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# Thermal Correction to the Kinnersley Black Hole in a Lorentz-Violating Dirac Field Theory

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According to Lorentz-violating theory, the dynamical equation of Dirac particles in the Kinnersley black hole with variably accelerated linear motion is modified. The Hawking quantum tunneling radiation characteristics of Kinnersley black hole are obtained by solving the modified equation. The expression of the Hawking temperature of Kinnersley black hole has been updated.

**Keywords: Kinnersley black hole, quantum tunneling radiation, Hawking temperature, Lorentz symmetry violating, Dirac particle**

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## 1 INTRODUCTION

During past years, people have made many meaningful researches on the quantum tunneling radiation of black holes [1–15]. These researches have enriched us with knowledge about the thermodynamic evolution of black holes. Using the Hawking quantum tunneling radiation theory, Kraws et al calculated the Hawking temperature and entropy of black hole. The research of quantum tunneling radiation for black hole has been increased greatly by the semiclassical Hamilton-Jacobi method [8, 9]. Kerner and Mann used the semiclassical theory to study the quantum tunneling radiation characteristic of Dirac field particles in black hole [16, 17]. They divided the fermion spin into up and down and decomposed the Dirac equations into two groups, then obtained the tunneling rate of Dirac particles at the event horizon and the Hawking temperature of black hole.

In 2009, Lin and Yang proposed a new method to study the quantum tunneling radiation of black hole. They transformed the Dirac equation in curved space-time into a matrix equation by using the semiclassical approximation theory, and then the resulting matrix equation was further converted to the Hamilton-Jacobi equation for Dirac particles in curved space-time by using the commutation relation of gamma matrices. Finally they derived the Hawking quantum tunneling rate of fermions and other important physical quantities of black hole based on the Hamilton-Jacobi equation [11, 14, 18–21]. Their work showed that the Hamilton-Jacobi equation and its Hamilton principal function $S$ can be applied to the study of quantum tunneling radiation of fermions in curved space-time. The developing Hamilton-Jacobi method can effectively solve the problems related to fermion tunneling radiation and unify the expressions of quantum tunneling radiation theory in curved space-time.

The study on modifications to the Hawking radiation of black holes can help for understanding the information loss paradox [22–26]. Banerjee and Majhi extended the beyond semiclassical approximation to include all quantum corrections [27–32]. Lin and Yang investigate tunnelling of charged black holes based on Klein-Gordon scalar particle theory to derive corrections to the tunnelling rate and temperature in Reissner-Nordström space-time and Reissner Nerdström-de Sitter space-time, respectively [33]. Beyond semiclassical theory and semiclassical theory are both modified theories, for which the former can give more accurate correction to the quantum tunneling rate of black hole.

With research of string theory and quantum gravity theory, people have realized that the Lorentz relation should be modified at high energy, which will lead to the Dirac equation in curved space-time to be modified. The application of Lorentz-violating theory in curved space-time is a Frontier subject worthy of attention [34–40]. Cruz et al introduced the Lorentz symmetry violating term in the scalar field Lagrangian, namely the Lorentz-violating scalar field theory, to research thermal corrections to the Casimir energy [41]. The Dirac particle action and Dirac equation with Lorentz symmetry violating in flat space-time are introduced in [42]. The influence of Lorentz violating theory on the quantum tunneling radiation of Dirac particles in the Vaidya black hole is investigated in [43].

However, for the stationary and dynamic axisymmetric black holes, the effect of Lorentz violating theory on their quantum tunneling radiation has not been studied. In this paper, Lorentz violating theory is considered to correct the quantum tunneling rate, the temperature and entropy of a dynamic Kinnersley black hole with variably accelerated linear motion, resulting in some new conclusion.

In the second section, we introduce the modification of Lorentz violating Dirac field theory to Dirac particle dynamics equation in the Kinnersley space-time. The third section studies the thermal and entropy corrections to Kinnersley black hole in a Lorentz violating Dirac field theory. The last section gives a discussion on the results obtained in this paper.

