New Soliton Applications in Earth's Magnetotail Plasma at Critical Densities

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New plasma wave solutions of the modified Kadomtsev Petviashvili (MKP) equation are presented. These solutions are written in terms of some elementary functions, including trigonometric, rational, hyperbolic, periodic, and explosive functions. The computational results indicate that these solutions are consistent with the MKP equation, and the numerical solutions indicate that new periodic, shock, and explosive forms may be applicable in layers of the Earth's magnetotail plasma. The method employed in this paper is influential and robust for application to plasma fluids. In order to depict the propagating soliton profiles in a plasma medium, the MKP equation must be solved at critical densities. In order to achieve this, the Riccati-Bernoulli sub-ODE technique has been utilized in solutions. The research findings indicate that a number of MKP solutions may be applicable to electron acoustics appearing in the magnetotail.

1. INTRODUCTION

The existence of electron acoustic solitary excitations (EAs) in plasmas has been noticed in laboratories [1, 2]. Different observations in space have confirmed propagations of EAs in magnetospheres, auroral zones, broadband electrostatic noise (BEN), heliospheric shock, and geomagnetic tails [3–10]. The concept of EAs was generated by Fried and Gould [11]. It is principally an acoustic-type of wave with inertia given by the mass of cold electrons and restoring force expressed by hot electron thermal pressure [12]. Abdelwahed et al. [10] inspected the modulation of characteristics of EAs in non-isothermal electron plasmas [13] using a time-fractional modified non-linear equation. Pakzad studied [14] cylindrical EAs by hot non-extensive electrons, and found through numerical simulations that the spherical amplitude is greater than the cylindrical in EAs. Non-thermal critical geometrical EA plasmas were studied using a Gardner-type equation in Shuchy et al. [15]. Contributions of solitons to science have been discussed in many research works, some of which may be listed as [16–23]. The observed BEN emission bursts in auroras and the Earth's magnetotail regions indicate small and large amplitude electric fields with some explosive and rational domains at critical density. These wave structures appear to be prevalent in some parts of these regions [16, 17]. Therefore, we aim to obtain the solutions that confirm the existence of the electrostatic field in our model.

Let us consider the non-linear partial differential equation

$$H(\varphi, \varphi_x, \varphi_t, \varphi_{xx}, \varphi_{xt}, \varphi_{tt}, ...) = 0, \qquad (1.1)$$

where $\varphi(x, t)$ is an unknown function. Using the wave transformation

$$\varphi(x, t) = \varphi(\xi), \qquad \xi = kx - ct, \qquad (1.2)$$

Equation (1.1) is converted to an ODE:

$$E(\varphi, \varphi', \varphi'', \varphi''', ...) = 0. \qquad (1.3)$$

Many models in physics, fluid mechanics, and engineering are written in the form of (1.1), and this form may be transformed into the ODE:

$$\alpha_1 \varphi'' + \alpha_2 \varphi^3 + \alpha_3 \varphi = 0, \qquad (1.4)$$

(see for instance [24–35], and so on). Equation (1.3) is quite significant and useful in our computations, and we employ a robust and unified method known as the Riccatti-Bernoulli (RB) sub-ODE method [36]. The RB sub-ODE method has been used as a box solver for many systems of equations arising in applied science and physics. There are other powerful analytical methods that solve such ODEs; an important example is the Lie algebra method (see [37, 38]).

Next, we describe the RB sub-ODE method briefly.

2. THE RB SUB-ODE METHOD

According to the RB sub-ODE method [36], the solution of Equation (1.3) is

$$\varphi' = a\varphi^{2-m} + b\varphi + c\varphi^m, \qquad (2.1)$$

where a, b, c, and n are constants that will be determined later. From Equation (2.1), we get

$$\varphi'' = ab(3-m)\varphi^{2-m} + a^2(2-m)\varphi^{3-2m} + mc^2\varphi^{2m-1}$$
$$+ bc(m+1)\varphi^m + (2ac+b^2)\varphi, \qquad (2.2)$$

$$\varphi''' = \varphi' [ab(3-m)(2-m)\varphi^{1-m} + a^2(2-m)(3-2m)\varphi^{2-2m}$$
$$+ m(2m-1)c^2\varphi^{2m-2} + bcn(m+1)\varphi^{m-1} + (2ac+b^2)]. \qquad (2.3)$$

