Evolution of Global Cooperation in Multi-Level Threshold Public Goods Games With Income Redistribution

Jinming Du [1,2]* and Baokui Wang [3]

[1] Liaoning Engineering Laboratory of Operations Analytics and Optimization for Smart Industry, Northeastern University, Shenyang, China, [2] Institute of Industrial and Systems Engineering, Northeastern University, Shenyang, China, [3] Joint Exercises and Training Center, Joint Operations College, National Defense University, Beijing, China

Income redistribution is a feasible means to adjust the income among individuals, which could reduce the gap between the rich and the poor and realize the social equity. By means of taxation and public services, the income could be transferred from some individuals to others directly or indirectly. We study how income redistribution affects the evolution of global cooperation through proposing a multi-level threshold public goods game model and introducing two kinds of income redistribution mechanisms. We find that both of the income redistribution mechanisms promote global cooperation. Furthermore, the global income redistribution is more in favor of the emergence of global cooperative behaviors than the local income redistribution mechanism. On the other hand, the fixation time of global cooperation is sharply shortened after introducing income redistribution mechanisms. In threshold public goods games, only when the amount of collected public goods reaches a certain threshold, the income of individuals can be guaranteed. Hence, the influences of thresholds of different levels on strategies are investigated in the paper.

Keywords: evolutionary game theory, public goods game, stochastic processes, human cooperation, income redistribution

1. INTRODUCTION

Collective actions, such as a group of neighborhood residents donating money to construct a public project, require voluntary contributions to collect public goods [1]. Voluntary contributing activity is widespread and substantial [2–5]. It is beneficial for the group, however, costly for individuals. Performing an altruistic act can weigh heavily on individual wellbeing and prosperity. Selfish individuals always have an advantage over cooperators. Such social dilemma can be represented as the public goods game (PGG) [6–8]. In human societies, people are often required to sacrifice personal benefits for the common goods and work together to achieve what they are unable to achieve alone [9, 10]. Especially when it comes to the situation that people are faced with the option of voluntary contribution to achieve a collective goal, where public goods cannot be provided in part, but only in whole after a certain cost (threshold) is covered. Threshold public goods game (TPGG) models nicely capture the main features of the above described social phenomena [11–18]. In the typical TPGG, the size of a proposed project and the associated total cost (threshold) are predetermined. The public goods are provided if the total contributions meet or exceed the threshold; otherwise, no goods are provided and all individuals suffer with nothing irrespective of whether they contributed or not. Since cooperation forms the bedrock of our efforts

for a sustainable and better future, understanding cooperative behaviors in complex interactive systems has been one of the grand scientific challenges of the global society [19–24]. The problem is in many ways unnatural. Now that free riders can enjoy the same benefits for free, what kind of mechanism can motivate individuals to care for and contribute to the public goods, if only the fittest survive?

Most governments devote considerable resources to the provision of public goods available for all citizens to consume, such as national defense, environmental protection, health insurance and highways. Such universal provision schemes can redistribute income from the rich to the poor [25], and further realizing the fairness of the society [26–28]. Redistribution of income may provide a nonexcludable benefit to those who give, and many such schemes are universal in the sense that everyone is eligible and the provision is free [29]. One of the classic forms of income redistribution is the tax system, in which people are taxed at fixed rates. People who make more money pay higher taxes, thereby forfeiting more of their income to the government. Tax funds are used to benefit the society as a whole by providing a variety of public and social services by the government, and the direct transfer of income may occur in the case of welfare payments and other forms of cash assistance made to low-income members of society [30]. Previous works on physical models of collective dilemmas, however, seldom theoretically analyze how income redistribution influences the evolution of cooperation in the complex social-economic system.

Motivated by this, we propose a multi-level threshold public goods game model, where global and local public goods are clearly distinguished. Although pure public goods are defined as being non-rival in consumption and non-excludable [31], however, there exist impure public goods in reality. Owing to geographic space, some classes of goods are globally public, and others are only locally public. Global public goods are available to the entire population while local public goods may be available only to the residents of a very small neighborhood [32, 33]. Thus, players in the model can choose among selfishness, contributing to global public goods, or local public goods. In particular, the global public goods and local public goods both involve the threshold. If the collected public goods are not enough, dangers would happen. For example, coastal inhabitants may be inundated owing to failure of fundraising for a dam [34], disease may spread caused by inadequate voluntary vaccination [35–39] and regional defense system may collapse due to insufficient finance [40].

We then respectively consider two kinds of income redistribution mechanisms in the multi-level TPGG. In the model with local income redistribution, players have to pay part of their idealized income to the focal group according to a given income expenditure proportion after each round. Distinguished from the contribution action during the PGG process, such compulsory payment is named as the second-order payment. Subsequently, the accumulated income are redistributed to all the members of this group regardless of their strategies and the quantity of their second-order payments. On the other hand, for the global income redistribution, players pay part of their idealized income to the whole population.

Similarly, the accumulated income is then redistributed to all the players in the whole population. In reality, the local income redistribution seems like a special transaction tax in economic system, which is collected according to the definite quantity of the volume of trade. And then, the revenue is redistributed to the group members uniformly, which amounts to the fiscal subsidy for a particular industry. While for the global income redistribution, the processes of second-order payment and income redistribution can be classified as the process of collecting and redistributing the gross income of personal income tax for the whole country. Based on this model, we theoretically investigate the evolution of cooperations of different levels and free-riders under collective risks, and focus on the influence of diverse income redistribution mechanisms on the global cooperation.

2. MODEL

In this paper, we study a finite population of N players. The whole population is divided into M groups, then there are $m = N/M$ players in each of the group [41, 42]. Player x can choose a strategy $S_x \in \{G, L, S\}$, where G, L and S represent global cooperation, local cooperation and selfishness, respectively. Each player has one unit of money at the beginning of the game. They should decide whether put their money into Global account, Local account or Personal account. If the money is put into the Personal account, it is saved without multiplication. The player will finally own the single unit of money. The money put into the Local account are added and multiplied by a local gain-factor r_1 $(1 < r_1 < m)$. Then it will be equally distributed to the players in the focal group. The money put into the Global account are summed and multiplied by a global gain-factor r_2 $(1 < r_2 < N)$. Then it is distributed to all the players in the whole population irrespective of whether they are global cooperators or not.