## 2 LORENTZ'S VIOLATING THEORY AND DIRAC EQUATION IN DYNAMICAL CURVED SPACE-TIME

According to Hamilton principle, the dynamic equation of Dirac particle in flat space-time can be acquired from the action of Dirac particle in flat space-time [41–43]. Adding the Lorentz symmetry violating term into the action and using Hamilton principle, we can obtain the Dirac equation of Lorentz symmetry violating in a flat space-time. It only needs to pay attention to two points for generalizing the particle dynamics equation from the flat space-time to the curved space-time: one is to generalize the gamma matrix $\gamma^\mu$ from the flat space-time to the curved space-time, with different curved space-time having different $\gamma^\mu$; the other is to generalize the ordinary derivative to the covariant derivative related to connection. Therefore, in the Kinnersley curved space-time, the dynamics equation of spin 1/2 fermion with Lorentz symmetry violating can be expressed as [42].

$$\left\{\gamma^\mu D_\mu\left[1+\hbar^2\frac{a}{m^2}(\gamma^\mu D_\mu)^2\right]+\frac{b}{\hbar}\gamma^5+c\hbar\,(u^\alpha D_\alpha)^2 -\frac{m}{\hbar}\right\}\Psi=0 \tag{1}$$

where $\Psi$ is the wave function. For Dirac particles, the wave function $\Psi$ and the action $S$ are linked by

$$\Psi=\psi_0 e^{\frac{i}{\hbar}S} \tag{2}$$

where $\psi_0$ is a column matrix. For Dirac particles the non-stationary Kinnersley black hole

$$S=S(v,r,\theta,\phi), \tag{3}$$

where $v$ is the advanced Eddington coordinate. The covariant derivative $D_\mu$ in **Eq. 1** is defined by

$$D_\mu=\partial_\mu+\frac{i}{2}\Gamma_\mu^{\alpha\beta}\Pi_{\alpha\beta}, \tag{4}$$

where $\Gamma_\mu^{\alpha\beta}$ is the connection in Riemannian geometry, and $\Pi_{\alpha\beta}$ is expressed as

$$\Pi_{\alpha\beta}=\frac{i}{4}\left[\gamma^\alpha,\gamma^\beta\right]. \tag{5}$$

$\frac{i}{\hbar}\Gamma_\mu^{\alpha\beta}\Pi_{\alpha\beta}$ is the spin connection term that characterizes the spinor covariant derivative in curved space-time. In **Eq.1**, $a$, $b$ and $c$ are all small quantities that satisfy $a, b, c \ll m$, where $m$ is particle mass. The Gamma matrices $\gamma^\mu$ or $\gamma^\nu$ meet the following anticommutation relation:

$$\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu=2g^{\mu\nu}I, \tag{6}$$

$$\gamma^5\gamma^\mu+\gamma^\mu\gamma^5=0. \tag{7}$$

The 4D ether-like field vector $u^\alpha$ is not a constant vector in curved space-time, but it must satisfy the following condition

$$u^\alpha u_\alpha=\mathrm{c}(constant). \tag{8}$$

We substitute **Eq. 2** into **Eq. 1**, and keep the terms associated with the lowest order of $\hbar$, then **Eq. 1** is reduced to

$$\left\{i\gamma^\mu\partial_\mu S\left[1-\frac{a}{m^2}\gamma^\alpha\gamma^\beta\partial_\alpha S\partial_\beta S\right]-cu^\alpha u^\beta\partial_\alpha S\partial_\beta S +b\gamma^5-m\right\}\Psi=0. \tag{9}$$

Considering **Eq. 6**, the following equality holds:

$$\gamma^\alpha\gamma^\beta\partial_\alpha S\partial_\beta S=g^{\alpha\beta}\partial_\alpha S\partial_\beta S. \tag{10}$$

