



OPEN ACCESS

EDITED BY
Pavan Kumar Aluri,
Indian Institute of Technology (BHU),
India

REVIEWED BY
Mohammad Atazadeh,
Azarbaijan Shahid Madani
University, Iran
Christian Corda,
International Institute for Applicable
Mathematics and Information Sciences,
India

*CORRESPONDENCE
Mårten Wikström,
marten.wikstrom@helsinki.fi

SPECIALTY SECTION
This article was submitted to
Cosmology,
a section of the journal
Frontiers in Physics

RECEIVED 16 July 2022
ACCEPTED 27 September 2022
PUBLISHED 14 October 2022

CITATION
Annala A and Wikström M (2022), Dark
matter and dark energy denote the
gravitation of the expanding universe.
Front. Phys. 10:995977.
doi: 10.3389/fphy.2022.995977

COPYRIGHT
© 2022 Annala and Wikström. This is an
open-access article distributed under
the terms of the [Creative Commons
Attribution License \(CC BY\)](https://creativecommons.org/licenses/by/4.0/). The use,
distribution or reproduction in other
forums is permitted, provided the
original author(s) and the copyright
owner(s) are credited and that the
original publication in this journal is
cited, in accordance with accepted
academic practice. No use, distribution
or reproduction is permitted which does
not comply with these terms.

Dark matter and dark energy denote the gravitation of the expanding universe

Arto Annala¹ and Mårten Wikström^{2*}

¹Department of Physics, University of Helsinki, Helsinki, Finland, ²Institute of Biotechnology, University of Helsinki, Helsinki, Finland

We reason that it is the gravitation of all ordinary matter, extending from the dense distant past to the sparse present, rather than dark matter, that shows up in galaxy rotation and velocity dispersion. Likewise, we argue that it is this gradient in the gravitational energy due to the expansion, rather than dark energy, that explains Type 1a supernovae brightness vs. redshift data. Our conclusions follow from statistical mechanics, the thermodynamic theory based on the atomistic axiom that everything comprises quanta. In line with the Einstein field equations, the vacuum quanta embodying gravitation, geometrized as spacetime, equate in dynamic balance to the quanta embodying the substance of the stress–energy tensor. In accordance with quantum field theory, the proposed ground-state field of paired light quanta complies with Bose–Einstein statistics and assumes an excited state around a particle.

KEYWORDS

dark energy, dark matter, flatness problem, graviton, horizon problem, photon, quantum of action, vacuum

1 Introduction

Astronomical observations have revealed more gravitation than expected. Most notably, spiral galaxies rotate faster [1], galaxies in clusters move faster [2], and light rays passing by galaxies bend more than is accounted for by visible matter in the galaxies [3,4]. Consequently, the dark matter parameter was added to the 1980s concordance model to match calculations with data [5,6]. However, the composition of dark matter remains unknown.

In contrast, other astronomical observations have shown less gravitation than expected. Most importantly, distant supernovae, dimmer than projected from their redshifts, entail that the gravitation of all matter, luminous and dark, cannot hold back the universe from expanding at an accelerating rate [7]. Hence, the dark energy parameter was included in the concordance model at the turn of the millennium to square calculations with data. However, the quintessence of dark energy also remains enigmatic.

Despite disclosing such deviations from expectations, astronomical data display unexpected unity. The same patterns emerge at all scales of the evolving universe. For example, baryonic mass vs. orbital velocity and vs. velocity dispersion of galaxies follow a power law over 10 orders [8]. Likewise, the Type 1a supernovae luminosity vs. redshift

follows a straight line over 10 orders of magnitude before deviating noticeably [9,10]. Also, cumulative curves of void size and galaxy mass follow power laws closely [11].

Moreover, the mass-to-light ratio for the main sequence stars is nearly scale-free over seven magnitudes in luminosity [12]. In other words, bands of the renowned Hertzsprung–Russell diagram, luminosity vs. temperature, span straight lines across a log–log plot [13]. The Tully–Fisher relation, spiral galaxy mass vs. luminosity [14,15], covers several magnitudes in the same manner. So, the Faber–Jackson relation, luminosity vs. central stellar velocity dispersion of elliptical galaxies, also extends as a straight line over many magnitudes [16–18]. Also, the initial mass function for a group of stars decreases in a power law manner over three magnitudes [19]. Similarly, stellar velocities correlate with supermassive black hole mass in a galaxy bulge [20]. The cosmic ray flux vs. energy stretches 12 orders [21] and cosmic microwave background brightness vs. frequency stretches about four orders of magnitude.

