Joule-Thomson Expansion of the Quasitopological Black Holes

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In this paper, we investigate the thermal stability and Joule-Thomson expansion of some new quasitopological black hole solutions. We first study the higher-dimensional static quasitopological black hole solutions in the presence of Born-Infeld, exponential, and logarithmic nonlinear electrodynamics. The stable regions of these solutions are independent of the types of the nonlinear electrodynamics. The solutions with horizons relating to the positive constant curvature, $k = +1$, have a larger region in thermal stability, if we choose positive quasitopological coefficients, $\mu_i > 0$. We also review the power Maxwell quasitopological black hole. We then obtain the five-dimensional Yang-Mills quasitopological black hole solution and compare it with the quasitopological Maxwell solution. For large values of the electric charge, q, and the Yang-Mills charge, e, we showed that the stable range of the Maxwell quasitopological black hole is larger than the Yang-Mills one. This is while thermal stability for small charges has the same behavior for these black holes. Thereafter, we obtain the thermodynamic quantities for these solutions and then study the Joule-Thomson expansion. We consider the temperature changes in an isenthalpic process during this expansion. The obtained results show that the inversion curves can divide the isenthalpic ones into two parts in the inversion pressure, P_i. For $P < P_i$, a cooling phenomenon with positive slope happens in $T - P$ diagram, while there is a heating process with a negative slope for $P > P_i$. As the values of the nonlinear parameter, β, the electric and Yang-Mills charges decrease, the temperature goes to zero with a small slope and so the heating phenomena happens slowly.

Keywords: qusitopological, Joule-Thomson, Yang-Mills, higher dimensional black holes, born-infeld

1 INTRODUCTION

Black holes as thermodynamic systems have been one of the most interesting subjects in theoretical physics for several decades. The main motivation for studying thermodynamics of AdS black holes originates from the AdS/CFT correspondence. In this correspondence, the dynamics of the quantum field theory in $(d-1)$-dimensions are related to the dynamics of an AdS black hole in d-dimensions [1]. The first-order phase transition of the AdS Schwarzschild black holes was studied by Hawking and Page [2] and prompted many physicists to study the phase structure of black holes. For example, the thermodynamic phase structures of the charged AdS black holes have been studied in Refs. [3, 4]. In this paper, the cosmological constant has been considered as the dynamical pressure, while its conjugate is the thermodynamic volume of the black hole. P-V diagram of charged AdS black holes is similar to the van der Waals liquid-gas phase transition.

Joule-Thomson expansion is another thermodynamic issue which has attracted many people. For the van der Waals gases, the Joule-Thomson expansion is an isenthalpic process in which we can probe the temperature changes as the gas expands from the high pressure to the low one, through porous plugs. In fact, the Joule-Thomson expansion is a tool used to know whether a cooling or heating process is happening for a gas. The zero value of the Joule-Thomson coefficient is called the inversion point in which the two cooling to heating processes intersect. Considering the mass of a black hole as an enthalpy [5], the Joule-Thomson expansion of charged AdS black holes was studied for the first time in Ref. [6]. The Joule-Thomson expansion was studied in many papers [7–14]. Joule-Thomson expansion of the higher dimensional charged AdS and Gauss-Bonnet AdS black holes have been broadly probed in Refs. [7, 8]. Joule-Thomson expansion of charged AdS black holes in rainbow gravity has also been studied in Ref. [15]. In Refs. [16, 17], the Joule-Thomson expansion of the usual and regular (Bardeen)-AdS black holes have been investigated. Joule-Thomson expansion of Born-Infeld AdS black holes has been investigated in Ref. [18]. Born-Infeld nonlinear electrodynamics was introduced by Born and Infeld with the main aim to remove the divergence of the electric field [19]. Other types of the nonlinear electrodynamics such as logarithmic nonlinear (LN) and exponential nonlinear (EN) were introduced in Refs. [20, 21]. The main purpose of this paper is to obtain the Joule-Thomson expansion for the nonlinear quasitopological black holes.

Quasitopological gravity is a higher curvature modified theory in d dimensions. This gravity has some advantages which has attracted us to investigating it further. Based on the AdS/CFT correspondence, this gravity can provide a one-to-one duality between .the central charges of the conformal field theory and the parameters in the gravitational side [22–24]. Also, Einstein's gravity is a low energy limit of the string theory which predicts higher dimensions. As the terms of the quasitopological gravity are not true topological invariants, so they can generate nontrivial gravitational terms in lower dimensions. This is the benefit of this gravity to the other modified gravities such as Lovelock. This gravity can also provide a dual CFT which respects the causality [25]. Quasitopological black holes and $P − V$ criticality behavior have been studied in Refs. [26, 27]. Thermodynamics of the AdS black hole and holography in generalized quasi-topological gravity was investigated in Refs. [28, 29].

We also obtained the five-dimensional Yang-Mills (YM) quasitopological black hole solutions and compared them with the ones in quasitopological Maxwell theory. The YM theory is one of the attractive non-abelian gauge theories that comes from the low energy limit of the string theory modelâ€™s spectrum. Non-abelian gauge fields beside the gravitational ones can be an effective subject in the physical phenomena of the results of superstring models. The analytic black hole solution of the Einstein-YM (EYM) was first developed by Yasskin in Ref. [30]. Black hole solutions in the presence of nonabelian Yang-Miils theory have been obtained in Refs. [31–34]. Black holes in a non-abelian Born-Infeld theory and supersymmetric EYM theories were studied in [35, 36], respectively. Using the Wu-Yang ansatz [37], black hole solutions of the various gravities coupled to the YM field have been explored in Refs. [38–44]. It is interesting to look at the Joule-Thomson expansion of the Yang-Mills quasitopological solutions.

This paper is arranged as follows: We start with the quasitopological gravity and the nonlinear electrodynamic theory in **Sec. 2** and find the related static solutions. We then obtain the thermodynamic quantities and study the thermal stability of the related solutions in the sections III and IV, respectively. We also probe the Joule-Thomson expansion of the Power Maxwell quasitopological black hole in Sec.VI. In Sec. VII, we obtain the solution of the Yang-Mills black hole in the presence of the quasitopological gravity and then probe the thermal stability and Joule-Thomson expansion for this black hole. Lastly, we provide a conclusion of the paper in **Sec. 8**.

2 THE STATIC SOLUTIONS OF THE $(n + 1)$-DIMENSIONAL NONLINEAR QUASITOPOLOGICAL GRAVITY

The main structure of the $(n + 1)$-dimensional quasitopological gravity starts from the action [45–47]

$$I_{bulk} = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \{ -2\Lambda + R + \hat{\mu}_2 \mathcal{L}_2 + \hat{\mu}_3 \mathcal{L}_3 + \hat{\mu}_4 \mathcal{L}_4 + \mathcal{L}(F) \}, \tag{1}$$

where $\mathcal{L}(F)$ is the matter source that for the nonlinear electrodynamics is considered as follows:

$$\mathcal{L}(F) = \begin{cases} 4\beta^2 \left[1 - \sqrt{1 + \dfrac{F^2}{2\beta^2}} \right], & BI \\ 4\beta^2 \left[\exp\left(-\dfrac{F^2}{4\beta^2} \right) - 1 \right], & EN \\ -8\beta^2 \ln\left[1 + \dfrac{F^2}{8\beta^2} \right], & LN \end{cases} \tag{2}$$

where BI, EN, and LN are the abbreviations of the Born-Infeld, exponential, and logarithmic forms, respectively [19–21].

