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Solitary wave solutions of Fitzhugh–Nagumo-type equations with conformable derivatives

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The Fitzhugh–Nagumo equation is an important non-linear reaction–diffusion equation used to model the transmission of nerve impulses. This equation is used in biology as population genetics; the Fitzhugh–Nagumo equation is also frequently used in circuit theory. In this study, we give solutions to the fractional Fitzhugh–Nagumo (FN) equation, the fractional Newell–Whitehead–Segel (NWS) equation, and the fractional Zeldovich equation. We found the exact solutions of these equations by conformable derivatives. We have obtained the exact solutions within the time-fractional conformable derivative for these equations.

KEYWORDS

exact solutions, conformable derivative, the Fitzhugh-Nagumo equation, solitary wave solution, solitons

1 Introduction

Fractional differential equations (FDEs) are generalizations of known differential equations (ODEs). FPDEs are used effectively in many fields of science [1–4]. These equations are significant models to interpret plasma physics, relativistic physics, quantum mechanics, non-linear optics, etc. So, FPDE studies are getting more and more important. Recent developments and applications in fractional calculus have been discussed by many authors [5–9]. Many fractional models can be converted to a FODE, allowing us to use the power-series technique to find all open-series analytical solutions.

The fractional Fitzhugh–Nagumo equation is an important non-linear reaction–diffusion equation used to model the transmission of nerve impulses. This equation is used in biology as population genetics; the fractional Fitzhugh–Nagumo equation is also frequently used in circuit theory.

There are numerous effective ways to find solutions of PDEs. These methods are the $(\frac{G'}{G})$-expansion [10], the sub-equation (11) and (12), the exp-function [13, 14], the first integral [15], the functional variable [16, 17], the modified simplest [18, 19], the Kudryashov [20, 21], the extended simple [22], and the extended tanh–coth methods

[23, 24]. These techniques allow for the calculation of PDE solutions in a variety of different formats.

These equations we deal with in this study are effective equations that play a fundamental role in many phenomena such as plasma physics and optics. The non-linear phenomena of wave happen in different fields such as optical fiber, physics, and biology. It is necessary to gain the exact solutions of such models for the better understanding of non-linear wave phenomena.

In the second part, the conformable derivative is introduced. In the third part, the extended tanh–coth method is given. In the other sections, we found the exact solutions of the (1 + 1) dimensional time-fractional Fitzhugh–Nagumo (FN) equation, the Newell–Whitehead (NW) equation, and the Zeldovich equation *via* this method.

2 Conformable derivative

Definition 1. The basic limit definition of this derivative is [25]:

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\xi)^{-\alpha} \big(f(\xi) - f(0)\big) d\xi \;, & 0 < \alpha < 1, \\ \big(f^{(n)}(t)\big)^{(\alpha-n)} \;, & n \le \alpha < n+1, \quad n \ge 1. \end{cases} \tag{1}$$

Some properties of this derivative are given in [26, 27].

Definition 2. Let g: (0, ∞) → R be a function. The conformable derivative of g for order α is defined by

$$T_\alpha(g(k)) = \lim_{\varepsilon \to 0} \frac{g(k + \varepsilon k^{1-\alpha}) - g(k)}{\varepsilon}, \tag{2}$$

for all $k > 0$, $\alpha \in (0, 1)$.

Theorem 3. If a function g: $[0, \infty) \to R$ is $\alpha -$ differentiable at $t_0 > 0$, $\alpha \in (0, 1]$ and g is continuous at t_0.

Theorem 4. Let f and g be $\alpha -$ differentiable at a point $t > 0$, $\alpha \in (0, 1]$:

$$\begin{aligned} T_\alpha(\alpha f + bg) &= aT_\alpha(f) + bT_\alpha(g), \; for\, all\, a, b \in R. \\ T_\alpha(t^p) &= pt^{p-\alpha}, \; for\, all\, p \in R. \\ T_\alpha(\lambda) &= 0, \; for\, constant\, functions\, f(t) = \lambda. \\ T_\alpha(fg) &= fT_\alpha(g) + gT_\alpha(f). \\ T_\alpha\left(\frac{f}{g}\right) &= \frac{gT_\alpha(f) - fT_\alpha(g)}{g^2}. \end{aligned} \tag{3}$$

