



Approximate Simulations for the Non-linear Long-Short Wave Interaction System

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This research paper studies the semi-analytical and numerical solutions of the non-linear long-short wave interaction system. This represents an optical field that does not change through multiplication due to a sensitive balance being struck between linear and non-linear impacts in an elastic medium, defined as a medium that can adjust its shape as a consequence of deforming stress and return to its original form when the force is eliminated. In this medium, a wave is produced by vibrations that are a consequence of acoustic power, known as a sound wave or acoustic wave. The Adomian decomposition method and the cubic and septic B-spline methods are applied to the suggested system to obtain distinct types of solutions that are used to explain the novel physical properties of this system. These novel features are described by different types of figures that show more of the physical properties of this model. Also, the convergence between the obtained solutions is discussed through tables that show the values of absolute error between them.

Keywords: nonlinear long-short wave interaction system, adomian decomposition method, cubic B-spline method, septic B-spline method, semi-analytical and numerical solutions

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1. INTRODUCTION

Optical study is considered as one of the most important methodologies in this age due to its different and important applications in several fields. To develop a deeper understanding of this type of study, mathematicians have derived many analytical, semi-analytical, and numerical schemes to obtain distinct types of solutions that are used to characterize the physical properties of optical soliton waves. The optical soliton constitutes an optical field that does not alter through multiplication due to a sensitive balance being struck between linear and non-linear impacts in the medium [1–5]. Optical soliton can be of two types:

- Spatial solitons: the non-linear influence balances the diffraction. The electromagnetic field can alter the refraction index of the medium while propagating, thus establishing an architecture identical to a graded-index fiber [6–10].
- Temporal solitons: if the electromagnetic field is already spatially restricted, it is feasible to transmit pulses that will not alter their form, as the non-linear impacts will be in equilibrium with the dispersion [11–15].

The non-linear long-short wave interaction system describes the interaction between one short transverse wave and one long longitudinal wave propagating in a generalized elastic medium. This system has the following form:

$$\begin{cases} i \Phi_t + \Phi_{xx} - \Phi \Psi = 0, \\ \Psi_t + \Psi_x + (|\Phi|^2)_x. \end{cases} \quad (1)$$

where $\Phi(x, t)$ represents the slowly varying envelope of the short transverse wave, $\Psi(x, t)$ discriminates the long longitudinal wave, (x) is the locational harmonization, and (t) is the time. Waves in plasmas are defined as an interrelated set of particles and fields that disseminate in a periodically duplicating fashion. A plasma is a quasi-neutral, electrically conductive fluid. Plasma waves have an EM character of two types, electrostatic and electromagnetic. Electrostatic and electromagnetic waves have oscillating species in electrons and ions. Some examples of the dispersion relationships of plasma waves in electrostatic and electromagnetic terms are as follows:

- Plasma oscillation: rapid oscillations of the electron intensity in conducting media such as plasmas or metals in the ultraviolet zone
- Upper hybrid oscillation: a form of oscillation of magnetized plasma
- Ion acoustic wave: one kind of longitudinal oscillation of the ions and electrons in a plasma
- Electrostatic ion cyclotron wave: a longitudinal wobble of the ions in a magnetized plasma, with dissemination nearly perpendicular to the magnetic field
- Langmuir wave
- Lower hybrid oscillation: a longitudinal fluctuation of ions and electrons in a magnetized plasma
- Light wave: a wave made of oscillating magnetic and electric fields; comprises radio waves, microwaves, ultraviolet, visible light, infrared, gamma rays, and X-rays
- O wave
- X wave
- R wave (whistler-mode)
- L wave
- Alfvén wave: a kind of magnetohydrodynamic wave in which ions oscillate in response to a restoration strength presented by an effective tension on the magnetic field lines; this kind of wave was named after Hannes Alfvén
- Magnetosonic wave: a longitudinal wave of ions in a magnetized plasma disseminating perpendicular to the stationary magnetic field.