We define $F^2 = F_{\mu\nu}F^{\mu\nu}$, where the electromagnetic field tensor is described as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with A_μ as the vector potential. The Lagrangians $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_4$ are respectively referred to as the second-order Lovelock (Gauss-Bonnet), cubic and quartic quasitopological gravity with the constant coefficients $\hat{\mu}_2$, $\hat{\mu}_3$ and $\hat{\mu}_4$. It should be noted that a hat () is not the sign of an operator. $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_4$ are defined as [45]

$$\mathcal{L}_2 = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2, \tag{3}$$

$$\mathcal{L}_3 = R_a{}^c b{}^d R_c{}^e d{}^f R_e{}^a f{}^b + \frac{1}{(2n-1)(n-3)}$$

$$\left[\frac{3(3n-5)}{8} R_{abcd} R^{abcd} R - 3(n-1) R_{abcd} R^{abc}{}_e R^{de} \right.$$

$$+ 3(n+1) R_{abcd} R^{ac} R^{bd} + 6(n-1) R_a{}^b R_b{}^c R_c{}^a - \frac{3(3n-1)}{2} R_a{}^b R_b{}^a R$$

$$\left. + \frac{3(n+1)}{8} R^3 \right] \tag{4}$$

$$\mathcal{L}_4 = c_1 R_{abcd} R^{cdef} R^{hg}{}_{ef} R^{ab}{}_{hg} + c_2 R_{abcd} R^{abcd} R^{ef}{}_{ef} + c_3 RR_{ab} R^{ac} R_c^b$$

$$+ c_4 \left(R_{abcd} R^{abcd} \right)^2 + c_5 R_{ab} R^{ac} R_{cd} R^{db} + c_6 RR_{abcd} R^{ac} R^{db}$$

$$+ c_7 R_{abcd} R^{ac} R^{be} R_e^d + c_8 R_{abcd} R^{acef} R^b{}_e R_f^d$$

$$+ c_9 R_{abcd} R^{ac} R_{ef} R^{bedf} + c_{10} R^4 + c_{11} R^2 R_{abcd} R^{abcd}$$

$$+ c_{12} R^2 R_{ab} R^{ab} + c_{13} R_{abcd} R^{abef} R^c{}_{efg} R^{dg}$$

$$+ c_{14} R_{abcd} R^{aecf} R_{gehf} R^{gbhd}, \tag{5}$$

where the coefficients c_i's are written in the **Supplementary Appendix A**. To find a handle of the static topological solutions, we use the metric.

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{k,n-1}^2, \tag{6}$$

where $d\Omega_{k,n-1}^2$ represents the line element of a $(n-1)$-dimensional hypersurface Σ with the constant curvature $k = 1, 0, -1$ as the spherical, flat, and hyperbolic geometries, respectively. By varying the action (1) with respect to A_μ and solving the related equation, the electromagnetic field tensor is obtained as

$$F_{tr} = \begin{cases} \dfrac{q}{r^{n-1}} \left(\sqrt{1+\eta} \right)^{-1}, & BI \\[2ex] \beta \sqrt{L_W(\eta)}, & EN \\[2ex] \dfrac{2q}{r^{n-1}} \left(1 + \sqrt{1+\eta} \right)^{-1}, & LN \end{cases} \tag{7}$$

where $\eta = \frac{q^2}{\beta^2 r^{2n-2}}$ and q is an integration constant. If we vary the action (1) with respect to $g_{\mu\nu}$ and redefine the quasitopological gravity coefficients as

$$\mu_2 = (n-2)(n-3) \hat{\mu}_2, \tag{8}$$

$$\mu_3 = \frac{(n-2)(n-5)(3n^2 - 9n + 4)}{8(2n-1)} \hat{\mu}_3, \tag{9}$$

$$\mu_4 = n(n-1)(n-3)(n-7)(n-2)^2 \left(n^5 - 15n^4 + 72n^3 - 156n^2 \right.$$
$$\left. + 150n - 42 \right) \hat{\mu}_4, \tag{10}$$

so the gravitational field equation is gained as

$$\mu_4 \Psi^4 + \mu_3 \Psi^3 + \mu_2 \Psi^2 + \Psi + \xi = 0, \tag{11}$$

where we have the definitions $\Psi(r) = [k - f(r)]/r^2$ and

$$\xi = -\frac{2\Lambda}{n(n-1)} - \frac{m}{r^n} +$$

$$\frac{4\beta^2}{n(n-1)} \left\{ 1 - {}_2F_1 \left\{ \left[-\frac{1}{2}, -\frac{n}{2(n-1)} \right], \right. \right.$$

$$\begin{cases} \left[\frac{n-2}{2(n-1)} \right], -\eta \right\} \right\}, \ BI \\[2ex] -\dfrac{4\beta^2}{n(n-1)} + \dfrac{4(n-1)\beta q}{n(n-2)r^n} \left(\dfrac{q}{\beta} \right)^{\frac{1}{n-1}} \left[L_W(\eta) \right]^{\frac{n-2}{2(n-1)}} \\[2ex] \times {}_2F_1 \left\{ \left[\dfrac{n-2}{2(n-1)} \right], \left[\dfrac{3n-4}{2(n-1)} \right], -\dfrac{1}{2(n-1)} L_W(\eta) \right\}, \ EN \\[2ex] -\dfrac{4\beta q}{(n-1)r^{n-1}} \left[L_W(\eta) \right]^{\frac{1}{2}} \times \left\{ 1 - \dfrac{1}{n} \left[L_W(\eta) \right]^{-1} \right\}, \ EN \\[2ex] \dfrac{8(2n-1)}{n^2(n-1)} \beta^2 \left[1 - \sqrt{1+\eta} \right] + \dfrac{8(n-1)q^2}{n^2(n-2)r^{2n-2}} {}_2F_1 \\[2ex] \left\{ \left[\dfrac{n-2}{2(n-1)}, \dfrac{1}{2} \right], \left[\dfrac{3n-4}{2(n-1)} \right], -\eta \right\} \\[2ex] -\dfrac{8}{n(n-1)} \beta^2 \ln \left[\dfrac{2\sqrt{1+\eta} - 2}{\eta} \right], \ LN \end{cases} \tag{12}$$

where m is an integration constant relating to the mass of the black hole. In the above relation, $L_W(x)$ and ${}_2F_1[(a,b),(c),z]$ are, respectively, the Lambert and hypergeometric functions. We get to the solution for **Eq. 11**

$$f(r) = k - r^2 \times \begin{cases} -\dfrac{\mu_3}{4\mu_4} + \dfrac{-W + \sqrt{-\left(3A + 2y - \dfrac{2B}{W} \right)}}{2}, & \mu_4 > 0, \\[3ex] -\dfrac{\mu_3}{4\mu_4} + \dfrac{W - \sqrt{-\left(3A + 2y + \dfrac{2B}{W} \right)}}{2}, & \mu_4 < 0, \end{cases} \tag{13}$$

where for brevity reasons, we have described W, A, y, and B in the **Supplementary Appendix B**.

