature) [50–55], and superior energy resolution can also now be achieved with hard X-rays [56, 57] for certain specific cases such as the iridium L-edge.

5. Shake-up RIXS and Shallow Core Holes

5.1. Classes of Shake-up Excitation, and their Influence on RIXS Excitation Line Shapes

The existence of a core hole can dramatically change valence orbital energies and cause the entire atomic wavefunction of a resonance site to contract, resulting in so-called “shake-up” excitations observed in RIXS. Shake-up excitations include double magnons (see Discussion in Bisogni et al. [28]) and charge transfer excitations at the M- and L- edges (see **Figure 4** below), and can have a distinctive pattern of incident energy dependence in RIXS spectra [8]. These excitations have a fractional contribution to scattering intensity in the RIXS spectrum that scales roughly as $I \sim K^2/\Gamma^2$ when core hole lifetime is short ($|K/\Gamma| \ll 1$) due to destructive interference in the numerator of the Kramers-Heisenberg scattering equation (see discussion of “indirect” RIXS in Ament et al. [4]). Here, K is a factor determined by the kinetics through which the resonance states evolve to create a particular excitation, and Γ is the intermediate state lifetime.

When low energy shake-up modes such as phonons are coupled to with high probability, they can cause RIXS features to shift in energy, and to adopt new line shapes that are non-trivially convoluted with the shake-up excitation profile. Multi-phonon shake-up has been associated with incident energy dependent shifts of < 10 meV [7], ~ 50 meV [23] and > 0.1 eV [22] in excitation energies at the M-, L-, and K-edges of 3d transition metals, respectively, showing that EUV resonant scattering at the M-edge has a particularly mild shake-up effect, and may give the most accurate representation of intrinsic excitation energies and line shapes.

Hartree-Fock calculations can qualitatively explain the degree of shake-up occurring at each edge, as they predict that the radial wavefunction of d-electrons in NiO will contract by (M) 0.6%, (L) 6%, and (K) 11% when core holes of these symmetries are present. The reason the EUV M-edge has the least contraction is that the radial part of the core orbital wavefunction for shallow core holes is quite similar to the radial wavefunction of valence electrons, meaning that the total radial charge density distribution surrounding the scattering site is not greatly disrupted in resonance states. A weak core hole potential has also been proposed in Chiuzaian et al. [16] to explain the lack of charge transfer excitations in M-edge RIXS on CoO.

Single magnon excitations are also technically shake-up excitations with a kinetic parameter “ K ” set by the strength of core hole spin orbit coupling. In the soft X-ray RIXS, single magnon excitations are usually not created in the $|K/\Gamma| \ll 1$ regime, so it is not necessary to consider their shake-up nature. In the EUV however, core hole spin orbit coupling tends to be weak, creating a close link between the intensity of single magnon excitations and core hole lifetime.

5.2. Looking for Charge Transfer Excitations Outside the AM Window

To evaluate the degree to which charge transfer shake-up excitations are reduced in EUV RIXS, we examine M-edge scattering data on the model Mott insulators NiO and CoO. Previous experimental studies performed with soft X-rays at the transition metal L-edge (700–900 eV) have noted that charge transfer features from these materials have significant intensity at 4–7 eV energy loss [14, 16–18]. M-edge studies have noted that charge transfer excitations are weak, but stopped short of presenting data that covered the Mott gap feature [15, 16]. Broad-range M-edge RIXS resonance profiles of CoO and NiO are shown in **Figure 3**, including energies above the insulating gap. All features visible below the insulating band gap energy can be accounted for from AM models (**Figure 3B**) that do not allow charge transfer, indicating that the materials are qualitatively well described with only a strongly correlated d-orbital basis.

Focusing on energies above the insulating gap (dashed line in **Figure 3A**) shows that there is remarkably little intensity that could be attributed to charge transfer excitations and other high energy shake-up features. (data at even higher energy loss can be found in **Figure 6**) A feature that disperses linearly with incident energy, with a slope of ~ 1 , is indicated with a dashed red line on the NiO profile, and has intensity of $I_{CT} \sim 0.02$. Intensity fluctuations in the CoO profile also achieve an amplitude of ~ 0.02 , but cannot be definitively identified as RIXS features because of worse signal to noise and due to a large background slope from the elastic line. The 1 eV t_{2g} to e_g multiplet excitation

achieves a peak intensity more than 50 times larger than features above the insulating gap ($I_{\sim 1\text{ eV}} > 1 \gtrsim 50 I_{CT}$) for both materials.

