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Optical Solutions of Schrödinger Equation Using Extended Sinh–Gordon Equation Expansion Method

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In this paper, we investigated the non-linear Schrödinger equation (NLS) to extract optical soliton solutions by implementing the extended Sinh–Gordon equation expansion method (ShGEEM). Optical soliton solutions included bright, dark, combined bright-dark, singular soliton combined singular soliton solutions, and singular periodic wave solutions. Our new results have been compared to these in the literature. Also, graphical analysis was presented with 3D and contour graphs to understand the physics of obtained solutions.

Keywords: extended Sinh-Gordon equation expansion method (ShGEEM), optical soliton, non-linear Schrödinger equation, exact solutions, singular soliton solution

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INTRODUCTION

In recent years, soliton propagation in non-linear optical fiber has become the most extensive topic of research in the field of non-linear sciences. In non-linear optical fiber, the study of the non-linear Schrödinger equation (NLS) plays an important role in order to understand the dynamical behavior of optical soliton. NLS helps to provide exact soliton solutions in non-linear fiber optics. During the last few years, in the study of optical solitons, many new research developments have taken place, which is a great achievement in the field of soliton [1–15]. However, there are a lot of problems that need to be solved.

Many new methods have been developed to tackle complicated problems in a very smooth manner and provide exact soliton solutions of these problems such as the modified simple equation method [16, 17], the extended trial equation method [18, 19], the $\tan(\frac{\phi(\xi)}{2})$-expansion method [20, 21], and many others.

In this paper, our main focus is the study of NLS [22]. This equation has large physical importance in non-linear optics.

$$iV_t - V_{xx} + 2|V|^2V - 2\sigma^2V = 0, \quad i = \sqrt{-1}, \tag{1}$$

where $V(x,t)$ is a complex function and σ is a constant. It should also be noted that, for $\sigma = 0$, Equation (1) reduces to the non-Kerr law non-linearity as

$$V_t - V_{xx} + 2|V|^2V = 0, \quad i = \sqrt{-1} \tag{2}$$

To study Equation (1), we consider the following wave transformation:

$$V(x,t) = p(\xi)\, e^{i\phi(x,t)}, \xi = \rho x - \upsilon t,\ \varphi = -kx + \varpi t + \theta \quad (3)$$

where $\varphi(x,t)$ is the phase component, and k, ϖ, θ, *and* υ represent the frequency, wave number, phase constant, and velocity of the soliton. By substituting Equation (3) into Equation (1), we obtain the following real and imaginary equations:

$$\left(\frac{d^2}{d\xi^2}\varphi(\xi)\right)\rho^2 + \phi(\xi)\left(k^2 - 2\sigma^2 - \varpi\right) - 2(\phi(\xi))^3 = 0, \quad (4)$$

$$\upsilon = 2k\rho, \quad (5)$$

ALGORITHM OF EXTENDED ShGEEM

To describe the mechanism of the extended Sinh–Gordon equation method (SGEM) for differential equations, we consider the equation [23]

$$\Upsilon_{xt} = \varrho \sinh(\Upsilon), \quad (6)$$

where $\Upsilon = \Upsilon(x,t)$ and ϱ is a nonzero constant.

Applying the traveling wave transformation $\Upsilon(x,t) = \Phi(\zeta)$, $\zeta = \lambda(x - \mu t)$, to Equation (6), we acquire the following form of non-linear ODE:

$$\Phi'' = -\frac{\varrho}{\lambda^2\mu}\sinh(\Phi), \quad (7)$$

where $\Phi = \Phi(\zeta)$, λ is a wave number, and μ is the velocity of the traveling wave. By applying the integration procedure, Equation (7) can be found in a simplified form:

$$\left[\frac{\left(\Phi'\right)}{2}\right]^2 = -\frac{\varrho}{\lambda^2\mu}\sinh^2\left(\frac{\Phi}{2}\right) + r, \quad (8)$$

where r is the constant of integration. Setting $v(\zeta) = \frac{\Phi}{2}$, and $\theta = -\frac{\varrho}{\lambda^2\mu}$, into Equation (8) yields

$$v'(\zeta) = \sqrt{\theta \sinh^2(v) + r}, \quad (9)$$

Equation (9) has the following set of solutions, by substituting different values for given parameters θ and r.

