frontiers | Frontiers in Physics

TYPE Brief Research Report
PUBLISHED 06 October 2022
DOI 10.3389/fphy.2022.1041100

Check for updates

OPEN ACCESS

EDITED BY
Yunqing Yang,
Zhejiang Ocean University, China

REVIEWED BY
Xing Lue,
Beijing Jiaotong University, China
Xiazhi Hao,
Zhejiang University of Technology, China

*CORRESPONDENCE
Shuang Li,
liangsh222@gmail.com

SPECIALTY SECTION
This article was submitted to Interdisciplinary Physics, a section of the journal Frontiers in Physics

RECEIVED 10 September 2022
ACCEPTED 15 September 2022
PUBLISHED 06 October 2022

CITATION
Qi F-H, Li S, Li Z and Wang P (2022), Multiple lump solutions of the (2+1)-dimensional sawada-kotera-like equation.
Front. Phys. 10:1041100.
doi: 10.3389/fphy.2022.1041100

COPYRIGHT
© 2022 Qi, Li, Li and Wang. This is an open-access article distributed under the terms of the Creative Commons Attribution License (CC BY). The use, distribution or reproduction in other forums is permitted, provided the original author(s) and the copyright owner(s) are credited and that the original publication in this journal is cited, in accordance with accepted academic practice. No use, distribution or reproduction is permitted which does not comply with these terms.

Multiple lump solutions of the (2+1)-dimensional sawada-kotera-like equation

Feng-Hua Qi[1], Shuang Li[1]*, Zhenhuan Li[1] and Pan Wang[2]

[1]School of Information, Beijing Wuzi University, Beijing, China, [2]Sports Business School, Beijing Sport University, Beijing, China

In this paper, 1-lump solution and 2-lump solution of a (2 + 1)-dimensional Sawada-Kotera-like equation are obtained by means of the Hirota's bilinear method and long wave limit method. The propagation orbits, velocities and the collisions among waves are analyzed. By setting the parameter values, the dynamic characteristics of the obtained solutions are shown in 3D and density plots. These conclusions enrich the dynamical theory of higher-dimensional nonlinear dispersive wave equations.

KEYWORDS

multiple lump solution, long wave limit, sawada-kotera-like equation, hirota bilinear, partial differential equations

1 Introduction

Nonlinear evolution equations can be used to simulate various nonlinear phenomena in the real world, which appear in fluid mechanics [1–3], optical fibers[4], applied mathematics[5–7], chemistry and biology[8–10], etc. In recent years, searching for exact solutions of nonlinear evolution equations has attracted considerable attention, such as lump solutions[11–16], soliton solutions[17–21] and breather solutions[22–25].

The (2 + 1)-dimensional Sawada–Kotera equation:

$$5u_xu^2 + 5uu_y + 5u_{xxx}u + 5u_x\partial_x^{-1}u_y + 5u_{xx}u_x - u_t + 5u_{xxy} + u_{xxxxx} - 5\partial_x^{-1}u_{yy} = 0, \quad (1)$$

has important and wide applications in conformal field theory, quantum gravity field theory and conserved current of Liouville equation[26–28]. Soliton solutions[29–31], lump solutions[32,33], travelling wave solutions[34] and some other exact solutions[35] of Eq. 1 have been detailed. In this paper, we mainly consider the (2 + 1)-dimensional Sawada-Kotera-like equation[36]:

$$\begin{aligned} &-180\partial_x^{-1}u_{yy} - 36u_t + 180u_{xxy} + 30u_{xx}uw + 15u_{xxx}w^2 + 180u_xv + 180uu_y + 135u_xu^2 \\ &+90u_{xxx}u - 30u_{xx}u_x + 15u^3w + 35u_xuw^2 + 5u^2w^3 + \frac{5}{36}uw^5 + \frac{20}{3}u_{xx}w^3 + \frac{5}{4}u_xw^4 = 0, \end{aligned} \quad (2)$$

in which ∂^{-1} represents the partial integration operator. Eq. 2 is gained from Eq. 1 by the generalized bilinear method[36]. When $v_x = u_y$ and $\omega_x = u$, Eq. 2 can be reduced to Eq. 1. And Eq. 2 is different from the Sawada-Kotera-like equations which have been mentioned by [32,37].As far as we known, multiple lump solutions of Eq. 2

have not been presented in any existing articles. Classic lump, generalized lump solutions and new rogue wave solutions of Eq. 2 have been obtained by [36]. In this paper, we will study multiple lump solutions of Eq. 2. In Section 2, we construct 1-lump solution and 2-lump solution of Eq. 2 by employing the Hirota's bilinear method and long wave limit method. The dynamical behaviors of the solutions are analyzed in Section 3. Section 4 is our conclusions.