The solitary solutions $\varphi_i(\xi)$ of Equation (2.1) are given by

1. At $m = 1$

$$\varphi(\xi) = \varsigma e^{(a+b+c)\xi}. \qquad (2.4)$$

2. At $m \neq 1$, $b = 0$, and $c = 0$

$$\varphi(\xi) = \left(a(m-1)(\xi + \varsigma)\right)^{\frac{1}{m-1}}. \qquad (2.5)$$

3. At $m \neq 1$, $b \neq 0$, and $c = 0$

$$\varphi(\xi) = \left(\frac{-a}{b} + \varsigma e^{b(m-1)\xi}\right)^{\frac{1}{m-1}}. \qquad (2.6)$$

4. At $m \neq 1$, $a \neq 0$, and $b^2 - 4ac < 0$

$$\varphi(\xi) = \left(\frac{-b}{2a} + \frac{\sqrt{4ac - b^2}}{2a} \tan\left(\frac{(1-m)\sqrt{4ac - b^2}}{2}(\xi + \varsigma)\right)\right)^{\frac{1}{1-m}} \qquad (2.7)$$

and

$$\varphi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{4ac - b^2}}{2a} \cot\left(\frac{(1-m)\sqrt{4ac - b^2}}{2}(\xi + \varsigma)\right)\right)^{\frac{1}{1-m}}. \qquad (2.8)$$

5. At $m \neq 1$, $a \neq 0$, and $b^2 - 4ac > 0$

$$\varphi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \coth\left(\frac{(1-m)\sqrt{b^2 - 4ac}}{2}(\xi + \varsigma)\right)\right)^{\frac{1}{1-m}} \qquad (2.9)$$

and

$$\varphi(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a} \tanh\left(\frac{(1-m)\sqrt{b^2 - 4ac}}{2}(\xi + \varsigma)\right)\right)^{\frac{1}{1-m}}. \qquad (2.10)$$

6. At $m \neq 1$, $a \neq 0$, and $b^2 - 4ac = 0$

$$\varphi(\xi) = \left(\frac{1}{a(m-1)(\xi + \varsigma)} - \frac{b}{2a}\right)^{\frac{1}{1-m}}. \qquad (2.11)$$

2.0.1. Bäcklund Transformation

If $\varphi_{r-1}(\xi)$ and $\varphi_r(\xi)(\varphi_r(\xi) = \varphi_r(\varphi_{r-1}(\xi)))$ are the solutions of Equation (2.1), we have

$$\frac{d\varphi_r(\xi)}{d\xi} = \frac{d\varphi_r(\xi)}{d\varphi_{r-1}(\xi)} \frac{d\varphi_{r-1}(\xi)}{d\xi}$$
$$= \frac{d\varphi_r(\xi)}{d\varphi_{r-1}(\xi)}(a\varphi_{r-1}^{2-m} + b\varphi_{r-1} + c\varphi_{r-1}^m),$$

namely

$$\frac{d\varphi_r(\xi)}{a\varphi_r^{2-m} + b\varphi_r + c\varphi_r^m} = \frac{d\varphi_{r-1}(\xi)}{a\varphi_{r-1}^{2-m} + b\varphi_{r-1} + c\varphi_{r-1}^m}. \qquad (2.12)$$

Integrating Equation (2.12) once with respect to ξ, we get the Bäcklund transformation of Equation (2.1) as follows:

$$\varphi_r(\xi) = \left(\frac{-cL_1 + aL_2\left(\varphi_{r-1}(\xi)\right)^{1-m}}{bL_1 + aL_2 + aL_1\left(\varphi_{r-1}(\xi)\right)^{1-m}}\right)^{\frac{1}{1-m}}, \qquad (2.13)$$

where L_1 and L_2 are arbitrary constants. Equation (2.13) gives the infinite solutions of Equations (2.1) and (1.1).