Set I:

If we substitute $r = 0$, $\theta = 1$ in Equation (9), we obtain

$$v'(\zeta) = \sin h(v), \quad (10)$$

Simplifying Equation (10), we acquire the following solutions:

$$\sin h(v(\zeta)) = \pm csch(\zeta), \quad or \quad \sin h(v(\zeta)) = \pm isech(\zeta), \quad (11)$$

and

$$\cos h(v(\zeta)) = \pm coth(\zeta), \ or\ \cos h(v(\zeta)) = \pm tanh(\zeta)!, \quad (12)$$

where $i = \sqrt{-1}$.

Set II:

If we substitute $r = 1$, $\theta = 1$ in Equation (9), we have the following equation:

$$v'(\zeta) = \cos h(v), \quad (13)$$

After simplification in Equation (13), we have the following solutions:

$$\sin h(v(\zeta)) = \tan(\zeta), \quad or \quad \sin h(v(\zeta)) = -cot(\zeta), \quad (14)$$

and

$$\cos h(v(\zeta)) = \pm sec(\zeta), \quad or \quad \cos h(v(\zeta)) = \pm tan(\zeta), \quad (15)$$

To obtain the different wave solutions of non-linear partial differential equations (NPDEs), we consider the equation in the following form:

$$\complement(\Upsilon, \Upsilon_t, \Upsilon_x, \Upsilon_{xx}, \Upsilon_{xt}, \Upsilon_{tt}, \ldots) = 0, \quad (16)$$

Step I: By using wave transformation $\Upsilon(x,t) = \Phi(\zeta)$, $\zeta = \lambda(x - \mu t)$, we first transform Equation (16) into the following NODE:

$$H\left(\Phi, \Phi', \Phi'', \Phi^2\Phi', \ldots\right) = 0, \quad (17)$$

Step II: We suppose that Equation (17) has a new ansatz solution in the following form:

$$\Phi(v) = \sum_{\kappa=1}\left[\mathcal{B}_\kappa sinh(v(\zeta)) + A_\kappa cosh(v(\zeta))\right]^\kappa + \mathring{A}_0, \quad (18)$$

where $\mathring{A}_0$, $\mathring{A}_\kappa$, $\mathcal{B}_\kappa$, $(\kappa = 1, \ldots, n)$ are constants to be determined later. The value of can be determined by balancing the highest order dispersive term with the non-linear term in Equation (17).

Step III: We substitute Equation (18) for the fixed value of in Equation (17) to obtain a polynomial form of equation in $v'^f sinh^g(v) cosh^\iota(v), \left(f = 0, 1\ and\ g, \iota = 0, 1, 2 \ldots\ldots\right)$. We get the system of algebraic equations by equating the coefficients of $v'^f sinh^g(v) cosh^\iota(v)$ to be all zero. We extract the values of coefficients $\mathring{A}_0$, $\mathring{A}_\kappa$, $\mathcal{B}_\kappa, \lambda$, μ by solving the system of algebraic equations with the help of MAPLE 2016.

Step IV: Substituting the values of $\mathring{A}_0$, $\mathring{A}_\kappa$, $\mathcal{B}_\kappa$, μ in Equations (19)–(22), we obtain the following wave solutions to the non-linear Equation (16):

$$\Phi(\zeta) = \sum_{\kappa=1}^{N}\left[\pm\mathcal{B}_\kappa isech(\zeta) \pm \mathring{A}_\kappa \tanh(\zeta)\right]^\kappa + \mathring{A}_0, \quad (19)$$

$$\Phi(\zeta) = \sum_{\kappa=1}^{N}\left[\pm\mathcal{B}_\kappa csch(\zeta) \pm \mathcal{B}_\kappa \coth(\zeta)\right]^\kappa + \mathring{A}_0, \quad (20)$$

$$\Phi(\zeta) = \sum_{\kappa=1}^{N}\left[\mathcal{B}_\kappa \sec(\zeta) + \mathring{A}_\kappa \tan(\zeta)\right]^\kappa + \mathring{A}_0, \quad (21)$$

and

$$\Phi(\zeta) = \sum_{\kappa=1}^{N} \left[\mathcal{B}_\kappa \csc(\zeta) - \mathring{A}_\kappa \cot(\zeta)\right]^\kappa + \mathring{A}_0, \quad (22)$$

APPLICATION OF EXTENDED ShGEEM TO EQUATION (1)

In this section, Extended ShGEEM [24–29] is implemented to Equation (1).