After game interaction, players are asked to take part in the process of income redistribution. We respectively consider two cases: one is local redistribution, the other one is global redistribution. For local redistribution, each player has to pay part of their income to its group according to the given income expenditure proportion p_1. The parameter p_1 denotes the proportion of the second-order income obtained in the PGG of the focal group. Subsequently, the accumulated income expenditure of players in this group is redistributed to the m group members irrespective of their strategies and the amount of their second-order payments. Thus, the actual income of player x in this group after local redistribution of income can be calculated as

$$\pi_x^l = \pi_x \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x, \qquad (1)$$

where π_x denotes the income of a player after one PGG, $p_1 \sum_{x=1}^{m} \pi_x$ is the sum of the accumulated second-order payments of the group members. Denote $\pi_G^1(i)$, $\pi_L^1(l)$ and $\pi_S^1(m - i - l)$ as the final payoff of each G, L and S player, respectively, when there are i G players, l L players in the local group and

the other $N - i - l$ players all hold S strategy in the whole population [43]:

$$\pi_S^1(m - i - l) = (\frac{i \times r_2}{N} + \frac{l \times r_1}{m} + 1) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x \tag{2}$$

$$\pi_G^1(i) = (\frac{i \times r_2}{N} + \frac{l \times r_1}{m}) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x \tag{3}$$

$$\pi_L^1(l) = (\frac{i \times r_2}{N} + \frac{l \times r_1}{m}) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x, \tag{4}$$

where $\pi_x = ir_2/N + lr_1/m + 1$ for S player and $\pi_x = ir_2/N + lr_1/m$ for G and L players.

On the other hand, for global redistribution, players are mandatory to pay the part of their income to the whole population according to a fixed proportion p_2, which is the second-order payment in global redistribution. We emphasize that p_2 denotes the income expenditure proportion of all the second-order income player obtained in the PGG of the whole population. Subsequently, the accumulated income is redistributed to all the players irrespective of their strategies or the quantity of their second-order payments. Thus, the payoff of each player after the global income redistribution can be calculated as

$$\pi_x^g = \pi_x \times (1 - p_2) + \frac{p_2}{N} \times \sum_{x=1}^{N} \pi_x, \tag{5}$$

where π_x is the income of a player after PGG, $p_2 \sum_{x=1}^{N} \pi_x$ is the sum of the accumulated second-order payments of all the players.

We consider the two-level TPGG, thus the payoffs of players are threatened by two-level risks. Here, we denote s_1 as the local threshold and s_2 as the global threshold. Then we introduce the threshold functions:

$$\theta_1(l) = \begin{cases} q_1 & for \ l \times r_1 < s_1 \\ 0 & for \ l \times r_1 \geq s_1 \end{cases} \tag{6}$$

$$\theta_2(i) = \begin{cases} q_2 & for \ i \times r_2 < s_2 \\ 0 & for \ i \times r_2 \geq s_2 \end{cases} \tag{7}$$

where i is the number of global cooperators and l is the number of local cooperators in the focal group. If the amount of collected global public goods is less than s_2, a world-wide danger is on the way with a probability q_2. Once such danger happens, the payoffs of all the individuals are zero. If the amount of public goods in the Global account is more than s_2, the collective target achieves and disasters are not going to happen. In this case, all the players will get their payoffs in the public goods game. However, if the amount of local public goods in a group is less than s_1, a potential risk could happen with a probability q_1. Once suffered such risk, the payoffs of players in the focal group would be lost.

We use imitation process to describe the evolution of strategies. Players are likely to learn the strategies of their successful counterparts'. Firstly, we randomly select a player A from the N population. Then, another player B should be chosen. With a probability φ, B will be selected from the N population. Otherwise, with probability $1 - \varphi$, B is chosen only from A's local group. In other words, the larger φ is, the more likely players interact with each other globally. In our daily life, the interaction within a group is much more frequent than that between groups. A would learn B's strategy with a probability $1/[1 + e^{-\omega(\pi_B - \pi_A)}]$ [44–48], where π_x is the payoff of individual x. ω denotes the imitation intensity [49–52], measuring the dependence of decision making on the payoff comparison. Here, we define two different imitation intensities. We denote ω_1 as the imitation intensity within a group and ω_2 as the imitation intensity between groups. During the evolutionary process of strategies, each player has the chance of switching its strategy to a different one with a probability μ. In this paper, we assume the exploration rate $\mu \to 0$. The parameters in the model are listed in **Table 1**.

3. METHODS

We are interested in how global cooperation evolves. To this end, we study the stationary distribution and the fixation time. It is common that the interaction within a group is much more frequent than that between groups [53, 54], thus the fixation process of a single mutant in the population goes through two steps: the fixation of this mutant in its local group and the fixation of such group in the whole population. We theoretically analyze the two kinds of income redistributions respectively.

3.1. Local Income Redistribution
We consider a single local group composed of $m - i$ S players and i G players. All the other groups in the whole population are full of S players. Based on Equations (2, 3), the payoffs of each G

TABLE 1 | Parameters in the model.

Symbols	The meaning of the parameters
N	Population size
M	Number of groups
m	Number of players in each group
r_1	Local gain-factor
r_2	Global gain-factor
p_1	Local income expenditure proportion
p_2	Global income expenditure proportion
s_1	Local threshold
s_2	Global threshold
q_1	Local risk probability
q_2	Global risk probability
φ	Globally learning probability
μ	Exploration rate
ω_1	Imitation intensity within a group
ω_2	Imitation intensity between groups

player and each S player in the focal group are:

$$\pi_G^1(i) = \frac{i \times r_2}{N} \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x \tag{8}$$

$$\pi_S^1(m - i) = (\frac{i \times r_2}{N} + 1) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x. \tag{9}$$

The number of G players changes from i to $i \pm 1$ in one time step with a probability:

$$T^{\pm}(i) = (1 - \varphi) \times \frac{i}{m} \times \frac{m - i}{m} \times \left\{ 1 + e^{\pm \omega_1 [\pi_S^1(m-i) - \pi_G^1(i)]} \right\}^{-1}, \tag{10}$$

where ω_1 is the imitation intensity within a group. The fixation probability of a single G mutant invading a group of S players is denoted by P_{SG}^1, which is given by Traulsen et al. [45] and Wu et al. [54]:

$$P_{SG}^1 = \left[1 + \sum_{j=1}^{m-1} \prod_{i=1}^{j} \frac{T^-(i)}{T^+(i)} \right]^{-1}$$

$$= \left\{ 1 + \sum_{j=1}^{m-1} e^{\omega_1 \sum_{i=1}^{j} [\pi_S^1(m-i) - \pi_G^1(i)]} \right\}^{-1}. \tag{11}$$

We define the fixation probability of a local group full of G players invading the whole population full of S players as P_{SG}^2. The payoff of each G player is denoted by $\pi_G^2(i)$ and that of S player is $\pi_S^2(M - i)$ when there are i local groups full of G players and the other $M - i$ groups full of S players.

$$\pi_G^2(i) = \frac{i \times r_2}{M} \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \frac{i \times r_2}{M} = \frac{i \times r_2}{M} \tag{12}$$

$$\pi_S^2(M - i) = (\frac{i \times r_2}{M} + 1) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} (\frac{i \times r_2}{M} + 1)$$

$$= \frac{i \times r_2}{M} + 1. \tag{13}$$

A new group full of G players arises when two players with different strategies from different local groups are chosen, and the S player alters its strategy through imitation, then it takes over its local group. Thus, the probability to increase the number of local groups full of G players by one is given by:

$$\Gamma^+(i) = \varphi \times \frac{i}{M} \times \frac{M - i}{M} \times \left\{ 1 + e^{\omega_2 [\pi_S^2(M-i) - \pi_G^2(i)]} \right\}^{-1} \times P_{SG}^1(k), \tag{14}$$

where ω_2 is the imitation intensity between groups. $P_{SG}^1(k)$ represents the fixation probability of a single G mutant invading a group of S players when there already exist k groups full of G.