3. UNIFIED SOLVER

In this section, we will describe the practical implementation of the concept of a unified solver.

$$\alpha_1 \varphi'' + \alpha_2 \varphi^3 + \alpha_3 \varphi = 0, \qquad (3.1)$$

Substituting Equation (2.2) into Equation (3.1), we obtain

$$\alpha_1 \left(ab(3-m)\varphi^{2-m} + a^2(2-m)\varphi^{3-2m} + mc^2\varphi^{2m-1}\right.$$
$$\left. + bc(m+1)\varphi^m + (2ac+b^2)\varphi\right) + \alpha_2\varphi^3 + \alpha_3\varphi = 0. \quad (3.2)$$

Making $m = 0$, Equation (3.2) is reduced to

$$\alpha_1(3abu^2 + 2a^2\varphi^3 + bc + (2ac+b^2)\varphi) + \alpha_2\varphi^3 + \alpha_3\varphi = 0. \quad (3.3)$$

Setting each coefficient of $\varphi^i (i = 0, 1, 2, 3)$ to zero, we get

$$\alpha_1 bc = 0, \quad (3.4)$$
$$\alpha_1(2ac + b^2) + \alpha_3 = 0, \quad (3.5)$$
$$3\alpha_1 ab = 0, \quad (3.6)$$
$$2\alpha_1 a^2 + \alpha_2 = 0. \quad (3.7)$$

Solving Equations (3.4)–(3.7) yields

$$b = 0, \quad (3.8)$$
$$c = \mp \frac{\alpha_3}{\sqrt{-2\alpha_1\alpha_2}}, \quad (3.9)$$
$$a = \pm\sqrt{\frac{-\alpha_2}{2\alpha_1}}. \quad (3.10)$$

Hence, we present the following possible cases for solutions of Equations (3.1) and (1.1).

1. When $b = 0$ and $c = 0$ ($\alpha_3 = 0$), the solution of Equation (3.1) is

$$\varphi_1(x, t) = \left(\mp\sqrt{\frac{-\alpha_2}{2\alpha_1}}(\xi + \varsigma)\right)^{-1}, \quad (3.11)$$

   where ς is an arbitrary constant.
2. When $\frac{\alpha_3}{\alpha_1} < 0$, substituting Equations (3.8)–(3.10) and (1.2) into Equations (2.7) and (2.8), the trigonometric function solutions of Equation (1.1) are then given by

$$\varphi_{2,3}(x, t) = \pm\sqrt{\frac{\alpha_3}{\alpha_2}} \tan\left(\sqrt{\frac{-\alpha_3}{2\alpha_1}}(\xi + \varsigma)\right) \quad (3.12)$$

$$\varphi_{4,5}(x, t) = \pm\sqrt{\frac{\alpha_3}{\alpha_2}} \cot\left(\sqrt{\frac{-\alpha_3}{2\alpha_1}}(\xi + \varsigma)\right), \quad (3.13)$$

   where ς is an arbitrary constant.
3. When $\frac{\alpha_3}{\alpha_1} > 0$, substituting Equations (3.8)–(3.10) and (1.2) into Equations (2.9) and (2.10), the hyperbolic function solutions of Equation (1.1) are,

$$\varphi_{6,7}(x, t) = \pm\sqrt{\frac{-\alpha_3}{\alpha_2}} \tanh\left(\sqrt{\frac{\alpha_3}{2\alpha_1}}(\xi + \varsigma)\right) \quad (3.14)$$

and

$$\varphi_{8,9}(x, t) = \pm\sqrt{\frac{-\alpha_3}{\alpha_2}} \coth\left(\sqrt{\frac{\alpha_3}{2\alpha_1}}(\xi + \varsigma)\right), \quad (3.15)$$

where ς is an arbitrary constant.

