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Narrow-Linewidth Laser Linewidth Measurement Technology

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A narrow-linewidth laser with excellent temporal coherence is an important light source for microphysics, space detection, and high-precision measurement. An ultranarrow-linewidth output with a linewidth as narrow as subhertz has been generated with a theoretical coherence length over millions of kilometers. Traditional grating spectrum measurement technology has a wide wavelength scanning range and an extended dynamic range, but the spectral resolution can only reach the gigahertz level. The spectral resolution of a high-precision Fabry–Pérot interferometer can only reach the megahertz level. With the continuous improvement of laser coherence, the requirements for laser linewidth measurement technology are increasing, which also promotes the rapid development of narrow-linewidth lasers and their applications. In this article, narrow-linewidth measurement methods and their research progress are reviewed to provide a reference for researchers engaged in the development, measurement, and applications of narrow-linewidth lasers.

Keywords: narrow-linewidth, laser, measurement, high coherence, beat note

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INTRODUCTION

Narrow-linewidth lasers with extremely low phase noise and a large coherence length have been widely used as a high-spectral-purity light source in gravitational wave detection [1, 2], optical atomic clocks [3, 4], lidar [5, 6], high-speed coherent optical communication [7, 8], and distributed optical fiber sensing [9, 10]. The main reason for the linewidth generation is the phase fluctuation caused by spontaneous radiation [11] and the noise induced by mechanical and temperature factors [12, 13]. Therefore, the laser linewidth reflects the physical and frequency stability of the laser. Scully and Lamb [14] proposed the laser quantum theory. They deduced that the spectral profile of the laser is Lorentzian and calculated its width (full width at half height). The linewidth value, an essential parameter of the laser, directly affects the accuracy of the narrow-linewidth laser in detection [15], sensitivity in sensing [16], and bit error rate in communication [17, 18]. Therefore, precise measurement of the linewidth value is a prerequisite for the application of narrow-linewidth lasers.

Different types of laser produce a broad coverage of linewidths, as large as tens of gigahertz [19, 20] and as small as subhertz [21, 22]. At present, the resolution of a commercial optical spectrum analyzer based on diffraction gratings is approximately 0.05 nm (gigahertz level)[1], and the resolution of Fabry–Pérot interferometers can reach a few megahertz[2]. The rapid development and application

[1]https://tmi.yokogawa.com.
[2]https://www.thorlabs.com.