Similarly, the probability to decrease the number of G groups by one is:

$$\Gamma^-(i) = \varphi \times \frac{i}{M} \times \frac{M - i}{M} \times \left\{ 1 + e^{\omega_2 [\pi_G^2(i) - \pi_S^2(M-i)]} \right\}^{-1} \times P_{GS}^1(k). \tag{15}$$

Hence, the fixation probability of a G group in the whole population is obtained as follows:

$$P_{SG}^2 = \left(1 + \sum_{j=1}^{M-1} \left\{ e^{\omega_2 \sum_{i=1}^{j} [\pi_S^2(M-i) - \pi_G^2(i)]} \prod_{i=1}^{j} \left[\frac{P_{GS}^1(k)}{P_{SG}^1(k)} \right] \right\} \right)^{-1}. \tag{16}$$

We aim to analyze the multi-level TPGG. Thus, the payoffs above are conditional. Once global danger happens, all the individuals will lose their wealth. On the other hand, if local danger strikes, the players in the focal group lose their wealth. By utilizing the threshold functions, Equations (6, 7), the revised payoffs are as follows:

$$\pi_G^1(i) = \left[\frac{i \times r_2}{N} \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x \right]$$

$$\times (1 - q_1) \times [1 - \theta_2(i)] \tag{17}$$

$$\pi_S^1(m - i) = \left[(\frac{i \times r_2}{N} + 1) \times (1 - p_1) + \frac{p_1}{m} \times \sum_{x=1}^{m} \pi_x \right]$$

$$\times (1 - q_1) \times [1 - \theta_2(i)] \tag{18}$$

$$\pi_G^2(i) = (\frac{i \times r_2}{M}) \times (1 - q_1) \times [1 - \theta_2(i \times m)] \tag{19}$$

$$\pi_S^2(M - i) = (\frac{i \times r_2}{M} + 1) \times (1 - q_1) \times [1 - \theta_2(i \times m)]. \tag{20}$$

Inserting Equations (17, 18) into Equation (11) and Equations (19, 20) into Equation (16), we can get the following equations when there are k groups full of G players:

$$P_{SG}^1(k) = \left\{ 1 + \sum_{j=1}^{m-1} e^{\omega_1 (1-q_1)(1-p_1) \sum_{i=1}^{j} [1 - \theta_2(i + mk)]} \right\}^{-1} \tag{21}$$

$$P_{SG}^2 = \left(1 + \sum_{j=1}^{M-1} \left\{ e^{\omega_2 (1-q_1) \sum_{i=1}^{j} [1 - \theta_2(mi)]} \prod_{k=1}^{j} \left[\frac{P_{GS}^1(k)}{P_{SG}^1(k)} \right] \right\} \right)^{-1}. \tag{22}$$

The fixation probability of a single G mutant invading the whole global population full of S players is defined as ρ_{SG}. Hence, we have:

$$\rho_{SG} \approx P_{SG}^1(0) \times P_{SG}^2. \tag{23}$$

Accordingly, we can get the fixation probability ρ_{SG}, and also ρ_{GS}, ρ_{SL}, ρ_{LS}, ρ_{LG}, ρ_{GL}, which are given as follows:

$$\rho_{SG} \approx \left[1 + \sum_{j=1}^{m-1} e^{\omega_1 (1-p_1)(1-q_1)(1-q_2) j} \right]^{-1}$$

$$\times \left(1 + \sum_{j=1}^{M-1}\left\{e^{\omega_2\,(1-q_1)\,\sum_{i=1}^{j}[1-\theta_2(m\,i)]}\prod_{k=1}^{j}\left[\frac{P_{GS}^1(k)}{P_{SG}^1(k)}\right]\right\}\right)^{-1} \quad (24)$$

$$\rho_{GS} \approx \left[1 + \sum_{j=1}^{m-1}e^{-\omega_1\,(1-p_1)\,(1-q_1)\,(1-q_2)\,j}\right]^{-1}$$

$$\times \left(1 + \sum_{j=1}^{M-1}\left\{e^{-\omega_2\,(1-q_1)\,\sum_{i=1}^{j}[1-\theta_2(m\,i)]}\prod_{k=1}^{j}\left[\frac{P_{SG}^1(k)}{P_{GS}^1(k)}\right]\right\}\right)^{-1} \quad (25)$$

$$\rho_{SL} \approx \left\{1 + \sum_{j=1}^{m-1}e^{\omega_1\,(1-p_1)\,(1-q_2)\,\sum_{i=1}^{j}[1-\theta_1(i)]}\right\}^{-1}$$

$$\times \left\{1 + \sum_{j=1}^{M-1}e^{[\omega_2\,(1-q_1-r_1)+\omega_1\,(m-1)\,(1-p_1)]\,(1-q_2)\,j}\right\}^{-1} \quad (26)$$

$$\rho_{LS} \approx \left\{1 + \sum_{j=1}^{m-1}e^{-\omega_1\,(1-p_1)\,(1-q_2)\,\sum_{i=1}^{j}[1-\theta_1(i)]}\right\}^{-1}$$

$$\times \left\{1 + \sum_{j=1}^{M-1}e^{[-\omega_2\,(1-q_1-r_1)-\omega_1\,(m-1)\,(1-p_1)]\,(1-q_2)\,j}\right\}^{-1} \quad (27)$$

$$\rho_{LG} \approx \frac{1}{m} \times \left\{1 + \sum_{j=1}^{M-1}e^{\omega_2\,\sum_{i=1}^{j}[1-\theta_2(m\,i)]\,(r_1+q_1\,\frac{i\,r_2}{M})}\right\}^{-1} \quad (28)$$

$$\rho_{GL} \approx \frac{1}{m} \times \left\{1 + \sum_{j=1}^{M-1}e^{-\omega_2\,\sum_{i=1}^{j}[1-\theta_2(m\,i)]\,(r_1+q_1\,\frac{i\,r_2}{M})}\right\}^{-1} . \quad (29)$$

