



Exact Soliton Solutions to the Cubic-Quartic Non-linear Schrödinger Equation With Conformable Derivative

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The research paper aims to investigate the space-time fractional cubic-quartic non-linear Schrödinger equation in the appearance of the third, and fourth-order dispersion impacts without both group velocity dispersion, and disturbance with parabolic law media by utilizing the extended sinh-Gordon expansion method. This method is one of the strongest methods to find the exact solutions to the non-linear partial differential equations. In order to confirm the existing solutions, the constraint conditions are used. We successfully construct various exact solitary wave solutions to the governing equation, for example, singular, and dark-bright solutions. Moreover, the 2D, 3D, and contour surfaces of all obtained solutions are also plotted. The finding solutions have justified the efficiency of the proposed method.

Keywords: the non-linear cubic-quartic Schrödinger equation, conformable derivative, analytical solutions, the extended sinh-Gordon expansion method, solitary wave solutions

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1. INTRODUCTION AND MOTIVATION

Non-linear partial differential equations have different types of equations, one of them is the non-linear Schrödinger equation (NLSE) that relevant to the classical and quantum mechanics. The non-linear Schrödinger equation is a generalized (1+1)-dimensional version of the Ginzburg-Landau equation presented in 1950 in their study on supraconductivity and has been specifically reported by Chiao et al. [1] in their research of optical beams. In the past several years, various methods have been proposed to obtain the exact optical soliton solutions of the non-linear Schrödinger equation [2–12]. Dispersion and non-linearity are two of the essential components for the distribution of solitons across inter-continental regions. Usually, group velocity dispersion (GVD) level with self-phase modulation in a sensible manner allows these solitons to sustain tall range travel. In fact it might happen that the GVD is tiny and thus totally ignored, in this case the dispersion effect is determined by third and fourth order dispersion effects. Subsequently, this equation has been studied in a variety of ways, such as the Lie symmetry [13], both the $(m + \frac{G'}{G})$ -improved expansion, and the $\exp(-\varphi(\xi))$ -expansion methods [14], and the semi-inverse variation principle method [4]. In this study, the extended sinh-Gordon expansion method (ShGEM) is applied to the non-linear cubic-quartic Schrödinger equations with the Parabolic law of fractional order, which is given by

$$iD_t^\alpha u + i\beta D_x^{3\alpha} u + \gamma D_x^{4\alpha} u + cF(|u|^2)u = 0, \quad (1)$$

where $u(x, t)$ is the complex valued wave function. The operator D^α of order α , where $\alpha \in (0, 1]$ is the fractional derivative, the parameters γ and β are real constants, a real-valued algebraic function $F(|u|^2)$ is p -times continuously differentiable, then

$$F(|u|^2) \in \bigcup_{m, n=1}^{\infty} C^p((-n, n) \times (-m, m) : \mathbb{R}^2). \quad (2)$$

By using the relation of

$$F(u) = c_1 u + c_2 u^2,$$

on Equation (1), we obtain the fractional non-linear Schrödinger equations with Parabolic law as follows:

$$iD_t^\alpha u + i\beta D_x^{3\alpha} u + \gamma D_x^{4\alpha} u + (c_1 |u|^2 + c_2 |u|^4) u = 0. \quad (3)$$

The extended sinh-Gordon expansion method is intended to a generalization of the sine-Gordon expansion equation because it is based on an auxiliary equation namely the sine-Gordon equation (see previous studies [15, 16] for details). Moreover, different computational and numerical methods have been utilized to constructed new solutions to the non-linear partial differential equations, such as the variable separated method [17], the auxiliary parameters and residual power series method [18], the Bernoulli sub-equation method [19, 20], the modified auxiliary expansion method [21], the homotopy analysis transform method [22–26], the homotopy perturbation sumudu transform method [27], the shooting method with the explicit Runge-Kutta scheme [28, 29], and the Adomian decomposition method [30]. Recently, several fractional operators have been applied to the mathematical models in order to seek their exact solutions, such as the Laplace transform [31, 32], the Nabla operator [33–35].

The outline of paper are organize the paper as follows: A short review of the conformable derivative is presented in section 2. Section 3 deals with the analysis of the ShGEM. In section 4, the method is applied to solve the non-linear Schrödinger equation involving the fractional derivatives with the parabolic law. Eventually, in section 5, we presented our conclusion of this paper.