If g is differentiable,

$$T_\alpha(g)(t) = t^{1-\alpha} \frac{dg}{dt}(t). \tag{4}$$

Non-linear conformable partial differential equations (NCPDEs) with one independent variable are as follows:

$$P\left(\frac{\partial^\alpha u}{\partial t^\alpha}, \frac{\partial u}{\partial x}, \frac{\partial^{2\alpha} u}{\partial t^{2\alpha}}, \frac{\partial^2 u}{\partial x^2}, \ldots\right) = 0, \; 0 < \alpha \le 1, \tag{5}$$

$$u(x,t) = U(\xi), \; \xi = cx - v\frac{t^\alpha}{\alpha}, \tag{6}$$

$$\frac{\partial^\alpha}{\partial t^\alpha} = -v\frac{\partial}{\partial \xi}, \; \frac{\partial^{2\alpha}}{\partial t^{2\alpha}} = v^2 \frac{\partial^2}{\partial \xi^2}, \; \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \; \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \ldots \tag{7}$$

The non-linear fPDE (5) can be converted to a non-linear ODE using Eq. 6:

$$Q\left(U, U', U'', U''', \ldots\right) = 0. \tag{8}$$

Let us now discuss how we will approach solving the Fitzhugh–Nagumo equation, the Newell–Whitehead equation, and the Zeldovich equation.

3 The extended tanh–coth method

The tanh–coth approach can be summed up as follows in [28]:

The wave variable $\xi = cx - v\frac{t^\alpha}{\alpha}$ into a PDE

$$P(u, u_t, u_x, u_{xx}, \ldots) = 0. \tag{9}$$

$u(x, t) = U(\xi)$ is a traveling wave solution. A non-linear ODE can be converted from the non-linear PDE (9):

$$Q\left(U, U', U'', U''', \ldots\right) = 0. \tag{10}$$

From the derivatives of the independent variable Y,

$$Y = tanh(\xi), \; Y' = 1 - Y^2, \tag{11}$$

$$\begin{aligned} \frac{d}{d\xi} &= (1 - Y^2)\frac{d}{dY}, \\ \frac{d^2}{d\xi^2} &= (1 - Y^2)\left(-2Y\frac{d}{dY} + (1 - Y^2)\frac{d^2}{dY^2}\right). \end{aligned} \tag{12}$$

The tanh method can be expressed as follows:

$$U(\xi) = S(Y) = \sum_{k=0}^{m} a_k Y^k. \tag{13}$$

The following is an extension of Eq. 13 [29]:

$$U(\xi) = S(Y) = \sum_{k=0}^{m} a_k Y^k + \sum_{k=1}^{m} b_k Y^{-k}. \tag{14}$$

4 Solutions of the Fitzhugh–Nagumo equation

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u(1-u)(u-\mu), \; t > 0, \; 0 < \alpha \le 1, \; x \in R. \tag{15}$$

The FN equation is an important non-linear reaction–diffusion equation used to model the transmission of nerve impulses. Also used in biology as population genetics, this equation is also frequently used in circuit theory, where x is the space variable and t is a time variable.

Since the FN equation is (1 + 1)-dimensional and time-fractional, using the wave variable $\xi = cx - v\frac{t^\alpha}{\alpha}$, Eq. 15 can be converted into a non-linear ODE (Eq. 16).

$$vU' + c^2U'' + U(1-U)(U-\mu) = 0, \tag{16}$$

where c and v are constants. By balancing $(U)^3$ with U'' in Eq. 16, we obtain the following:

$$3m = m + 2, \tag{17}$$

$$m = 1. \tag{18}$$

The solution form is as follows:

$$U(\xi) = a_0 + a_1 Y + b_1 Y^{-1},\ Y = \tanh(\xi),\ Y' = 1 - Y^2. \tag{19}$$

Here a_0 and a_1, b_1 are arbitrary constants. Eq. 19 is substituted into Eq. 16, and if the coefficients of Y are used to set the system of equations to zero, then

$$\begin{aligned}
-a_1^3 + 2a_1c^2 &= 0.\\
-3a_0a_1^2 + a_1^2\mu + a_1^2 - a_1v &= 0.\\
-3a_0^2a_1 + 2a_0a_1\mu - 3a_1^2b_1 - 2a_1c^2 + 2a_0a_1 - a_1\mu &= 0.\\
-a_0^3 + a_0^2\mu - 6a_0a_1b_1 + 2a_1b_1\mu + a_0^2 - a_0\mu + 2a_1b_1 + a_1v + b_1v &= 0.\\
-3a_0^2b_1 + 2a_0b_1\mu - 3a_1b_1^2 - 2b_1c^2 + 2a_0b_1 - b_1\mu &= 0.\\
-3a_0b_1^2 + b_1^2\mu + b_1^2 - b_1v &= 0.\\
-3a_0b_1^2 + b_1^2\mu + b_1^2 - b_1v &= 0.
\end{aligned} \tag{20}$$

By solving this system of equations, we get the following cases.