Considering a homogeneous balance between Φ'' and Φ^3 in Equation (4) yields $N = 1$. And setting the value of N in Equations (18)–(22), we obtain

$$\Phi(v) = \mathcal{B}_1 sinh(v(\zeta)) + \mathring{A}_1 cosh(v(\zeta)) + \mathring{A}_0, \quad (23)$$
$$\Phi(\zeta) = \pm\mathcal{B}_1 isech(\zeta) \pm \mathring{A}_1 tanh(\zeta) + \mathring{A}_0, \quad (24)$$
$$\Phi(\zeta) = \pm\mathcal{B}_1 csch(\zeta) \pm \mathring{A}_1 coth(\zeta) + \mathring{A}_0, \quad (25)$$
$$\Phi(\zeta) = \pm\mathcal{B}_1 sec(\zeta) + \mathring{A}_1 tan(\zeta) + \mathring{A}_0, \quad (26)$$
$$\Phi(\zeta) = \pm\mathcal{B}_1 csc(\zeta) - \mathring{A}_1 cot(\zeta) + \mathring{A}_0, \quad (27)$$

Substituting Equation (23) together with its derivatives in Equation (4), we get a polynomial equation in $v'^f sinh^g(v) cosh^\iota(v), \left(f = 0, 1 \text{ and } g, \iota = 0, 1, 2 \ldots\ldots\right)$. Using some hyperbolic identities, we acquire a system of algebraic equations by setting the coefficients of $v'^f sinh^g(v) cosh^\iota(v)$ equal to zero. After simplifying the system of equations, we obtain the values of $\mathring{A}_0$, $\mathring{A}_\kappa$, $\mathcal{B}_\kappa, \rho, k, \lambda$ with the help of Maple 16. Subsisting all the values of $\mathring{A}_0$, $\mathring{A}_\kappa$, $\mathcal{B}_\kappa, \rho, k, \lambda$ in any of Equations (24)–(27), we found numerous different types of soliton solutions of Equation (1).

Result I:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = \pm\frac{1}{2}\rho, \mathcal{B}_1 = \pm\frac{1}{2}\rho, \varpi = k^2 + \frac{1}{2}\rho^2 - 2\sigma^2, \quad (28)$$

Result II:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = 0, \ \mathcal{B}_1 = \pm\rho, \ \varpi = k^2 - \rho^2 - 2\sigma^2, \quad (29)$$

Result III:

$$\sigma^2 \mathring{A}_0 = 0, \ \mathring{A}_1 = \pm\rho, \ \mathcal{B}_1 = 0, \ \varpi = k^2 + 2\rho^2 - 2\sigma^2, \quad (30)$$

Result IV:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = 0 \ , \ \mathcal{B}_1 = \pm\rho, \quad \rho = \sqrt{k^2 - 2\sigma^2 - \varpi}, \quad (31)$$

Result V:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = \rho \ , \mathcal{B}_1 = 0, \ \rho = \frac{1}{2}\sqrt{-2k^2 + 4\sigma^2 + 2\varpi}, \quad (32)$$

Result VI:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = \frac{1}{2}\rho, \mathcal{B}_1 = \frac{1}{2}\rho, \rho = \sqrt{-2k^2 + 4\sigma^2 + 2\varpi}, \quad (33)$$

Result VII:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = \mp\frac{1}{2}\rho, \mathcal{B}_1 = \pm\frac{1}{2}\rho, \varpi = k^2 - \frac{1}{2}\rho^2 - 2\sigma^2, \quad (34)$$

Result VIII:

$$\mathring{A}_0 = 0, \ \mathring{A}_1 = \frac{1}{2}\rho, \ \mathcal{B}_1 = \frac{1}{2}\rho, \ \rho = \sqrt{2k^2 - 4\sigma^2 - 2\varpi}, \quad (35)$$

Substituting the values of the above given results in Equations (24)–(27), we get the following solutions.

Case I: Bright Optical Solitons

Substituting the values of the parameters given in Results II and IV into Equation (24):

$$V_1(x,t) = \pm i\rho sech\left(\rho x - 2tk\rho\right) e^{i\left(-kx + t\left(k^2 - \rho^2 - 2\sigma^2\right) + \theta\right)}, \quad (36)$$
$$V_2(x,t) = \pm i\sqrt{k^2 - 2\sigma^2 - \varpi}\ \text{sech}\left(-2tk\sqrt{k^2 - 2\sigma^2 - \varpi} + \sqrt{k^2 - 2\sigma^2 - \varpi}\,x\right) \times e^{i(-kx + t\varpi + \theta)} \quad (37)$$

where $\left(k^2 - 2\sigma^2 - \varpi\right) > 0$, for valid solutions.