All of the properties and abilities of the non-linear partial differential equations are used to describe these natural phenomena. According to these properties, many mathematicians have developed methods and are still trying to find new general methods to obtain exact and single traveling wave solutions for these models. For more details about these methods, please see [16–36].

The rest of this paper is arranged as follows. In section 2, the Adomian decomposition method [37–40] and Cubic and

septic B-spline method [41–50] are used to obtain approximate solutions of the non-linear long-short wave interaction system. In section 4, the conclusion is given.

2. APPLICATION

This section applies the Adomian decomposition method as the semi-analytical scheme and the cubic & septic B-spline methods as numerical schemes to the non-linear long-short wave interaction system [51–55] that is given by:

$$\begin{cases} i \Phi_t + \Phi_{xx} - \Phi \Psi = 0, \\ \Psi_t + \Psi_x + (|\Phi|^2)_x. \end{cases} \quad (2)$$

Using the wave transformation $\Phi(x, t) = e^{i\eta} \Lambda(\varepsilon)$, $\Psi(x, t) = \varphi(\varepsilon)$ where $\eta = (\rho x + ct)$, $\varepsilon = (ax + bt)$ transforms the non-linear partial differential equation (2) into the following ordinary differential equation:

$$\begin{cases} (b + 2a\rho) i \Lambda - (\rho^2 + c) \Lambda + a^2 \Lambda'' - \Lambda \varphi = 0, \\ (a + b) \varphi' + a (\Lambda^2)' = 0. \end{cases} \quad (3)$$

Equating the complex term to zero leads to

$$b = -2a\rho. \quad (4)$$

Integrating the second equation of the system (3) with zero constant of integration yields:

$$\varphi = \frac{-a}{a+b} \Lambda^2. \quad (5)$$

Substituting (4) and (5) into the first equation in the system (3) yields:

$$a^2 \Lambda'' - (\rho^2 + c) \Lambda + \frac{1}{1-2\rho} \Lambda^3 = 0. \quad (6)$$

According to the analytical solutions obtained in Raghda et al. [Submitted], the exact solution of Equation (6) takes the following formula

$$\Lambda(\varepsilon) = 8 \tanh\left(\frac{\varepsilon}{2}\right). \quad (7)$$

2.1. Semi-analytical Solution

This section applies the Adomian decomposition method to Equation (6) by using its exact solution (6) with the following conditions:

$$\Lambda(0) = 0, \quad \Lambda'(0) = 4,$$

where $\left[\sigma = 6, a = 4, \alpha = 1, \beta = 5, \rho = 4.5 \right]$. Implementation of the Adomian decomposition method on Equation (6) yields

$$\Lambda_0 = 4\varepsilon, \quad (8)$$

$$\Lambda_1 = 0.025 \varepsilon^5 - 1.17708 \varepsilon^3, \tag{9}$$

$$\Lambda_2 = 0.000416667 \varepsilon^{10} - 0.031529 \varepsilon^8 - 0.00105097 \varepsilon^7 + 0.103914 \varepsilon^5, \tag{10}$$

where λ_i, β_i fulfill the conditions:

$$L \Lambda(\varepsilon) = \emptyset(\varepsilon_i, \Lambda(\varepsilon_i)) \text{ where } (i = 0, 1, \dots, n)$$

and

$$\beta_i(\varepsilon) = \frac{1}{6h^3} \begin{cases} (\varepsilon - \varepsilon_{i-2})^3, & \varepsilon \in [\varepsilon_{i-2}, \varepsilon_{i-1}], \\ -3(\varepsilon - \varepsilon_{i-1})^3 + 3h(\varepsilon - \varepsilon_{i-1})^2 + 3h^2(\varepsilon - \varepsilon_{i-1}) + h^3, & \varepsilon \in [\varepsilon_{i-1}, \varepsilon_i], \\ -3(\varepsilon_{i+1} - \varepsilon)^3 + 3h(\varepsilon_{i+1} - \varepsilon)^2 + 3h^2(\varepsilon_{i+1} - \varepsilon) + h^3, & \varepsilon \in [\varepsilon_i, \varepsilon_{i+1}], \\ (\varepsilon_{i+2} - \varepsilon)^3, & \varepsilon \in [\varepsilon_{i+1}, \varepsilon_{i+2}], \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