3 THERMODYNAMIC BEHAVIOR OF THE $(n+1)$-DIMENSIONAL STATIC NONLINEAR QUASITOPOLOGICAL BLACK HOLE

Via the AdS/CFT correspondence, the thermodynamic behaviors of an AdS black hole can provide a set of knowledge for a certain dual conformal field theory (CFT). So, in this section, we are eager to obtain the thermodynamic quantities of the static nonlinear quasitopological black hole. Using the subtraction method [48], the mass of this black hole is gained as

$$M = \frac{(n-1)}{16\pi} m, \tag{14}$$

where m can be obtained from **Eq. 11** by the fact that $f(r_+) = 0$. The electric charge of the black hole can be determined from the Gauss law

$$Q = \frac{1}{4\pi} \int F_{tr} r^{n-1} d\Omega_k = \frac{q}{4\pi}, \tag{15}$$

and the electric potential U is defined by the formula $U = A_\nu \chi^\nu|_\infty - A_\nu \chi^\nu|_{r=r_+}$, where χ^ν is the killing vector. So, we can obtain the electric potential as follows

$$U = \begin{cases} \frac{q}{(n-2)r_+^{n-2}} {}_2F_1 \left\{ \left[\frac{1}{2}, \frac{n-2}{2(n-1)} \right], \left[\frac{3n-4}{2(n-1)} \right], -\eta_+ \right\}, & BI \\[8pt] \frac{n-1}{n-2} \beta \left(\frac{q}{\beta} \right)^{\frac{1}{n-1}} [L_W(\eta_+)]^{\frac{n-2}{2(n-1)}} {}_2F_1 \\ \quad \left\{ \left[\frac{n-2}{2(n-1)} \right], \left[\frac{3n-4}{2(n-1)} \right], -\frac{1}{2(n-1)} L_W(\eta_+) \right\} \\ \quad -\beta r_+ \sqrt{L_W(\eta_+)}, & EN \\[8pt] \frac{q}{(n-2)r_+^{n-2}} F_2 \left\{ \left[\frac{n-2}{2(n-1)}, \frac{1}{2}, 1 \right], \left[\frac{3n-4}{2(n-1)}, 2 \right], -\eta_+ \right\}, & LN \end{cases} \tag{16}$$

where $\eta_+ \equiv \eta(r = r_+)$. The entropy [49] and the Hawking temperature of the static nonlinear quasitopological black hole can be obtained by

$$S = \frac{n-1}{4} r_+^{n-1} \left[\frac{1}{n-1} + \frac{2k\mu_2}{(n-3)r_+^2} + \frac{3k^2\mu_3}{(n-5)r_+^4} + \frac{4k^3\mu_4}{(n-7)r_+^6} \right], \tag{17}$$

$$T = \frac{f'(r_+)}{4\pi}$$

$$= \frac{1}{4\pi r_+ (4\mu_4 k^3 + 3\mu_3 k^2 r_+^2 + 2\mu_2 k r_+^4 + r_+^6)} \times \left[\mu_4 k^4 (n-8) \right.$$

$$+ \mu_3 k^3 (n-6) r_+^2 + \mu_2 k^2 (n-4) r_+^4 + k(n-2) r_+^6 - \frac{2\Lambda}{n-1} r_+^8$$

$$+ \left\{ \begin{array}{ll} \frac{4\beta^2}{n-1} r_+^8 (1 - \sqrt{1+\eta_+}), & BI \\[8pt] -\frac{4\beta^2}{n-1} r_+^8 \left\{ 1 - [1 - L_W(\eta_+)] \exp\left[\frac{L_W(\eta_+)}{2} \right] \right\}, & EN \\[8pt] \frac{8\beta^2}{n-1} r_+^8 \left\{ 1 - \sqrt{1+\eta_+} - \ln\left[\frac{2(\sqrt{1+\eta_+}-1)}{\eta_+} \right] \right\}. & LN \end{array} \right] \tag{18}$$

If we consider the thermodynamic volume and pressure as below [50]

$$V = \frac{r_+^n}{n}, \qquad\qquad P = -\frac{\Lambda}{8\pi}, \tag{19}$$

therefore, the first law of the thermodynamics in the extended phase space follows from the formula

$$dM = TdS + UdQ + VdP + Bd\beta + \Psi_2 d\mu_2 + \Psi_3 d\mu_3 + \Psi_4 d\mu_4, \tag{20}$$

where B and Ψ_i's, $(i = 1, 2, 3)$ are denoted, respectively, as the potentials conjugate to the nonlinear parameter β and couplings μ_i's, respectively. They are defined as follows

$$B = \frac{\partial M}{\partial \beta},$$

$$\Psi_2 = \frac{\partial M}{\partial \mu_2} = \frac{(n-1)k^2 r_+^{n-4}}{16\pi} - \frac{(n-1)k r_+^{n-3}}{2(n-3)} T,$$

$$\Psi_3 = \frac{\partial M}{\partial \mu_3} = \frac{(n-1)k^3 r_+^{n-6}}{16\pi} - \frac{3(n-1)k^2 r_+^{n-5}}{4(n-5)} T,$$

$$\Psi_4 = \frac{\partial M}{\partial \mu_4} = \frac{(n-1)k^4 r_+^{n-8}}{16\pi} - \frac{(n-1)k^3 r_+^{n-7}}{(n-7)} T, \tag{21}$$

that the relations $T = \frac{\partial M}{\partial S}$, $U = \frac{\partial M}{\partial Q}$, $V = \frac{\partial M}{\partial P}$ are established. In the extended phase space, we can write the Smarr-type formula of this black hole as

$$M = \frac{1}{n-2} \left[(n-1)TS - 2PV + (n-2)UQ - \beta B + 2\mu_2\Psi_2 \right.$$
$$\left. + 4\mu_3\Psi_3 + 6\mu_4\Psi_4 \right]. \tag{22}$$

If we determine the specific volume $v = \frac{4r_+}{n-1}$ and use the pressure (III) in Eq. (III), we can specify the equation of state for the static nonlinear quasitopological black hole as $P(v, T)$. The critical points of the static nonlinear quasitopological black hole can be derived from the following conditions

$$\frac{\partial P}{\partial v}\Big|_{v_C} = 0, \qquad \frac{\partial^2 P}{\partial v^2}\Big|_{v_C} = 0. \tag{23}$$

Critical behavior of the cubic quasitopological black hole has been investigated in Ref. [27]. As the critical behavior of the quartic quasitopological black hole is similar to the cubic one, we refrain from repeating them here.