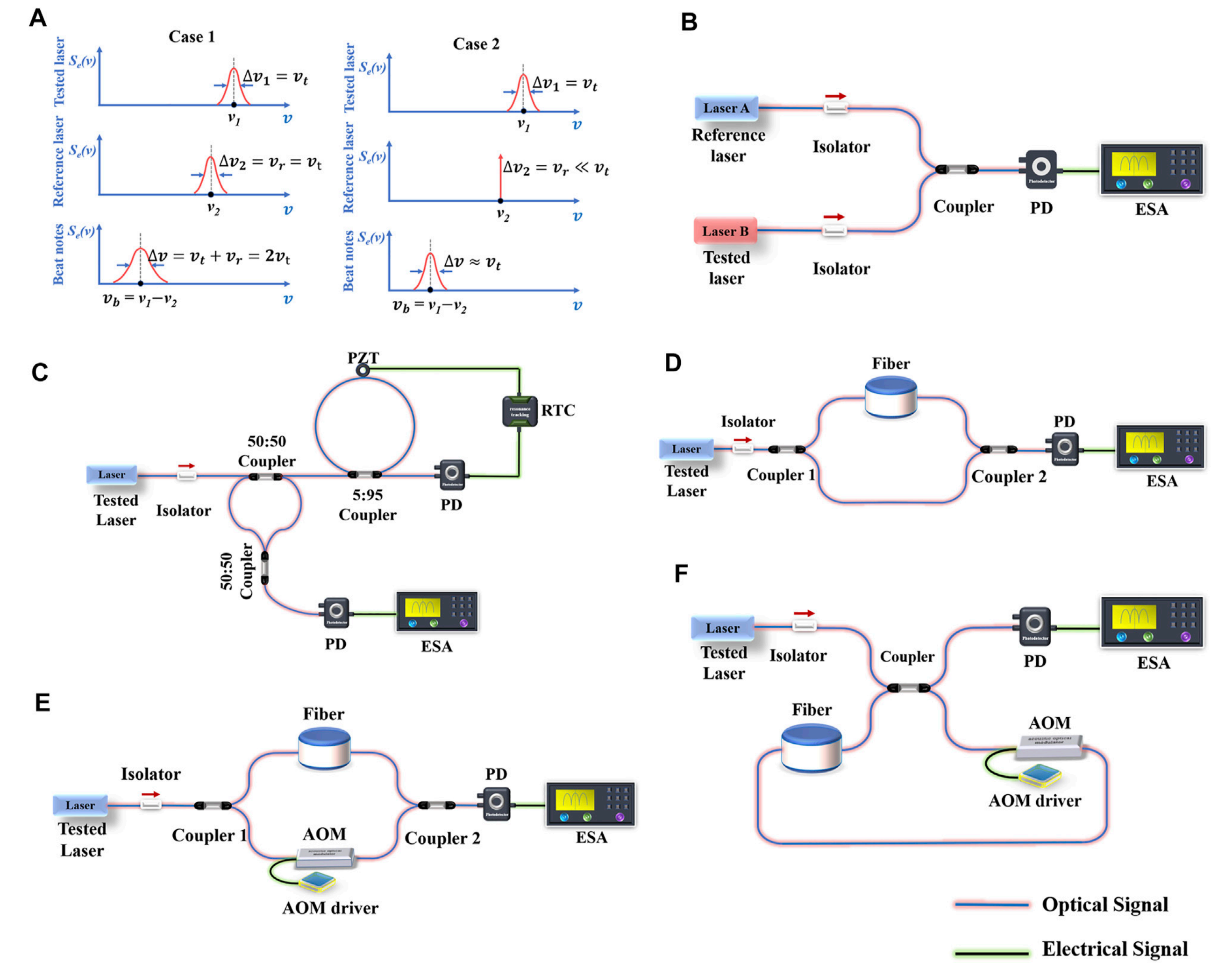

FIGURE 1 | (A) Principle of optical beat notes, **(B)** beating note with a reference laser, **(C)** beating note with the Stokes wave of a Brillouin laser, **(D)** delayed self-homodyne interferometric detection, **(E)** delayed self-heterodyne interferometric detection, and **(F)** recirculating delayed self-heterodyne interferometer detection (PD, photodetector; ESA, electrical spectrum analyzer; RTC, resonance tracking circuit; PZT, piezoelectric transducer; AOM, acoustic optical modulator).

of narrow-linewidth lasers have resulted in higher requirements for laser linewidth measurement technology. Specific devices must be built for lasers with a narrower linewidth (sub-megahertz) to measure the linewidth. In the past few decades, laser frequency stabilization [23, 24] and mode selection [25] have matured, and many narrow-linewidth measurement schemes are constantly being updated.

In this article, typical methods for measuring narrow-linewidth lasers are reviewed, and the characteristics of each method, as well as the status of its development, are summarized. Finally, a summary and an outlook for the future development of narrow-linewidth measurements are provided.

Narrow-Linewidth Laser Measurement Method

Two methods are mainly used for linewidth measurement: directly calculating the laser linewidth using the power spectrum density (PSD) of the laser and deducing the linewidth indirectly based on the relationship between the phase noise and linewidth. The power spectrum contains more-intuitive linewidth information, and it is relatively easy to obtain; therefore, a large proportion of linewidth measurement experiments focus on the former. Optical beat notes are necessary to obtain the PSD. The mixed signal of two incoherent lasers, each with a Lorentzian line shape, still has a Lorentzian line shape, and the PSD of the beat notes can be expressed as [26].