$$\Lambda_3 = 0.000618538 \varepsilon^{10} + 0.00196005 \varepsilon^9 - 0.0148781 \varepsilon^7 - 0.0000501598 \varepsilon^{11} - 5.5733112373737385 \times 10^{-6} \varepsilon^{12} + 3.7560096153846164 \times 10^{-7} \varepsilon^{13}, \tag{11}$$

where $i \in [-2, n + 2]$, so that the numerical formula of the solution is given as

$$\Lambda_i(\varepsilon) = \lambda_{i-1} + 4\lambda_i + \lambda_{i+1}. \tag{15}$$

According to (8–11), we get

$$\Lambda_{\text{Semi-analytical}} = 3.7560096153846164 \times 10^{-7} \varepsilon^{13} - 5.5733112373737385 \times 10^{-6} \varepsilon^{12} - 0.0000501598 \varepsilon^{11} + 0.0010352 \varepsilon^{10} + 0.00196005 \varepsilon^9 - 0.031529 \varepsilon^8 - 0.0159291 \varepsilon^7 + 0.128914 \varepsilon^5 - 1.17708 \varepsilon^3 + 4\varepsilon + \dots \tag{12}$$

Substituting Equation (15) into (6), leads to a system of equations. Solving this system of equations gives the value of λ_i . Replacing the values of λ_i, β_i into Equation (13) gives the data shown in **Table 2**.

2.2. Numerical Solutions

This section studies the numerical solutions of the modified BBM equation by applying the cubic and septic B-spline techniques, which are considered as the most accurate numerical tools for getting this type of solution.

2.2.1. Cubic-Spline

According to the cubic B-spline, the numerical solution of the modified BBM equation (6) is given by

$$\Lambda(\varepsilon) = \sum_{i=-1}^{n+1} \lambda_i \beta_i, \tag{13}$$

TABLE 1 | Computational, semi-analytical, and absolute error values obtained by using the Adomian decomposition method.

Value of ε	Analytical value	Semi-analytical value	Value of absolute error
0.000	0.000	0.000	0.0000000000
0.001	0.004	0.004	8.4375×10^{-10}
0.002	0.008	0.008	6.75×10^{-9}
0.003	0.012	0.012	2.27812×10^{-8}
0.004	0.0160000	0.0159999	5.39999×10^{-8}
0.005	0.0200000	0.0199999	1.05468×10^{-7}
0.006	0.0239999	0.0239997	1.82249×10^{-7}
0.007	0.0279999	0.0279996	2.89405×10^{-7}
0.008	0.0319998	0.0319994	4.31997×10^{-7}
0.009	0.0359998	0.0359991	6.15088×10^{-7}
0.010	0.0399997	0.0399988	8.4374×10^{-7}

TABLE 2 | Computational, numerical, and absolute error values obtained by using the cubic B-spline scheme.

Value of ε	Val. Com.	Val. Nu.	Value of abs. error
0.000	0.0000000	0.0000000000	0.0000000000
0.001	0.004	0.0040001	8.35327×10^{-8}
0.002	0.008	0.0080002	1.62003×10^{-7}
0.003	0.012	0.0120002	2.30348×10^{-7}
0.004	0.0160000	0.0160003	2.83505×10^{-7}
0.005	0.0200000	0.0200003	3.16411×10^{-7}
0.006	0.0239999	0.0240003	3.24004×10^{-7}
0.007	0.0279999	0.0280002	3.01222×10^{-7}
0.008	0.0319998	0.0320001	2.43003×10^{-7}
0.009	0.0359998	0.0359999	1.44283×10^{-7}
0.010	0.0399997	0.0399997	6.93889×10^{-18}

TABLE 3 | Computational, numerical, and absolute error value obtained by using the septic B-spline scheme.