Then the dynamics **Eq. 9** becomes

$$\begin{aligned} i\gamma^\mu\partial_\mu S\Psi &= \left(1-\frac{a}{m^2}g^{\alpha\beta}\partial_\alpha S\partial_\beta S\right)^{-1}\left(cu^\alpha u^\alpha\partial_\alpha S\partial_\beta S-b\gamma^5+m\right)\Psi \\ &\approx \left[1+\left(\frac{c}{m}u^\alpha u^\beta+\frac{a}{m^2}g^{\alpha\beta}\right)\partial_\alpha S\partial_\beta S-\frac{b}{m}\gamma^5\right]m\Psi, \end{aligned} \tag{11}$$

Taking square for both sides of **Eq. 11**, and omitting the $\frac{b}{m}\gamma^5$ term, we transform it into

$$\begin{aligned} &-\gamma^\mu\gamma^\nu\partial_\mu S\partial_\nu S\Psi \\ &= m^2\psi+2\left(cmu^\alpha u^\beta+ag^{\alpha\beta}\right)\partial_\alpha S\partial_\beta S\Psi+\mathcal{O} \end{aligned} \tag{12}$$

where $\mathcal{O}$ is a high order small quantity. Then we use **Eq. 6** to simplify **Eq. 12**, resulting in

$$\left[g^{\mu\nu}\partial_\mu S\partial_\nu S+2\left(cmu^\mu u^\nu+ag^{\mu\nu}\right)\partial_\mu S\partial_\nu S+m^2\right]\Psi=0 \tag{13}$$

Obviously,$\Psi \neq 0$, making it necessary that

$$g^{\mu\nu}\partial_\mu S\partial_\nu S + 2\left(cmu^\mu u^\nu + ag^{\mu\nu}\right)\partial_\mu S\partial_\nu S + m^2 = 0. \tag{14}$$

In this equation $S$ is the action, also called Hamiltonian principal function. In the process of derive this equation, we ignore the term $\frac{b}{m}\gamma^5$ for two reasons. One is that $b$ is a small quantity; the other is that the term $\frac{b}{m}\gamma^5$ in the square bracket of **Eq. 11** only indicates the correction in quantity and thus can be ignored. However, the terms other than $\frac{b}{m}\gamma^5$ in the square bracket of **Eq. 11** are related to the metric tensor or ether-like field vector, thus can not be ignored. In fact, **Eq. 14** is completely equivalent to **Eq. 1**, since **Eq. 14** is also the dynamic equation of Dirac particles. **Eq. 14** is a new form of modified Hamilton-Jacobi equation about the action $S$ of Dirac particles. Starting from **Eq. 14**, we can conveniently study the tunneling radiation characteristics of fermion with mass $m$ in the curved non-stationary Kinnersley black hole. This is an innovation that has not been reported yet.

For a Dirac particle with mass of $m$ and charge $q$, its dynamic equation is very complicated. Firstly, **Eq. 4** must be modified to

$$D_\mu = \partial_\mu - \frac{\mathrm{i}}{\hbar}qA_\mu + \frac{\mathrm{i}}{2}\Gamma^{\alpha\beta}_\mu\Pi_{\alpha\beta}. \tag{15}$$

Substituting **Eq. 15** into **Eq. 1** and using the same method as deriving **Eq. 14**, we can get the dynamic equation of Dirac particle with mass $m$ and charge $q$ as follows

$$\begin{aligned}&\left[g^{\mu\nu}(1+2a) + 2cmu^\mu u^\nu\right](\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu)\\&+m^2 = 0.\end{aligned} \tag{16}$$

During the derivation of **Eqs 14**, **16**, we ignored higher order quantities in terms of $a$ and $c$. From the point of view of mathematics and physics, this is a reasonable and effective approximation. In the next section, we will study the tunneling radiation of Dirac particles in the non-stationary Kinnersley black hole according to **Eq. 16**.