$$s(v) = \frac{\Delta v}{2\pi\left[(v - v_b)^2 + (\Delta v/2)^2\right]} \tag{1}$$

where $\Delta v = v_t + v_r$, v_t is the linewidth of the tested laser, v_r is the linewidth of the reference laser, and v_b is the difference between the output frequencies of the two lasers mentioned above (also the center frequency of the beat notes). Two lasers with the same or similar frequencies interfere and produce a beat signal with a lower frequency. The linewidth of the beat signal is the sum of the widths of the two lasers participating in the beat. There are usually two cases of beat notes suitable for linewidth measurement, as shown in **Figure 1A**. If $\nu_t = \nu_r$, the linewidth of the beat notes is twice the linewidth of the tested laser ($\Delta\nu = 2\nu_t$, case 1). If $\nu_t \gg \nu_r$, (that is, the output spectrum of the reference laser is exceptionally narrow and the linewidth can be ignored compared with the tested laser), the linewidth value of the beat signal is approximately the value of the linewidth of the tested laser ($\Delta\nu \approx \nu_t$, case 2). These two cases are flexibly used in different measurement structures according to the specific conditions of the tested lasers. In this section, the focus is on four measurement structures based on the above two cases.

Beating Note With a Reference Laser

Figure 1B shows the measurement structure of using an additional laser to generate a reference laser beam and beating with the tested laser. The photodetector (PD) receives the mixed signal and transmits it to an electrical spectrum analyzer (ESA). Based on case 1, a laser with the same linewidth as the test is used to serve as the reference laser. In this case, any linewidth can be measured, but obtaining, measuring, and calibrating a reference laser is difficult. Based on case 2, using a reference laser whose linewidth value is negligible compared with the tested laser can meet the linewidth measurement requirements of most lasers; however, it is difficult to measure the ultranarrow linewidth in this case accurately. In addition, the two lasers participating in the beating should have the same amplitude, the same optical frequency (or a slight difference), and constant phase difference [27], which requires a stable experimental environment, increasing the difficulty and cost of testing.

Although this method has high requirements for the stability of the two lasers and the experimental environment, scientific researchers still favor it. In particular, ultranarrow-linewidth measurement based on case 1 does not require extra algorithm design, such as measuring the ultranarrow linewidth (see *Delayed self-heterodyne interferometric detection*), when analyzing beat notes. This avoids calculation errors, and the results are more convincing. In 1999, Young et al. [28] used a high-precision Fabry–Pérot cavity to realize a narrow-linewidth output. They built a second similar cavity for laser stability measurement (including linewidth measurement) and adjusted the laser frequency difference ν_b between the two cavities to 400 MHz to avoid low-frequency noise. They designed an independent vacuum chamber, temperature control system, and vibration isolation platform to ensure the stability of the two lasers. The measured linewidth was 0.6 Hz. In 2012, Kessler et al. [29] used two traditional cavity-stabilized lasers as reference laser sources to study a test laser. By analyzing the heterodyne signals among the three lasers, they concluded that the stability of the tested laser was the best, so the linewidth of the laser under test was the narrowest. The narrowest linewidth value obtained by the heterodyne signals between the tested laser and the other two lasers was 49 mHz, so they judged the linewidth of the tested laser to be less than 40 mHz. In 2013, Lee et al. [30] reported a chip-based resonator in the form of a spiral. It had a strong resistance to thermal noise and mechanical noise interference. Two identical fiber lasers were locked to the spiral resonator using a Pound–Drever–Hall locking system. The linewidth obtained by analyzing the beat notes of the two locked lasers was less than 100 Hz. In 2015, Liang et al. [31] built a laser identical to the tested laser to measure the narrow linewidth. They designed a thermal package and a sealed environment for the reference resonator to avoid technical noise caused by environmental and temperature fluctuations, and they concluded that the laser has an integrated linewidth of 30 Hz and a subhertz instantaneous linewidth. In 2018, Pavlov et al. [32] achieved a single-frequency output by using self-injection locking. They used a narrow-linewidth tunable fiber laser to perform heterodyne measurement with the tested laser and obtained a 340-Hz Lorentzian width and 1.7-kHz Gaussian width. According to Voigt linewidth theory [33], it was concluded that the laser linewidth was less than 1 kHz.