2. BASIC DEFINITIONS

The basic definitions of the conformable derivative of order α are given as follows [36–41]:

Definition 2.1. Assume the function $h : (0, \infty) \rightarrow \mathbb{R}$ then, the conformable derivative of h of order α is defined as $D_t^\alpha h(t) = \lim_{\varepsilon \rightarrow 0} \frac{h(t + \varepsilon t^{1-\alpha}) - h(t)}{\varepsilon}$, $\forall t > 0$, and $0 < \alpha \leq 1$.

Definition 2.2. Assume that $c \geq 0$ and $t \geq c$, let h be a function defined on $(c, t]$ as well as $\alpha \in \mathbb{R}$. Then, the α -fractional integral of h is given by

$${}_t I_c^\alpha h(t) = \int_c^\alpha \frac{h(x)}{x^{1-\alpha}} dx, \quad (4)$$

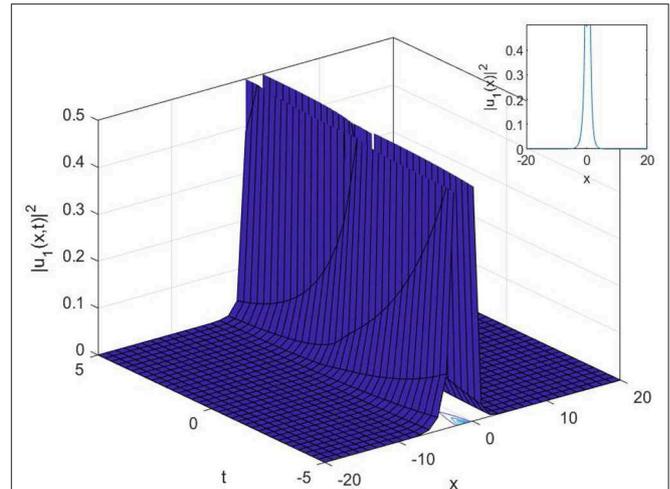


FIGURE 1 | 3D, 2D, and contour surfaces of Equation (26) where $\omega = 0.1, c_2 = 0.1, \kappa = 2, \alpha = 0.8$.