2 1-lump solution and 2-lump solution

The long wave limit method is an effective method to generate M-lump solutions from N-soliton solutions[38–44]. In this section, we will construct the 1-lump solution and 2-lump solution of Eq. 2. As a preparation for constructing 1-lump solution and 2-lump solution of Eq. 2, we first study the N-soliton solutions[45]. With the aid of the variable transformation

$$u = 6(\ln f)_{xx}, \omega = 6(\ln f)_{x}, v = 6(\ln f)_{xy}, \tag{3}$$

Eq. 2 can be transformed into a bilinear form[36]:

$$\begin{aligned}&\left(D_{5,x}^{6} + 5D_{5,y}D_{5,x}^{3} - 5D_{5,y}^{2} - D_{5,x}D_{5,t}\right)f\cdot f = -2f_{tx}f - 10f_{yy}f + 10f_{xxxy}f\\&\quad +2f_{x}f_{t} - 30f_{xxy}f_{x} + 10f_{y}^{2} - 10f_{y}f_{xxx} + 30f_{xy}f_{xx} + 30f_{xxxx}f_{xx}\\&-20f_{xxx}^{2} = 0,\end{aligned} \tag{4}$$

where D_t, D_x, and D_y are the bilinear derivative operators, which can be defined by generalized D operator[46]:

$$D_{p,x_1}^{n_1}\cdots D_{p,x_M}^{n_M}(f\cdot f) = \prod_{i=1}^{M}\left(\frac{\partial}{\partial x_i} + \alpha\frac{\partial}{\partial x_i'}\right)^{n_i} f(x_1,\ldots,x_M)f(x_1',\ldots,x_M')\bigg|_{x_1'=x_1,\ldots,x_M'=x_M}. \tag{5}$$

It means that Eq. 3 are solutions of Eq. 2 if and only if f is a solution of Eq. 4. Based on the Hirota's bilinear method, the N-soliton solutions of Eq. 4 have been obtained[45]:

$$f = f_N = \sum_{\mu=0,1}\exp\left(\sum_{i=1}^{N}\mu_i\eta_i + \sum_{i<j}^{N}\mu_i\mu_jA_{ij}\right), \tag{6}$$

where

$$\eta_i = k_i(x + p_iy + \omega_it) + \eta_i^0, \omega_i = k_i^4 + 5k_i^2p_i - 5p_i^2,$$
$$\exp A_{ij} = \frac{k_i^4 - 3k_i^3k_j + k_j^4 + (p_i - p_j)^2 - 3k_ik_j(k_j^2 + p_i + p_j) + k_i^2(4k_j^2 + 2p_i + p_j) + k_j^2(p_i + 2p_j)}{k_i^4 + 3k_i^3k_j + k_j^4 + (p_i - p_j)^2 + 3k_ik_j(k_j^2 + p_i + p_j) + k_i^2(4k_j^2 + 2p_i + p_j) + k_j^2(p_i + 2p_j)}, \tag{7}$$

with k_i, p_i and η_i^0 are arbitrary constants, $\sum_{\mu=0,1}$ is the summation of possible combinations of $\mu_i = 0, 1(i = 1, 2\cdots N)$.By taking a limit of $\eta_i^0 = -1, k_i \to 0\,(i = 1, 2, \cdots N)$ and considering all the k_i in the same asymptotic order in Eq. 6, we have