During the evolutionary process, players have the chance of exploring strategies with a probability μ. Since we assume the exploration rate $\mu \to 0$, it assures that a single mutant vanishes or fixates in the population before the next one appears [55, 56]. Thus, the evolutionary process can be approximated by a Markov chain where the state space is composed of homogeneous states full of each type of players (G, L or S). The corresponding transition probability matrix T is:

$$T = \begin{pmatrix} T_{SS} & \frac{\mu}{2}\,\rho_{SL} & \frac{\mu}{2}\,\rho_{SG} \\ \frac{\mu}{2}\,\rho_{LS} & T_{LL} & \frac{\mu}{2}\,\rho_{LG} \\ \frac{\mu}{2}\,\rho_{GS} & \frac{\mu}{2}\,\rho_{GL} & T_{GG} \end{pmatrix} . \quad (30)$$

Here, $T_{ii} = 1 - \sum_{k\neq i}(\frac{\mu}{2}\,\rho_{ik})$, where $i,k \in \{G,L,S\}$. Stationary distribution describes the percentage of time spent by the population in each homogeneous state in the long run, which is determined by the normalized left eigenvector corresponding to the eigenvalue 1 of the transition matrix. The stationary distribution for Equation (30) can be calculated as follows:

$$X_S = \frac{\rho_{GS}\,\rho_{LG} + \rho_{GS}\,\rho_{LS} + \rho_{LS}\,\rho_{GL}}{\Delta} \quad (31)$$

$$X_L = \frac{\rho_{GS}\,\rho_{SL} + \rho_{SL}\,\rho_{GL} + \rho_{SG}\,\rho_{GL}}{\Delta} \quad (32)$$

$$X_G = \frac{\rho_{SG}\,\rho_{LS} + \rho_{SL}\,\rho_{LG} + \rho_{SG}\,\rho_{LG}}{\Delta}, \quad (33)$$

where X_S, X_L, and X_G represent the probability to find the population in the homogeneous state consisting entirely of selfish ones, local cooperators, and global cooperators, respectively. The normalization factor Δ assures $X_S + X_L + X_G = 1$.

On the other hand, the average time to reach a certain state for the first time can be derived analytically in the limit of rare explorations. For example, we denote fixation time τ_{GS} as the average time starting in pure state of G to reach S. This fixation time satisfies:

$$\tau_{GS} = 1 + r_{GL}\,\tau_{LS} + r_{GG}\,\tau_{GS}, \quad (34)$$

where $r_{ij} = \delta_{ij} + \frac{\mu N}{2}(\rho_{ij} - \delta_{ij})$. It represents the transition probability from the homogeneous state i to the homogeneous state j. ρ_{ij} expresses the fixation probability. δ_{ij} denotes the Kronecker delta. $\frac{\mu N}{2}$ means the rate at which mutants of type j are born (as only two types of mutants can be produced with equal probability), since on average it takes the time of $\frac{1}{\mu N}$ for per mutation. Then, we can get the average time of reaching the homogeneous state S from the initial pure states G and L:

$$\tau_{GS} = 1 + \frac{\mu N}{2}\,\rho_{GL}\,\tau_{LS} + [1 - \frac{\mu N}{2}\,(\rho_{GS} + \rho_{GL})]\,\tau_{GS} \quad (35)$$

$$\tau_{LS} = 1 + \frac{\mu N}{2}\,\rho_{LG}\,\tau_{GS} + [1 - \frac{\mu N}{2}\,(\rho_{LS} + \rho_{LG})]\,\tau_{LS}. \quad (36)$$

Solving Equations (35)-(36), we have:

$$\tau_{GS} = \frac{2\,(\rho_{GL} + \rho_{LG} + \rho_{LS})}{\mu N\,(\rho_{GS}\,\rho_{LG} + \rho_{GL}\,\rho_{LS} + \rho_{GS}\,\rho_{LS})} \quad (37)$$

$$\tau_{LS} = \frac{2\,(\rho_{GL} + \rho_{GS} + \rho_{LG})}{\mu N\,(\rho_{GS}\,\rho_{LG} + \rho_{GL}\,\rho_{LS} + \rho_{GS}\,\rho_{LS})}. \quad (38)$$

Similarly, expressions for other fixation time can be shown as follows:

$$\tau_{SL} = \frac{2\,(\rho_{GL} + \rho_{GS} + \rho_{SG})}{\mu N\,(\rho_{GL}\,\rho_{SG} + \rho_{GL}\,\rho_{SL} + \rho_{GS}\,\rho_{SL})} \quad (39)$$

$$\tau_{GL} = \frac{2\,(\rho_{GS} + \rho_{SG} + \rho_{SL})}{\mu N\,(\rho_{GL}\,\rho_{SG} + \rho_{GL}\,\rho_{SL} + \rho_{GS}\,\rho_{SL})} \quad (40)$$

$$\tau_{SG} = \frac{2\,(\rho_{LG} + \rho_{LS} + \rho_{SL})}{\mu N\,(\rho_{LG}\,\rho_{SG} + \rho_{LS}\,\rho_{SG} + \rho_{LG}\,\rho_{SL})} \quad (41)$$

$$\tau_{LG} = \frac{2\,(\rho_{LS} + \rho_{SG} + \rho_{SL})}{\mu N\,(\rho_{LG}\,\rho_{SG} + \rho_{LS}\,\rho_{SG} + \rho_{LG}\,\rho_{SL})}. \quad (42)$$

Based on the solved fixation probabilities, Equations (24–29), we can deduce the stationary distribution and the fixation time with a complete form.

3.2. Global Income Redistribution

Similarly with the former analysis for local income redistribution, we consider a single local group in which there are i G players and $m - i$ S players, and assume that all the other groups are full of S players. Compared with Equations (17–20) for local income redistribution, the payoffs of each G and S player for global income redistribution are:

$$\pi_G^1(i)' = \left[\frac{i \times r_2}{N} \times (1-p_2) + \frac{p_2}{N} \times \sum_{x=1}^{N}\pi_x\right] \times (1-q_1)$$

$$\times [1 - \theta_2(i)] \tag{43}$$

$$\pi_S^1(m-i)' = \left[(\frac{i \times r_2}{N} + 1) \times (1 - p_2) + \frac{p_2}{N} \times \sum_{x=1}^{N} \pi_x \right] \times (1 - q_1)$$

$$\times [1 - \theta_2(i)] \tag{44}$$

$$\pi_G^2(i)' = \left[(\frac{i \times r_2}{M}) \times (1 - p_2) + \frac{p_2}{N} \times \sum_{x=1}^{N} \pi_x \right]$$

$$\times (1 - q_1) \times [1 - \theta_2(m \times i)] \tag{45}$$

$$\pi_S^2(M-i)' = \left[(\frac{i \times r_2}{M} + 1) \times (1 - p_2) + \frac{p_2}{N} \times \sum_{x=1}^{N} \pi_x \right]$$

$$\times (1 - q_1) \times [1 - \theta_2(m \times i)]. \tag{46}$$

Based on the payoffs, we can get the fixation probability ρ'_{SG}, ρ'_{GS}, ρ'_{SL}, ρ'_{LS}, ρ'_{LG}, and ρ'_{GL} for global income redistribution, which are given as follows:

$$\rho'_{SG} \approx \left[1 + \sum_{j=1}^{m-1} e^{\omega_1 (1-p_2)(1-q_1)(1-q_2)j} \right]^{-1}$$

$$\times \left(1 + \sum_{j=1}^{M-1} \left\{ e^{\omega_2 (1-p_2)(1-q_1) \sum_{i=1}^{j}[1-\theta_2(mi)]} \right. \right.$$

$$\left. \left. \prod_{k=1}^{j} \left[\frac{P_{GS}^1(k)}{P_{SG}^1(k)} \right] \right\} \right)^{-1} \tag{47}$$

$$\rho'_{GS} \approx \left\{ 1 + \sum_{j=1}^{m-1} e^{[-\omega_1 (1-p_2)(1-q_1)(1-q_2)j]} \right\}^{-1}$$

$$\times \left(1 + \sum_{j=1}^{M-1} \left\{ e^{-\omega_2 (1-p_2)(1-q_1) \sum_{i=1}^{j}[1-\theta_2(mi)]} \right. \right.$$

$$\left. \left. \prod_{k=1}^{j} \left[\frac{P_{SG}^1(k)}{P_{GS}^1(k)} \right] \right\} \right)^{-1} \tag{48}$$

$$\rho'_{SL} \approx \left[1 + \sum_{j=1}^{m-1} e^{\omega_1 (1-p_2)(1-q_2) \sum_{i=1}^{j}[1-\theta_1(i)]} \right]^{-1}$$

$$\times \left\{ 1 + \sum_{j=1}^{M-1} e^{[\omega_2 (1-q_1-r_1)+\omega_1 (m-1)](1-p_2)(1-q_2)j} \right\}^{-1} \tag{49}$$

$$\rho'_{LS} \approx \left\{ 1 + \sum_{j=1}^{m-1} e^{-\omega_1 (1-p_2)(1-q_2) \sum_{i=1}^{j}[1-\theta_1(i)]} \right\}^{-1}$$

$$\times \left\{ 1 + \sum_{j=1}^{M-1} e^{[-\omega_2 (1-q_1-r_1)-\omega_1 (m-1)](1-p_2)(1-q_2)j} \right\}^{-1} \tag{50}$$

$$\rho'_{LG} \approx \frac{1}{m} \times \left[1 + \sum_{j=1}^{M-1} e^{\omega_2 (1-p_2) \sum_{i=1}^{j}[1-\theta_2(mi)](r_1+q_1 \frac{ir_2}{M})} \right]^{-1} \tag{51}$$

$$\rho'_{GL} \approx \frac{1}{m} \times \left(1 + \sum_{j=1}^{M-1} e^{\{-\omega_2 (1-p_2) \sum_{i=1}^{j}[1-\theta_2(mi)](r_1+q_1 \frac{ir_2}{M})\}} \right)^{-1} \tag{52}$$

Based on these fixation probabilities, it is easy to deduce the corresponding stationary distribution and the fixation time with a complete form for the case of global income redistribution.

4. RESULTS AND DISCUSSION

Sustainable development calls for more and more global cooperation. Former collective risk dilemma models, however, seldom distinguish global cooperators from local ones. In this paper, we explicitly consider different cooperators arising from the group structured population to address how global cooperative behavior is affected by collective risk and income redistribution mechanisms. Income redistribution is a means of adjusting the income among individuals, which could make full use of social capital. We explore how income redistribution of different levels influence the evolution of global contribution in multi-level threshold public goods games.
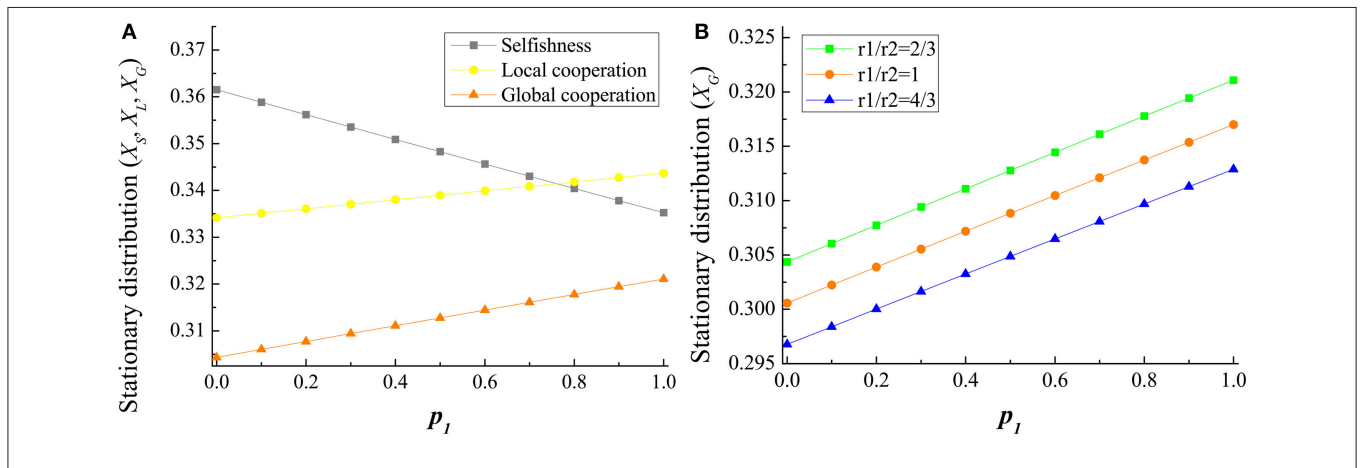
We first study the local income redistribution mechanism. The stationary distribution of three strategies are compared in **Figure 1A**. With the increment of the local income expenditure proportion p_1, X_G (the stationary distribution of G) and X_L show an ascending trend while X_S descends. It is found that the global cooperation is promoted by local income redistribution compared with typical TPGG, which is shown as $p_1 = 0$. In PGG model, the Nash equilibrium predicts zero provision. Thus, the selfishness is the dominate strategy, while global cooperation is inferior. When public goods can only be provided if global contributions reach a minimum threshold, this creates an advantage in that Pareto efficient outcomes can be Nash equilibria. In TPGG, however, we still see significant under-provision of the global public goods. After introducing the local income redistribution, players share part of their payoffs. The mechanism changes the comparison between different strategies in the local group, which reduces the inferiority of global cooperators. Especially under the high risk circumstance, the global cooperation becomes a Nash equilibrium (if the collective target is so large which requires almost all the players to contribute). For different gain-factors, it is shown in **Figure 1B** that the global cooperative behavior is promoted with the rise of p_1. It is well-known that, in the context of PGG, small values of gain-factor favor defectors and large values benefit cooperators. In our work, larger r_1/r_2 indicates much worse condition for global contributors. Even though in such situations, compared with the frequencies of global cooperation at $p_1 = 0$, global cooperators still have a much better chance for survival when $p_1 > 0$. That is because the wealth gap between global cooperators and others is narrowed with increasing p_1, which makes the cooperative behaviors have more chance to prevail in the multi-level TPGG rather than without such mechanism. Local income redistribution balances the income difference among individuals in the same group. Suffered as a consequence, the final payoff of each player is strongly dependent on the quality of its group. Obviously, more local cooperators make larger contributions in the group with a fixed number of participants. Thereinto, a player in the group with more local cooperators has a

competitive advantage over the other players. Thus, local income redistribution, which acts as a driving force for promoting cooperation in the local group, especially local cooperation, gives prominence to the role of groups on the evolution of cooperation.