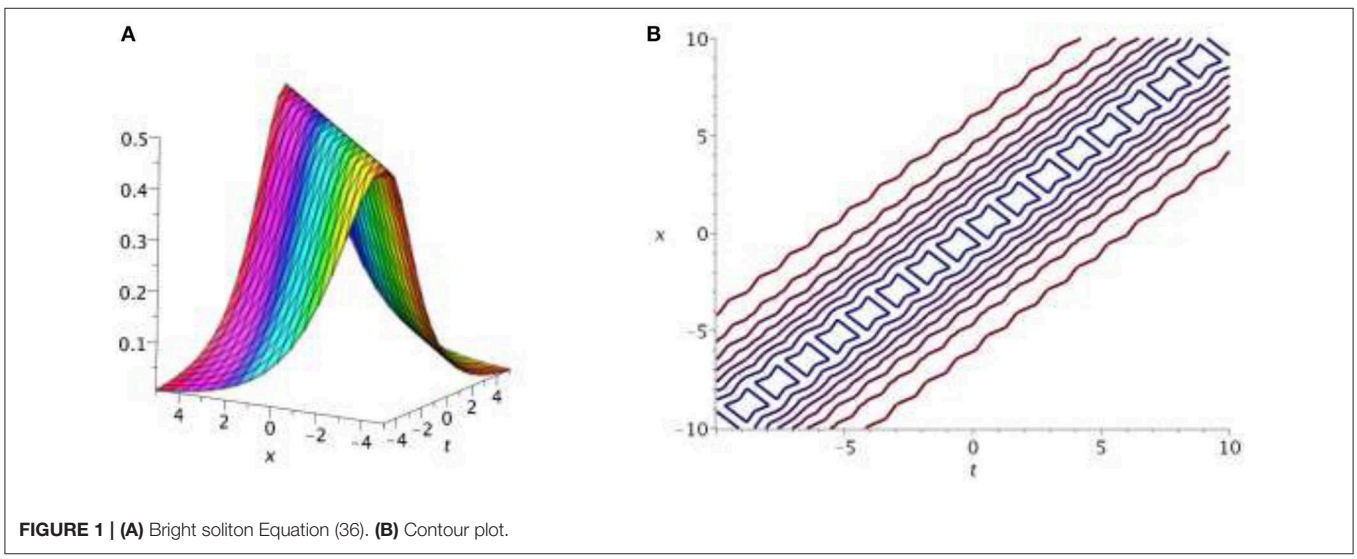

FIGURE 1 | (A) Bright soliton Equation (36). **(B)** Contour plot.

Case II: Dark Optical Solitons

Substituting the values of the parameters given in Results III and V into Equation (24):

$$V_3(x,t) = \pm\rho \tanh\left(-2tk\rho + \rho x\right) e^{i\left(-kx+t\left(k^2+2\rho^2-2\sigma^2\right)+\theta\right)}, \quad (38)$$

$$V_4(x,t) = \left(\tanh\left(-tk\sqrt{-2k^2+4\sigma^2+2\varpi} + \frac{\frac{1}{2}\sqrt{-2k^2+4\sigma^2+2\varpi}}{2\sqrt{-2k^2+4\sigma^2+2\varpi}\,x}\right)\right) e^{i(-kx+t\varpi+\theta)} \quad (39)$$

where $\left(-2k^2+4\sigma^2+2\varpi\right) > 0$, for valid solutions.

Case III: Combined Dark-Bright Optical Soliton Solutions

Using the values of the parameters given in Results I and VI into Equation (24):

$$V_5(x,t) = \pm\frac{1}{2}\rho\left(isech\left(\rho x - 2tk\rho\right) + \tanh\left(\rho x - 2tk\rho\right)\right) e^{i\left(-kx+t\left(k^2+\frac{1}{2}\rho^2-2\sigma^2\right)+\theta\right)}, \quad (40)$$

$$V_6(x,t) = \left(\operatorname{sech}\left(-2tk\sqrt{2\varpi-2k^2+4\sigma^2} + \sqrt{2\varpi-2k^2+4\sigma^2}\,x\right)\right) + \left(\tanh\left(-2tk\sqrt{2\varpi-2k^2+4\sigma^2} + \sqrt{2\varpi-2k^2+4\sigma^2}\,x\right)\right) \times e^{i(-kx+t\varpi+\theta)}. \quad (41)$$

where $\left(2\varpi - 2k^2 + 4\sigma^2\right) > 0$, for valid solutions.