Case 1:

$$\begin{aligned}
a_0 &= \frac{1}{2}, \quad a_1 = 0,\ b_1 = \pm\frac{1}{2},\\
c &= \pm\frac{\sqrt{2}}{4},\ \mu = \mu,\ v = \pm\left(\frac{\mu}{2} - \frac{1}{4}\right).
\end{aligned} \tag{21}$$

Therefore, the exact solution is as follows:

$$u_1(x,t) = \frac{1}{2} \pm \frac{1}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}x - \left(\pm\frac{\mu}{2} - \frac{1}{4}\right)\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \tag{22}$$

Case 2:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = 0, \quad b_1 = \pm a_0,\\
c &= \pm\frac{\sqrt{2}}{2}a_0,\ \mu = 2a_0,\ v = \pm\left(a_0 - a_0^2\right).
\end{aligned} \tag{23}$$

The exact solution is as follows:

$$u_2(x,t) = a_0 \pm a_0\left(\tanh\left(\pm\frac{\sqrt{2}}{2}a_0x \pm \left(a_0 - a_0^2\right)\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \tag{24}$$

Case 3:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = 0, \quad b_1 = \pm(a_0 - 1),\\
c &= \pm\frac{\sqrt{2}}{2}(a_0 - 1),\ \mu = 2a_0 - 1,\ v = \pm a_0(a_0 - 1).
\end{aligned} \tag{25}$$

The exact solution is as follows:

$$u_3(x,t) = a_0 \pm a_0\left(\tanh\left(\pm\frac{\sqrt{2}}{2}(a_0 - 1)x \pm a_0(a_0 - 1)\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \tag{26}$$

Case 4:

$$\begin{aligned}
a_0 &= \frac{1}{2}, \quad a_1 = \pm\frac{1}{2},\ b_1 = 0,\\
c &= \pm\frac{\sqrt{2}}{4},\ \mu = \mu, \quad v = \pm\left(\frac{\mu}{2} - \frac{1}{4}\right).
\end{aligned} \tag{27}$$

The exact solution is as follows:

$$u_4(x,t) = \frac{1}{2} \pm \frac{1}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}x \pm \left(\frac{\mu}{2} - \frac{1}{4}\right)\frac{t^\alpha}{\alpha}\right)\right). \tag{28}$$

Case 5:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = \pm a_0,\ b_1 = 0,\\
c &= \pm\frac{\sqrt{2}}{2},\ \mu = 2a_0, \quad v = \pm\left(a_0^2 - a_0\right).
\end{aligned} \tag{29}$$

The exact solution is as follows:

$$u_5(x,t) = a_0 \pm a_0\left(\tanh\left(\pm\frac{\sqrt{2}}{2}x \pm \left(a_0^2 - a_0\right)\frac{t^\alpha}{\alpha}\right)\right). \tag{30}$$

Case 6:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = \pm(a_0 - 1),\ b_1 = 0,\\
c &= \pm\frac{\sqrt{2}}{2}(a_0 - 1),\ \mu = 2a_0 - 1, \quad v = \pm\left(a_0^2 - a_0\right).
\end{aligned} \tag{31}$$

The exact solution is as follows:

$$\begin{aligned}
u_6(x,t) &= a_0 \pm (a_0 - 1)\\
&\quad \times\left(\tanh\left(\pm\frac{\sqrt{2}}{2}(a_0 - 1)x \pm \left(a_0^2 - a_0\right)\frac{t^\alpha}{\alpha}\right)\right).
\end{aligned} \tag{32}$$

Case 7:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = \pm\frac{a_0}{2},\ b_1 = \pm\frac{a_0}{2},\\
c &= \pm\frac{\sqrt{2}}{4}a_0,\ \mu = 2a_0, \quad v = \pm\frac{1}{2}\left(a_0^2 - a_0\right).
\end{aligned} \tag{33}$$

The exact solution is as follows:

$$\begin{aligned}
u_7(x,t) &= a_0 \pm \frac{a_0}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}a_0x \pm \frac{1}{2}\left(a_0^2 - a_0\right)\frac{t^\alpha}{\alpha}\right)\right)\\
&\quad \pm \frac{a_0}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}a_0x \pm \frac{1}{2}\left(a_0^2 - a_0\right)\frac{t^\alpha}{\alpha}\right)\right)^{-1}.
\end{aligned} \tag{34}$$