A very rough upper limit to the spectral intensity of charge transfer features relative to AM RIXS features can be obtained from these data, if we make several assumptions about how charge transfer features manifest in the data. First, we assume that the incident energy dependence of M-edge charge transfer features is not very different from that of the 1 eV feature. Single impurity Anderson model calculations for the M-edge support this assumption. Second, we focus only on the region within 4 eV of the band gap, as this is where most charge transfer intensity is found at the L-edge, with the exception of the high energy emission line. The emission line is far weaker at the M-edge than at the L-edge, and only low energy parts of the emission line are associated with simple metal-ligand charge transfer in recent models [17, 18]. Finally, we assume that the peak intensity of charge transfer excitations is 50% greater than the average intensity of the 4 eV region and, based on the data quality in **Figure 3**, that a feature with peak intensity greater than 0.01 in several consecutive RIXS curves would be recognizable in the data.

The greatest total charge transfer intensity that could be hidden in the spectra under these assumption is $7 \pm 1\%$ and $20 \pm 5\%$ of the total intensity of visible inelastic AM RIXS features for NiO and CoO, respectively. Error bars are much larger for CoO, due to inaccuracy in subtracting the elastic tail. Additionally, the dispersive feature observed above the band gap in NiO (**Figure 3A**) has an integrated intensity that is 3% of

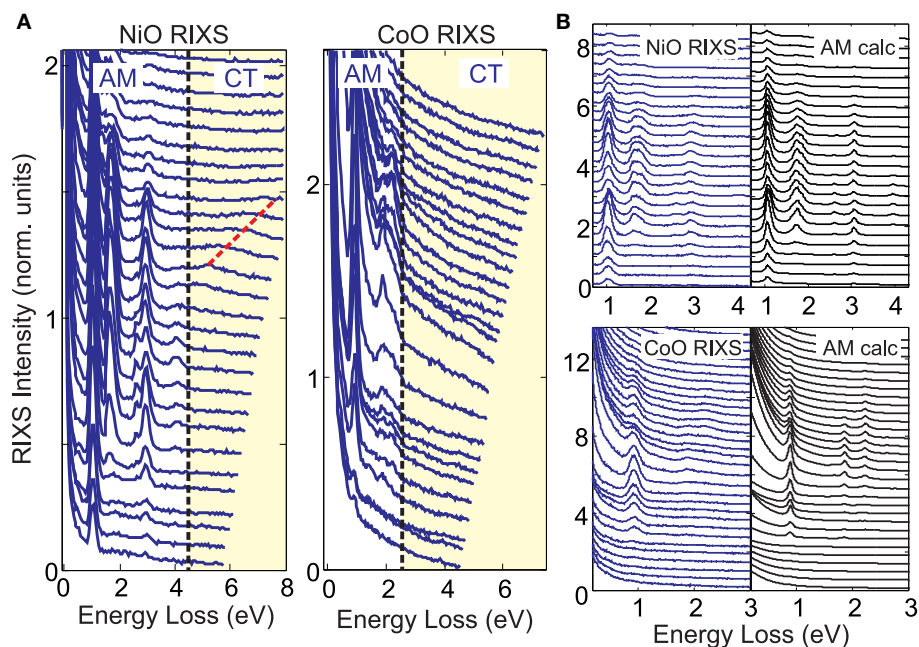


FIGURE 3 | Non-local excited states vanish. (A) (left) NiO RIXS curves measured with $h\nu = 62.5\text{--}76$ eV photons and (right) CoO curves measured at $h\nu = 56.5\text{--}69.5$ eV (0.5 eV step) are shown on an expanded scale that emphasizes the yellow-shaded high energy loss region. The band gap is indicated with a dashed line. **(B)** RIXS data beneath the charge transfer gap

have good correspondence with a single-atom atomic multiplet (AM) model ($h\nu$ is (NiO) 63–75.5 and (CoO) 55–70 eV). Intensity units are defined so that peak features have identical height in **(A,B)**. The relative scattering intensity from NiO vs. CoO cannot be directly compared due to uncontrolled experimental factors (e.g., beam slit widths).