4. MATHEMATICAL MODEL

We use stretched $\tau = \epsilon^3 t$, $\xi = \epsilon(x - \lambda t)$, $\eta = \epsilon^2 y$, where ϵ is an arbitrarily small number and λ is the speed of EA. Elwakil et al. [17] examined two-dimensional propagation of EAs in plasma with cold fluid of electrons and two different ion temperatures within the framework of Poisson equations:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = (n_e - n_{il} - n_{ih}), \quad (4.1)$$

$$n_{il} = \mu\exp(\frac{-\phi}{\nu\beta + \mu}), n_{ih} = \gamma\exp(\frac{-\beta\phi}{\nu\beta + \mu}). \quad (4.2)$$

where T_l is the low ion temperature at equilibrium density μ, T_h is the high ion temperature at equilibrium density γ, and $\beta = \frac{T_l}{T_h}$. The computational results indicate that the system reaches critical density μ_c which makes non-linearity vanish. At $\mu = \mu_c$, the modified KP equation was given:

$$\frac{\partial}{\partial\xi}\left(\frac{\partial}{\partial\tau}\phi + G\phi^2\frac{\partial}{\partial\xi}\phi + R\frac{\partial^3}{\partial\xi^3}\phi\right) + Q\frac{\partial^2}{\partial\eta^2}\phi = 0$$

with

$$\mu_c = \frac{\beta^2\lambda^4 - \lambda^4 \pm (\beta - 1)\lambda^2\sqrt{\beta^2\lambda^4 + 2\beta\lambda^4 + \lambda^4 - 12\beta} - 6\beta^2 + 6\beta}{2(-3\beta^2 + 6\beta - 3)}, \quad (4.3)$$

$$G = \frac{1}{2}\lambda(-\frac{3\nu\beta^2}{2(\mu + \beta\nu)^2} - \frac{3\mu}{2(\mu + \beta\nu)^2} - \frac{3}{\lambda^4}), \quad (4.4)$$

$$R = \frac{\lambda^3}{2}, Q = \frac{\lambda}{2}. \quad (4.5)$$

We use a similarity transformation in the form:

$$\chi = L\xi + M\eta - \tau(\upsilon_1 + \upsilon_2), \quad (4.5)$$
$$\phi(\chi) = \phi(x, y, t) \quad (4.6)$$
$$\tau = t, \quad (4.7)$$

where L and M are *directional cosines* of x and y axes.

The MKP equation transformed to the ODE form is:

$$-3(\nu - s)\phi + \delta\phi^3 + 3\sigma\frac{d^2\phi}{d\chi^2} = 0. \quad (4.8)$$

Equation (4.8) gives a stationary soliton in the form of

$$\phi_c = \sqrt{6(\frac{\nu - S}{\delta})}\text{sech}\left(\frac{\sqrt{\frac{\nu - S}{\delta}}}{\sqrt{\frac{\sigma}{\delta}}}\chi\right), \quad (4.9)$$

$$S = \frac{M^2 Q}{L} - u, \quad (4.10)$$
$$\delta = GL, \sigma = RL^3, \quad (4.11)$$

where u and ν are traveling speeds in both directions.
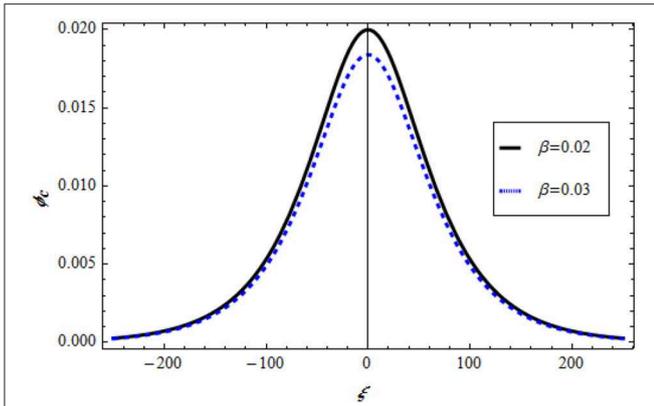
FIGURE 1 | Variation of ϕ_c against χ, β for $u = 0.01, v = 0.01, L = 0.95$.

FIGURE 2 | Variation of rational ϕ_c against χ, β for $u = 0.01, v = 0.01, L = 0.95$.

FIGURE 3 | Variation of periodic ϕ_c against χ for $\beta = 0.05, u = 0.02, v = 0.5, L = 0.92$.