Beating Note With the Stokes Wave of Brillouin Laser

This approach does not require an additional reference laser, which solves the structural limitation that an additional laser must be used to provide a reference laser beam for linewidth measurement (see *Beating note with a reference laser*). As shown in **Figure 1C**, the tested laser was divided into two beams by a coupler. One beam is injected into the fiber ring cavity to form a Brillouin laser, and the other couples with the first-order Stokes wave propagating backward in the Brillouin laser. The coupling signal enters the PD to beat, and the beat note signal is fed into the ESA. The measurement principle is based on case 2. The tested laser is also the pump used to generate the Brillouin laser. The first-order Stokes beam linewidth ν_r of the Brillouin laser is usually much smaller than the pump linewidth, so the linewidth of the beat notes is approximately the linewidth of the tested laser ($\Delta\nu \approx \nu_r$). Compared with the structure discussed in *Beating note with a reference laser*, this scheme only requires the tested laser to complete the linewidth measurement and makes testing easier. Generally, the Brillouin frequency shift reaches 10–20 GHz [34] in the optical fiber. This frequency shift is the frequency difference ν_b between the Stokes wave and the tested laser. It is also the center frequency of the beat notes. Therefore, a large Brillouin frequency shift causes the center frequency of the beat signal to exceed the measurement range of the ESA, which causes difficulties in the measurement. In principle, this scheme is only suitable for 1–500-kHz linewidth measurement [35]. When the linewidth of the tested laser is extremely narrow, this scheme is no longer applicable, and significant errors occur.

In 1991, Smith [36] verified the good linewidth compression effect of the Brillouin laser by building a Brillouin fiber ring laser. In 1994, Boschung [37] realized a 3.84-Hz linewidth Brillouin laser output in a fiber ring cavity by using an incident laser with a linewidth of 100 kHz. These works provide a theoretical and experimental foundation for the beating frequency with the Stokes wave of the Brillouin laser. In 1996, Kueng [35] built the measurement structure shown in **Figure 1C** for the first time and obtained a linewidth of 4.2 kHz. In 2005, Dong et al. [38]

proposed a scheme using a second-order Stokes wave in the fiber ring as the reference beam, which further improved the accuracy of the Brillouin laser first-order Stokes optical beat method.

Delayed Self-Homodyne Interferometric Detection

The delayed self-homodyne interferometric structure is shown in **Figure 1D**, and the measurement principle is based on case 1. This structure is based on the unbalanced Mach–Zehnder interferometer (UMZI) [39]. It does not have the measurement range limitation of the structure in *Beating note with the Stokes wave of Brillouin laser* and has the advantages of a simple structure, wide measurement range, and low optical transmission loss. However, the center frequency v_b of the beat notes is 0 Hz; therefore, the low-frequency noise in the environment interferes with the measurement results. Considering the sensing characteristics of the optical fiber [40], the disturbance of environmental noise, temperature, pressure, and other factors can seriously affect the test results obtained by the optical fiber [41].

In 1986, Ryu [42] found that this method has the advantages of simple setting and high resolution when measuring narrow-linewidth lasers. In 1989, Nazarathy [43] derived the photocurrent power spectral density for this structure. In 1998, Ludvigsen [44] reported an optimized delay homodyne method in which the photocurrent signal was amplified by a low-noise amplifier and mixed with a stable 200 MHz local oscillator in a double-balanced mixer. They obtained a 460-kHz linewidth value with only 71.7 m of fiber. Because of its poor stability, this structure has rarely been used for direct linewidth measurement in recent years, but it often appears in optical frequency discriminators (OFDs) for linewidth measurement. The measurement principle of OFDs is no longer based on case 1 or case 2, and the linewidth information is calculated from the noise spectrum. In 2019, Gundavarapu [45] et al. reported a narrow-linewidth Brillouin laser output on an integrated Si_3N_4 waveguide platform. They measured the linewidth based on an OFD using a fiber-based UMZI and a balanced PD (the PD in **Figure 1D** is changed to a balanced PD), and the linewidth value was calculated to be 0.7 Hz. In 2021, Chauhan [46] et al. reported a visible-light photonic integrated Brillouin laser, and the measured laser linewidth value was 269 Hz using an OFD.