For a long time, scale-free data have been thought to present a universal law [22–26]; however, it was only recently related to thermodynamics [27–30], which, in turn, derives from statistical mechanics of evolving systems [31–34]. Hence, we draw insight from statistical mechanics into the cosmological-scale evolution and find dark matter and dark energy to be unnecessary hypotheses because the gravitation of all ordinary matter across the expanding universe accounts for the observations.

As derived below, our conclusions stem from the thermodynamic distributions of quanta embodying both matter and vacuum. In the dynamic balance, the energy of matter quanta equals the energy of vacuum quanta. The Einstein field equations express this balance locally and universally so that the stress–energy tensor equates to the metric tensor of spacetime. However, this geometrization of gravitation leaves the substance of gravitation subject to speculations. Instead of matching the cosmological model with accumulating data by introducing unsubstantiated parameters, we derive a consistent theory from a universal axiom to make it falsifiable by all data.

To put our proposal in perspective, we stress that the dark matter and dark energy problems can be tackled in principle in the framework of extended gravity [35,36].

2 Distribution of quanta in matter

In their attempts to derive thermodynamics from a statistical theory, Boltzmann [37] and Gibbs [38] devised statistical mechanics and deduced macroscopic *equilibrium* properties from microscopic constituents such as atoms. However, the ubiquitous scale-free patterns emerge from *non-equilibrium* processes, heading toward thermodynamic balance. Therefore, we rederive the thermodynamic theory for evolving systems, most notably the expanding universe.

The thermodynamic balance between all matter and all space displays itself in $Mc^2 = GM^2/R$, equating the total energy of all mass, M , to the total gravitational potential of the universe within its radius, R , [39]. The balance implies exchange between matter and space. Newton had already inferred that matter and light are interconvertible [40]. Later, Einstein identified the sources of gravitation with density, flux of energy, and momentum and equated the corresponding stress–energy tensor to metric tensors.

Presently, we know that annihilation produces photons and that pair production converts photons to particles. All known constituents of matter transform by emitting or absorbing quanta [41]; hence, we adopt the ancient atomistic axiom favored by Boltzmann, but in a modern form, where everything comprises quanta of actions [42–44]. Explicitly, the quantum of light carrying energy, E , on its period, t , measures the universal invariant, Planck’s constant, $h = Et$, qualifying the photon as the fundamental constituent.

2.1 Entropy

The all-inclusive axiom allows us to describe any system with a general energy-level diagram, where all entities comprise quanta (Figure 1), and to derive the thermodynamics of dissipative systems constituting the evolving universe [31,33,45].

The equation of state can be derived by considering what it takes for an entity, indexed with j , for example, a grain of dust, to exist. Its probability, ${}_1P_j = \phi_1\phi_2\phi_3\cdots = \prod_k\phi_k$, is a product, \prod_k , of the substrates, say atoms, indexed with k . If any k -substrate is missing, the product form ensures that ${}_1P_j = 0$. The factor, density in energy [38], $\phi_k = N_k \exp[(-\Delta G_{jk} + i\Delta Q_{jk})/k_B T]$, denotes the k -substrate in numbers, N_k , having an energy difference, $\Delta G_{jk} = G_j - G_k$, with respect to the j -product. The energy, $i\Delta Q_{jk}$, in the form of the vector potential, absorbed or emitted in the jk -transformation, is distinguished by the prefactor, i , from the scalar potential bound in matter, for example, to deal with decay that violates the presumed, stationary-state conservation of probability [46]. The self-similar exponential form, $de^x/dt = e^x$, applies for a statistical system, where a transformation perturbs the system’s average energy, $k_B T$, insignificantly [38].