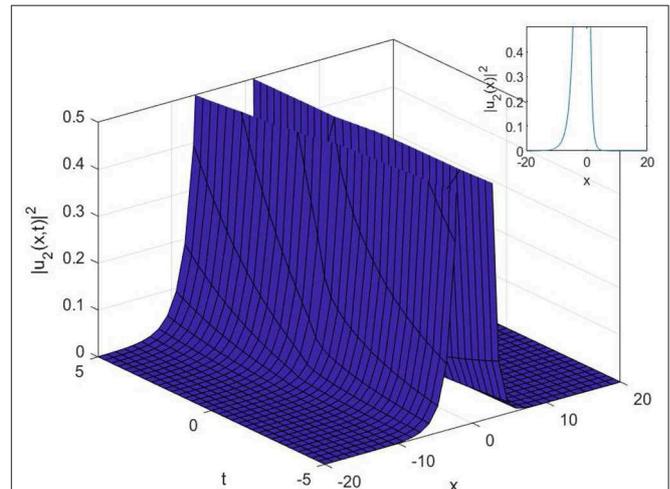


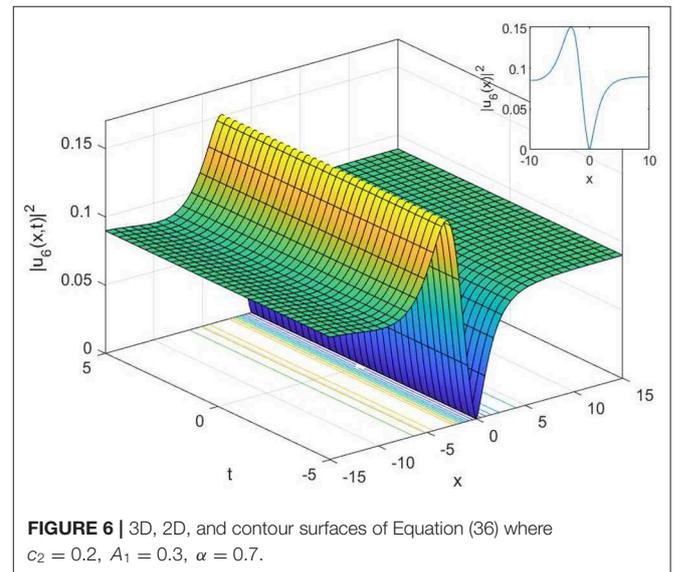
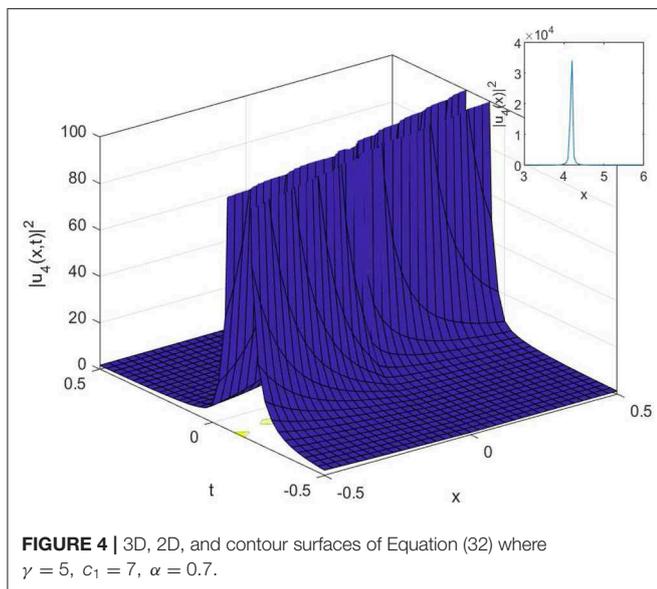
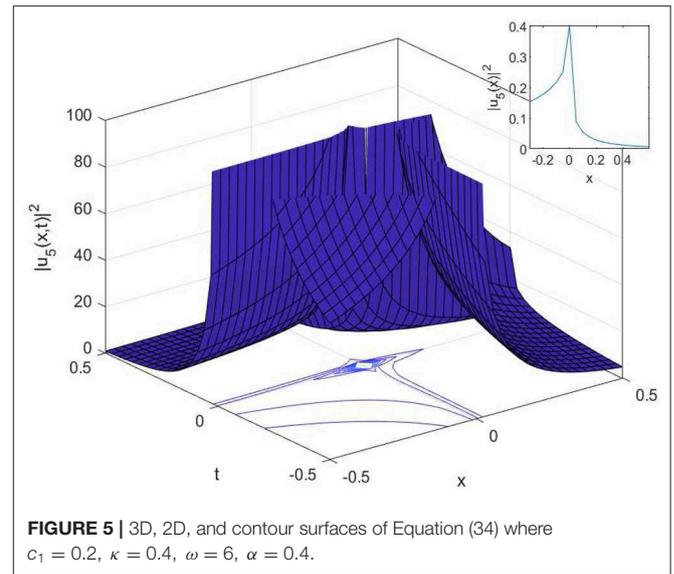
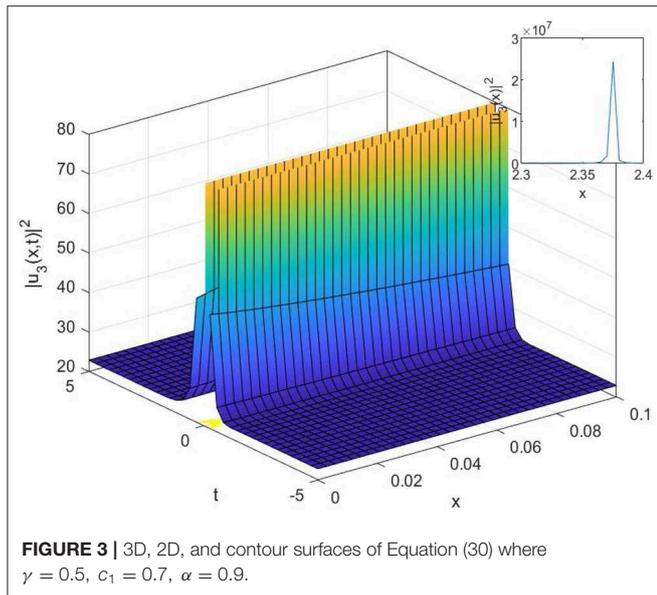
FIGURE 2 | 3D, 2D, and contour surfaces of Equation (28) where $\gamma = 0.5, c_2 = 0.2, \kappa = 0.4, \alpha = 0.7$.

if the Riemann improper integral exists.