## 3 THERMAL CORRECTION TO THE KINNERSLEY BLACK HOLE IN THE LORENTZ-VIOLATING THEORY

According to Kinnersley's research on the metric of accelerating black holes, the space-time line element of a linearly moving black hole with variable acceleration described by the advanced Eddington coordinate $v$ is [44].

$$\begin{aligned}\mathrm{d}s^2 = &\left(1 - 2a_k r\cos\theta - r^2 f^2 - 2Mr^{-1}\right)\mathrm{d}v^2 - 2\mathrm{d}v\mathrm{d}r\\&-2r^2 f\mathrm{d}v\mathrm{d}\theta - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2,\end{aligned} \tag{17}$$

$$f = -a_k\sin\theta, \tag{18}$$

where $a_k = a_k(v)$ is the acceleration of the Kinnersley black hole, $\theta$ and $\phi$ are spherical coordinates, and $M$ is the mass of the black hole. The north pole $\theta = 0$ of the black hole always points in the direction of acceleration. Therefore, the covariant and contravariant metric tensors are

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & -1 & -r^2 f & 0\\ -1 & 0 & 0 & 0\\ -r^2 f & 0 & -r^2 & 0\\ 0 & 0 & 0 & -r^2\sin^2\theta\end{pmatrix} \tag{19}$$

$$g^{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 & 0\\ -1 & g^{11} & f & 0\\ 0 & f & -r^{-2} & 0\\ 0 & 0 & 0 & -r^{-2}\sin^{-2}\theta\end{pmatrix} \tag{20}$$

where

$$g00 = 1 - 2a_k r\cos\theta - r^2 f^2 - 2Mr^{-1} \tag{21}$$

$$g^{11} = -\left(1 - 2a_k r\cos\theta - 2Mr^{-1}\right) \tag{22}$$

Obviously, this black hole is uncharged. According to the zero hypersurface equation

$$g^{\mu\nu}\frac{\partial F}{\partial x^\mu}\frac{\partial F}{\partial x^\nu} = 0, \tag{23}$$

The event horizon of the black hole should satisfy the following equation:

$$2\dot{r}_H - \left(1 - 2a_k r\cos\theta - 2Mr^{-1}\right) - 2fr'_H - \left(\frac{r'_H}{r}\right)^2 = 0, \tag{24}$$

where $\dot{r}_H = \frac{\partial r_H}{\partial v}$, $r'_H = \frac{\partial r_H}{\partial\theta}$. As one of fundamental characteristics, the Kinnersley black hole is axial symmetric and has Killing vector $(\frac{\partial}{\partial\phi})^\alpha$. Now, by using **Eqs 16**, **20** is reduced to

$$\begin{aligned}&(1+2a)\Big[g^{11}(\partial_r S - qA_r)^2 - 2(\partial_r S - qA_r)(\partial_v S - qA_v)\\&+2f(\partial_r S - qA_r)(\partial_\theta S - qA_\theta) - \frac{1}{r^2}(\partial_\theta S - qA_\theta)^2\\&-\frac{1}{r^2\sin^2\theta}(\partial_\phi S - qA_\phi)^2\Big]\\&+2cmu^\mu u^\nu(\partial_\mu S - qA_\mu)(\partial_\nu S - qA_\nu) + m^2 = 0.\end{aligned} \tag{25}$$

Note that $A_\mu$ is not the electromagnetic potential of the Kinnersley black hole itself, but the electromagnetic potential of the cosmological space around the moving non-charged Kinnersley black hole. According to **Eqs 17**, **19**, **20**, we construct the ether-like field vector $u^\mu$ such that

$$u^v = \frac{c_v}{\sqrt{g_{00}}}, u^r = \frac{c_r}{g_{01}}\sqrt{g_{00}}, u^\theta = \frac{c_\theta}{g_{02}}\sqrt{g_{00}}, u^\phi = \frac{c_\phi}{\sqrt{g_{33}}} \tag{26}$$

where $c_v$, $c_r$, $c_\theta$, $c_\phi$ are all constants. It is easy to show that $u^v u_v = c_v^2, u^r u_r = c_v c_r, u^\theta u_\theta = c_v c_\theta, u^\phi u_\phi = c_\phi^2$ and thus condition (8) is met. Substituting **Eq. 26** into **Eq. 25** will result in