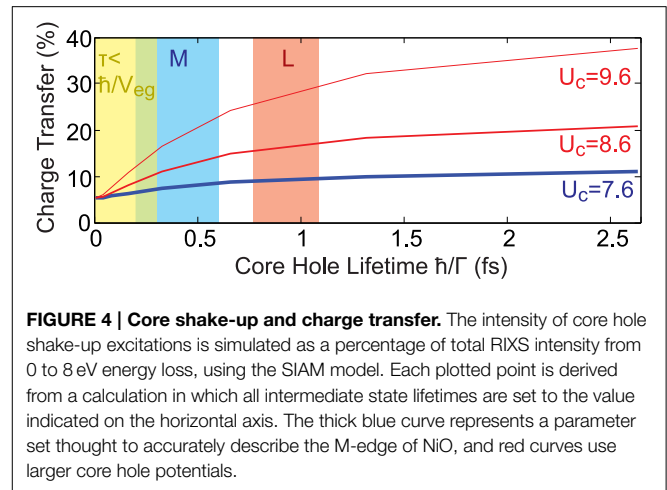
the AM features, giving an estimated range of 3–10% for the fraction of total scattering intensity that can be attributed to charge transfer features in the 3–8 eV energy loss range for NiO. The underlying reasons for decreased charge transfer intensity at the M-edge relative to the L-edge requires some nuance to parse. **Figure 4** below will address this question with respect to specific modeling parameters.

5.3. Time and Energy Considerations Governing Charge Transfer

Charge transfer can be considered in numerics by augmenting AM calculations with the single Anderson impurity model (SIAM) [58]. We have chosen to implement this model only for NiO in this paper, owing to the detailed study of NiO modeling parameters in Matsubara et al. [17] and other analysis justifying the approach [18]. Modeling parameters follow Matsubara et al. [17], and are described in the Appendix. The SIAM model is well suited to NiO due to factors such as a large semi-classical local spin moment, and a large band gap which is likely to have cleaner charge transfer character than the CoO band gap. By construction in the model, a hole localized next to the scattering site is described by a phase coherent superposition of delocalized states, and the speed at which holes delocalize away from the scattering site in the absence of any binding potential is given by the inverse of the bandwidth factor W . The model does not attempt to define the detailed symmetries and dynamics through which holes delocalize beyond these qualitative parameters, and is not meant to be quantitatively compared with the insulating band gap or the actual distribution of oxygen orbital DOS in band structure.

The intensity of charge transfer excitations in the 0–8 eV energy range as a function of core hole lifetime and the core hole potential is modeled in **Figure 4**. When core hole lifetime approaches zero, charge transfer excitations shrink to a direct RIXS component that is 5% of the total spectral intensity. The $I \sim K^2/\Gamma^2$ time scale that characterizes the onset of shake-up RIXS scattering is very short. A positive second derivative is only found for core hole lifetime values $\tau < 100\text{as}$ that are much smaller than any of the M- or L-edge intermediate state lifetimes calculated for NiO. Intensity of charge transfer modes grows quickly as core hole lifetime increases from 0as to 300as, corresponding to the inverse metal-ligand hopping parameter $\hbar/V_{eg} = 300\text{as}$. Describing the monopole part of the Coulomb potential with a core hole term U_C estimated previously [17] for the M-edge of NiO and d-orbital Hubbard parameter U_d gives a weak shake-up potential of about $U_C - U_d = 0.4\text{eV}$, resulting in M-edge predictions that have only $\sim 50\%$ more charge transfer intensity than the $\tau = 0$ direct RIXS value. The weak manifestation of charge transfer excitations at the M-edge relative to the L-edge can be understood because the effective core hole U_C perturbation at the L-edge is larger (8.0 eV in Matsubara et al. [17]), and core hole lifetimes are longer than at the M-edge by a factor of $\gtrsim 2$.

Comparing **Figure 4** with the experimental upper bound of $\sim 10\%$ on charge transfer intensity in the 4–8 eV region suggests that the core hole potential U_C must be less than 1 eV larger than the Hubbard parameter U_d . Regardless of how precisely accurate this estimate may be, it is interesting to note that in the



indicated parameter range, the calculated contribution of shake-up RIXS mechanisms to the charge transfer excitation spectrum is weaker than the non-shake-up RIXS contribution. This is counterintuitive, and is not expected to be the case with soft X-rays at the L-edge. If it proves to be true, then experiments with better signal to noise may in the future use EUV RIXS to obtain spectroscopic information about charge transfer excitations that is not influenced by the incident energy dependence and core hole lifetime dependence that complicate shake-up RIXS spectra.