4 THERMAL STABILITY OF THE (n + 1)-DIMENSIONAL STATIC NONLINEAR QUASITOPOLOGICAL BLACK HOLE

In order to know where a black hole may exist physically or not, we should discuss its thermal stability. To study the thermal stability of the static nonlinear quasitopological black hole, we define the heat capacity C_P at the constant pressure as follows
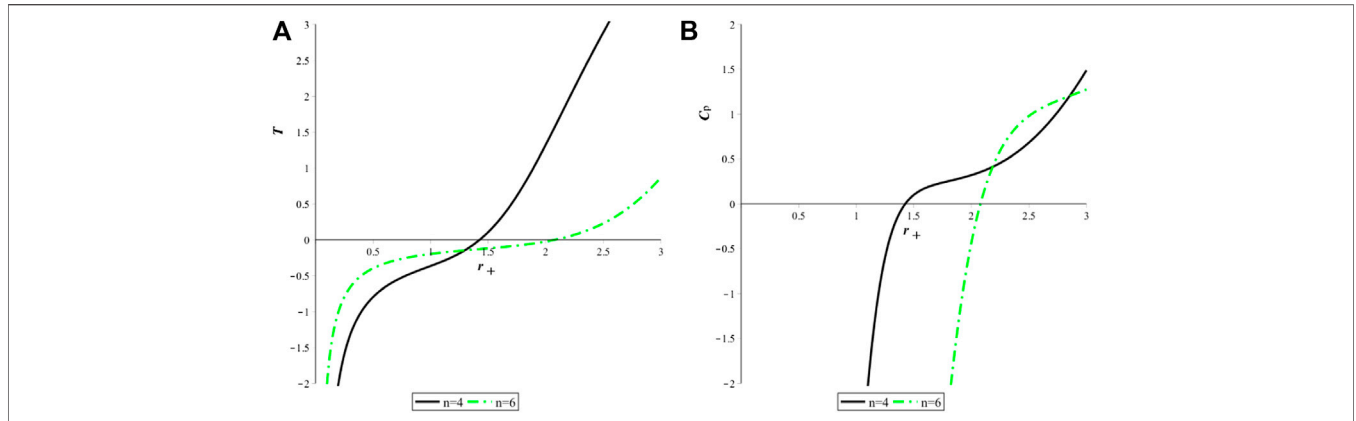
FIGURE 1 | Thermal stability of the BI quasitopological black hole with respect to r_+ for different values of dimension n with $\hat{\mu}_2 = 0.2$, $\hat{\mu}_3 = 0.1$, $\hat{\mu}_4 = 0.001$ $k = 1$, $q = 1$ and $\beta = 6$ **(A)** Temperature T, **(B)** Heat capacity C_P.
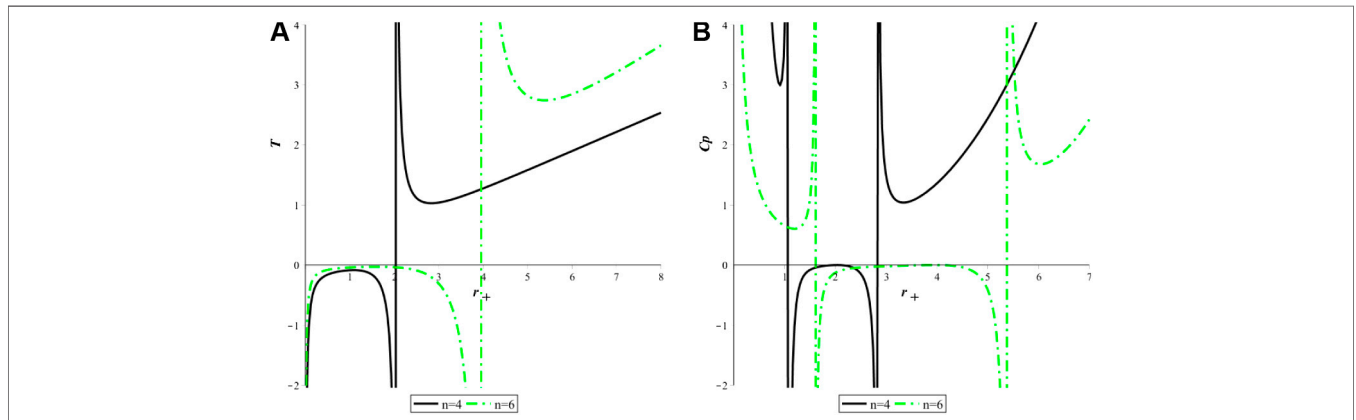
FIGURE 2 | Thermal stability of the BI quasitopological black hole with respect to r_+ for different values of dimension n with $\hat{\mu}_2 = 0.2$, $\hat{\mu}_3 = 0.1$, $\hat{\mu}_4 = -0.001$ $k = 1$, $q = 1$ and $\beta = 6$ **(A)** Temperature T, **(B)** Heat capacity C_P.

$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = T\frac{\left(\frac{\partial S}{\partial r_+}\right)_P}{\left(\frac{\partial T}{\partial r_+}\right)_P}. \qquad (24)$$

The positive value of C_P may lead to the thermal stability of the mentioned black hole, while the negative value shows the instability. We should note that the positive value of the temperature is a requirement to having physical solutions. To show the stability of the static nonlinear quasitopological black hole, we have plotted C_P and T for the BI black hole in **Figures 1**, **2** with $k = 1$ and $\hat{\mu}_2 > 0$ and $\hat{\mu}_3 > 0$. The obtained results show that the type of the nonlinear electrodynamics has a trivial effect on the thermal stability. Therefore, we have refrained from probing all of them and just included the stability of the BI theory. In **Figure 1** with $\hat{\mu}_4 > 0$, we can see a $r_{+\min}$ for each $n = 4$ and 6 dimensions, that C_P and T are both positive for $r_+ > r_{+\min}$. For $\hat{\mu}_4 < 0$ in **Figure 2B**, there are two $r_{+\min}$ and $r_{+\max}$ which C_P is positive for both regions $r_+ < r_{+\min}$ and $r_+ > r_{+\max}$. According the temperature diagram in **Figure 2A**, a unit positive region for both C_P and T can be gained for just $r_+ > r_{+\max}$. Comparing **Figures 1**, **2** shows that

for the same parameters, a black hole with $\hat{\mu}_4 > 0$ may have a larger region in thermal stability than the one with $\hat{\mu}_4 < 0$. We can also understand this result from **Eqs. 3, 17, 24**. The nonlinear quasitopological black holes with $k = 1$ and positive $\hat{\mu}_2$ and $\hat{\mu}_3$ can have larger positive regions for $\partial S/\partial r_+$, T and C_P, if we choose $\hat{\mu}_4 > 0$.