Delayed Self-Heterodyne Interferometric Detection

Delayed self-heterodyne interferometry (DSHI) overcomes the shortcoming that delayed self-homodyne interferometric structures are susceptible to low-frequency noise. The measurement principle is based on case 1. As shown in **Figure 1E**, an acousto-optic modulator (AOM) is introduced to shift the center frequency of the beat notes to a high frequency that is not affected by the environment to reduce system errors and improve measurement accuracy. Nevertheless, the DSHI method to measure narrow-linewidth lasers must be completed under the condition that the delay time is much longer than the coherence time. In theory, to test a 100-Hz linewidth, the fiber length required is as much as 1,590 km [47]. The long optical fiber increases the experimental volume and attenuation and introduces 1/f noise that can cause spectral line broadening [48], which results in larger measurement errors. Moreover, when the output power of the tested laser is large, the long-delay fiber generates stimulated Brillouin scattering, which is opposite to the direction of laser transmission; in this case, the incident pump energy is converted into Stokes wave and sound wave energy, increasing the transmission loss and even making the PD unable to detect the signal. The DSHI method was first proposed by Okoshi [49] in 1980. In 1986, Richter [50] derived the power spectral density function of the beat notes under the DSHI structure and reported that the measurement result would be more accurate with a delay time much longer than the coherence time of the tested laser. The spectral line of the beat signal is generally fitted by a Lorentzian function [51]. To offset the influence of 1/f noise introduced by long optical fiber, Chen [33] proposed a fitting scheme based on the Voigt profile. The Voigt function is the convolution of the Gaussian and Lorentz functions [52, 53]. Using this function to fit the collected data can effectively filter out the influence of 1/f noise.

To remove the 1/f noise from the root, researchers have proposed a short-fiber delayed self-heterodyne interferometer strategy for linewidth measurement. When the fiber is short, the PSD function is no longer expressed by **Eq. 1**, and the complete PSD function is expressed as [49, 54].

$$
\begin{aligned}
S(f) = \frac{P_0^2}{4\pi}\frac{\Delta f}{\Delta f^2 + (f - f_1)^2}\Bigg\{1 - exp(-2\pi\tau_d\Delta f) \\
\times\left[\cos(2\pi\tau_d(f - f_1)) + \Delta f\frac{\sin(2\pi\tau_d(f - f_1))}{f - f_1}\right]\Bigg\} \\
+ \frac{\pi P_0^2}{2}exp(-2\pi\tau_d\Delta f)\delta(f - f_1)
\end{aligned}
\tag{2}
$$

where P_0 is the power of the beat signal, Δf is the laser linewidth, f_1 is the AOM modulation frequency, τ_d is the delay time (proportional to the fiber length), and $\delta(f)$ is the impact function. According to the PSD function under a short optical fiber delay, different schemes have been designed to calculate the laser linewidth. In 2008, Jia [55] used polynomial fitting to eliminate the defect that caused the measurement accuracy to drop significantly when the delay time was insufficient and measured an 8-kHz linewidth with 25-km fiber. In 2015, Wei [56] obtained a numerical solution for the laser linewidth by measuring the frequency difference between the minimum points next to the maximum at the center frequency of the power spectrum. In principle, measuring a linewidth of 10 kHz requires only 300 m of delay fiber. In 2016, Huang [54, 57] reported an approach for contrasting the difference with the second peak and the second trough (CDSPST) of the coherence envelope to determine the laser linewidth. Only 3 km of fiber was used to measure a linewidth of 150 Hz. In 2018, Bai [58] successfully measured a laser with a linewidth of 98 Hz using

TABLE 1 | Comparison of laser linewidth measurement methods.