The probability of a population, as a product, $P_j = ({}_1P_j)({}_1P_j)({}_1P_j)\cdots/N_j! = ({}_1P_j)^{N_j}/N_j!$, enumerating N_j interchangeable entities, ensures that $P_j = 0$ if one entity is missing entirely. Since the indistinguishable configurations, i.e., microstates, are not proper states differing by energy, P_j is divided by permutations, $N_j!$

The system’s probability, P , is the product, \prod_j , of all P_j ,

$$P = \prod_j P_j = \prod_j \left(\prod_k \phi_k \right)^{N_j} / N_j!. \quad (1)$$

The logarithm of P multiplied by k_B is entropy:

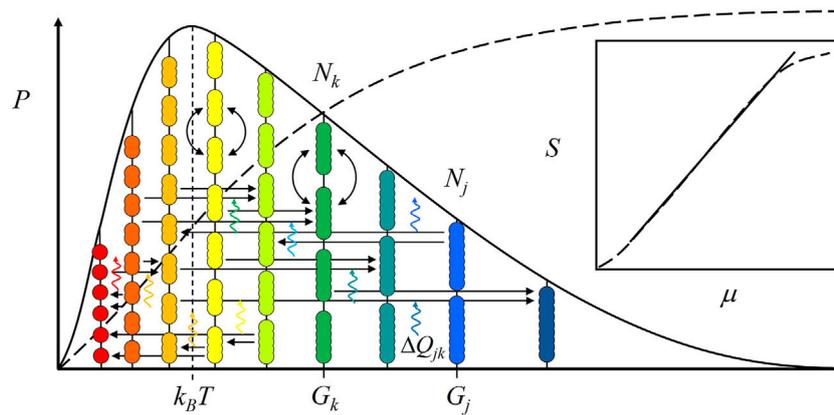


FIGURE 1

General energy-level diagram represents any system comprising quanta. Entities in numbers, N_k , of the same energy, G_k , relative to the average energy, $k_B T$, are on the same level. While their mutual exchange (bow arrows) causes no change, the system evolves toward balance with its surroundings through transformations (horizontal arrows) where quanta with energy, ΔQ_{jk} (wavy arrows) absorb into products, N_j , or emit from starting materials, N_k . The logarithm of the sigmoid, cumulative probability, P , (dashed line) is entropy, $S = k_B \ln P$. **Inset:** On the log–log scale, S vs. potential energy, μ , mostly follows a power law, i.e., a straight line.

$$S = k_B \ln P = k_B \sum_j \ln P_j = \frac{1}{T} \sum_{jk} N_j (-\Delta\mu_{jk} + i\Delta Q_{jk} + k_B T), \quad (2)$$

where $\ln N_j! \approx N_j \ln N_j - N_j$ approximates the factorial and $\Delta\mu_{jk} = \mu_j - \mu_k$ denotes the difference between the substrate potential, $\mu_k = k_B T \ln[N_k \exp(G_k/k_B T)]$, and the product potential, μ_j . When multiplied with T , S equals the system's total energy, TS . It comprises the bound, $\sum N_j k_B T$, and free, $\sum N_j (-\Delta\mu_{jk} + i\Delta Q_{jk})$, forms of energy.

As the system evolves with time, t , toward balance with its surroundings, free energy, $-\Delta\mu_{jk} + i\Delta Q_{jk}$, is consumed in dissipative transformations from N_k to N_j . For example, in stars, hydrogen is transformed into helium.

For a statistical system, the population changes can be approximated by continuous differentials, dN_j , to give the following:

$$T \frac{dS}{dt} = T \sum_j \frac{dS}{dN_j} \frac{dN_j}{dt} = \sum_{jk} \frac{dN_j}{dt} (-\Delta\mu_{jk} + i\Delta Q_{jk}), \quad (3)$$

where the variation in average energy, $k_B T$, is contained in dS . Since entropy is a function of energy (Eq. 2), S attains at $dS/dt = 0$ the steady-state maximum, $S = \sum k_B N_j$, when free energy attains its minimum, $-\Delta\mu_{jk} + i\Delta Q_{jk} = 0$.

Free energy drives the population changes

$$\frac{dN_j}{dt} = \frac{1}{k_B T} \sum_k \sigma_{jk} (-\Delta\mu_{jk} + i\Delta Q_{jk}), \quad (4)$$

through mechanistic factors, $\sigma_{jk} > 0$, facilitating jk -transformations [47]. For example, a high-density stellar core can be regarded as a mechanism enabling nuclear reactions.