5 JOULE-THOMSON EXPANSION OF THE $(n+1)$-DIMENSIONAL STATIC NONLINEAR QUASITOPOLOGICAL BLACK HOLE

In this section, we intend to study the Joule-Thomson expansion of the obtained quasitopological black hole coupled to the nonlinear electrodynamics. In the classical thermodynamics, the Joule-Thomson expansion is an isenthalpic process in which we can probe the temperature changes as the gas expands from the high pressure to the low one through porous plugs. The Joule-Thomson coefficient is obtained by the **Eq. 6**
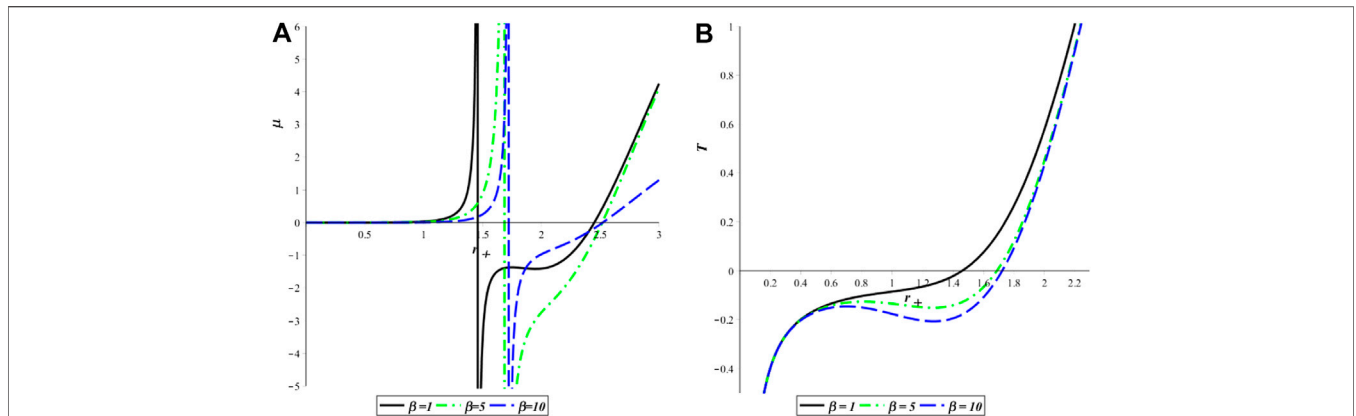
FIGURE 3 | The Joule-Thomson coefficient μ and temperature T of the BI quasitopological black hole with respect to r_+ for different values of the nonlinear parameter β with $\hat{\mu}_2 = 0.1$, $\hat{\mu}_3 = 0.1$, $\hat{\mu}_4 = 0.002$, $k = 1$, $Q = 2$ and $n = 4$ **(A)** Joule-Thomson coefficient μ, **(B)** Temperature T.

$$\mu = \left(\frac{\partial T}{\partial P} \right)_H = \frac{1}{C_P} \left[T \left(\frac{\partial V}{\partial T} \right)_P - V \right], \qquad (25)$$

where the enthalpy of the system, H, is fixed. In the gas expansion, the pressure always decreases. So, when the value of the coefficient μ is positive during the expansion, it means that the temperature decreases and therefore it is called a cooling phenomenon. However, when μ is negative, the temperature increases, and this is called a heating process. For $\mu = 0$, we can obtain the inversion temperature in which the process of the temperature changes vice versa. It can be obtained by the formula

$$T_i = V \left(\frac{\partial T}{\partial V} \right)_P. \qquad (26)$$

Because a black hole behaves like a thermodynamic system, we can consider the mass of a black hole as the enthalpy and probe the Joule-Thomson expansion for it.

Now, we would like to investigate the Joule-Thomson expansion of the higher-dimensional nonlinear quasitopological black hole and identify the region in which cooling, or heating occurs. Therefore in **Figure 3**, we have plotted the Joule-Thomson coefficient μ versus r_+ for different values of β and compared it with the temperature of the black hole. In **Figure 3A**, there is a $r_{+\text{ext}}$ for each value of β in which the coefficient μ diverges. This point is in accordance with $T = 0$ in **Figure 3B** where there is an extreme black hole. So, we can get some knowledge about the external black hole by recognizing the infinite points of μ. This figure also shows that by increasing the nonlinear parameter β, the value of $r_{+\text{ext}}$ increases. For $r_+ > r_{+\text{ext}}$ in **Figure 3A**, there is also an inversion phenomenon in $r_{+\text{inv}}$ in which the black hole goes from a heating process to a cooling one. For small β, inversion happens in a smaller r_+.

We have also plotted the isenthalpic curves and the inversion curve of the nonlinear quasitopological black hole for different values of Q and β in **Figure 4**. In each subfigure, we can see three isenthalpic curves with constant M and the related inversion curve happening at the maximum value of the isenthalpic curves. We define the inversion temperature and pressure of each isenthalpic as T_i and

P_i. The inversion curve divides the isenthalpic curves in to two parts where for $P < P_i$, the slope of the isenthalpic curve is positive and so cooling happens in the expansion. But, for $P > P_i$, the slope of the isenthalpic curve is negative, so there is heating for the black hole. For small values of parameter β in **Figures 4B,C**, the same behaviors are repeated, but for $P > P_i$, the temperature decreases to zero with a steeper slope. So, the heating process happens slowly. This is unlike the Einstein-Born-Infeld black hole for which the slope of the curve in the range $P > P_i$ is unchanged as β increases [18]. By decreasing the parameter β in **Figure 4B** with respect to the one in **Figure 4A**, the extreme black hole will happen in a larger pressure. In **Figure 4C** with a small electric charge, the heating happens very slowly and so the temperature gets to zero for a higher-pressure value than the **Figure 4B** with a larger charge. We have also checked the Joule-Thomson expansion of the obtained black hole for $\hat{\mu}_4 < 0$ in **Figure 5**. The result shows that we can face isenthalpic curves with $\hat{\mu}_4 < 0$ just for the hyperbolic geometry, $k = -1$.
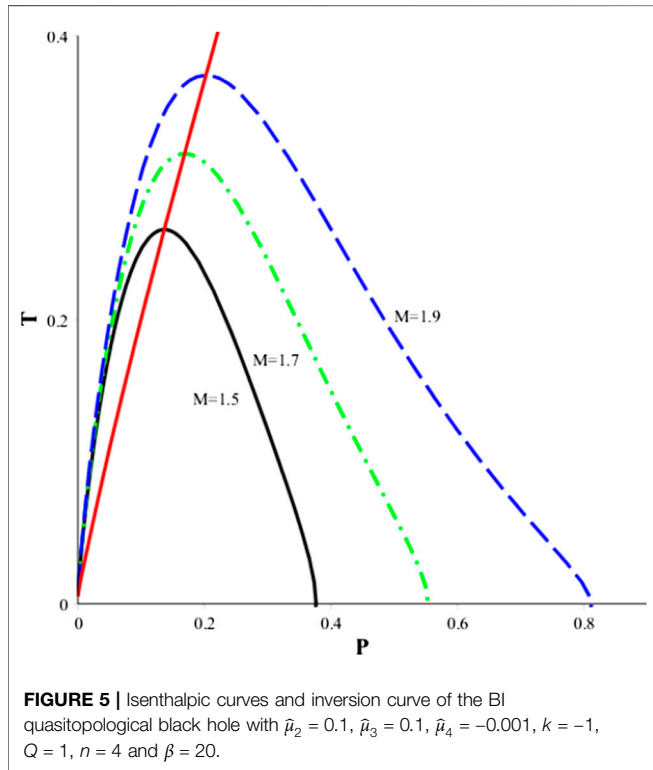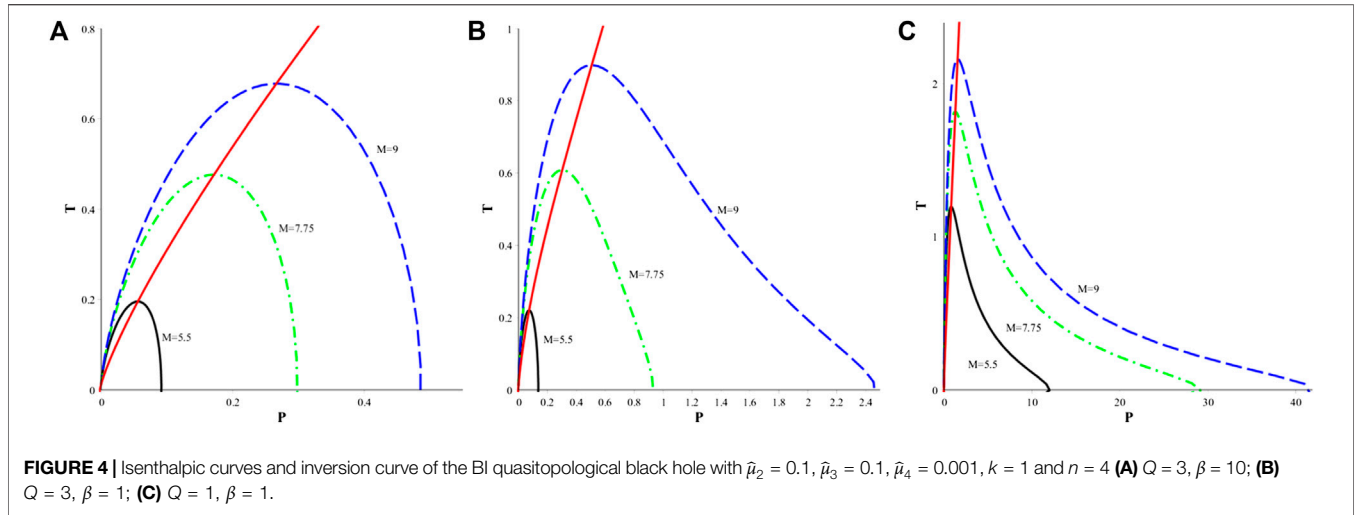
6 JOULE-THOMSON EXPANSION OF THE POWER MAXWELL QUASITOPOLOGICAL BLACK HOLE