We then probe how global income redistribution mechanism influences the evolution of different strategies in the multi-level TPGG model. As is illustrated in **Figure 2**, the stationary distribution of G (X_G) shows an ascending trend while X_S and X_L descend with the increment of p_2. Different from results in local income redistribution mechanism, only global cooperation is obviously improved. Under the mechanism of global income redistribution, the accumulated second-order payments are redistributed to the whole population irrespective

of their strategies and contributions. Thus, on the one hand, the payoff differences among strategies are reduced. On the other hand, the evolutionary advantage of compact cooperative clusters cannot spread to the whole population. Thus, the global income redistribution actually inhibit the heterogeneity of groups. When $p_2 \to 1$, almost all the players share all of their fortunes. Under such circumstance, the whole population is in a state of random drift. Each strategy holds a stationary distribution of $1/3$.

In the following, we study how long the population fixates at each state in both income redistribution mechanisms. We focus on the fixation time of each strategy, especially that of G strategy. With the increase of income expenditure proportion, both p_1 and p_2, the changes of average time that a mutant of each strategy

FIGURE 1 | The influence of the local income expenditure proportion on the stationary distribution of strategies. In **(A)**, the tendency of stationary distribution of selfishness, local contribution and global cooperation (X_S, X_L, and X_G) with respect to the increase of local income expenditure proportion p_1 are shown. X_G and X_L are promoted with the increasing p_1, while X_S decreases. It means that the effect of local income redistribution on promoting global cooperation becomes remarkably obvious with the increase of the proportion of redistribution in groups. Parameters are $m = 5$, $M = 20$, $N = 100$, $q_1 = q_2 = 0.8$, $r_1 = 2$, $r_2 = 3$, $s_1 = 2$, $s_2 = 160$, and $\omega_1 = \omega_2 = 0.005$. In **(B)**, the tendency of the stationary distribution of global cooperation with the increase of p_1 under different gain-factors are shown. We respectively study three r_1/r_2 ratios.
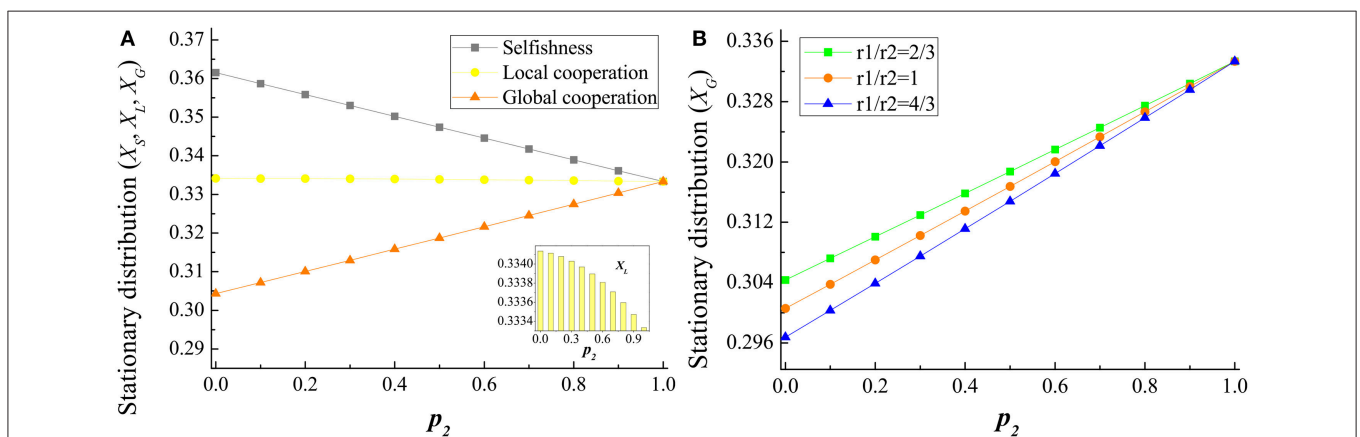

FIGURE 2 | The influence of the global income expenditure proportion on the stationary distribution of strategies. In **(A)**, the tendency of stationary distribution of S, L, and G (X_S, X_L, and X_G) with respect to the increase of global income expenditure proportion p_2 are shown. X_G is promoted with the increasing p_2, while X_S and X_L decrease. It means that the effect of global income redistribution on promoting global cooperation becomes remarkably obvious with the increase of the proportion of redistribution in the whole population. Parameters are $m = 5$, $M = 20$, $N = 100$, $q_1 = q_2 = 0.8$, $r_1 = 2$, $r_2 = 3$, $s_1 = 2$, $s_2 = 160$, and $\omega_1 = \omega_2 = 0.005$. In **(B)**, the tendency of the stationary distribution of global cooperation with the increase of p_2 under different gain-factors are shown. We respectively explore three r_1/r_2 ratios.

invades population full of the other two respectively are shown in **Figure 3**. After introducing an income redistribution (both global and local) into the multi-level TPGG model, the time for G invading the other two strategies are obviously shortened. The larger the p_1 (or p_2), the more likely global cooperation to be learned and adopted by other strategies' holders. Then G strategy could occupy the entire population more quickly. By comparison, the G strategy under global income redistribution mechanism fixates faster than local income redistribution. The change of the fixation time of S is on the contrary. The time for S invading the other two strategies remarkably prolong. It is known that the fixation time of L is shortened in TPGG compared with PGG. For the local income redistribution, the fixation time of L declines. The difficulty for other strategies to invade L increases. For the global income redistribution, however, although the time for L invading S is shortened, the time for L invading G becomes longer with the rise of p_2. Compared with the promotion of global cooperation and the inhibition of selfishness owing to the global income redistribution, to a certain extent, it only has little impact on local cooperation. For a limit case, when $p_2 = 1$, the fixation difficulty of all the strategies are the same.