$$
\begin{aligned}
&(1+2a)\Big[g^{11}(\partial_r S - qA_r)^2 - 2(\partial_r S - qA_r)(\partial_v S - qA_v) \\
&+2f(\partial_r S - qA_r)(\partial_\theta S - qA_\theta) - \frac{1}{r^2}(\partial_\theta S - qA_\theta)^2 \\
&-\frac{1}{r^2\sin^2\theta}(\partial_\phi S - qA_\phi)^2\Big] + 2cm\Big[\frac{c_v^2}{g_{00}}(\partial_v S - qA_v)^2 \\
&-2c_v c_r(\partial_r S - qA_r)(\partial_v S - qA_v) \\
&-2c_v c_\theta r^{-2} f^{-1}(\partial_v S - qA_v)(\partial_\theta S - qA_\theta) \\
&+2c_v c_\phi (g_{00}g_{33})^{-\frac{1}{2}}(\partial_v S - qA_v)(\partial_\phi S - qA_\phi) \\
&+c_r^2 g_{00}(\partial_r S - qA_r)^2 \\
&+2c_r c_\theta g_{00} r^{-2} f^{-1}(\partial_r S - qA_r)(\partial_\theta S - qA_\theta) \\
&-2c_r c_\phi g_{00}^{\frac{1}{2}} g_{33}^{-\frac{1}{2}}(\partial_r S - qA_r)(\partial_\phi S - qA_\phi) \\
&+c_\theta^2 g_{00} r^{-4} f^{-2}(\partial_\theta S - qA_\theta)^2 \\
&-2c_\theta c_\phi g_{00}^{\frac{1}{2}} g_{33}^{-\frac{1}{2}} r^{-2} f^{-1}(\partial_\theta S - qA_\theta)(\partial_\phi S - qA_\phi) \\
&+c_\phi^2 g_{33}^{-1}(\partial_\phi S - qA_\phi)^2\Big] + m^2 = 0.
\end{aligned} \tag{27}
$$

This is a time-dependent equation. To solve this equation, general tortoise coordinate transformation must be performed, i.e.

$$
\begin{aligned}
r_* &= r + \frac{1}{2\kappa(v_0,\theta_0)}\ln\frac{r - r_H(v,\theta)}{r_H(v_0,\theta_0)}, \\
v_* &= v - v_0, \\
\theta_* &= \theta - \theta_0.
\end{aligned} \tag{28}
$$

Therefore, we have

$$
\frac{\partial}{\partial r} = \frac{1 + 2\kappa(r - r_H)}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}, \tag{29}
$$

$$
\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta_*} - \frac{r'_H}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}, \tag{30}
$$

$$
\frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{\dot{r}_H}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}. \tag{31}
$$

Then, the following variable separation is performed for $S$

$$
S = R(v, r, \theta) + j\phi, \tag{32}
$$

and let

$$
\frac{\partial S}{\partial v_*} = -\omega, \tag{33}
$$

$$
\frac{\partial S}{\partial \phi} = j, \tag{34}
$$

$$
\frac{\partial S}{\partial \theta_*} = p_\theta. \tag{35}
$$

where $\omega$ is the particle energy, $p_\theta$ is a component of the particle generalized momentum in the $\theta$ direction, and the constant $j$ is a component of the particle generalized moment in the $\phi$ direction. Substituting **Eqs 29–31, 33–35** into **Eq. 27**, and considering the special condition of $r \to r_H$, $\theta \to \theta_0$, $v \to v_0$, we can get the dynamic equation of Dirac particles at the event horizon of the black hole as follows

$$
\lim_{\substack{r\to r_H \\ v\to v_0 \\ \theta\to\theta_0}} \frac{A}{B}\left(\frac{\partial S}{\partial r_*}\right)^2 - 2\omega\frac{\partial S}{\partial r_*} + 2\lim_{\substack{r\to r_H \\ v\to v_0 \\ \theta\to\theta_0}} \frac{C}{B}\frac{\partial S}{\partial r_*} = 0, \tag{36}
$$