Case 8:

$$\begin{aligned}
a_0 &= a_0, \quad a_1 = \pm\frac{1}{2}(a_0 - 1),\ b_1 = \pm\frac{1}{2}(a_0 - 1),\\
c &= \pm\frac{\sqrt{2}}{4}(a_0 - 1),\ \mu = 2a_0 - 1, \quad v = \pm\frac{1}{2}\left(a_0^2 - a_0\right).
\end{aligned} \tag{35}$$

The exact solution is as follows:

$$u_8(x,t) = a_0 \pm \frac{(a_0-1)}{2}\left[\left(\tanh\left(\pm\frac{\sqrt{2}}{4}(a_0-1)x \pm \frac{1}{2}(a_0^2-a_0)\frac{t^\alpha}{\alpha}\right)\right) + \left(\tanh\left(\pm\frac{\sqrt{2}}{4}(a_0-1)x \pm \frac{1}{2}(a_0^2-a_0)\frac{t^\alpha}{\alpha}\right)\right)^{-1}\right]. \quad (36)$$

Case 9:

$$\begin{aligned} a_0 &= \frac{1}{2}, \quad a_1 = \pm\frac{1}{4}, b_1 = \pm\frac{1}{4}, \\ c &= \pm\frac{\sqrt{2}}{8}, \quad \mu = \mu, \quad v = \pm\left(\frac{\mu}{4}-\frac{1}{8}\right). \end{aligned} \quad (37)$$

The exact solution is as follows:

$$u_9(x,t) = \frac{1}{2} \pm \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \left(\frac{\mu}{4}-\frac{1}{8}\right)\frac{t^\alpha}{\alpha}\right)\right) \pm \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \left(\frac{\mu}{4}-\frac{1}{8}\right)\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \quad (38)$$

5 Solutions of the Newell–Whitehead equation

If $\mu = -1$ in Eq. 15, the NW equation is obtained by

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u - u^3,\ t>0,\ 0<\alpha\le 1,\ x\in R. \quad (39)$$

Using the wave variable $\xi = cx - v\frac{t^\alpha}{\alpha}$, Eq. 39 can be converted to Eq. 40.

$$vU' + c^2U'' + U - U^3 = 0, \quad (40)$$

where c and v are constants. By balancing, we obtain

$$3m = m+2, \quad (41)$$
$$m = 1. \quad (42)$$

The solution form is as follows:

$$U(\xi) = a_0 + a_1Y + b_1Y^{-1},\ Y = \tanh(\xi),\ Y' = 1-Y^2. \quad (43)$$

Here, a_0 and a_1, b_1 are arbitrary constants. Eq. 43 is substituted into Eq. 41, and if the coefficients of Y are used to set the system of algebraic equations to zero, then

$$\begin{aligned} -a_1^3 + 2a_1c^2 &= 0. \\ -3a_0a_1^2 - a_1v &= 0. \\ -3a_0^2a_1 - 3a_1^2b_1 - 2a_1c^2 + a_1 &= 0. \\ -a_0^3 - 6a_0a_1b_1 + a_1v + b_1v + a_0 &= 0. \\ -3a_0^2b_1 - 3a_1b_1^2 - 2b_1c^2 + b_1 &= 0. \\ -3a_0b_1^2 - b_1v &= 0. \\ -b_1^3 + 2b_1c^2 &= 0. \end{aligned} \quad (44)$$

By solving this system of equations, we get the following cases.

Case 1:

$$\begin{aligned} a_0 &= \pm\frac{1}{2}, \quad a_1 = 0, b_1 = \frac{1}{2}, \\ c &= \pm\frac{\sqrt{2}}{4}, \quad v = \pm\frac{3}{4}. \end{aligned} \quad (45)$$

Therefore, the exact solution is as follows:

$$u_1(x,t) = \pm\frac{1}{2} + \frac{1}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}x \pm \frac{3}{4}\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \quad (46)$$

Case 2:

$$\begin{aligned} a_0 &= \pm\frac{1}{2}, \quad a_1 = \frac{1}{2}, b_1 = 0, \\ c &= \pm\frac{\sqrt{2}}{4}, \quad v = \pm\frac{3}{4}. \end{aligned} \quad (47)$$