$$\begin{aligned}f_N &= \prod_{i=1}^{N}\theta_i + \frac{1}{2}\sum_{i,j}^{N}B_{ij}\prod_{s\neq i,j}^{N}\theta_s + \cdots + \frac{1}{M!2^M}\sum_{i,j,\ldots,p,q}^{N}\\&\times\overbrace{B_{ij}B_{kl}\ldots B_{pq}}^{M}\prod_{m\neq i,j,k,l,\ldots,p,q}^{N}\theta_m + \cdots,\end{aligned} \tag{8}$$

where

$$\theta_i = x + yp_i - 5tp_i^2, B_{ij} = -\frac{6(p_i + p_j)}{(p_i - p_j)^2}, \tag{9}$$

and $\sum_{i,j,\cdots N}^{N}$ represents the summation over all possible compositions of i, j, $\cdots p$, q, which are taken different values from 1, 2, $\cdots N$. Taking $N = 2M$ in Eq. 8 and $p_{M+i} = p_i^{\prime}(i = 1, 2, \ldots, M)$ in Eq. 9, where "*" denotes the complex conjugation, the M-lump solutions to Eq. 4 can be obtained.In the case of $N = 2$, Eq. 8 is changed into

$$f_2 = \theta_1\theta_2 + B_{12}, \tag{10}$$

where $\theta_1 = x + yp_1 - 5tp_1^2$, $\theta_2 = x + yp_2 - 5tp_2^2$, $B_{12} = -\frac{6(p_1+p_2)}{(p_1-p_2)^2}$. If we take $p_2 = p_1^* = a - b{*}I$, where a and b are real constants, I is an imaginary number unit, Eq. 10 is changed into

$$\begin{aligned}f_2 = {}&\frac{3a}{b^2} + 25a^4t^2 + 50a^2b^2t^2 + 25b^4t^2 - 10a^2tx + 10b^2tx + x^2\\&- 10a^3ty - 10ab^2ty + 2axy + a^2y^2 + b^2y^2,\end{aligned} \tag{11}$$

then we can obtain the 1-lump solution of Eq. 2:

$$\begin{aligned}u &= 6(\ln f_2)_{xx}\\&= -\{12b^2[25a^4b^2t^2 + 25b^6t^2 + b^2x^2 - 10a^3b^2ty + a(-3 + 30b^4ty + 2b^2xy)\\&\quad +b^4(10tx - y^2) + a^2b^2(-150b^2t^2 - 10tx + y^2)]\}[25a^4b^2t^2 + 25b^6t^2 + b^2x^2\\&-10a^3b^2ty + a(3 - 10b^4ty + 2b^2xy) + a^2b^2(50b^2t^2 - 10tx + y^2) + b^4(10tx + y^2)]^{-2}.\end{aligned} \tag{12}$$

This wave keeps moving on the line $y = -\frac{2ax}{a^2+b^2}$, the velocity along the x-axis is $v_x = x + 5b^2t + 5a^2t$, and the velocity along the y-axis is $v_y = y - 10\,at$.In the case of $N = 4$, Eq. 8 is changed into

$$\begin{aligned}f4 = {}&\theta_1\theta_2\theta_3\theta_4 + B_{12}\theta_3\theta_4 + B_{13}\theta_2\theta_4 + B_{14}\theta_2\theta_3 + B_{23}\theta_1\theta_4\\&+ B_{24}\theta_1\theta_3 + B_{34}\theta_1\theta_2 + B_{12}B_{34} + B_{13}B_{24} + B_{14}B_{23},\end{aligned} \tag{13}$$

where $\theta_i = x + yp_i - 5tp_i^2$, $B_{ij} = -\frac{6(p_i+p_j)}{(p_i-p_j)^2}$, $(1 \le i < j \le 4)$ and $p_1 = p_2^* = a_1 + b_1I$ $p_3 = p_4^* = a_2 + b_2I$. Substituting Eq. 13 into Eq. 3, we can obtain 2-lump solution of Eq. 2:

$$\begin{aligned}u &= 6\ln(f_4)_{xx}\\&= [-6(\beta_1 + \beta_2 + \beta_3 + \beta_4 + \beta_5 + \beta_6 + \beta_7)^2 + 12(\beta_8 + \beta_9 + \beta_{10} + \beta_{11})\\&(\beta_{12} + \beta_{13} + \beta_{14} + \beta_{15} + \beta_{16})] \quad\times(-\beta_{12} - \beta_{13} - \beta_{14} - \beta_{15} - \beta_{16})^{-2},\end{aligned} \tag{14}$$