Value of ε	Val. Com.	Val. Nu.	Value of abs. error
0.000	0.0000000	0.0000000000	0.0000000000
0.001	0.0040000	0.0040001	7.5153×10^{-8}
0.002	0.0080000	0.0080002	1.70905×10^{-7}
0.003	0.0120000	0.0120002	2.31487×10^{-7}
0.004	0.0160000	0.0160003	2.88889×10^{-7}
0.005	0.0200000	0.0200003	3.19377×10^{-7}
0.006	0.0239999	0.0240003	3.3083×10^{-7}
0.007	0.0279999	0.0280002	3.01294×10^{-7}
0.008	0.0319998	0.0320001	2.59145×10^{-7}
0.009	0.0359998	0.0359999	1.26976×10^{-7}
0.010	0.0399997	0.0399997	6.93889×10^{-18}

2.2.2. Septic-Spline

Based on the septic B-spline, the suggested solution of the ordinary differential form of the modified BBM equation (6) is given as follows:

$$\Lambda(\varepsilon) = \sum_{i=-1}^{n+1} \lambda_i \beta_i, \tag{16}$$

where λ_i, β_i satisfies the conditions

$$L \Lambda(\varepsilon) = \vartheta(\varepsilon_i, \Lambda(x_i)) \text{ where } (i = 0, 1, \dots, n)$$

and

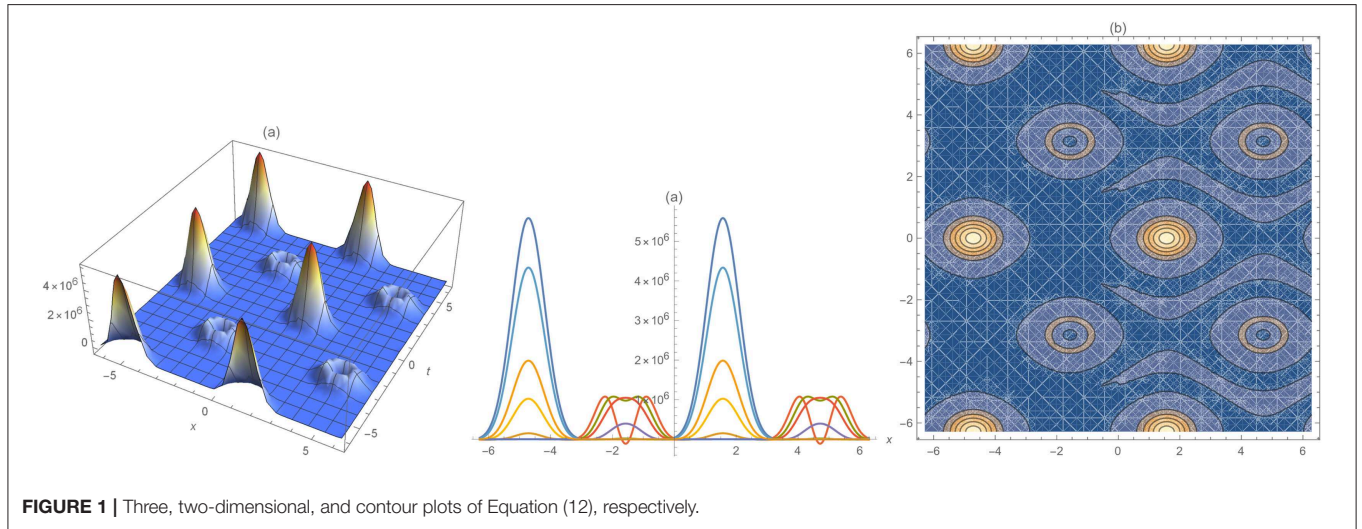


FIGURE 1 | Three, two-dimensional, and contour plots of Equation (12), respectively.