Case IV: Singular Soliton Solutions

Using the values of the parameters given in Results II, III, IV, and V into Equation (25):

$$V_7(x,t) = \pm\rho csch(-2tk\rho + \rho x) e^{i\left(-kx+t\left(k^2-\rho^2-2\sigma^2\right)+\theta\right)}, \quad (42)$$

$$V_8(x,t) = \pm\rho \coth(-2tk\rho + \rho x) e^{i\left(-kx+t\left(k^2+2\rho^2-2\sigma^2\right)+\theta\right)} \quad (43)$$

$$V_9(x,t) = \pm\sqrt{k^2-2\sigma^2-\varpi}\, csch(-2tk\sqrt{k^2-2\sigma^2-\varpi}$$

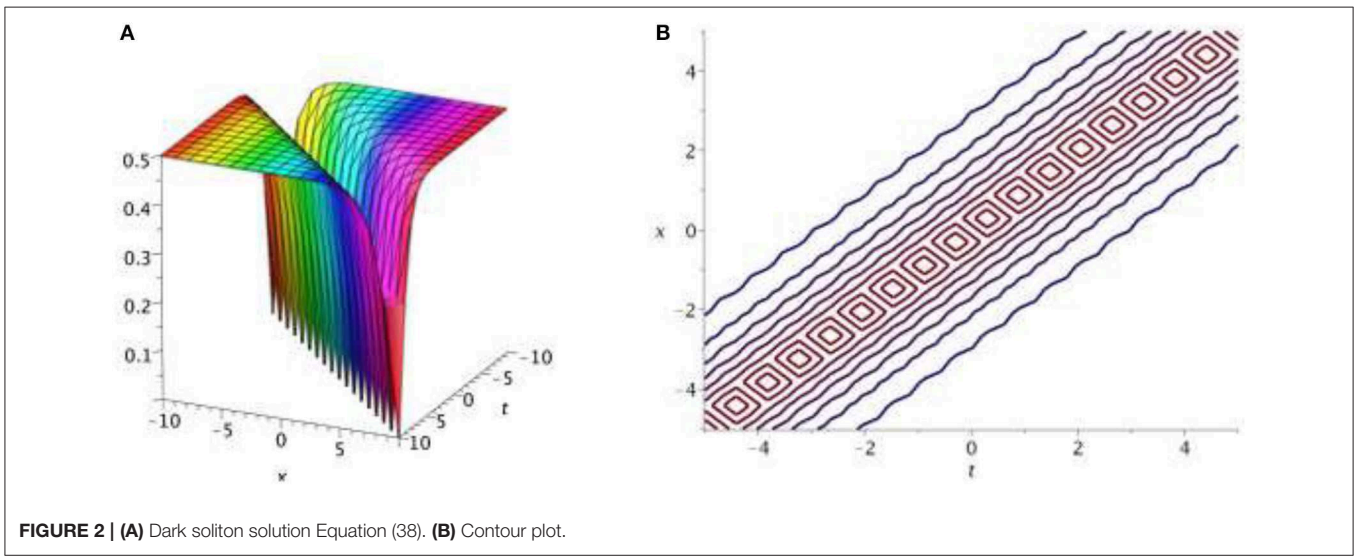

FIGURE 2 | (A) Dark soliton solution Equation (38). **(B)** Contour plot.