Power Maxwell is another nonlinear electrodynamics which can preserve the conformal invariance of the theory in higher dimensions. It has the form

$$\mathcal{L}(F) = \left(-F_{\mu\nu} F^{\mu\nu} \right)^s, \qquad (27)$$

where for the nonlinear parameter $s = 1$, it is reduced to the linear Maxwell theory. In order to have an $(n + 1)$-dimensional conformal invariant action, the energy-momentum tensor should be traceless which leads to the value, $s = (n + 1)/4$. For a general study, we consider an arbitrary value for the parameter s. Quasitopological black hole solutions in the presence of the power Maxwell matter field have been obtained in Refs. [51, 52]. In this section, we aim to investigate the Joule-Thomson expansion of these solutions. The temperature of this black hole follows from [51, 52]

FIGURE 4 | Isenthalpic curves and inversion curve of the BI quasitopological black hole with $\hat{\mu}_2 = 0.1$, $\hat{\mu}_3 = 0.1$, $\hat{\mu}_4 = 0.001$, $k = 1$ and $n = 4$ **(A)** $Q = 3$, $\beta = 10$; **(B)** $Q = 3$, $\beta = 1$; **(C)** $Q = 1$, $\beta = 1$.

FIGURE 5 | Isenthalpic curves and inversion curve of the BI quasitopological black hole with $\hat{\mu}_2 = 0.1$, $\hat{\mu}_3 = 0.1$, $\hat{\mu}_4 = -0.001$, $k = -1$, $Q = 1$, $n = 4$ and $\beta = 20$.

$$
\begin{aligned}
T = &\frac{1}{4\pi r_+ \left(4\mu_4 k^3 + 3\mu_3 k^2 r_+^2 + 2\mu_2 k r_+^4 + r_+^6\right)} \\
&\times \begin{bmatrix} \mu_4 k^4 (n-8) + \mu_3 k^3 (n-6) r_+^2 + \mu_2 k^2 (n-4) r_+^4 \\ +k(n-2) r_+^6 - \dfrac{2\Lambda}{n-1} r_+^8 \end{bmatrix} \\
&-\frac{q^{2s} 2^s (n-2s)^{2s} r_+^{2s(1-n)/(2s-1)}}{4\pi \left(4\mu_4 k^3 r_+^{-6} + 3\mu_3 k^2 r_+^{-4} + 2\mu_2 k r_+^{-2} + 1\right)(n-1)(2s-1)^{2s-1}} r_+,
\end{aligned}
$$
(28)

where r_+ has a range between $r_0 \leq r_+ < \infty$ that $r_0 \neq 0$. Using **Eq. 26**, we plot the isenthalpic and inversion curves of the power Maxwell quasitopological solutions in **Figure 6**. It is clear that for a general parameter s, the inversion curve has divided the isenthalpic curves into two cooling and heating parts. In the cooling/heating process, the temperature decreases/increases as the pressure decreases in the isenthalpic process. If we compare our results with the Joule-Thomson expansion of the power Maxwell black holes in Einstein gravity [53], they show that the quasitopological gravity cannot make an enhancement for the isenthalpic curves. It is also clear from **Figure 6** that by increasing the parameter value, s, an extreme black hole happens at lower pressure. This is unlike the Einstein-power-Maxwell black hole, in which the extreme black hole with larger s has a larger pressure.