The exact solution is as follows:

$$u_2(x,t) = \pm\frac{1}{2} + \frac{1}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}x \pm \frac{3}{4}\frac{t^\alpha}{\alpha}\right)\right). \quad (48)$$

Case 3:

$$\begin{aligned} a_0 &= \pm\frac{1}{2}, \quad a_1 = \frac{1}{4}, b_1 = \frac{1}{4}, \\ c &= \pm\frac{\sqrt{2}}{8}, \quad v = \pm\frac{3}{8}. \end{aligned} \quad (49)$$

The exact solution is as follows:

$$u_3(x,t) = \pm\frac{1}{2} + \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \frac{3}{4}\frac{t^\alpha}{\alpha}\right)\right) + \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \frac{3}{4}\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \quad (50)$$

6 Solutions of the Zeldovich equation

If $\mu = 0$ in Eq. 15, the Zeldovich equation is obtained:

$$\frac{\partial^\alpha u}{\partial t^\alpha} = \frac{\partial^2 u}{\partial x^2} + u - u^3,\ t>0,\ 0<\alpha\le 1,\ x\in R. \quad (51)$$

Using the wave variable $\xi = cx - v\frac{t^\alpha}{\alpha}$, Eq. 51 can be converted to Eq. 52.

$$vU' + c^2U'' + U^2 - U^3 = 0, \quad (52)$$

where c and v are constants. From Eq. 52, we obtain

$$3m = m+2, \quad (53)$$
$$m = 1. \quad (54)$$

The solution form is as follows:

$$U(\xi) = a_0 + a_1Y + b_1Y^{-1},\ Y = \tanh(\xi),\ Y' = 1-Y^2. \quad (55)$$

Here, a_0 and a_1, b_1 are the arbitrary constants. Eq. 55 is substituted into Eq. 52, and if the coefficients of Y are used to set the system of equations to zero, then

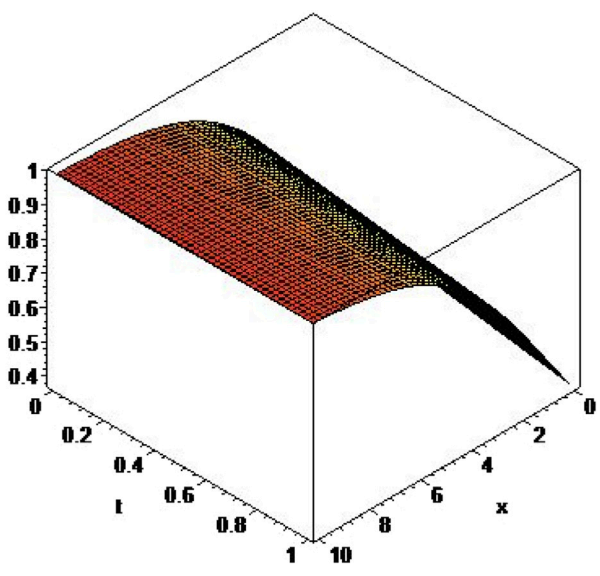

FIGURE 1
Figure of u4 in (Eq. 4.14) with μ = 2, 0 < t < 1, and 0 < x < 10.

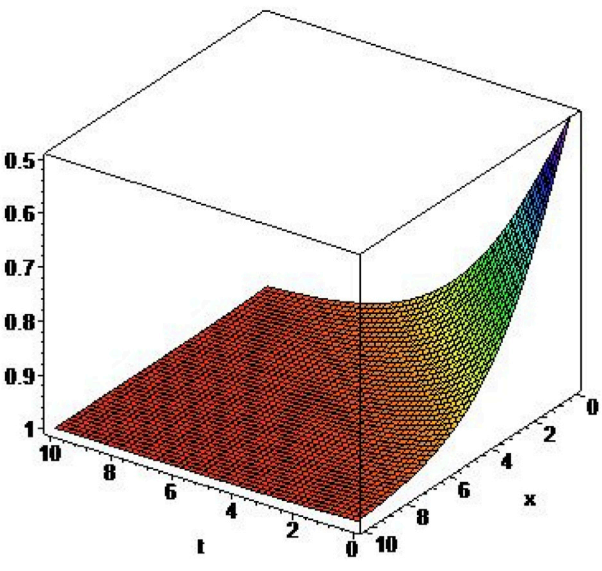

FIGURE 2
Figure of u5 in (Eq. 4.16) with a0 = 1/2, 0 < t < 10, and 0 < x < 10.