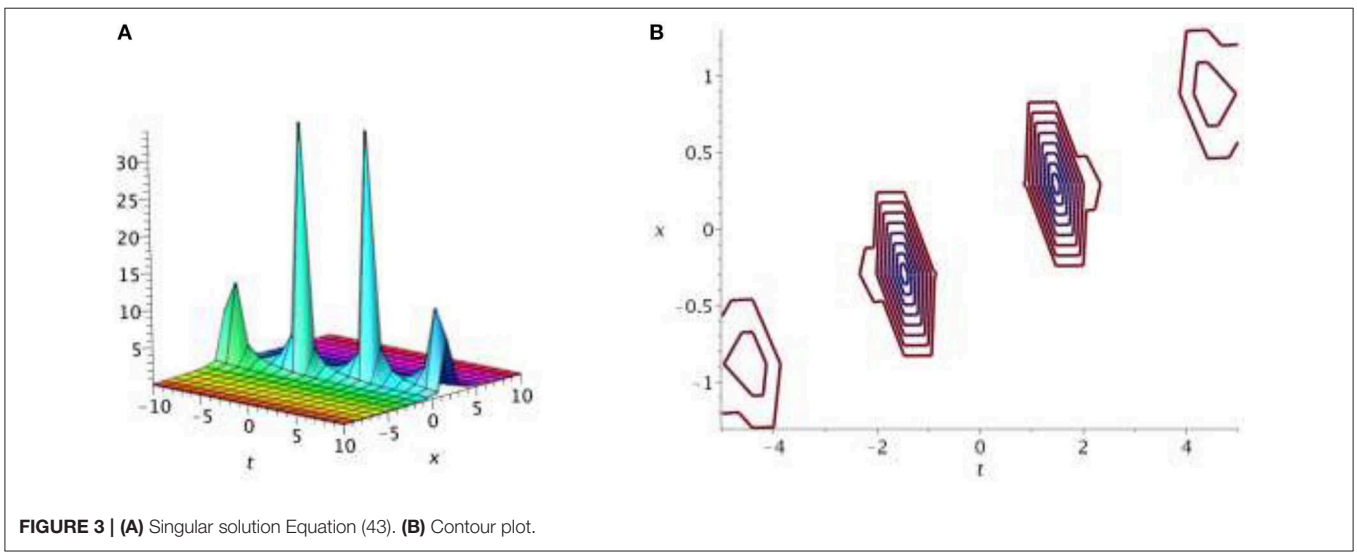

FIGURE 3 | (A) Singular solution Equation (43). **(B)** Contour plot.

$$+\sqrt{k^2-2\sigma^2-\varpi}\,x)\;\times e^{i(-kx+t\varpi+\theta)} \tag{44}$$

where $\left(k^2-2\sigma^2-\varpi\right)>0$, for valid solutions.

$$V_{10}(x,t) = \left(\begin{array}{c}\frac{1}{2}\sqrt{2\varpi-2k^2+4\sigma^2}\\ \coth\left(-tk\sqrt{2\varpi-2k^2+4\sigma^2}+\frac{1}{2}\sqrt{2\varpi-2k^2+4\sigma^2}x\right)\end{array}\right) \times e^{i(-kx+t\varpi+\theta)}, \tag{45}$$

where $\left(2\varpi-2k^2+4\sigma^2\right)>0$, for valid solutions.

Case V: Combined Singular Solitons

Substituting the values of the parameters given in Results I and VI into Equation (25):

$$V_{11}(x,t) = \pm\left(\frac{1}{2}\rho csch\left(-2tk\rho+\rho x\right)-\frac{1}{2}\rho\coth\left(-2tk\rho+\rho x\right)\right) \times e^{i(-kx+t(k^2+\frac{1}{2}\rho^2-2\sigma^2)+\theta)}, \tag{46}$$

$$V_{12}(x,t) = \left(\begin{array}{c}\frac{1}{2}\sqrt{-2k^2+4\sigma^2+2\varpi}\\ \times csch\left(-2tk\sqrt{-2k^2+4\sigma^2+2\varpi}+\sqrt{-2k^2+4\sigma^2+2\varpi}x\right)\\ +\frac{1}{2}\sqrt{-2k^2+4\sigma^2+2\varpi}\\ \times\coth\left(-2tk\sqrt{-2k^2+4\sigma^2+2\varpi}+\sqrt{-2k^2+4\sigma^2+2\varpi}x\right)\end{array}\right) \times e^{i(-kx+t\varpi+\theta)}, \tag{47}$$

where $\left(-2k^2+4\sigma^2+2\varpi\right)>0$, for valid solutions.

Case VI: Singular Periodic Wave Solitons

Substituting the values of the parameters given in Result VII into Equations (26), (27):

$$V_{13}(x,t) = \frac{1}{2}\rho\left(\pm\sec\left(-2tk\rho+\rho x\right)\mp\tan\left(-2tk\rho+\rho x\right)\right) e^{i(-kx+t(k^2-\frac{1}{2}\rho^2-2\sigma^2)+\theta)}, \tag{48}$$