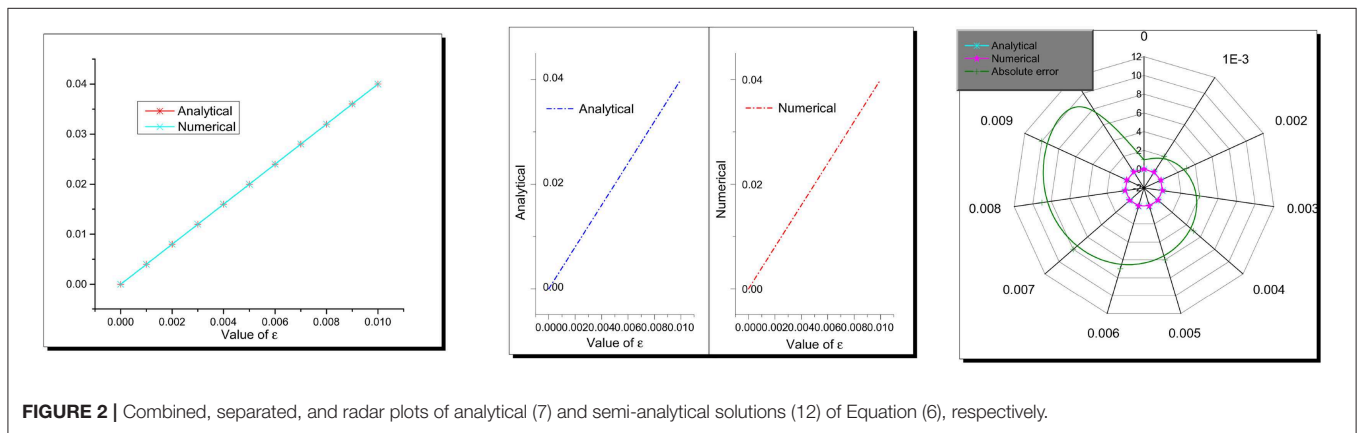


FIGURE 2 | Combined, separated, and radar plots of analytical (7) and semi-analytical solutions (12) of Equation (6), respectively.

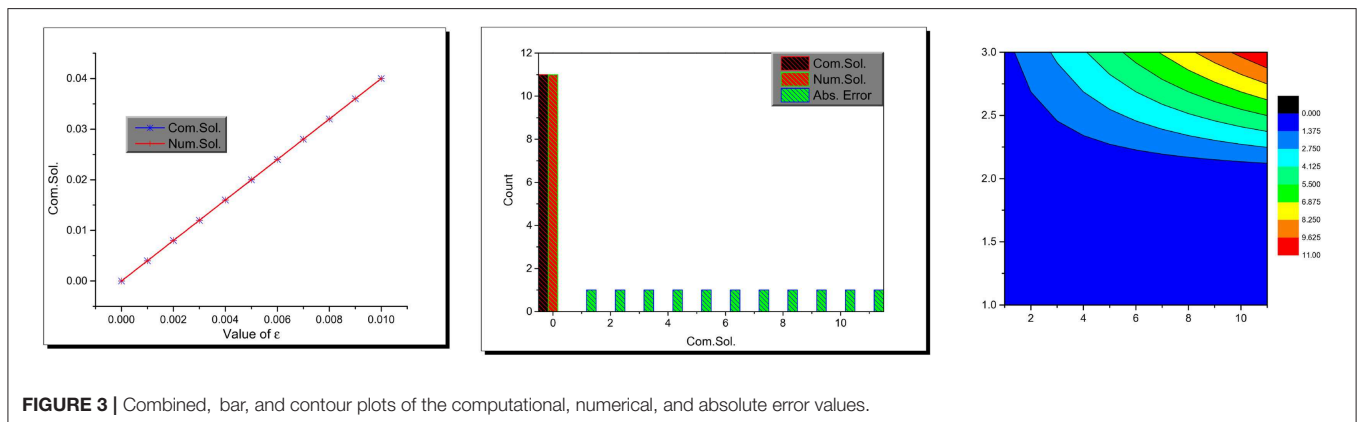


FIGURE 3 | Combined, bar, and contour plots of the computational, numerical, and absolute error values.