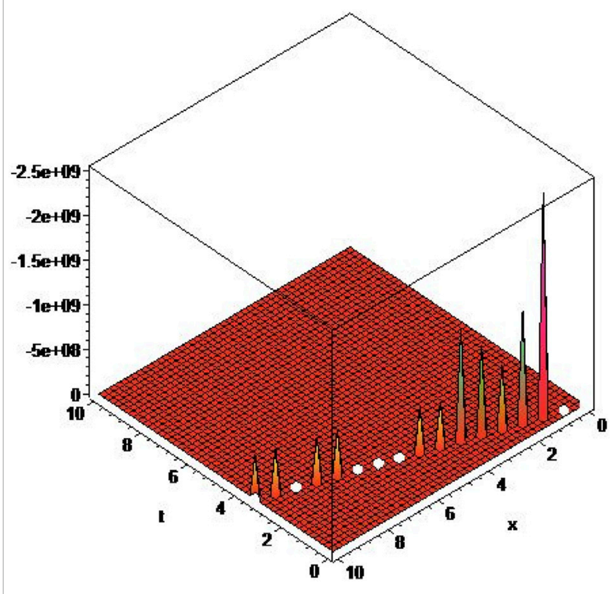

FIGURE 3
Figure of u7 in (Eq. 4.20) with a0 = 1/2, 0 < t < 10, and 0 < x < 10.

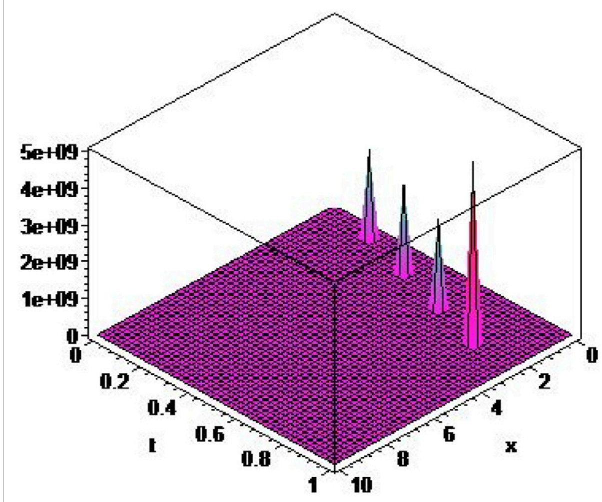

FIGURE 4
Figure of u9 in (Eq. 4.24) with μ = 2, 0 < t < 1, and 0 < x < 10.

$$\begin{aligned} a_1^3 - 2a_1c^2 &= 0. \\ 3a_0a_1^2 - a_1^2 - a_1\nu &= 0. \\ 3a_0^2a_1 + 3a_1^2b_1 + 2a_1c^2 - 2a_0a_1 &= 0. \\ a_0^3 + 6a_0a_1b_1 - a_0^2 - 2a_1b_1 + a_1\nu + b_1\nu &= 0. \\ 3a_0^2b_1 + 3a_1b_1^2 + 2b_1c^2 - 2a_0b_1 &= 0. \\ 3a_0b_1^2 - b_1^2 - b_1\nu &= 0. \\ b_1^3 - 2b_1c^2 &= 0. \end{aligned} \tag{56}$$

By solving this system of equations, we get the following cases.

Case 1:

$$\begin{aligned} a_0 = a_0,\ a_1 = 0,\ b_1 &= \pm\, a_0\sqrt{\frac{1-a_0}{3a_0-1}}, \\ c = c, \qquad \nu &= \pm\, 3a_0^2\sqrt{\frac{1-a_0}{3a_0-1}} - a_0\sqrt{\frac{1-a_0}{3a_0-1}}. \end{aligned} \tag{57}$$

Therefore, the exact solution is as follows:

$$u_1(x,t) = a_0 \pm a_0\sqrt{\frac{1-a_0}{3a_0-1}}\left(\tanh\left(cx \pm 3a_0^2\sqrt{\frac{1-a_0}{3a_0-1}} - a_0\sqrt{\frac{1-a_0}{3a_0-1}}\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \tag{58}$$

Case 2:

$$\begin{aligned} a_0 &= \frac{1}{2}, & a_1 &= \pm\frac{1}{2}, b_1 = 0,\\ c &= \pm\frac{\sqrt{2}}{4}, & v &= \pm\frac{1}{4}. \end{aligned} \tag{59}$$