7 JOULE-THOMSON EXPANSION OF THE FIVE-DIMENSIONAL YANG-MILLS QUASITOPOLOGICAL BLACK HOLE

In this section, we consider the non-abelian Yang-Mills theory with the quasitopological gravity and obtain the related five-dimensional solutions. We also obtain the thermodynamic quantities of this black hole and then probe the Joule-Thomson expansion for it. In five dimensions, we can just consider the six-parameters gauge groups, $SO(4)$ and $SO(3,1)$, where the action is defined by the relation (1) with the matter source
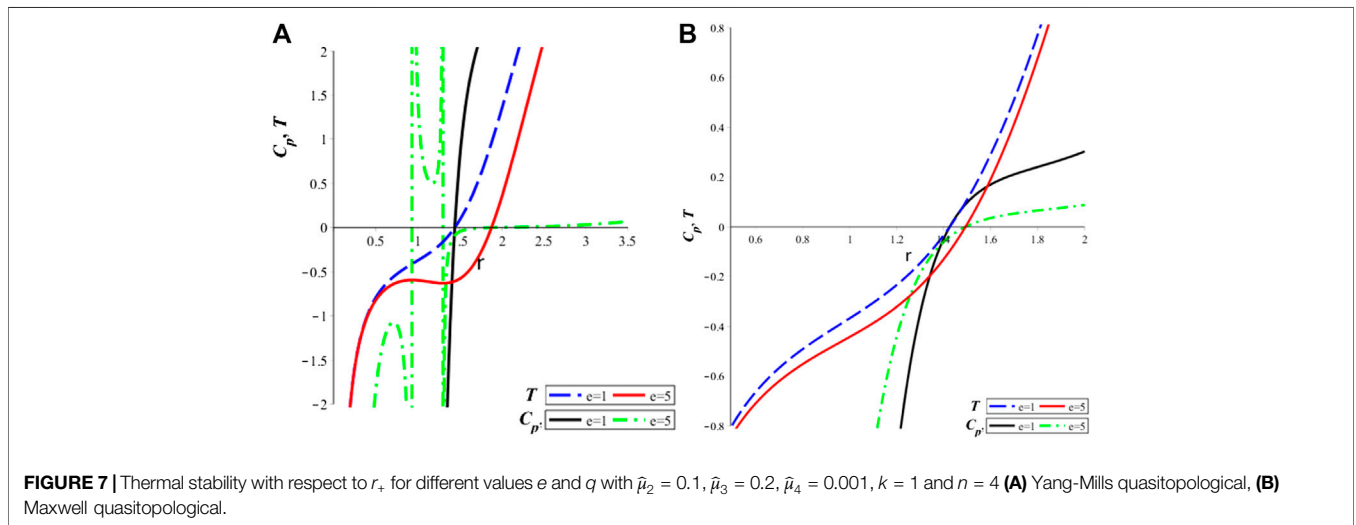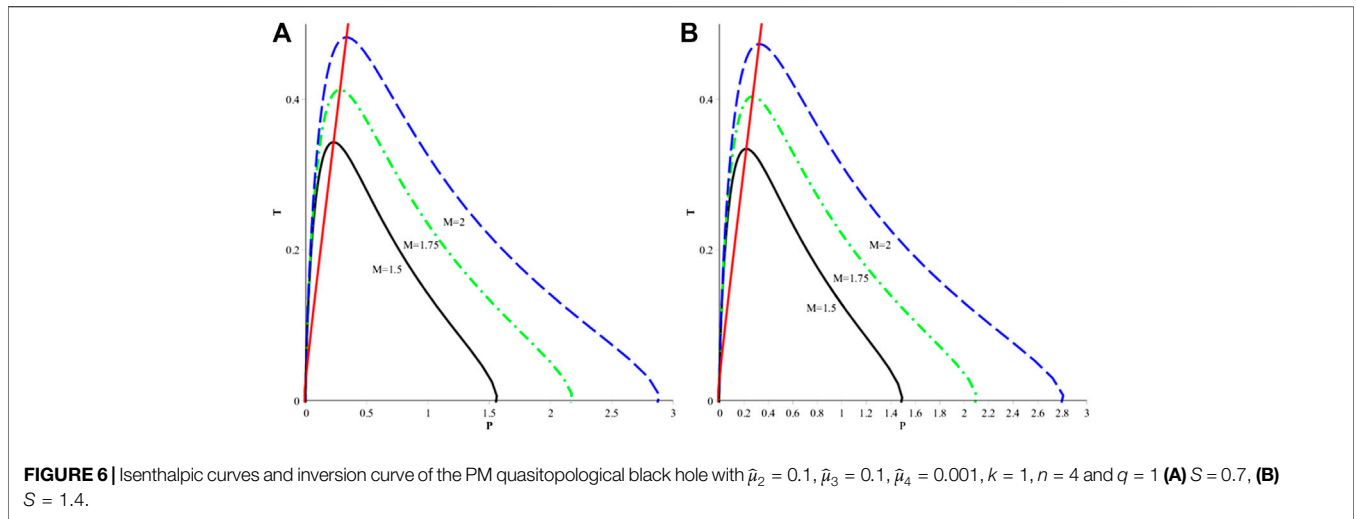
$$\mathcal{L}(F) = -\gamma_{ab} F_{\mu\nu}^{(a)} F^{(b)\mu\nu}. \tag{29}$$

The gauge field tensor $F_{\mu\nu}$ is described as follows

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + \frac{1}{e} C_{bc}^a A_\mu^{(b)} A_\nu^{(c)}, \tag{30}$$

where e is a coupling constant and C_{bc}^a's are the structure constants of the gauge groups that a, b go from 1 to 6. In order to have analytical solutions, we use the Wu-Yang ansatz

FIGURE 6 | Isenthalpic curves and inversion curve of the PM quasitopological black hole with $\hat{\mu}_2 = 0.1, \hat{\mu}_3 = 0.1, \hat{\mu}_4 = 0.001, k = 1, n = 4$ and $q = 1$ **(A)** $S = 0.7$, **(B)** $S = 1.4$.

FIGURE 7 | Thermal stability with respect to r_+ for different values e and q with $\hat{\mu}_2 = 0.1, \hat{\mu}_3 = 0.2, \hat{\mu}_4 = 0.001, k = 1$ and $n = 4$ **(A)** Yang-Mills quasitopological, **(B)** Maxwell quasitopological.

[37] and obtain the gauge potentials of the gauge groups. Using the appropriate coordinates, we have written the gauge potentials and the structure constants of the groups $SO(4)$ and $SO(3, 1)$ in the **Supplementary Appendix C**. If we vary the Yang-Mills quasitopological action with respect to $g_{\mu\nu}$, we get to **Eq. 11**, where ξ obeys from

$$\xi = -\frac{\Lambda}{6} + \frac{m}{r^4} - \frac{2e^2}{r^4} \ln\left(\frac{r}{r_0}\right), \tag{31}$$

and r_0 is a constant that for simplicity, we chose, $r_0 = 1$. In order to have the static Yang-Mills quasitopological solutions, we use the metric (6) with $k = -1, +1$ that leads to the solutions (13) with the ξ defined in **Eq. 31**. The mass and entropy of the five-dimensional Yang-Mills quasitopological black hole can be gained as **Eqs. 14Eqs. 17** where m is obtained as below

$$m = \mu_4 \frac{k^4}{r_+^4} + \mu_3 \frac{k^3}{r_+^2} + \mu_2 k^2 + k r_+^2 - \frac{\Lambda}{6} r_+^4 - 2e^2 \ln(r_+). \tag{32}$$