We further investigate how decision-makings are affected by the change of the thresholds. As shown in **Figure 4**, the global cooperative behavior is promoted with the increasing global threshold. By adding a threshold in global PGG, the game is turned from a social dilemma into a sort of coordination game. In particular, with a large threshold, players are facing a sufficiently severe potential crisis. Such risk indicates that all the players probably lose their wealth. Higher global threshold means a bigger target which has to be reached to avoid the risk. Global cooperation is necessary for public safety, and becomes more and more important with the increasing risk. Once the disaster happens, all the individuals are equally wealthy. Thus, global cooperators can gain a foothold. Because income redistribution

could narrow the payoff differences among strategies, global cooperation has more opportunity to be adopted in high risk cases. This paves the way for them to emerge in the population. In comparison, for the same income expenditure proportion, $p_1 = p_2$, the growth of global cooperation is more obvious in the global income redistribution mechanism than in the local one. It hints that when sharing more wealth with the whole population, the relationship among individuals becomes more close. Hence, individuals are more inclined to cooperate globally
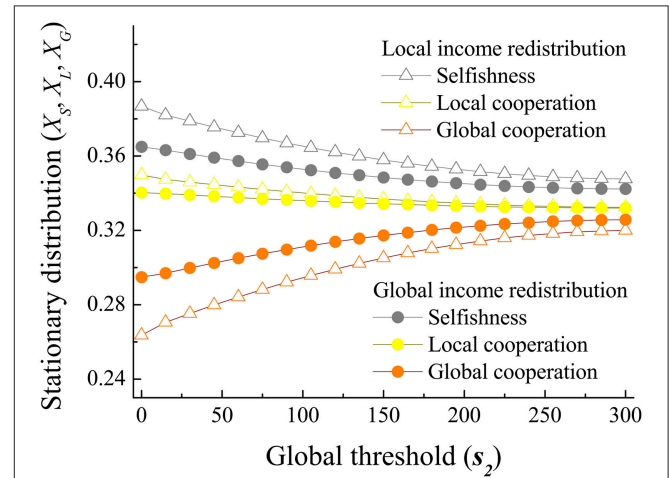
FIGURE 4 | The stationary distribution changes with the global threshold s_2. Global cooperation is promoted with the increase of the global threshold s_2, while selfishness and local cooperation decrease. It hints that players are more apt to cooperate globally under high global risks. We investigate both of the two income redistribution mechanisms, and the stationary distributions of strategies are calculated respectively in each mechanism under the same parameter values. Parameters are: $m = 5$, $M = 20$, $N = 100$, $r_1 = 2$, $r_2 = 3$, $s_1 = 2$, $q_1 = q_2 = 0.8$, $p_1 = 0.5$, $p_2 = 0.5$, and $\omega_1 = \omega_2 = 0.005$.
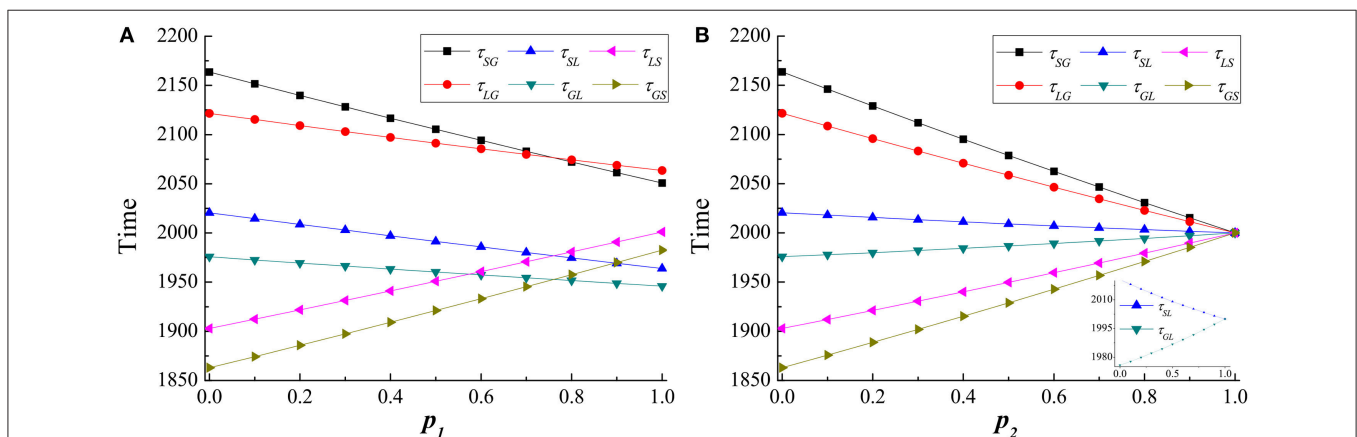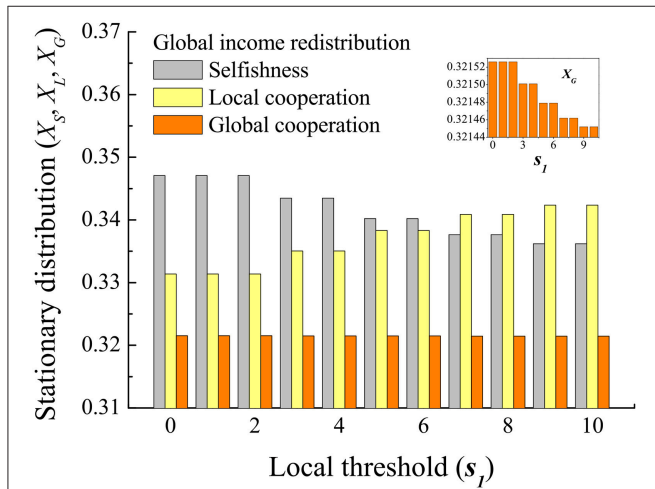

FIGURE 3 | The fixation time changes with the income expenditure proportions. **(A)** reflects the local income redistribution; **(B)** reflects the global income redistribution. In each panel, average fixation time of each strategy invading the others are respectively shown. When a mutant G invades S population, denoting τ_{SG} as the average time starting in pure state of S to reach G. When a mutant G invades L population, denoting τ_{LG} as the average time starting in pure state of L to reach G. Both τ_{SG} and τ_{LG} decline with the increase of p_2 and p_1. Likely, a mutant L invades S population, a mutant L invades G population, a mutant S invades L population, and a mutant S invades G population are respectively shown as τ_{SL}, τ_{GL}, τ_{LS} and τ_{GS}. It shows that the larger p_2 and p_1 benefit the fixation of global cooperation. Parameters are: $m = 5$, $M = 20$, $N = 100$, $r_1 = 2$, $r_2 = 3$, $s_1 = 2$, $s_2 = 160$, $q_1 = q_2 = 0.8$, and $\omega_1 = \omega_2 = 0.005$.

for collecting global public goods to resist the disaster. Moreover, we study the influence of local threshold on the results. As shown in **Figure 5**, the increase of s_1 obviously promotes local

FIGURE 5 | The stationary distribution changes with the local threshold s_1. Local cooperation is promoted with the increase of s_1, while selfishness decreases. It has little influence on global cooperation, which slightly drops with the increase of s_1. Local risk makes the L strategy become a better behavior to be chosen. The results are obtained under global income redistribution mechanisms, while similar results can be found under the local income redistribution case. Parameters are: $m = 5$, $M = 20$, $N = 100$, $r_1 = 2$, $r_2 = 3$, $s_2 = 160$, $q_1 = q_2 = 0.8$, $p_1 = 0.5$, $p_2 = 0.5$, and $\omega_1 = \omega_2 = 0.005$.

cooperation, while inhibits selfishness. Meanwhile, it has only a little impact on global cooperation, which slightly drops with the rise of s_1. The results are obtained under global income redistribution mechanisms, while similar results can be found under the local income redistribution too. Compared with the global threshold, local thresholds have much less effects on the global cooperation. Since we focus on the global cooperation, we mainly study the impacts of global threshold on the results in this paper.