Theorem 2.1. Let $\alpha \in (0, 1]$, and $h = h(t), g = g(t)$ be α -conformable differentiable at a point $t > 0$, then:

$$\begin{aligned} D_t^\alpha (ah + bg) &= aD_t^\alpha h + bD_t^\alpha g, \text{ for all } (a, b \notin \mathbb{R}). \\ D_t^\alpha (t^\lambda) &= \lambda t^{\lambda-\alpha}, \text{ for all } (\lambda \in \mathbb{R}). \\ D_t^\alpha (hg) &= gD_t^\alpha (h) + hD_t^\alpha (g). \\ D_t^\alpha \left(\frac{h}{g}\right) &= \frac{gD_t^\alpha (h) - hD_t^\alpha (g)}{g^2}. \end{aligned} \quad (5)$$

Furthermore, if function h is differentiable, then $D_t^\alpha (h(t)) = t^{1-\alpha} \frac{dh}{dt}$.



Theorem 2.2. (see for details pervious research [40]): Let h be a differentiable function and α is order of the conformable derivative. Let g be a differentiable function defined in the range of h , then

$$D_t^\alpha (f \circ g) (t) = t^{1-\alpha} g(t)^{\alpha-1} g'(t) D_t^\alpha (f(t))_{t=g(t)}, \quad (6)$$

here “prime” is the classical derivatives with respect to t .

3. THE EXTENDED ShGEM

In the current section, we presented the main steps of the e ShGEM (see previous study [42, 43]).

Consider the following fractional non-linear PDE:

$$W (D_x^\sigma p, p^2 D_x^{2\sigma} p, D_t^\nu p, D_t^\nu D_x^\sigma p, \dots) = 0, \quad (7)$$

where $p = p(x, t)$.

Consider the wave transformation

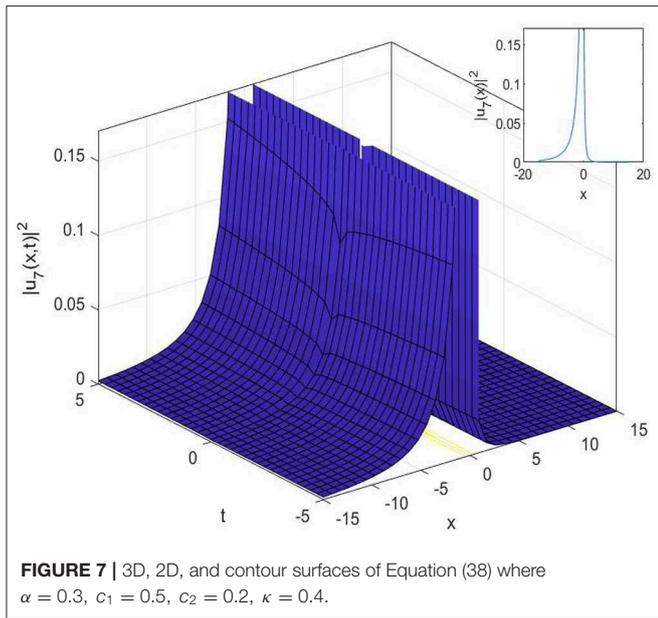
$$p(x, t) = \psi(\zeta), \zeta = \frac{x^\sigma}{\sigma} - c \frac{t^\nu}{\nu}, \quad (8)$$

by substitute relation Equation (8) into Equation (7), we obtain the following non-linear ODE:

$$P(\psi, \psi', \psi'', \psi^2 \psi', \dots) = 0. \quad (9)$$

Consider the trial solution of Equation (9) of the form

$$\psi(\theta) = \sum_{j=1}^k [A_j \sinh(\theta) + B_j \cosh(\theta)]^j + A_0. \quad (10)$$



The parameters A_j, B_j , for $(j = 1, 2, \dots, k)$ and A_0 are real constants, and θ is a function of η that hold the following ODE:

$$\theta' = \sinh(\theta). \tag{11}$$

The homogeneous balance principle is applied on Equation (9) to find the value of k . From the space-time fractional the sinh-Gordon equation, we have (see previous study [15, 16]).

