



Corrigendum: Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File

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A Corrigendum on

Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File by Lapolla, A., and Godec, A. (2019). Front. Phys. 7:182. doi: 10.3389/fphy.2019.00182

In the original article, there was an error. In section 2.1 the diffusion matrix **D** the in-line equation was defined with a factor of 2 instead of 1/2, i.e., $\mathbf{D} = 2\boldsymbol{\sigma}\boldsymbol{\sigma}^T$ instead of $\mathbf{D} = \boldsymbol{\sigma}\boldsymbol{\sigma}^T/2$.

In section 2.1 in the paragraph following Equation (4), a copy-paste error occurred in the sentence "... where for reversible system (i.e., those obeying detailed balance) we have $\hat{\mathcal{L}}\hat{\mathcal{L}}^{\dagger} = \hat{\mathcal{L}}^{\dagger}\hat{\mathcal{L}} = 0$."

In section 2.2. in the paragraph following Equation (13) there is an obvious redundant additional factor $\Psi_{00}^{-1}(\mathbf{q})d\mathbf{q}$ present immediately after the in-line equation: $\langle \Psi_{l0}|\Psi_{k0}\rangle_{\Psi_{00}^{-1}} \equiv$

 $\int_{\Xi} d\mathbf{q}' \Psi_{00}(\mathbf{q}')^{-1} \Psi_{0l}(\mathbf{q}') \Psi_{k0}(\mathbf{q}').$

A correction has been made to section 2.1 [paragraph following Equation (1)]. The paragraph now reads:

"where **D** is the symmetric positive-definite diffusion matrix. $\hat{\mathcal{L}}$ propagates probability measures $\mu_t(\mathbf{x})$ in time, which will throughout be assumed to posses well-behaved probability density functions $P(\mathbf{x}, t)$, i.e., $d\mu_t(\mathbf{x}) = P(\mathbf{x}, t)d\mathbf{x}$ [thereby posing some restrictions on $\mathbf{F}(\mathbf{x})$]. On the level of individual trajectories Equation (1) corresponds to the Itô equation $d\mathbf{x}_t = \mathbf{F}(\mathbf{x}_t)dt + \boldsymbol{\sigma} d\mathbf{W}_t$ with \mathbf{W}_t being a *d*-dimensional vector of independent Wiener processes whose increments have a Gaussian distribution with zero mean and variance dt, i.e., $\langle dW_{t,i}dW_{t',j}\rangle = \delta_{ij}\delta(t-t')dt$, and where $\boldsymbol{\sigma}$ is a $d \times d$ symmetric noise matrix such that $\mathbf{D} = \boldsymbol{\sigma} \boldsymbol{\sigma}^T/2$. Moreover, we assume that $\mathbf{F}(\mathbf{x})$ admits the following decomposition into a potential (irrotational) field $-\mathbf{D}\nabla\varphi(\mathbf{x})$ and a non-conservative component $\vartheta(\mathbf{x})$, $\mathbf{F}(\mathbf{x}) = -\mathbf{D}\nabla\varphi(\mathbf{x}) + \vartheta(\mathbf{x})$ with the two fields being mutually orthogonal $\nabla\varphi(\mathbf{x}) \cdot \vartheta(\mathbf{x}) = 0$ [73]. By insertion into Equation (1) one can now easily check that $\hat{\mathcal{L}}e^{-\varphi(\mathbf{x})} = 0$, such that the stationary solution of the Fokker-Planck equation (also referred to as the steady state [74, 75], which is the terminology we adopt here) by construction does not depend on the non-conservative part $\vartheta(\mathbf{x})$."

A correction has been made to the aforementioned sentence in section 2.1, in the paragraph following Equation (4), which now reads:

"such that the conditional probability density starting from a general initial condition $|p_0\rangle$ becomes $P(\mathbf{x}, t|p_0, 0) = \langle \mathbf{x} | \hat{U}(t) | p_0 \rangle \equiv \int d\mathbf{x}_0 p_0(\mathbf{x}_0) G(\mathbf{x}, t | \mathbf{x}_0, 0)$. Moreover, as $\mathbf{F}(\mathbf{x})$ is assumed to be sufficiently confining (i.e., $\lim_{\mathbf{x}\to\infty} P(\mathbf{x}, t) = 0$, $\forall t$ sufficiently fast), such that $\hat{\mathcal{L}}$ corresponds to a coercive and densely defined operator on V (and $\hat{\mathcal{L}}^{\dagger}$ on W, respectively) [76–78]. Finally, $\hat{\mathcal{L}}$ is throughout assumed to be *normal*, i.e., $\hat{\mathcal{L}}^{\dagger}\hat{\mathcal{L}} - \hat{\mathcal{L}}\hat{\mathcal{L}}^{\dagger} = 0$ and thus henceforth V = W, where for reversible system (i.e., those obeying detailed balance) we have $\hat{\mathcal{L}} \Leftrightarrow \hat{\mathcal{L}}^{\dagger}$ ".

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Lapolla A and Godec A (2020) Corrigendum: Manifestations of Projection-Induced Memory: General Theory and the Tilted Single File. Front. Phys. 8:7. doi: 10.3389/fphy.2020.00007 Finally, the redundant factor $\Psi_{00}^{-1}(\mathbf{q})d\mathbf{q}$ has been deleted in section 2.2 in the paragraph following Equation (13).

"can be equal to $Q_{p_{ss}}(\mathbf{q}, t|\mathbf{q}_0, 0)$. As this will generally not be the case this essentially means that the projected dynamics is in general non-Markovian. The proof is established by noticing that $\Psi_{kl}(\mathbf{q}') = \Psi_{lk}^{\dagger}(\mathbf{q}')$ such that $\langle \Psi_{l0}|\Psi_{k0}\rangle_{\Psi_{00}^{-1}} \equiv \int_{\Xi} d\mathbf{q}' \Psi_{00}(\mathbf{q}')^{-1} \Psi_{0l}(\mathbf{q}') \Psi_{k0}(\mathbf{q}')$."

The authors apologize for this error and state that this does not change the scientific conclusions

of the article in any way. The original article has been updated.

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