where

$$\beta_1 = 6(p_1 + p_3)\alpha_2(p_1 - p_3)^{-2} + 6(p_2 + p_3)\alpha_1(p_2 - p_3)^{-2},$$
$$\beta_2 = 6(p_1 + p_2)\alpha_3(p_1 - p_2)^{-2} - \alpha_1\alpha_2\alpha_3,$$
$$\beta_3 = 6(p_1 + p_4)(\alpha_2 + \alpha_3)(p_1 - p_4)^{-2},$$

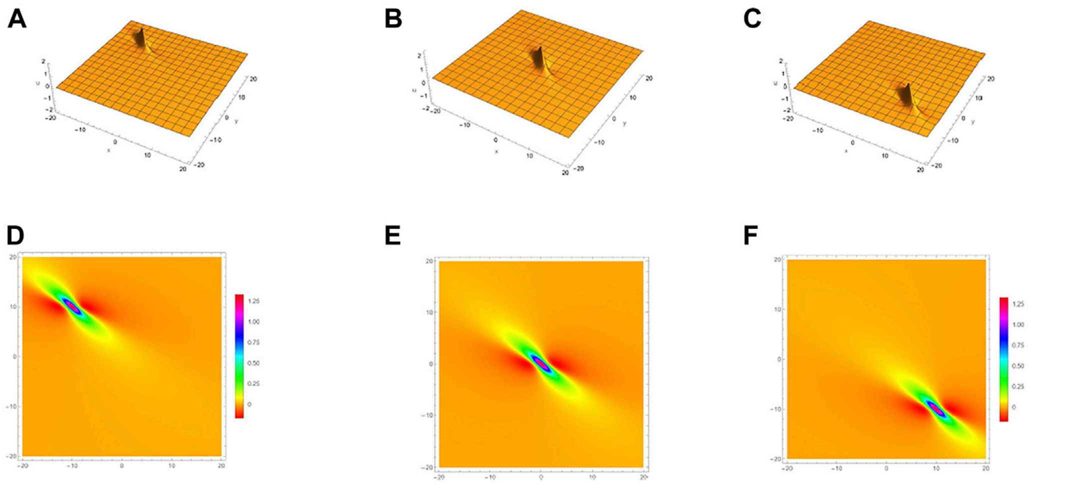

FIGURE 1
1-lump solution Eq. 12 for Eq. 2 with $a = 1$, $b = 1$: **(A)** $t = -1$; **(B)** $t = 0$; **(C)** $t = 1$; **(D)**, **(E)**, **(F)** are the density plot of **(A)**, **(B)**, **(C)** respectively.