The exact solution is as follows:

$$u_2(x,t) = \frac{1}{2} \pm \frac{1}{2}\left(\tanh\left(\pm\frac{\sqrt{2}}{4}x \pm \frac{1}{4}\frac{t^\alpha}{\alpha}\right)\right). \tag{60}$$

Case 3:

$$\begin{aligned} a_0 &= \frac{1}{2}, & a_1 &= \pm\frac{1}{4}, b_1 = \pm\frac{1}{4},\\ c &= \pm\frac{\sqrt{2}}{8}, & v &= \pm\frac{1}{8}. \end{aligned} \tag{61}$$

The exact solution is as follows:

$$u_3(x,t) = \frac{1}{2} \pm \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \frac{1}{8}\frac{t^\alpha}{\alpha}\right)\right) \pm \frac{1}{4}\left(\tanh\left(\pm\frac{\sqrt{2}}{8}x \pm \frac{1}{8}\frac{t^\alpha}{\alpha}\right)\right)^{-1}. \tag{62}$$

7 Conclusion

In this article, we present the extended tanh–coth method for solving non-linear space–time conformable PDEs. We found the exact and traveling wave solutions of some important space–time fPDEs *via* the extended tanh–coth method. We know many of the results obtained are new solutions that do not exist in the literature. The hyperbolic and trigonometric function solutions are significant to explain a variety of physical phenomena. This suggests that the extended tanh–coth method is more effective in finding the solutions of non-linear fPDEs. The 3D plots of the acquired solutions are presented by choosing appropriate values to the parameters in Figures 1–4. These are the advantages of the extended tanh–coth method. The offered method can be utilized to assist complicated models applicable to a wide variety of physical situations. We hope that the telecommunications industry and other such forms of waveguides will find this study to be beneficial.

Moreover, we come to the understanding that the newly obtained hyperbolic function and trigonometric function solutions in this study may help explain some complex physical aspects in non-linear physical sciences and are related to such physical properties. In future studies, these approaches can be easily used for other NFDEs, NFDE systems, fractional complex equations, fractional difference equations, etc.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

ACC: Original Draft, Methodology, Validation; AB: Investigation, Supervision, Writing—Review and Editing; OAA: Conceptualization, Methodology, Writing; MA: Original Draft, Validation, Founding acquisition.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

References

1. Miller KS, Ross B. *An introduction to the fractional calculus and fractional differential equations*. New York: Wiley (1993).

2. Podlubny I. *Fractional differential equations*. California: Academic Press (1999).

3. Hilfer R. *Applications of fractional calculus in physics*. River Edge, NJ, USA: World Scientific Publishing (2000).

4. Kilbas AA, Srivastava HM, Trujillo JJ. *Theory and applications of fractional differential equations*. Amsterdam: Elsevier (2006).

5. Jajarmi A, Baleanu D, Vahid ZK, Pirouza HM, Asad JH. A new and general fractional Lagrangian approach: A capacitor microphone case study. *Results Phys* (2021) 31:104950. doi:10.1016/j.rinp.2021.104950

6. Acay B, Inc M, Mustapha UT, Yusuf A. Fractional dynamics and analysis for a lana fever infectious ailment with Caputo operator, Chaos. *Solitons & Fractals* (2022) 153:111605. doi:10.1016/j.chaos.2021.111605

7. Erturk VS, Godwe E, Baleanu D, Kumar P, Asad J, Jajarmi A. Novel fractional-order Lagrangian to describe motion of beam on nanowire. *Acta Phys Pol A* (2021) 140(3):265–72. doi:10.12693/aphyspola.140.265

8. Rawya A, Alquran M, Ali M, Yusuf A, Momani S. On group of Lie symmetry analysis, explicit series solutions and conservation laws for the time-fractional (2+1)-dimensional Zakharov-Kuznetsov (q,p,r) equation. *J Geometry Phys* (2022) 176:104512. doi:10.1016/j.geomphys.2022.104512

9. Baleanu D, Abadi MH, Jajarmi A, Vahid KZ, Nieto JJ. A new comparative study on the general fractional model of COVID-19 with isolation and quarantine effects. *Alexandria Eng J* (2022) 61(6):4779–91. doi:10.1016/j.aej.2021.10.030

10. Bekir A, Güner Ö, Ayhan B, Cevikel AC. Exact solutions for fractional differential-difference equations by (G'/G)-Expansion method with modified Riemann-Liouville derivative. *Adv Appl Math Mech* (2016) 8(2):293–305. doi:10.4208/aamm.2014.m798