where

$$
\begin{aligned}
&A = \\
&[2\kappa(r - r_H)]^{-1}\left\{(1+2a)\left[g^{11} + 2\dot{r}_H - 2fr'_H - \left(\frac{r'_H}{r}\right)^2\right]\right\} \\
&-2cm\Big[\dot{r}_H^2 c_v^2 g_{00}^{-1} + 2c_v c_r \dot{r}_H - 2c_v c_\theta r^{-2} f^{-1}\dot{r}_H r'_H \\
&+c_r^2 g_{00} + 2c_v c_\theta r^{-2} f^{-1} g_{00} r'_H + c_\theta^2 g_{00} r'_H\Big],
\end{aligned} \tag{37}
$$

$$
\begin{aligned}
B|_{\substack{r\to r_H \\ v\to v_0 \\ \theta\to\theta_0}} &= B' = (1+2a) \\
&+2cm\left[c_v^2\dot{r}_H\left(1 - 2a_k\cos\theta_0 - r_H^2 a_k^2\sin^2\theta_0\right.\right. \\
&\left.\left.-2Mr_H^{-1}\right)^{-1} + c_v c_r + c_v c_\theta r_H^{-2} a_k^{-1}\sin^{-1}\theta_0 r'_H\right],
\end{aligned} \tag{38}
$$

$$
\begin{aligned}
&C|_{\substack{r\to r_H \\ v\to v_0 \\ \theta\to\theta_0}} = C' = (1+2a)q\{A_0 + A_1[1 - 2a_k r_H\cos\theta_0 \\
&-2Mr_H^{-1} - \dot{r}_H + fr'_H] - A_2(r'_H r^{-2} - a_k\sin\theta_0)\} \\
&+(1+2a)p_\theta(r_H^{-2}r'_H - a_k\sin\theta_0) + 2cm\{qA_0 c_v[c_r \\
&+c_v\dot{r}_H(1 - 2a_k\cos\theta_0 - r_H^2 a_k^2\sin^2\theta_0 - 2Mr_H^{-1})^{-1} \\
&+c_\theta r_H^{-2} a_k^{-1}\sin^{-1}\theta_0 r'_H] - c_r qA_1[c_v\dot{r}_H + (1 \\
&-2a_k r_H\cos\theta_0 - r_H^2 a_k^2\sin^2\theta_0 - 2Mr_H^{-1})c_r \\
&-c_\theta r_H^{-2} a_k^{-1}\sin^{-1}\theta_0 r'_H)] + c_\theta qA_2 r_H^{-2} a^{-1}\sin^{-1}\theta_0 \\
&(1 - 2a_k r_H\cos\theta_0 - r_H^2 a_k^2\sin\theta_0 - 2Mr_H^{-1}) \\
&(c_\theta r_H^{-2} r'_H a_k^{-1}\sin^{-1}\theta_0 + c_r) + qc_\phi A_3 r_H^{-1}\sin^{-1}\theta_0 \\
&[-c_v\dot{r}_H(2a_k r_H\cos\theta_0 - 1 + r_H^2 a_k^2\sin^2\theta_0 + 2Mr_H^{-1})^{-\frac{1}{2}} \\
&a_k^{-1}\sin^{-1}\theta_0 + (2a_k r_H\cos\theta_0 - 1 + r_H^2 a_k^2\sin^2\theta_0 \\
&+2Mr_H^{-1})^{\frac{1}{2}}(c_r + c_\theta r_H^{-2} a_k^{-1}\sin^{-1}\theta_0)] + C''\},
\end{aligned} \tag{39}
$$