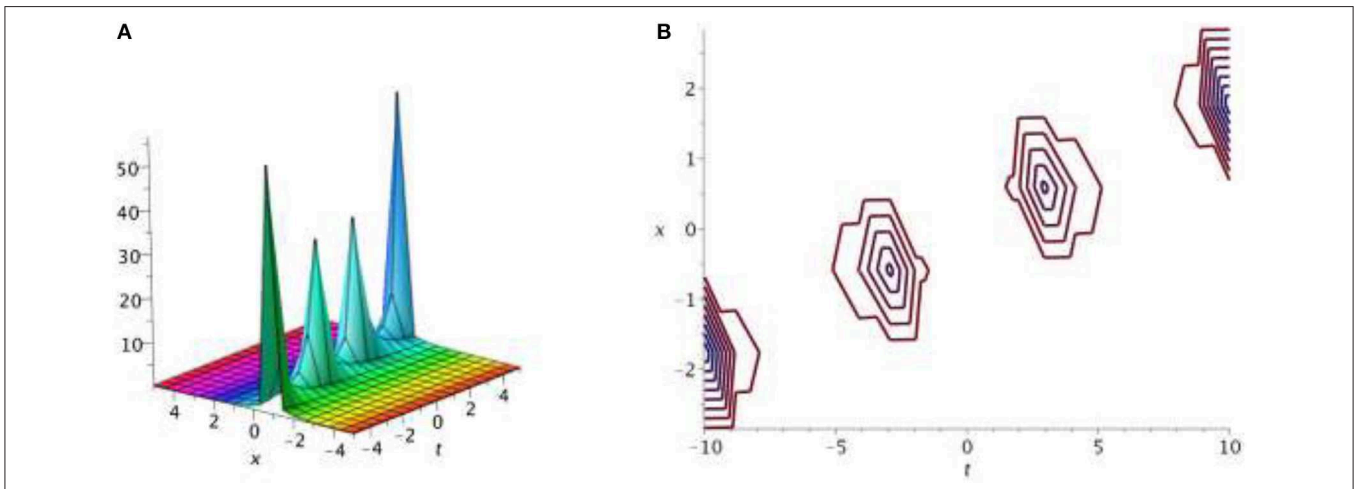

FIGURE 4 | (A) Combined singular solution Equation (47). **(B)** Contour plot.

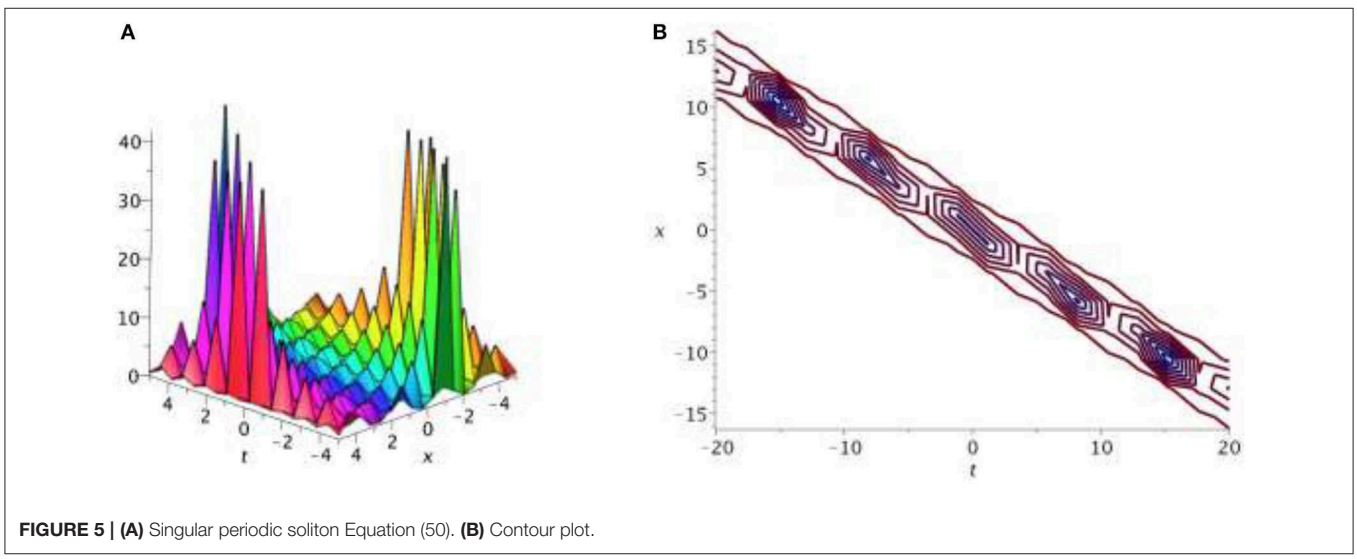

FIGURE 5 | (A) Singular periodic soliton Equation (50). **(B)** Contour plot.

$$V_{14}(x,t) = \left(\begin{pmatrix} \frac{1}{2}\sqrt{2k^2-4\sigma^2-2\varpi} \\ \times \sec\left(-2tk\sqrt{2k^2-4\sigma^2-2\varpi}+\sqrt{2k^2-4\sigma^2-2\varpi}x\right) \\ +\frac{1}{2}\sqrt{2k^2-4\sigma^2-2\varpi} \\ \times \tan\left(-2tk\sqrt{2k^2-4\sigma^2-2\varpi}+\sqrt{2k^2-4\sigma^2-2\varpi}x\right) \end{pmatrix}\right) \times e^{i(-kx+t\varpi+\theta)}, \quad (49)$$

where $\left(2k^2-4\sigma^2-2\varpi\right) > 0$, for valid solutions.