$$D_t^\nu D_x^\sigma p = \lambda \sinh(p). \tag{12}$$

The exact solutions of Equation (12) may be given as

$$\sinh(\theta) = \pm \operatorname{csch}(\zeta) \text{ or } \sinh(\theta) = \pm \operatorname{sech}(\zeta), \tag{13}$$

and

$$\cosh(\theta) = \pm \operatorname{coth}(\zeta) \text{ or } \cosh(\theta) = \pm \tanh(\zeta). \tag{14}$$

Letting solutions of Equation (10) along with Equations (13) and (14) as the form

$$\psi(\zeta) = \sum_{j=1}^k [\pm i A_j \operatorname{sech}(\zeta) \pm B_j \tanh(\zeta)]^j + A_0, \tag{15}$$

$$\psi(\zeta) = \sum_{j=1}^k [\pm A_j \operatorname{csch}(\zeta) \pm B_j \operatorname{coth}(\zeta)]^j + A_0. \tag{16}$$

Finding the value of k and then inserting Equations (10) and (12) into Equation (9), we get a system of terms of:

$$\sinh^i(\theta) \cosh^j(\theta), \tag{17}$$

we gather a group of over-defined non-linear algebraic equations in A_0, A_j, B_j , putting the coefficients of $\sinh^i(\theta) \cosh^j(\theta)$ to

zero, and finding the solutions of acquired system, we gain the values of $A_0, A_j, B_j, c_1, c_2, \kappa$, and ω . Putting the values of $A_0, A_j, B_j, c_1, c_2, \kappa$, and ω into Equations (15) and (16), we can find the solutions of Equation (7).

4. IMPLEMENT OF THE EXTENDED ShGEM

The implementation of the extended ShGEM to the cubic-quartic non-linear Schrödinger equation with conformable derivative is provided in this section.

Consider the wave transformation

$$u(x, t) = U(\xi) e^{i\theta}, \quad \xi = \frac{x^\alpha}{\alpha} - v \frac{t^\alpha}{\alpha}, \quad \theta = -\frac{\kappa x^\alpha}{\alpha} + \frac{\omega t^\alpha}{\alpha}. \tag{18}$$

In Equation (18), $\theta(x, t)$ represents the phase component of the soliton. The ω, κ, v are the wave number, the soliton frequency, and the soliton velocity, respectively. Substituting wave transformation into Equation (2) and splitting the outcomes equation into real and imaginary parts, we gain

$$-(\beta \kappa^3 - \gamma \kappa^4 + \omega) U + c_1 U^3 + c_2 U^5 + 3\beta \kappa U'' - 6\gamma \kappa^2 U'' + \gamma U^{(4)} = 0, \tag{19}$$

$$-(3\beta \kappa^2 - 4\gamma \kappa^3 + \nu) U' + \beta U^{(3)} - 4\gamma \kappa U^{(3)} = 0. \tag{20}$$

Multiply both sides of Equation (19) by U' and integrate it, we obtain

$$\gamma \left(-\frac{(U'')^2}{2} + U''' U' \right) + \frac{c_1 U^4}{4} + \frac{c_2 U^6}{6} + 3\gamma \kappa^2 (U')^2 + \frac{1}{2} U^2 (-4\gamma \kappa^4 + \gamma \kappa^4 - \omega) = 0. \tag{21}$$

From Equation (20), we get constraint conditions $\nu = 4\gamma \kappa^3 - 3\beta \kappa^2$ and $\beta = 4\gamma \kappa$. Balancing the terms $U''' U'$ and U^6 yields $\kappa = 1$. With $\kappa = 1$, Equations (10), (16), and (17) change to

$$\psi(\theta) = [A_1 \sinh(\theta) + B_1 \cosh(\theta)] + A_0, \tag{22}$$

$$\psi(\zeta) = [\pm i A_1 \operatorname{sech}(\zeta) \pm B_1 \tanh(\zeta)] + A_0, \tag{23}$$

and

$$\psi(\zeta) = [\pm m_1 \operatorname{csch}(\zeta) \pm n_1 \operatorname{coth}(\zeta)] + n_0, \tag{24}$$

respectively.

Inserting Equation (22) along with Equation (12) into Equation (21), and using constraint conditions provides a non-linear algebraic system. Equating each coefficient of $\sinh^i(\theta) \cosh^j(\theta)$ with the same power to zero, and finding the obtained system of algebraic equations, we gain the values of the parameters. Putting the obtained values of the parameters into Equations (23) and (24), give the solutions of Equation (3).