In the global income redistribution mechanism, we further investigate the mutual influence of global threshold s_2 and p_2 on the evolution of strategies. As shown in **Figure 6**, with the increase of s_2 and p_2, global cooperation shows an increasing trend, while selfishness declines correspondingly. It is worth noting that the trend for local cooperation with the change of p_2 is related to the size of global threshold s_2. For smaller s_2, local cooperation decreases with the rise of p_2; for larger s_2, local cooperation increases with the rise of p_2. While for intermediate s_2, local cooperation rises under lower p_2 and then reduces under higher p_2. It is found that there exists a most adaptable value of p_2 for local cooperators under global income redistribution. This phenomenon means that, on one hand, the mechanism of global income redistribution reduces income inequality within the whole population, which is generally regarded to be a positive improvement to society. But on the other hand, it may negatively affect the efficiency of social-economic system (local economic development). Thereby, the income expenditure proportion, which can be described as the social tolerance, should be limited under such case. Beyond these limits, the
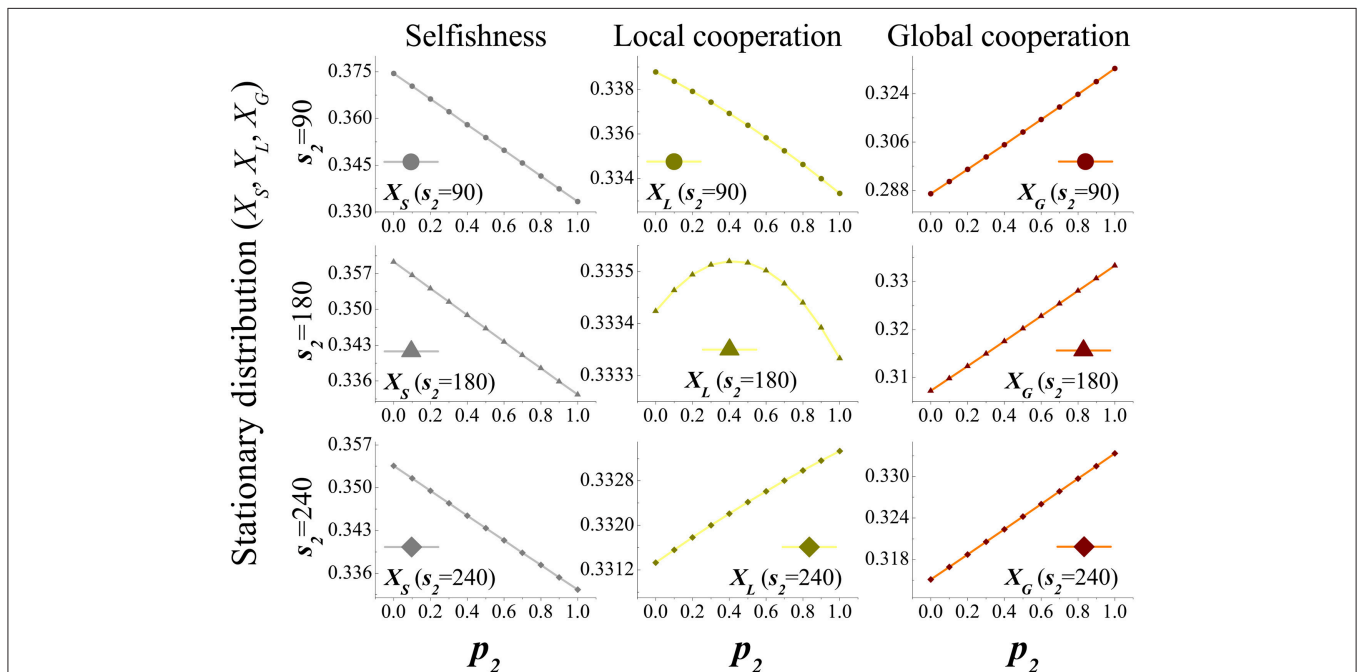
FIGURE 6 | The stationary distribution changes with p_2 under different global thresholds. Global cooperation is promoted with the increase of p_2 as well as s_2, while selfishness decreases. It hints that players are more apt to cooperate globally under high global risks. As to local cooperation, it depends on s_2. For lower s_2, local cooperation decreases with the rise of p_2. For higher s_2, local cooperation increases with the rise of p_2. For moderate s_2, local cooperation first rises and then drops with the increase of p_2. Parameters are: $m = 5$, $M = 20$, $N = 100$, $r_1 = 2$, $r_2 = 3$, $s_1 = 2$, $q_1 = q_2 = 0.8$, and $\omega_1 = \omega_2 = 0.005$.

enthusiasm of individual investment and rapid development of social-economic system may be on the brink of collapse.

5. CONCLUSION

In this paper, we have studied the evolution of strategies in the multi-level threshold public goods games, where global and local cooperation are clearly distinguished. By introducing two kinds of income redistribution mechanisms, we investigate how income expenditure proportions (p_1 and p_2) and risks (thresholds) influence the average abundance of strategies and fixation time. It is shown that with larger income redistribution proportions, players are more apt to cooperate globally, especially under high collective risks. When individuals are conscious of an even greater calamity, they are apt to form an alliance to prevent the risk through cooperation globally. The more disruptive the danger is, the more likely they succeed in collective target. Selfishness is effectively inhibited under both income redistribution mechanisms. It implies that an income redistribution mechanism may be effective for solving the social dilemma of free-riders and promoting social equity. We further compare the influences of the local and global income redistribution on the global cooperation and local cooperation. It is found that compared with local income redistribution, global income redistribution is more in favor of global cooperation. On the contrary, local income redistribution is more beneficial for local cooperation. Our model is relatively simple compared with the actual situations, but it characterizes some main features of the systems with income redistribution, and show results that the frequency of global cooperation may be promoted in some cases. This study may provide some useful implications for investors, fundraisers and also government officials. The theoretical analysis in this work is only a first step toward the models of learning process. Since learning and interaction between players should be on the same scale, we hope more accurate theoretical methods on this kind of models could be explored in the future.

AUTHOR CONTRIBUTIONS

JD and BW designed and performed the research as well as wrote the paper.

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