FIGURE 4 | Variation of shock ϕ_c against χ for $\beta = 0.02, u = 0.02, v = 0.5, L = 0.5$.

FIGURE 5 | Variation of explosive shock ϕ_c against χ for $\beta = 0.02, u = 0.02, v = 0.5, L = 0.5$.

5. RESULTS AND DISCUSSION

Comparing Equation (4.8) with the general form (3.1) gives $\alpha_1 = 3\sigma$, $\alpha_2 = \delta$, and $\alpha_3 = -3(v - s)$. According to the unified solver given in section 3, solutions of Equation (4.8) are expressed as follows.

5.1. Rational Function Solutions: (When $v = s$)

The rational solutions of Equation (4.8) are.

$$\phi_{1,2}(x,t) = \left(\mp \sqrt{\frac{-\delta}{6\sigma}} \, (\chi + \varsigma) \right)^{-1}. \tag{5.1}$$

5.2. Trigonometric Function Solution: (When $\frac{v-s}{\sigma} > 0$)

The trigonometric solutions of Equation (4.8) are

$$\phi_{3,4}(x,t) = \pm \sqrt{\frac{-3(v-s)}{\delta}} \, tan\left(\sqrt{\frac{v-s}{2\sigma}} (\chi + \varsigma) \right) \tag{5.2}$$

and

$$\phi_{5,6}(x,t) = \pm\sqrt{\frac{-3(\upsilon - s)}{\delta}} \cot\left(\sqrt{\frac{\upsilon - s}{2\sigma}}(\chi + \varsigma)\right). \quad (5.3)$$

5.3. Hyperbolic Function Solution: (When $\frac{\upsilon - s}{\sigma} < 0$)

The hyperbolic solutions of Equation (4.8) are.

$$\phi_{7,8}(x,t) = \pm\sqrt{\frac{3(\upsilon - s)}{\delta}} \tanh\left(\sqrt{\frac{s - \upsilon}{2\sigma}}(\chi + \varsigma)\right) \quad (5.4)$$

and

$$\phi_{9,10}(x,t) = \pm\sqrt{\frac{3(\upsilon - s)}{\delta}} \coth\left(\sqrt{\frac{s - \upsilon}{2\sigma}}(\chi + \varsigma)\right). \quad (5.5)$$

Two-dimensional propagation of solitary non-linear EAs has been examined in a plasma mode using parameters related to sheet layers of plasmas of the Earth's magnetotail [16, 17]. At a certain ion density value called the criticality value, the equation obtained cannot describe the mode. Hence, the new stretching produced by the MKP equation describes the critical system under investigation. Equation (4.9) represents a soliton with stationary behavior, as shown in **Figure 1**. At the critical point, many solitary forms are concerned with the behavior of EAs using the Riccati-Bernoulli solver for the MKP equation.

Solution (5.1) is a solitary wave type called explosive type, which has rapidly increasing amplitude, as depicted in **Figure 2**. Solution (5.2) has a blow-up periodic shape, as shown in **Figure 3**. Dissipative behaviors are also produced in **Figures 4**, **5**. In the solution of (5.4), the shock wave is propagated in the medium, as shown in **Figure 4**. Finally, the explosive shock profile is obtained for solution (5.5), as shown in **Figure 5**.

6. CONCLUSIONS

We have devoted major effort to examining the adequate description of the new type solutions at critical density in plasma layers of the Earth's magnetotail. The application of perturbation theory leads to the modified MKP equation. An RB sub-ODE solver gives new solitary excitations for the MKP equation, including periodic, explosive, and shock types. The new explosive shocks represent the wave motion of plasma solitons. Moreover, these new exact solitonic and other solutions to the MKP equation supply guidelines for the classification of the new types of waves according to the model parameters and can introduce the following types: (a) solitary and hyperbolic solutions, (b) periodic solutions, (c) explosive solutions, (d) rational solutions, (e) shock waves, and (f) explosive shocks. The application of this model could be used in the verification of the broadband and observations of magnetotail electrostatic waves.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

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