6. Incident Energy Dependence of Core Hole Lifetime

At present, RIXS spectra are most commonly modeled with the Kramers–Heisenberg equation (see Appendix), which uses second order perturbation theory to limit explicit reference of time dynamics to a reduced set of inverse lifetime parameters (Γ_m). In these treatments, the lifetimes of core hole states are often empirically estimated and assumed to share a single value ($\Gamma_m \rightarrow \Gamma$). This fixed-lifetime approach produces very good results for most XAS calculations, and only coarse corrections are needed to optimize correspondence with experimental spectra [59–61]. However, the justification for applying the fixed-lifetime approximation to RIXS spectra is far weaker, and theoretical intuition from XAS does not always carry over nicely to RIXS. For example, the sum rule equating RIXS intensity (summed over energy loss and momentum) to the XAS spectrum is broken when core hole lifetimes are incorrect. At resonances in the EUV, a particularly broad range of core hole lifetimes can be observed, and core hole lifetime can vary significantly as a function of incident energy at a given resonance edge [6, 31–33].

Error from the fixed-lifetime approximation is amplified in RIXS spectra relative to other spectroscopies, due to the role lifetime plays in normalizing intensity in the Kramers–Heisenberg equation, and in mediating quantum interference throughout the RIXS spectrum. The qualitative nature of this problem for EUV RIXS studies can be seen by comparing data and theory for the Mott insulator NiO, studied at the nickel M-edge (**Figure 5**). The features simulated in **Figure 5A** (middle, right) are excitations of a single Ni^{2+} atom acted on by an

and numerical simulations presented for the Cu, Ni, and Co M-edges. These data are used to discuss a wide range of topics, including the origin and significance of the strong elastic line found in EUV RIXS, the scientific significance of superior energy resolution obtained in the EUV, and the matrix elements for RIXS scattering in the EUV. The origin of incident energy dependence in core hole lifetime for RIXS involving shallow core holes is discussed with respect to simulations and data that cover an unprecedented energy loss window for the materials studied. The tunability of core hole lifetime is a useful phenomenon for distinguishing between shake-up and non shake-up RIXS features, and for spectroscopic approaches that make use of the core hole as an approximate clock of ultrafast phenomena [30].

An accurate understanding of resonant scattering in the EUV is of increasing importance with the recent advances of short wavelength optics [39, 62, 63] and the availability of ultrashort UV and X-ray pulses [64]. As the state of the art in flux and energy

resolution at RIXS beamlines continues to improve, we expect that line shape effects recently identified via EUV RIXS (e.g., on the incident energy axis) will also be measured in the soft X-ray regime and charted as a function of momentum. The dependence of core hole decay rate on multiplet symmetries that vary within a given resonance edge (discussed in Section 6) will be smaller in the soft X-ray regime, but may also be identified within transition metal L-edges as high flux facilitates the study of incident energy dependence.

Acknowledgments

The Advanced Light Source is supported by the Director, Office of Science, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. The Cu K-edge RIXS experiments at SPring-8 were performed at BL11XU with the approval of the Japan Synchrotron Radiation Research Institute (JASRI) (Proposal No. 2007A3502 and 2007B3502).

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Conflict of Interest Statement: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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Appendix

Computational Details

The AM calculations in this paper closely resemble numerics in Wray et al. [6, 7, 9]. Scattering is evaluated using the Kramers-Heisenberg equation:

$$R_f(E, h\nu) \propto \sum_g \left| \sum_m \frac{\langle f|T^\dagger|m\rangle\langle m|T|g\rangle}{h\nu - E_m + i\Gamma_m/2} \right|^2 \times \frac{\frac{1}{2\pi}\Gamma_f}{(E - E_f)^2 + (\frac{1}{2}\Gamma_f)^2} \quad (1)$$

The quantity inside absolute value brackets represents the matrix elements for quantum paths from the ground state $|g\rangle$ to a final state $|f\rangle$, through short-lived intermediate states indexed by $|m\rangle$. Incident photon energy is written as $h\nu$, the excitation energy is E , and the inverse final state excitation lifetime is Γ_f . The incident photon operator T couples to the polarization of the incident photon beam, and the outgoing photon operator T^\dagger couples to the resultant polarization of scattered photons. Effects of quantum interference appear in the spectrum when inverse intermediate state lifetimes (Γ_m) are similar to the separation between intermediate state energies (E_m).