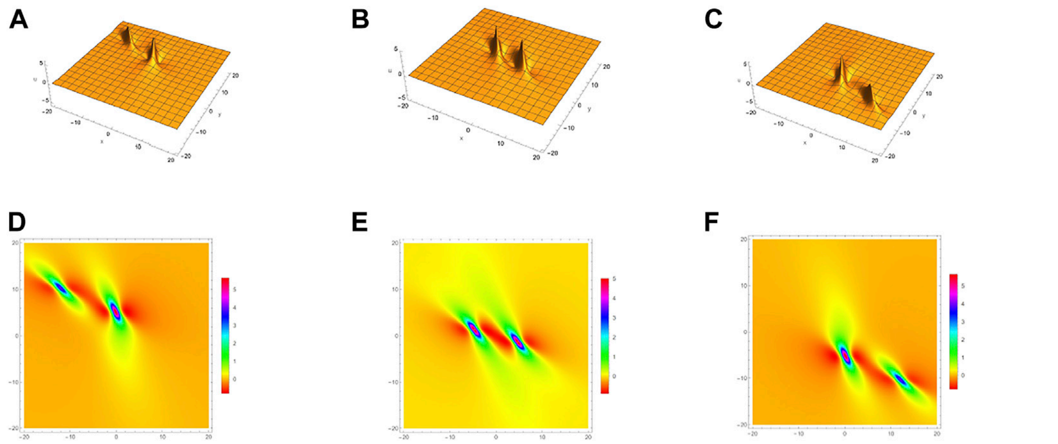

FIGURE 2
2-lump solution Eq. 14 for Eq. 2 with $a_1 = 1$, $a_2 = \frac{1}{3}$, $b_1 = 1$, $b_2 = \frac{1}{2}$: **(A)** $t = -1$; **(B)** $t = 0$; **(C)** $t = 1$; **(D)**, **(E)**, **(F)** are the density plot of **(A)**, **(B)**, **(C)** respectively.

$$\beta_4 = 6(p_2 + p_4)\alpha_1 + 6(p_2 + p_4)\alpha_3(p_2 - p_4)^{-2},$$
$$\beta_5 = 6(p_3 + p_4)(\alpha_1 + \alpha_2)(p_3 - p_4)^{-2},$$
$$\beta_6 = 6(p_1 + p_2)\alpha_4(p_1 - p_2)^{-2} - \alpha_1\alpha_2\alpha_4,$$
$$\beta_7 = 6(p_1 + p_3)\alpha_4(p_1 - p_3)^{-2} + 6(p_2 + p_3)\alpha_4(p_2 - p_3)^{-2},$$
$$\beta_8 = \alpha_1\alpha_3\alpha_4, \quad \beta_9 = \alpha_2\alpha_3\alpha_4, \quad \beta_{10} = \alpha_1\alpha_3 + \alpha_2\alpha_3,$$
$$\beta_{11} = \alpha_1\alpha_2 - 6(p_1 + p_2)(p_1 - p_2)^{-2} - 6(p_1 + p_3)(p_1 - p_3)^{-2} - 6(p_2 + p_3)(p_2 - p_3)^{-2},$$
$$\beta_{12} = \alpha_1\alpha_4 - 6(p_1 + p_4)(p_1 - p_4)^{-2} - 6(p_2 + p_4)(p_2 - p_4)^{-2} - 6(p_3 + p_4)(p_3 - p_4)^{-2},$$
$$\beta_{13} = \alpha_2\alpha_4 + \alpha_3\alpha_4,$$
$$\beta_{14} = 36(p_2 + p_3)(p_1 + p_4)[(p_2 - p_3)(p_1 - p_4)]^{-2} - 6(p_1 + p_4)\alpha_2\alpha_3(p_1 - p_4)^{-2},$$
$$\beta_{15} = 36(p_1 + p_3)(p_2 + p_4)[(p_1 - p_3)(p_2 - p_4)]^{-2} - 6(p_2 + p_4)\alpha_1\alpha_3(p_2 - p_4)^{-2},$$
$$\beta_{16} = 36(p_1 + p_2)(p_3 + p_4)[(p_1 - p_2)(p_3 - p_4)]^{-2} - 6(p_3 + p_4)\alpha_1\alpha_2(p_3 - p_4)^{-2},$$
$$\beta_{17} = -6(p_1 + p_3)\alpha_2\alpha_4(p_1 - p_3)^{-2},$$
$$\beta_{18} = -6(p_2 + p_3)\alpha_1\alpha_4(p_2 - p_3)^{-2},$$

$$\beta_{19} = -6(p_1 + p_2)\alpha_3\alpha_4(p_1 - p_2)^{-2}, \quad \beta_{20} = \alpha_1\alpha_2\alpha_3\alpha_4,$$

in which $\alpha_i = x + yp_i - 5tp_i^2$, $(i = 1, 2, 3, 4)$. The wave keeps moving along the line $y_1 = -\frac{2a_1 x}{a_1{}^2 + b_1{}^2}$, $y_2 = -\frac{2a_2 x}{a_2{}^2 + b_2{}^2}$.