Set 1

$$B_1 = \frac{2^{3/4} 3^{1/4} \omega^{1/4}}{(c_2(-1 - 6\kappa^2 + 3\kappa^4))^{1/4}}, c_1 = \frac{\sqrt{\frac{2}{3}} c_2 (5 + 3\kappa^2) \sqrt{\omega}}{\sqrt{c_2(-1 - 6\kappa^2 + 3\kappa^4)}},$$

$$A_0 = 0, \gamma = \frac{\omega}{1 + 6\kappa^2 - 3\kappa^4}, A_1 = 0, \tag{25}$$

we get

$$u_1(x, t) = \frac{2^{3/4} 3^{1/4} \omega^{1/4}}{(c_2(-1 - 6\kappa^2 + 3\kappa^4))^{1/4}}$$

$$\operatorname{csch}\left(\frac{x^\alpha}{\alpha} + \frac{8t^\alpha \kappa^3 \omega}{\alpha(1 + 6\kappa^2 - 3\kappa^4)}\right) e^{i\left(-\frac{x^\alpha \kappa}{\alpha} + \frac{t^\alpha \omega}{\alpha}\right)}. \tag{26}$$

Set 2

$$B_1 = \frac{(1 + i) 6^{1/4} \gamma^{1/4}}{c_2^{1/4}}, \omega = \gamma(1 + 6\kappa^2 - 3\kappa^4),$$

$$c_1 = i\sqrt{\frac{2}{3}} \sqrt{\gamma} \sqrt{c_2} (5 + 3\kappa^2), A_1 = 0, A_0 = 0, \tag{27}$$

we get

$$u_2(x, t) = \frac{(1 + i) 6^{1/4} a_2^{1/4}}{c_2^{1/4}}$$

$$\operatorname{csch}\left(\frac{x^\alpha}{\alpha} + \frac{8a_2 t^\alpha \kappa^3}{\alpha}\right) e^{i\left(-\frac{x^\alpha \kappa}{\alpha} + \frac{a_2 t^\alpha (1 + 6\kappa^2 - 3\kappa^4)}{\alpha}\right)}. \tag{28}$$

Set 3

$$A_0 = 0, A_1 = \frac{4\sqrt{2}\sqrt{\gamma}}{\sqrt{c_1}}, B_1 = 0, c_2 = -\frac{3c_1^2}{128\gamma},$$

$$\kappa = -\sqrt{\frac{2}{3}}, \omega = \frac{20\gamma}{3}, \tag{29}$$

we get

$$u_3(x, t) = -\frac{4\sqrt{2}\sqrt{\gamma}}{\sqrt{c_1}} \operatorname{coth}\left(\frac{16\sqrt{\frac{2}{3}}\gamma t^\alpha}{3\alpha} - \frac{x^\alpha}{\alpha}\right) e^{i\left(\frac{20\gamma t^\alpha}{3\alpha} + \frac{\sqrt{\frac{2}{3}} x^\alpha}{\alpha}\right)}. \tag{30}$$

Set 4

$$A_0 = 0, A_1 = -\frac{\sqrt{2}\sqrt{\gamma}}{\sqrt{c_1}}, B_1 = \frac{\sqrt{2}\sqrt{\gamma}}{\sqrt{c_1}}, c_2 = -\frac{3c_1^2}{8\gamma},$$

$$\kappa = -\frac{1}{\sqrt{6}}, \omega = \frac{5\gamma}{12}, \tag{31}$$

we get

$$u_4(x, t) = -\frac{\sqrt{2}\sqrt{\gamma}}{\sqrt{c_1}} \left(\operatorname{coth}\left(\frac{2\sqrt{\frac{2}{3}}\gamma t^\alpha}{3\alpha} - \frac{x^\alpha}{\alpha}\right) \right.$$

$$\left. + \operatorname{csch}\left(\frac{2\sqrt{\frac{2}{3}}\gamma t^\alpha}{3\alpha} - \frac{x^\alpha}{\alpha}\right) \right) \times e^{i\left(\frac{5a_2 t^\alpha}{12\alpha} + \frac{x^\alpha}{\sqrt{6}\alpha}\right)}. \tag{32}$$