The crystal field in AM calculations is described by (CoO) $10Dq = 0.96$ and (NiO) 1.03 eV. In the SIAM implementation for NiO, coupling with the ligand band reduces the effective $10Dq$ value to 0.53 eV. Values of $10Dq$ are chosen to match the sharp onset lineshape of the ~ 1 eV t_{2g} to e_g excitation in experimental spectra, as described in Chiuzaian et al. [15], Wray et al. [6, 7]. The calculations use atomic values of inter-d-orbital Slater-Condon interaction parameters ($F2(dd), F4(dd)$) for the SIAM, and scale them to 80% for AM simulations (64% of bare Hartree-Fock parameters). We have found that 3p-3d interactions ($G1(pd), G3(pd), F2(pd)$) require an unusually large correction at the M-edge, and are estimated at (NiO) 70% and (CoO) 65% of atomic values for the AM calculations, and 80% of nominal atomic values for the SIAM. The 3p spin orbit coupling strength is larger than first principles estimates, likely due to positive 4s-shell valence, and has been set to (CoO) 1.5 eV, (NiO) 1.6 eV, and (SrCuO₂) 1.8 eV.

Core hole lifetimes (Γ_m) are estimated from the dipole valence-to-core transition matrix element via the method

described in Wray et al. [6]. Using an empirical dipole decay rate constant that corresponds to a lifetime of $\Gamma \sim 2.5$ eV for d^{10} electron configurations gives very good correspondence with incident energy line shapes in cuprate, nickelate, and cobaltate compounds [6, 7, 9]. This approach is highly phenomenological, and does not attempt to comprehensively consider how decay occurs via Auger emission, which is numerically complex for shallow, radially-large core holes. Our preliminary calculations show that in the EUV, unconventional Auger channels involving electrons in ligand orbitals can have similar intensity to intraatomic Auger. Moreover, it is easy to show that ~ 1 Å scale electronic correlations in the ground state wavefunction that are not described by a d-orbital basis also have significant structure on the length scale of Auger electron wavelengths in the EUV, and are very important to consider (though numerically intractable). We hope that these issues will be addressed with theoretical rigor in future studies.

Parameters of the SIAM model mostly follow the treatment in Matsubara et al. [17]. We define a ligand band with $W = 3$ eV bandwidth, similar to the maximum dispersion observed for an oxygen derived band in recent analysis of NiO [65, 66]. Hopping between e_g orbitals and the nearest neighbor ligand states with corresponding symmetry is $V_{eg} = 2.2$ eV ($V_{t2g} = -V_{eg}/2$), and is reduced by 10% when a core hole is present. The core hole monopole potential (Slater Condon $F0$) is $U_C = 7.6$ eV, and the d-orbital Hubbard U parameter is $U_d = 7.2$ eV. The state basis is limited to allow a maximum of 1 ligand hole, as in Magnuson et al. [18], and the flat ligand band density of states distribution is approximated by 10 discrete energies ($N = 10$). The ground state density of oxygen holes within this model matches the experimentally based estimate of 0.2 [67].

Interatomic spin interactions are considered by incorporating an external exchange field of $J^* = 0.0126$ eV for CoO [68, 69] and a rough estimate of $J^* = 0.1$ eV for NiO. Type 2 phase transitions at (CoO) $T_N \sim 290$ K and (NiO) 524 K [70] preserve local spin correlations and do not have a pronounced effect on M-edge RIXS scattering features near room temperature [7]. The quasi-1D spin lattice and large spin interactions of SrCuO₂ result in corrections to the self energy contour of orbitons [20, 21], but do not behave as a static exchange field for most cuprate orbiton symmetries [71]. These self energy corrections are convoluted with Lorentzians representing orbiton lifetime in the **Figure 2** fit curves, following the implementation in Wray et al. [9] of the model in Wohlfeld et al. [20].