Figures 1–2, show the evolution of the 1-lump solution Eq. 12 and 2-lump solution Eq. 14 with the time variation. Figure 1 show the 1-lump waves for Eq. 2 under $a = 1$, $b = 1$ but with the different values of (a) and (d) $t = -1$, (b) and (e) $t = 0$, (c) and (f) $t = 1$. Figure 2 are the 2-lump waves for Eq. 2 with parameters $a_1 = 1, a_2 = \frac{1}{3}, b_1 = 1, b_2 = \frac{1}{2}$ with the different values of (a) and (d) $t = -1$, (b) and (e) $t = 0$, (c) and (f) $t = 1$. Figures 1D,E,F are the density plot of Figures 1A,B,C separately and Figures 2D,E,F are the density plot of Figures 2A,B,C respectively.

3 Conclusion

In this paper, we have presented the 1-lump solution Eq. 12 and 2-lump solution Eq. 14 of the (2 + 1)-dimensional Sawada-Kotera-like Eq. 2 by using a variable transformation. Dynamical features and density distributions of the presented solutions have been depicted through plots. It is expected that these results can be useful to understand the dynamical behavior of relevant fields in physics.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

FQ: work out the whole idea of this paper, including method and writing. SL: some calculations and writing of the paper. ZL: polish the whole paper. PW: check the English gramma.

Funding

The study is supported by the key project of Beijing Social Science Foundation “strategic research on improving the service quality of capital logistics based on big data technology (18GLA009)”.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher’s note

All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

References

1. Ablowitz MJ, Segur H. On the evolution of packets of water waves. *J Fluid Mech* (1979) 92:691–715. doi:10.1017/S0022112079000835

2. Liu F-Y, Gao Y-T, Yu X, Li L-Q, Ding C-C, Wang D. Lie group analysis and analytic solutions for a (2+1)-dimensional generalized bogoyavlensky–konopelchenko equation in fluid mechanics and plasma physics. *Eur Phys J Plus* (2021) 136:656–14. doi:10.1140/epjp/s13360-021-01469-x

3. Zhao Y-W, Xia J-W, Lü X. The variable separation solution, fractal and chaos in an extended coupled (2+1)-dimensional burgers system. *Nonlinear Dyn* (2022) 108: 4195–205. doi:10.1007/s11071-021-07100-z

4. Liu J-G, Osman M, Zhu W-H, Zhou L, Ai G-P. Different complex wave structures described by the hirota equation with variable coefficients in inhomogeneous optical fibers. *Appl Phys B* (2019) 125:175–9. doi:10.1007/s00340-019-7287-8

5. Xu T, Liu C, Qi F, Li C, Meng D. New double wronskian solutions of the whitham-broer-kaup system: Asymptotic analysis and resonant soliton interactions. *J Nonlinear Math Phys* (2021) 24:116. doi:10.1080/14029251.2017.1282248

6. Qi F-H, Huang Y-H, Wang P. Solitary-wave and new exact solutions for an extended (3+1)-dimensional jimbo–miwa-like equation. *Appl Math Lett* (2020) 100: 106004. doi:10.1016/j.aml.2019.106004

7. Lou S, Hao X. Linear superposition in the general heavenly equation. *Phys Lett A* (2022) 443:128203. doi:10.1016/j.physleta.2022.128203

8. Xu T, Chen Y, Li M, Meng D-X. General stationary solutions of the nonlocal nonlinear Schrödinger equation and their relevance to the pt-symmetric system. *Chaos* (2019) 29:123124. doi:10.1063/1.5121776

9. Xu T, Lan S, Li M, Li L-L, Zhang G-W. Mixed soliton solutions of the defocusing nonlocal nonlinear Schrödinger equation. *Physica D: Nonlinear Phenomena* (2019) 390:47–61. doi:10.1016/j.physd.2018.11.001

10. Epstein IR. Nonlinear oscillations in chemical and biological systems. *Physica D: Nonlinear Phenomena* (1991) 51:152–60. doi:10.1016/0167-2789(91)90228-2

11. Ma W-X. Lump solutions to the kadomtsev–petviashvili equation. *Phys Lett A* (2015) 379:1975–8. doi:10.1016/j.physleta.2015.06.061

12. Lü X, Wang J-P, Lin F-H, Zhou X-W. Lump dynamics of a generalized two-dimensional boussinesq equation in shallow water. *Nonlinear Dyn* (2017) 91: 1249–59. doi:10.1007/s11071-017-3942-y