Set 5

$$B_1 = -\frac{2\sqrt{(5 + 3\kappa^2)} \omega}{\sqrt{c_1(-1 - 6\kappa^2 + 3\kappa^4)}}, A_1 = 0, A_0 = 0,$$

$$c_2 = \frac{3c_1^2(-1 - 6\kappa^2 + 3\kappa^4)}{2(5 + 3\kappa^2)^2 \omega}, \gamma = \frac{\omega}{1 + 6\kappa^2 - 3\kappa^4}, \tag{33}$$

we obtain

$$u_5(x, t) = \frac{2\sqrt{(5 + 3\kappa^2)} \omega}{\sqrt{c_1(-1 - 6\kappa^2 + 3\kappa^4)}}$$

$$\operatorname{csch}\left(\frac{x^\alpha}{\alpha} + \frac{8t^\alpha \kappa^3 \omega}{\alpha(1 + 6\kappa^2 - 3\kappa^4)}\right) e^{i\left(-\frac{x^\alpha \kappa}{\alpha} + \frac{t^\alpha \omega}{\alpha}\right)}. \tag{34}$$

Set 6

$$B_1 = A_1, A_0 = 0, c_1 = -\frac{4A_1^2 c_2}{3}, \gamma = -\frac{2A_1^4 c_2}{3},$$

$$\omega = -\frac{5A_1^4 c_2}{18}, \kappa = \frac{1}{\sqrt{6}}, \tag{35}$$

we get

$$u_6(x, t) = \left(-A_1 \operatorname{coth}\left(\frac{4\sqrt{\frac{2}{3}} A_1^4 c_2 t^\alpha}{9\alpha} - \frac{x^\alpha}{\alpha}\right) \right.$$

$$\left. + A_1 \operatorname{csch}\left(\frac{4\sqrt{\frac{2}{3}} A_1^4 c_2 t^\alpha}{9\alpha} - \frac{x^\alpha}{\alpha}\right) \right)$$

$$\times e^{i\left(-\frac{5A_1^4 c_2 t^\alpha}{18\alpha} - \frac{x^\alpha}{\sqrt{6}\alpha}\right)}. \tag{36}$$

Set 7

$$B_1 = \frac{\sqrt{6}\sqrt{c_1}}{\sqrt{c_2(5 + 3\kappa^2)}}, \gamma = -\frac{3c_1^2}{2c_2(5 + 3\kappa^2)^2},$$

$$\omega = \frac{3c_1^2(-1 - 6\kappa^2 + 3\kappa^4)}{2c_2(5 + 3\kappa^2)^2}, A_1 = 0, A_0 = 0, \tag{37}$$

we get

$$u_7(x, t) = \frac{\sqrt{6}\sqrt{c_1}}{\sqrt{c_2(5 + 3\kappa^2)}} \operatorname{csch}\left(\frac{x^\alpha}{\alpha} - \frac{12c_1^2 t^\alpha \kappa^3}{c_2 \alpha (5 + 3\kappa^2)^2}\right)$$

$$e^{i\left(-\frac{x^\alpha \kappa}{\alpha} + \frac{3c_1^2 t^\alpha (-1 - 6\kappa^2 + 3\kappa^4)}{2c_2 \alpha (5 + 3\kappa^2)^2}\right)}. \tag{38}$$

5. CONCLUSION

In this article, we have successfully used the extended sinh-Gordon expansion method to solve the problem for the non-linear cubic-quartic Schrödinger equations involving fractional derivatives with the Parabolic law. A traveling wave

transforms in the sense of the comfortable derivative has been used to convert the governing equation into a NODE. The various optical solutions of the studied model have been constructed, for example, the singular soliton solutions as shown in **Figures 1–6**, and the dark-bright soliton solution as seen in **Figure 7**. Comparing our solutions to the results obtained in references [16–18], our findings solutions are new and different. To better analyze the dynamic attitude, and the characteristics of these solutions, the 2D, 3D and counter-surface of all obtained solutions are plotted. The study shows that this method is the effective and appropriate technique for finding the exact solution of the model considered in the paper.

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DATA AVAILABILITY STATEMENT

All datasets generated for this study are included in the article/supplementary material.

AUTHOR CONTRIBUTIONS

HD contributed in developing the proofs and edited the article for possible improvement. HG, KA, and RY contributed in developing the main results and proofs. All authors read the final version and approved it.

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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