11. Aksoy E, Cevikel AC, Bekir A. Soliton solutions of (2+1)-dimensional time-fractional Zoomeron equation. *Optik* (2016) 127(17):6933–42. doi:10.1016/j.ijleo.2016.04.122

12. Bekir A, Aksoy E, Cevikel AC. Exact solutions of nonlinear time fractional partial differential equations by sub-equation method. *Math Methods Appl Sci* (2015) 38(13):2779–84. doi:10.1002/mma.3260

13. Cevikel AC. New exact solutions of the space-time fractional KdV-Burgers and Non-Linear fractional Foam Drainage equation. *Therm Sci* (2018) 22:15–24. doi:10.2298/tsci170615267c

14. Bekir A, Güner Ö, Cevikel AC. The exp-function method for some time-fractional differential equations. *Ieee/caa J Autom Sinica* (2017) 4:315–21. doi:10.1109/jas.2016.7510172

15. Aksoy E, Bekir A, Cevikel AC. Study on fractional differential equations with modified Riemann-Liouville derivative via Kudryashov method. *Int J Nonlinear Sci Numer Simulation* (2019) 20(5):511–6. doi:10.1515/ijnsns-2015-0151

16. Rezazadeh H, Vahidi J, Zafar A, Bekir A. The functional variable method to find new exact solutions of the nonlinear evolution equations with dual-power-law nonlinearity. *Int J Nonlinear Sci Numer Simulation* (2020) 21(3-4):249–57. doi:10.1515/ijnsns-2019-0064

17. Cevikel AC, Bekir A, Akar M, San S. A Procedure to construct exact solutions of nonlinear evolution equations. *Pramana - J Phys* (2012) 79(3):337–44. doi:10.1007/s12043-012-0326-1

18. Taghizadeh N, Mirzazadeh M, Rahimian M, Akbari M. Application of the simplest equation method to some time-fractional partial differential equations. *Ain Shams Eng J* (2013) 4:897–902. doi:10.1016/j.asej.2013.01.006

19. Savaissou N, Gambo B, Rezazadeh H, Bekir A, Doka SY. Exact optical solitons to the perturbed nonlinear Schrodinger equation with dual-power law of nonlinearity. *Opt Quan Electron* (2020) 52:318. doi:10.1007/s11082-020-02412-7

20. Cevikel AC, Aksoy E. Soliton solutions of nonlinear fractional differential equations with their applications in mathematical physics. *Revista Mexicana de Fisica* (2021) 67(3):422–8. doi:10.31349/RevMexFis.67.422

21. Cevikel AC, Bekir A, Zahran EHM. Novel exact and solitary solutions of conformable Huxley equation with three effective methods. *J Ocean Eng Sci* (2022). doi:10.1016/j.joes.2022.06.010

22. Bekir A, Cevikel AC, Zahran EHM. New impressive representations for the soliton behaviors arising from the (2+1)-Boussinesq equation. *J Ocean Eng Sci* (2022). doi:10.1016/j.joes.2022.05.036

23. Cevikel AC. Traveling wave solutions of conformable Duffing model in shallow water waves. *Int J Mod Phys B* (2022) 36:2250164. doi:10.1142/s0217979222501648

24. Cevikel AC. New solutions for the high-dimensional fractional BLMP equations. *J Ocean Eng Sci* (2022). doi:10.1016/j.joes.2022.06.023

25. Khalil R, Al Horani M, Yousef A, Sababheh M. A new definition of fractional derivative. *J Comput Appl Math* (2014) 264:65–70. doi:10.1016/j.cam.2014.01.002

26. Abdeljawad T. On conformable fractional calculus. *J Comput Appl Math* (2015) 279:57–66. doi:10.1016/j.cam.2014.10.016

27. Eslami M, Rezazadeh H. The first integral method for Wu–Zhang system with conformable time-fractional derivative. *Calcolo* (2016) 53:475–85. doi:10.1007/s10092-015-0158-8

28. Cevikel AC, Bekir A. New solitons and Periodic solutions for (2+1)-dimensional Davey-Stewartson equations. *Chin J Phys* (2013) 51(1):1–13. doi:10.6122/CJP.51.1

29. Bekir A, Cevikel AC. Solitary wave solutions of two nonlinear physical models by tanh-coth method. *Commun Nonlinear Sci Numer Simulation* (2009) 14(5):1804–9. doi:10.1016/j.cnsns.2008.07.004