$$\beta_i(\varepsilon) = \frac{1}{h^5} \begin{cases} (\varepsilon - \varepsilon_{i-4})^7, & \varepsilon \in [\varepsilon_{i-4}, \varepsilon_{i-3}], \\ (\varepsilon - \varepsilon_{i-4})^7 - 8(\varepsilon - \varepsilon_{i-3})^7, & \varepsilon \in [\varepsilon_{i-3}, \varepsilon_{i-2}], \\ (\varepsilon - \varepsilon_{i-4})^7 - 8(\varepsilon - \varepsilon_{i-3})^7 + 28\varepsilon(\varepsilon - \varepsilon_{i-2})^7, & \varepsilon \in [\varepsilon_{i-2}, \varepsilon_{i-1}], \\ (\varepsilon - \varepsilon_{i-4})^7 - 8(\varepsilon - \varepsilon_{i-3})^7 + 28(\varepsilon - \varepsilon_{i-2})^7 + 56(\varepsilon - \varepsilon_{i-1})^7, & \varepsilon \in [\varepsilon_{i-1}, \varepsilon_i], \\ (\varepsilon_{i+4} - \varepsilon)^7 - 8(\varepsilon_{i+3} - \varepsilon)^7 + 28(\varepsilon_{i+2} - \varepsilon)^7 + 56(\varepsilon_{i+1} - \varepsilon)^7, & \varepsilon \in [\varepsilon_i, \varepsilon_{i+1}], \\ (\varepsilon_{i+4} - \varepsilon)^7 - 8(\varepsilon_{i+3} - \varepsilon)^7 + 28(\varepsilon_{i+2} - \varepsilon)^7, & \varepsilon \in [\varepsilon_{i+1}, \varepsilon_{i+2}], \\ (\varepsilon_{i+4} - \varepsilon)^7 - 8(\varepsilon_{i+3} - \varepsilon)^7, & \varepsilon \in [\varepsilon_{i+2}, \varepsilon_{i+3}], \\ (\varepsilon_{i+4} - \varepsilon)^7, & \varepsilon \in [\varepsilon_{i+3}, \varepsilon_{i+4}], \\ 0, & \text{otherwise,} \end{cases} \quad (17)$$

where $i \in [-3, n + 3]$. Thus, the approximate solution is given by:

$$v_i(\varepsilon) = \lambda_{i-3} + 120\lambda_{i-2} + 1191\lambda_{i-1} + 2416\lambda_i + 1191\lambda_{i+1} + 120\lambda_{i+2} + \lambda_{i+3}. \quad (18)$$

Substituting Equation (18) into Equation (6) produces a system of equations. Solving this system gives the data shown in Table 3.

to determine which one of them is the more accurate.

The comparison between the numerical solutions depends on showing which one of the schemes obtains the smallest value of the absolute value of error. To find these values, the obtained values of the total value of error in each method used are plotted in Figure 5, which shows that all the methods used are accurate and have almost the same amount of absolute failure.

3. RESULTS AND DISCUSSION

This section details a comparison between the numerical solutions obtained in our paper

4. CONCLUSION

This research paper succeeded in the application of the Adomian decomposition method and the cubic and septic B-spline