and

$$
\begin{aligned}
&C'' = -c_v c_\theta r_H^{-2}\dot{r}_H p_\theta a_k^{-1}\sin^{-1}\theta_0 + c_v c_\phi \dot{r}_H j \\
&(2a_k r_H\cos\theta - 1 + r_H^2 a_k^2\sin^2\theta_0 + 2Mr_H^{-1})^{-\frac{1}{2}} r_H^{-1} a_k^{-1}\sin^{-2}\theta_0 \\
&-c_r c_\theta p_\theta r_H^{-2} a_k^{-1}\sin^{-1}\theta_0(1 - 2a_k r_H\cos\theta - r_H^2 f^2 - 2Mr_H^{-1}) \\
&+c_\phi(2a_k r_H\cos\theta - 1 + r_H^2 a_k^2\sin^2\theta_0 + 2Mr_H^{-1})^{\frac{1}{2}} \\
&r^{-1}\sin^{-1}\theta_0(c_r j - c_\theta r^{-2} a_k^{-1}\sin^{-1}\theta_0 r'_H j) \\
&-c_\theta^2 r_H^{-4} a_k^{-2}\sin^{-2}\theta_0 p_\theta r'_H(1 - 2a_k r_H\cos\theta \\
&-r_H^2 a_k^2\sin^2\theta - 2Mr'_H).
\end{aligned} \tag{40}
$$

Let

$$\lim_{\substack{r\to r_H\\ v\to v_0\\ \theta\to\theta_0}} \frac{A}{B} = 1. \tag{41}$$

The first part of the expression (**37**) of $A$ will become an indeterminate formula of type $\frac{0}{0}$ as $r \to r_H$, $v \to v_0$, $\theta \to \theta_0$. So substituting **Eqs 37**, **38**, into **Eq. 41** and using L'Hospital rule, we get

$$\kappa(v_0,\theta_0) = \kappa = \frac{\dfrac{M}{r_H^2} + a_k\cos\theta_0 - \dfrac{(r_H')^2}{r_H^3}}{(1+2a) + 2cm(\dot r_H + c_v c_r + 2c_v c_\theta r_H^2 a_k^{-1}\sin^{-1}\theta_0)} \tag{42}$$

where $a_k = a_k\,(v_0)$. $\kappa(v_0, \theta_0)$ in **Eqs 29–31** is directly related to the small region $v_0, \theta_0$ at the event horizon $r_H$ of the black hole. In fact $\kappa$ is the surface gravity of the black hole. Let

$$\omega_0 = \lim_{\substack{r\to r_H\\ v\to v_0\\ \theta\to\theta_0}} \frac{C'}{B'}, \tag{43}$$

Be the chemical potential, also known as the maximal interleaving of Dirac energy levels of particles. Then **Eq. 36** is reduced to

$$\left(\frac{\partial S}{\partial r_*}\right)^2 - 2(\omega-\omega_0)\frac{\partial S}{\partial r_*} = 0. \tag{44}$$

By solving this equation, we get

$$\frac{\partial S}{\partial r_*} = (\omega-\omega_0) \pm (\omega-\omega_0). \tag{45}$$

According to **Eq. 29**, we have

$$\frac{\partial S}{\partial r} = \frac{1+2\kappa(r-r_H)}{2\kappa(r-r_H)}\frac{\partial S}{\partial r_*} \tag{46}$$

Therefore, the action $S$ of Dirac particle with mass $m$ and charge $Q$ can be obtained by integrating this equation and using the residue theorem, that is

$$S_\pm = \frac{\mathrm{i}\pi}{2\kappa}\left[(\omega-\omega_0)\pm(\omega-\omega_0)\right] \tag{47}$$

According to the quantum tunneling radiation theory and the semiclassical WKB approximation theory, the tunneling rate of Dirac particles at the event horizon of the non-stationary kennersley black hole should be

$$\Gamma \sim \exp(-2\mathrm{Im}(S_+ - S_-) = \exp\left(-\frac{\omega-\omega_0}{T_H}\right). \tag{48}$$

Here, $T_H$ is the Hawking temperature at the event horizon of the non-stationary Kennersley black hole, corrected by the Lorentz symmetry violating theory. It is linked to the surface gravity $\kappa$ at the event horizon by $T_H = \frac{\kappa}{2\pi}$. From **Eq. 42**, we get

$$\begin{aligned} T_H &= \frac{M + a_k r_H^2\cos\theta_0 - (r_H')^2 r_H^{-1}}{2\pi r_H^2\left[1+2a+2cm(\dot r_H + c_v c_r + 2cmc_v c_\theta a_k^{-1}\sin^{-1}\theta_0)\right]} \\ &= T_h[1 - 2(a + cm\dot r_H + cmc_v c_r \\ &\quad + 2cmc_v c_\theta a_k^{-1}\sin^{-1}\theta_0 r_H^{-2}) + \cdots], \end{aligned} \tag{49}$$