13. Ding L, Ma W-X, Chen Q, Huang Y. Lump solutions of a nonlinear pde containing a third-order derivative of time. *Appl Math Lett* (2021) 112:106809. doi:10.1016/j.aml.2020.106809

14. Chen S-J, Lü X. Lump and lump-multi-kink solutions in the (3+1)-dimensions. *Commun Nonlinear Sci Numer Simulation* (2022) 109:106103. doi:10.1016/j.cnsns.2021.106103

15. Lü X, Chen S-J. New general interaction solutions to the kpi equation via an optional decoupling condition approach. *Commun Nonlinear Sci Numer Simulation* (2021) 103:105939. doi:10.1016/j.cnsns.2021.105939

16. Lü X, Chen S-J. Interaction solutions to nonlinear partial differential equations via hirota bilinear forms: One-lump-multi-stripe and one-lump-multi-soliton types. *Nonlinear Dyn* (2021) 103:947–77. doi:10.1007/s11071-020-06068-6

17. Ma W-X. N-soliton solution and the Hirota condition of a (2+1)-dimensional combined equation. *Mathematics Comput Simulation* (2021) 190:270–9. doi:10.1016/j.matcom.2021.05.020

18. Dong J-J, Li B, Yuen M. Soliton molecules and mixed solutions of the (2+1)-dimensional bidirectional sawada–kotera equation. *Commun Theor Phys* (2020) 72: 025002. doi:10.1088/1572-9494/ab6184

19. Wang D, Huang Y, Yong X, Zhang J. Rational soliton solutions of the nonlocal nonlinear Schrödinger equation by the kp reduction method. *Int J Mod Phys B* (2019) 33:1950362. doi:10.1142/s0217979219503624

20. Chen S-J, Lü X. Observation of resonant solitons and associated integrable properties for nonlinear waves. *Chaos, Solitons & Fractals* (2022) 163:112543. doi:10.1016/j.chaos.2022.112543

21. Yin Y-H, Lü X, Ma W-X. Bäcklund transformation, exact solutions and diverse interaction phenomena to a (3+1)-dimensional nonlinear evolution equation. *Nonlinear Dyn* (2021) 108:4181–94. doi:10.1007/s11071-021-06531-y

22. Liu J-G, Zhu W-H. Multiple rogue wave, breather wave and interaction solutions of a generalized (3+1)-dimensional variable-coefficient nonlinear wave equation. *Nonlinear Dyn* (2021) 103:1841–50. doi:10.1007/s11071-020-06186-1

23. Tan W. Evolution of breathers and interaction between high-order lump solutions and N-solitons ($N \to \infty$) for Breaking Soliton system. *Phys Lett A* (2019) 383:125907. doi:10.1016/j.physleta.2019.125907

24. Zhao Z, He L. M-lump, high-order breather solutions and interaction dynamics of a generalized $$(2 + 1)$$-dimensional nonlinear wave equation. *Nonlinear Dyn* (2020) 100:2753–65. doi:10.1007/s11071-020-05611-9

25. Hao X. Nonlocal symmetries and molecule structures of the kdv hierarchy. *Nonlinear Dyn* (2021) 104:4277–91. doi:10.1007/s11071-021-06530-z

26. Sawada K, Kotera T. A method for finding n-soliton solutions of the kdv equation and kdv-like equation. *Prog Theor Phys* (1974) 51:1355–67. doi:10.1143/PTP.51.1355

27. Konopelchenko B, Dubrovsky V. Some new integrable nonlinear evolution equations in 2+1 dimensions. *Phys Lett A* (1984) 102:15–7. doi:10.1016/0375-9601(84)90442-0

28. An H, Feng D, Zhu H. General $${\varvec{M}}$$-lump, high-order breather and localized interaction solutions to the $$\varvec{2+1}$$-dimensional Sawada–Kotera equation. *Nonlinear Dyn* (2019) 98:1275–86. doi:10.1007/s11071-019-05261-6