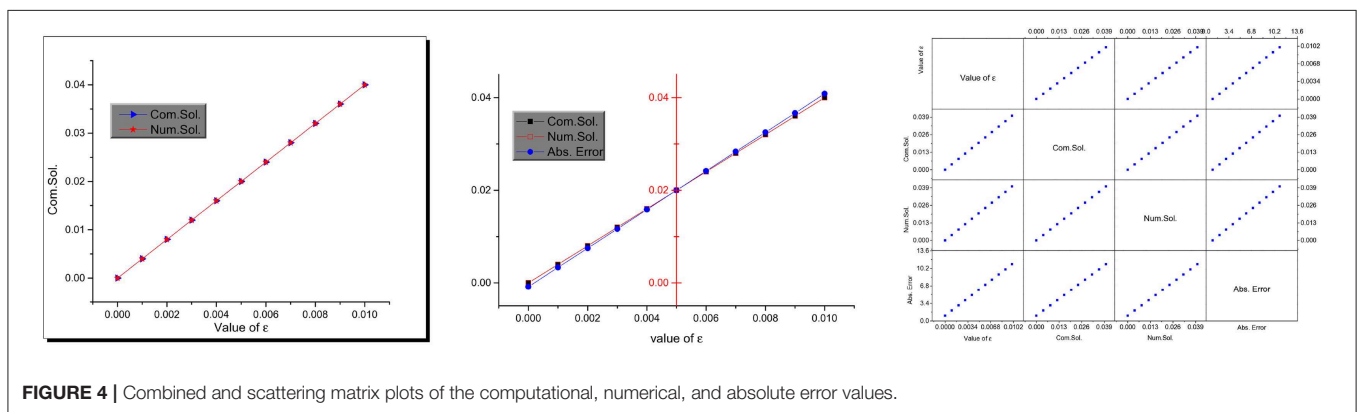


FIGURE 4 | Combined and scattering matrix plots of the computational, numerical, and absolute error values.

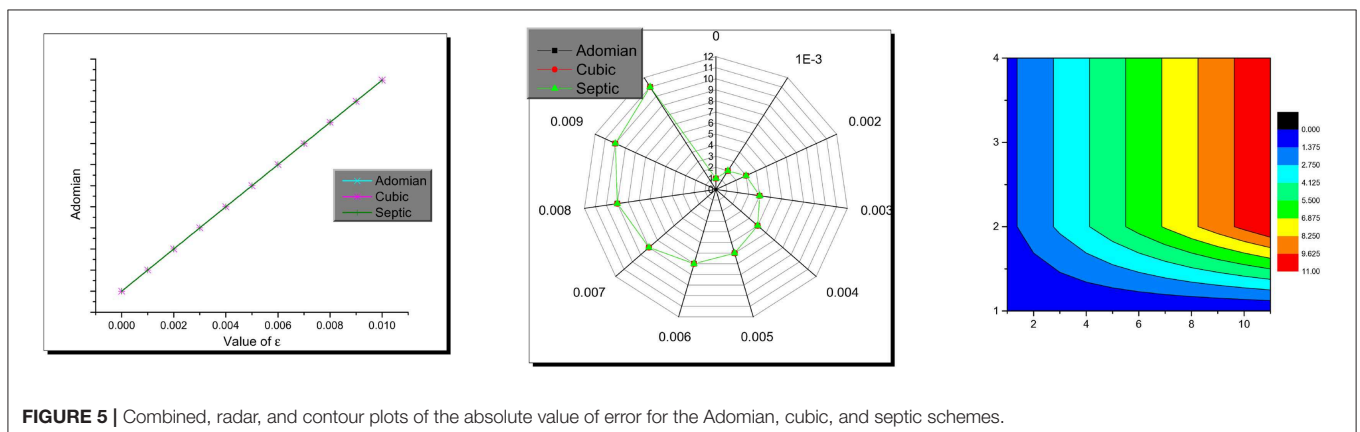


FIGURE 5 | Combined, radar, and contour plots of the absolute value of error for the Adomian, cubic, and septic schemes.

method to the non-linear long-short wave interaction system and in obtaining semi-analytical and numerical solutions for this system. Moreover, a comparison between the distinct types of solutions obtained is detailed, and the absolute values of error between them are shown in **Tables 1–3** and **Figures 1–5**. Both semi-computational and numerical schemes are shown to be powerful, effective, and able to be applied to many and various forms of non-linear evolution equations.

DATA AVAILABILITY STATEMENT

All datasets generated for this study are included in the article/supplementary material.

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AUTHOR CONTRIBUTIONS

All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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