Type	Advantages	Disadvantages	Linewidth	Ref
Beating note with a References laser	1. Easy to calculate 2. The measurement results are accurate (especially measuring ultranarrow linewidth according to case 1)	1. Two independent lasers are required 2. The References laser should be adjusted according to the tested laser	0.6 Hz 40 mHz 100 Hz 30 Hz	[28] [29] [30] [31]
		3. Obtaining, measuring, and calibrating the References laser is complicated	<1 kHz	[32]
Beating note with the Stokes wave of Brillouin laser	1. Only one laser can complete the linewidth measurement	1. The Brillouin frequency shift in the optical fiber exceeds the measurement range of some standard ESAs, which causes difficulties for the measurement	4.2 kHz	[35]
	2. The linewidth can be measured within a certain range without adjusting the structure	2. The Brillouin laser has a linewidth that makes the method unsuitable for the case where the linewidth of the tested laser is ultranarrow		
Delay self-homodyne interferometric detection	1. The structure is simple and easy to build 2. It can measure the linewidth within a certain range without adjusting the structure	The measurement results are extremely susceptible to low-frequency noise	460 kHz 0.7 Hz (OFD)	[44] [45]
			269 Hz (OFD)	[46]
Delayed self-heterodyne interferometric detection	1. The structure is simple and easy to build 2. It can measure the linewidth within a certain range without adjusting the structure	A long fiber introduces 1/f noise and causes spectral line broadening	8 kHz 150 Hz	[55] [54]
	3. The measurement structure is stable and not susceptible to external interference		98 Hz 2.53 kHz 458 Hz 151 Hz	[58] [59] [60] [61]

the CDSPST method with a fiber delay of 2,950 m. In 2019, He [59] reported a linewidth demodulation scheme that achieved linewidth measurement by demodulating the coherent envelope of a short-delay self-heterodyne interference spectrum, and used this method to demodulate a 2.53-kHz linewidth. In 2020, Wang [60] reported a dual-parameter acquisition method and used it to calculate a 458-Hz linewidth by obtaining the frequency difference and amplitude difference of the coherence envelope. In 2021, Xue [61] reported a linewidth measurement method that combined long and short fibers. The measurement result of the long-delay fiber was used as the initial value, and the short-fiber self-heterodyne measurement results were demodulated. Xue used this method to demodulate a laser linewidth of 151 Hz successfully.

Other Measurement Methods

In addition to the above approaches, many other structures and optimization algorithms for narrow-linewidth laser measurements have been developed. The recirculating delayed self-heterodyne interferometer (RDSHI) method (see **Figure 1F**) is also widely used [62–65]. The unique fiber ring structure of this approach permits the delay fiber to increase the delay time by several times between two laser beams [66]. It is also an approach for improving the traditional DSHI method. Moreover, there is a close relationship between frequency noise and linewidth. Domenico et al. [67] proposed an approximate formula to show the relationship between the linewidth and frequency noise. Zhou et al. [68] reported a method to estimate the laser linewidth from its frequency power spectral density, called the "power-area method." The β-separation line technique is an application of this relationship, which is convenient for calculating the laser linewidth after obtaining the laser frequency noise data [69]. Xu [70] et al.reported a method to measure the linewidth by using an unbalanced Michelson interferometer composed of a 3 × 3 optical fiber coupler. These structures can achieve a relatively accurate measurement of linewidth at a level of 1,000 Hz.

CONCLUSION AND OUTLOOK

Linewidth measurement technology is an essential part of the research and development process of narrow-linewidth lasers. Here, the typical methods for narrow-linewidth measurement were summarized, and the characteristics of various schemes were analyzed, as shown in **Table 1**. At present, measurement technology based on DSHI methods is developing rapidly, and its application is the most extensive among the methods. The method of beating notes with a reference laser is also used to test some ultranarrow-linewidth lasers owing to its excellent accuracy. The future renewal of linewidth compression technology is expected to place higher requirements on linewidth measurement technology. The RDSHI method has the advantages of the DSHI method. The multiorder beat signal measured by this scheme can avoid random errors and is expected to become the next research focus of narrow-linewidth measurement technology. In addition, Pollnau et al. [71, 72] questioned the traditional linewidth theory in recent studies, and they pioneered the theory that laser linewidth is a classical physics phenomenon, which may have an impact on linewidth measurement technology.

AUTHOR CONTRIBUTIONS

ZB: Investigation, Writing—original draft, Writing—review and editing, Supervision. ZZ: Investigation, Writing—original draft. YQ: Investigation, Writing—review and editing. JD: Investigation, Writing—review and editing. SL: Investigation, Writing—review and editing. XY: Writing—review and editing. YW: Writing—review and editing, Supervision. ZL: Writing—review and editing, Supervision.

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