We can also determine the temperature and the Yang-Mills charge of this black hole as

$$T = -\frac{6\mu_4 k^4 + 3\mu_3 k^3 r_+^2 - 3k r_+^6 + \Lambda r_+^8 + 3e^2 r_+^4}{6\pi r_+ \left(4\mu_4 k^3 + 3\mu_3 k^2 r_+^2 + 2\mu_2 k r_+^4 + r_+^6\right)}, \tag{33}$$

$$Q = \frac{1}{4\pi\sqrt{6}} \int d\Omega_3 \sqrt{Tr\left[F_{\mu\nu}^{(a)} F_{\mu\nu}^{(a)}\right]} = \frac{e}{4\pi}. \tag{34}$$
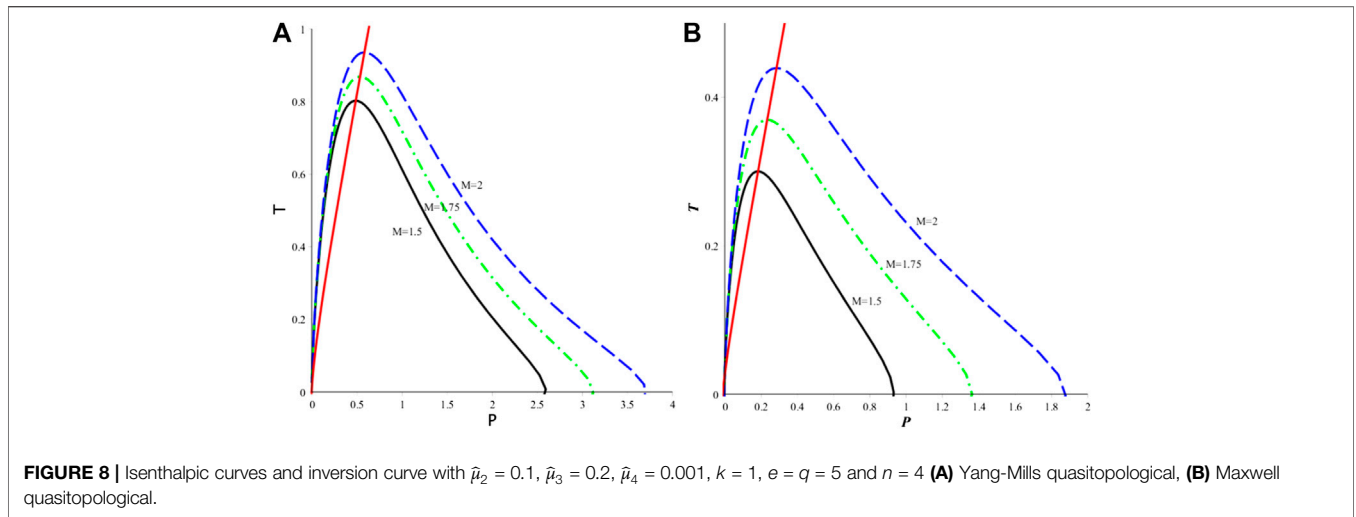
This black hole obeys the first law of thermodynamics

$$dM = TdS + UdQ, \tag{35}$$

where $T = \left(\frac{\partial M}{\partial S}\right)_Q$ is equal to the temperature (33) and the Yang-Mills potential U can be obtained by

$$U = \left(\frac{\partial M}{\partial Q}\right)_S = -12\pi Q \ln(r_+). \tag{36}$$

This relation restricts the range of the horizon value r_+ to $1 \leq r_+ < \infty$. To study the thermal stability of the Yang-Mills quasitopological black hole, we obtain the heat capacity from **Eq.**

FIGURE 8 | Isenthalpic curves and inversion curve with $\widehat{\mu}_2 = 0.1$, $\widehat{\mu}_3 = 0.2$, $\widehat{\mu}_4 = 0.001$, $k = 1$, $e = q = 5$ and $n = 4$ **(A)** Yang-Mills quasitopological, **(B)** Maxwell quasitopological.

24 and plot it in **Figure 7A**. We also compare the stability of this black hole with the Maxwell quasitopological black hole in **Figure 7B**. These figures show that for each value of e and q, there is a $r_{+\min}$ (the condition $r_{+\min} > 1$ is established for the Yang-Mills theory) that both C_P and T are positive for $r_+ > r_{+\min}$. For small charges e and q, $r_{+\min}$ has a same value in both Yang-Mills and Maxwell quasitopological theories, while for large charges, $r_{+\min}$ has a larger value in the Yang-Mills quasitopological gravity. Therefore, for large q, the Maxwell quasitopological black holes have a larger region in thermal stability than the Yang-Mills quasitopological black holes. We are also eager to investigate the Joule-Thomson expansion of the five-dimensional Yang-Mills quasitopological black hole in **Figure 8A** and to then compare it with the Maxwell quasitopological one in **Figure 8B**. In both **Figures 8A,B**, we can see a cooling/heating process for different mass values with a positive/negative slope in the isenthalpic curves. In fact, as the pressure decreases during this expansion, according to **Eq. 25**, a decrease/increase of the temperature is related to the positive/negative slope in the $T - P$ diagram. For each isenthalpic curve, there is an inversion temperature and pressure, T_i and P_i, which there is a cooling and heating process for $P < P_i$ and $P > P_i$, respectively. The heating process for the Yang-Mills quasitopological black hole in **Figure 8A** happens with a slower slope than the one in Maxwell quasitopological theory. So, the extreme Yang-Mills quasitopological black holes are described with larger pressures than the extreme Maxwell quasitopological black holes.

8 CONCLUDING REMARKS

In this paper, we reviewed some quasitopological black hole solutions and obtained their thermodynamic properties such as their thermal stability and Joule-Thomson expansion. At first, we achieved the $(n + 1)$-dimensional static quasitopological black hole solutions in the presence of three BI, EN, and LN forms of nonlinear electrodynamics. The obtained solutions are divided into two parts, for $\mu_4 > 0$ and $\mu_4 < 0$. We also obtained the thermodynamic quantities of this black hole and wrote the first law of thermodynamics in the

extended phase space. Then, we looked for the physical existence of the black hole by studying the thermal stability. The stable region of this black hole is independent of the types of the nonlinear electrodynamics. Also, for $k = 1$, the solutions with $\widehat{\mu}_i > 0$ may lead to a larger stable region than the one with negative $\widehat{\mu}_i$. Joule-Thomson expansion was the other goal we paid attention to for this black hole. So, we probed the temperature changes of this black hole in an isenthalpic process during the expansion in which the pressure decreases. For the nonlinear quasitopological black hole with $\widehat{\mu}_4 > 0$ and $k = 1$, we obtained an inversion curve which can divide the isenthalpic curves in to two parts. The part with the positive slope in the isenthalpic curve leads to a cooling process, while for the negative slope, heating may happen. For the black hole with a small charge q and nonlinear parameter β, the temperature reduces to zero with a very slow slope. It is possible to have an isenthalpic curve for $\widehat{\mu}_4 < 0$ just for $k = -1$.

We also studied the Joule-Thomson expansion of the power Maxwell quasitopological black holes. The results showed that for the large nonlinear parameter, s, the extreme black hole has a smaller pressure. This is while for the Einstein-power-Maxwell black hole, an extreme black hole with a small parameter s happens in low pressure. At the end, we looked at the Yang-Mills theory and gained the five-dimensional Yang-Mills solutions in the quasitopological gravity. We also carefully examined the thermodynamic quantities such as thermal stability and Joule-Thomson for this black hole and compared the results with the five-dimensional Maxwell quasitopological black hole. They show that there is a $r_{+\min}$, which the Yang-Mills quasitopological black hole is thermally stable for $r_+ > r_{+\min}$. For small values of the charges e and q, the value of $r_{+\min}$ is independent of the Yang-Mills or Maxwell theories, while, for large charges, $r_{+\min}$ has a larger value in Yang-Mills theory. Also, the heating process for the Yang-Mills quasitopological black hole happens more slowly than the one in the Maxwell black hole.

DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/**Supplementary Material**, further inquiries can be directed to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fphy.2021.628727/full#supplementary-material.

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