Substituting the values of the parameters given in Result VIII into Equations (26), (27):

$$V_{15}(x,t) = \frac{1}{2}\left(\pm\rho\csc\left(-2tk\rho+\rho x\right)\pm\rho\cot\left(-2tk\rho+\rho x\right)\right) e^{i(-kx+t(k^2-\frac{1}{2}\rho^2-2\sigma^2)+\theta)}, \quad (50)$$

$$V_{16}(x,t) = \left(\begin{pmatrix} \frac{1}{2}\sqrt{2k^2-4\sigma^2-2\varpi} \\ \csc\left(-2tk\sqrt{2k^2-4\sigma^2-2\varpi}+\sqrt{2k^2-4\sigma^2-2\varpi}x\right) \\ -\frac{1}{2}\sqrt{2k^2-4\sigma^2-2\varpi} \\ \cot(-2tk\sqrt{2k^2-4\sigma^2-2\varpi}+\sqrt{2k^2-4\sigma^2-2\varpi}x) \end{pmatrix}\right) \times e^{i(-kx+t\varpi+\theta)}, \quad (51)$$

where $\left(2k^2-4\sigma^2-2\varpi\right) > 0$, for valid solutions.

GRAPHS AND DISCUSSIONS

In this section, we presented some of our obtained solutions in the following figures.

Solutions V_1, V_2 of Equation (1) depict the bright optical soliton solutions. **Figure 1** represents the 3D surface of the bright soliton solution of Equation (36) with a contour plot for given parametric values $\rho = 0.5$, $\theta = 0.5$, $\sigma = 0.5$, $k = 0.5$.

Solutions V_3, V_4 of Equation (1) show the dark optical soliton solutions. **Figure 2** represents the 3D surface of the dark optical soliton solution of Equation (38) with a contour plot for given parametric values $\rho = 0.5$, $\theta = 0.5$, $\sigma = 0.5$, $k = 0.5$.

Figures 3, **4** represent the singular and combined singular soliton solutions of Equation (1), obtained from solutions of V_8 *and* V_{12}[Equations (38), (47)] for $\rho = 0.065$, $\theta = 1$, $\sigma = 0.09$, $k = 0.095$ and $\varpi = 0.05$, $\theta = 5$, $\sigma = 0.05$, $k = 0.09$.

Solutions V_{13}, V_{14}, V_{15}, V_{16} of Equation (1) represent the singular periodic wave solutions. **Figure 5** illustrates the 3D surface of the singular periodic wave solution of Equation (50) with a contour plot for given parametric values $\rho = 2.5$, $\theta = 0.2$, $\sigma = 0.2$, $k = 7.5$. For convenience, some other figures are not reported.

COMPARISONS

In Cheemaa and Younis [22], Nadia Cheema and Muhammad Younis investigated the traveling wave solutions of NLSE by applying the extended Fan sub-equation method. The obtained solutions V_3, V_4, V_8, V_{10}, V_{11}, V_{12}, V_{15}, V_{16} in this paper are equivalent to the solutions q_1, q_2, q_6, q_{15}, q_{16}. found in Cheemaa and Younis [22] for non-linear Schrödinger's equation. The extended Sinh–Gordon equation expansion method provides a large variety of optical soliton solutions [24–29]. By means of the extended Sinh–Gordon equation expansion method, we found some new more generalized exact solutions. Therefore, these new exact solutions are not reported before for this equation in the literature.

CONCLUSIONS

We have implemented the extended Sinh–Gordon equation expansion method to solve the non-linear Schrodinger equation for exact optical soliton solutions. The types of solutions we reported include singular periodic wave solutions, bright, dark, combined bright-dark, singular, and combined singular soliton solutions. The non-linear Schrodinger equation is one of the very major equations arising in the field of optic fibers. Its new solutions are expected to help engineers and scientists working in the field. It is worth mentioning that the solutions obtained by us are more generalized. That is, we have recovered not only many already existing solutions but also many unreported solutions. These new solutions are expected to help scientists working in the fields of optic fiber to understand the phenomenon governed by the non-linear Schrodinger equation. All the solutions have been verified for their exactness. Wherever the reported solutions have been recovered, they have been compared with their counterparts in the literature.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

The formulation of the problem was done by UK and AI. Non-dimensionalization of the nanofluid models by using invertible transformations done by NA. Mathematical analysis and the graphical results plotted and discussed by SM-D and IK. E-SS revised the whole manuscript and checked the typo mistakes.

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