where

$$T_h = \frac{M + a_k r_H^2\cos\theta_0 - (r_H')^2 r_H^{-1}}{2\pi r_H^2} \tag{50}$$

is the uncorrected Hawking temperature at the event horizon of the black hole. For clarity, only the terms of the zeroth and first order are showed in **Eq. 49**. It can be seen from **Eq. 49** that the coefficients $a$, $c$ of the correction term in **Eq. 1** and the components $u^v$, $u^r$, $u^\theta$ of ether-like field vector all have an effect on $T_H$. Since there is killing vector $(\frac{\partial}{\partial\phi})^\alpha$ in this space-time, $u^\phi$ has no effect on $T_H$. Moreover, according to **Eqs 38–40**, **43**, **48**, the tunneling rate $\Gamma$ and chemical potential $\omega_0$ of Dirac particles in this space-time are also corrected, and similarly, the quantities $a$, $c$, $u^v$, $u^r$, $u^\theta$, $u^\phi$ all have influence on $\Gamma$ and $\omega_0$. What needs to be further explained is that $A_0$, $A_1$, $A_2$, and $A_3$ in **Eqs 39**, **40** in fact correspond to $A_v$, $A_r$, $A_\theta$ and $A_\phi$ in **Eq. 27**. The only difference is that $r$ and $\theta$ have been replaced by $r_H$ and $\theta_0$ in $A_0$, $A_1$, $A_2$, and $A_3$. Obviously, $A_\mu$ has also an effect on $\Gamma$ and $\omega_0$. If $A_\mu = 0$, then

$$C' = C_1' = (1+2a)p_\theta(r_H^2 r_H' - a_k\sin\theta_0) + 2cmC'' \tag{51}$$

and

$$\omega_0 = \frac{C_1'}{B'}. \tag{52}$$

If the correction item is ignored, the chemical potential will be reduced to

$$\omega_0 = p_\theta\left(\frac{r_H'}{r_H^2} - a_k\sin\theta_0\right), \tag{53}$$

where $p_\theta$ has been defined in **Eq. 35**.

## 4 CONCLUSION

Based on the Lorentz symmetric violating theory, the semiclassical theory and the quantum tunneling radiation theory, we get the dynamical equation of Dirac particles by studying **Eq. 1**, namely the Dirac-Hamilton-Jacobi equation shown as **Eq. 14** or **Eq. 16**. After giving explicit formula of the ether-like field vector $u^\mu$ and solving **Eq. 16**, the corrected tunneling rate of Dirac particles and the corrected Hawking temperature at the event horizon of the Kinnersley black hole are obtained. These new results are of great significance for further studying the thermodynamic evolution of black holes. It is necessary to further note that the key to solve Dirac-Hamilton-Jacobi equation is to construct the ether-like field vector correctly. The specific form of $u^{\mu}$ must be selected according to the characteristics of the curved space-time to be investigated, so as to ensure the validity of the derivation. The reference time $v_0$ and the reference angle $\theta_0$ in **Eq. 28** are arbitrarily selected, so the results derived from the general tortoise coordinate transformation are of universal significance. In addition, the entropy $S$ of black hole is closely related to Hawking temperature of black hole. Using the change of Bekenstein-Hawking entropy $\Delta S_{\mathrm{BH}}$ to express the tunneling rate will give $\Gamma \sim e^{\Delta S_{\mathrm{BH}}}$. Therefore, the entropy of the black hole should also be corrected. If the Lorentz symmetric violating is not considered, the results in this paper will return to the uncorrected cases that have been known ubiquitously.

## DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, further inquiries can be directed to the corresponding author.

## AUTHOR CONTRIBUTIONS

Z-EL completed all the derivation and the paper of writing. JZ checked the errors in equations. S-ZY put forward the idea.

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