29. Lü X, Geng T, Zhang C, Zhu H-W, Meng X-H, Tian B. Multi-soliton solutions and their interactions for the (2+1)-dimensional sawada-kotera model with truncated painlevé expansion, hirota bilinear method and symbolic computation. *Int J Mod Phys B* (2009) 23:5003–15. doi:10.1142/S0217979209053382

30. Lü X. New bilinear bäcklund transformation with multisoliton solutions for the (2+1)-dimensional sawada-kotera model. *Nonlinear Dyn* (2014) 76:161–8. doi:10.1007/s11071-013-1118-y

31. Debin K, Rezazadeh H, Ullah N, Vahidi J, Tariq KU, Akinyemi L. New soliton wave solutions of a (2+1)-dimensional sawada-kotera equation. *J Ocean Eng Sci* (2022). doi:10.1016/j.joes.2022.03.007

32. Zhang H-Q, Ma W-X. Lump solutions to the ($$\mathbf 2+1 $$ 2 + 1)-dimensional Sawada–Kotera equation. *Nonlinear Dyn* (2016) 87:2305–10. doi:10.1007/s11071-016-3190-6

33. Huang L-L, Chen Y. Lump solutions and interaction phenomenon for (2+1)-dimensional sawada-kotera equation. *Commun Theor Phys* (2017) 67:473. doi:10.1088/0253-6102/67/5/473

34. Wang X, Zhao Q, Jia M, Lou S. Novel travelling wave structures for (2+1)-dimensional sawada-kotera equation. *Appl Math Lett* (2022) 124:107638. doi:10.1016/j.aml.2021.107638

35. Qi Z, Chen Q, Wang M, Li B. New mixed solutions generated by velocity resonance in the $$(2+1)$$-dimensional Sawada–Kotera equation. *Nonlinear Dyn* (2022) 108:1617–26. doi:10.1007/s11071-022-07248-2

36. Zhang R-F, Li M-C, Al-Mosharea E, Zheng F-C, Bilige S. Rogue waves, classical lump solutions and generalized lump solutions for sawada–kotera-like equation. *Int J Mod Phys B* (2022) 36. 2250044. doi:10.1142/s0217979222500448

37. Du Y-H, Yun Y-S, Ma W-X. Rational solutions to two sawada–kotera-like equations. *Mod Phys Lett B* (2019) 33:1950108. doi:10.1142/s0217984919501082

38. Ablowitz MJ, Satsuma J. Solitons and rational solutions of nonlinear evolution equations. *J Math Phys* (1978) 19:2180. doi:10.1063/1.523550

39. Satsuma J, Ablowitz MJ. Two-dimensional lumps in nonlinear dispersive systems. *J Math Phys* (1979) 20:1496–503. doi:10.1063/1.524208

40. Zhao Z, He L, Gao Y. Rogue wave and multiple lump solutions of the (2+1)-dimensional benjamin-ono equation in fluid mechanics. *Complexity* (2019) 2019: 1–18. doi:10.1155/2019/8249635

41. Chen S-J, Lü X, Li M-G, Wang F. Derivation and simulation of the m-lump solutions to two (2+1)-dimensional nonlinear equations. *Phys Scr* (2021) 96:095201. doi:10.1088/1402-4896/abf307

42. Zhao Z, He L. M-lump and hybrid solutions of a generalized (2+1)-dimensional Hirota–Satsuma–Ito equation. *Appl Math Lett* (2021) 111:106612. doi:10.1016/j.aml.2020.106612

43. Zhang W-J, Xia T-C. Solitary wave, m-lump and localized interaction solutions to the (4+1)-dimensional fokas equation. *Phys Scr* (2020) 95:045217. doi:10.1088/1402-4896/ab6a3f

44. He X-J, Lü X. M-lump solution, soliton solution and rational solution to a (3+1)-dimensional nonlinear model. *Math Comput Simulation* (2022) 197:327–40. doi:10.1016/j.matcom.2022.02.014

45. Satsuma J. N-soliton solution of the two-dimensional korteweg-devries equation. *J Phys Soc Jpn* (1976) 40:286–90. doi:10.1143/jpsj.40.286

46. Ma W-X. Generalized bilinear differential equations. *Stud Nonlinear Sci* (2011) 2:140–4. Available at: researchgate.net/publication/284931775.