Substituting dN_j/dt in Eq. 3 with Eq. 4 and squaring $-\Delta\mu_{jk} + i\Delta Q_{jk}$ in the orthogonal jk -basis show $dS \geq 0$. The fact that entropy cannot decrease follows from the conservation of quanta. If it were to be violated, the quanta emitted from matter would be lost into nowhere instead of being absorbed into the surrounding vacuum.

2.2 Continuum approximation

A continuous form of the equation of motion (Eq. 3) is useful for understanding the scale-free astronomical data. It is obtained by approximating the scalar, $\mu_j = (\partial U/\partial N_j)$, and vector, $Q_j = (\partial Q/\partial N_j)$, potentials by continuous scalar, U , and vector, Q , potentials:

$$T \frac{dN_j}{dt} = \sum_{jk} \frac{dN_j}{dt} \left(-\frac{\partial U}{\partial N_j} + i \frac{\partial Q}{\partial N_j} \right) = \left(-\frac{\partial U}{\partial t} + i \frac{\partial Q}{\partial t} \right) = \frac{d}{dt} 2K, \quad (5)$$

where TdS equals the change in kinetic energy $d2K$. As mentioned previously, the prefactor, i , serves to distinguish dissipated energy from the concomitant change in scalar potential [46].

The integral form of Eq. 5 with vanishing variation of the integrand $\mathbf{p} \cdot d\mathbf{x} = 2Kdt$ leads to Maupertuis' principle of least action where quanta with momenta, \mathbf{p} , propagate on geodesics, i.e., optimal paths, \mathbf{x} , [48]. Unlike the Lagrangian, based on the conservation of energy, Maupertuis' form is open for evolution. Mathematically speaking, the integration limit moves while integrating due to dissipation, $\partial Q/\partial t \neq 0$. Thus, Eq. 5 is non-integrable, yet not arbitrary, but bound by free energy. According

to Eqs 3–5, entropy does not only increase but also increase in the least time. Thus, bodies move along optimal, i.e., geodesics, rather than arbitrary paths. For example, a stone falls straight down.

Also multiplying Newton's second law, i.e., the change, $d_t \mathbf{p}$, in momentum, $\mathbf{p} = m\mathbf{v}$, with velocity, \mathbf{v} , gives the equation of motion:

$$\begin{aligned} \mathbf{F} &= \frac{d}{dt} \mathbf{p} = m\mathbf{a} + \mathbf{v} \frac{dm}{dt} \quad | \cdot \mathbf{v} \\ \mathbf{v} \cdot \mathbf{F} &= \mathbf{v} \cdot \frac{d}{dt} \mathbf{p} = \frac{d\mathbf{x}}{dt} \cdot m\mathbf{a} + \mathbf{v} \cdot \mathbf{v} \frac{dm}{dt} \\ &= -\frac{dU}{dt} + \frac{v^2}{c^2} \frac{dE}{dt} = -\frac{dU}{dt} + i \frac{dQ}{dt}, \end{aligned} \quad (6)$$

since kinetic energy, $2K = \mathbf{v} \cdot \mathbf{p} = \sum_j v_j m v_k$, vanishes for $j \neq k$ and $d_t \mathbf{v} \cdot \mathbf{p} = 0$ for acceleration $\mathbf{a} = d_t \mathbf{v} \perp \mathbf{v}$. The relativistic mass–energy equivalence, $E = mc^2$, derives from the total action, $nh = Et = mc^2 t = px$, of a system with n quanta. Defined in dissipative terms, the change in mass, $d_t m = d_t E/c^2 = i d_t Q/v^2$, denotes changes in geodesics, the quantized trajectories opening up for absorption of quanta from the surroundings or emission into the surroundings [49]. For example, in annihilation, quanta bound in matter transform into quanta of the vacuum.

When the system attains thermodynamic balance with its surroundings, Eqs. 5 and 6 reduce to the integrable virial theorem, $2K + U = 0$, relating the kinetic energy of a self-gravitating system, $2K$, to the gravitational potential energy, U , of the system. The time invariance, corresponding to constant energy by Noether's theorem, $2Kt = nh$, means that the quanta orbit geodesics with characteristic periods, t , of the motional modes [42,50].

2.3 The universality in data

The ubiquitous, nearly lognormal, skewed distributions summing up in a power-law manner can be deduced from Eq. 4 [32]. Initially, when there is a lot of free energy, e.g., for star formation, mechanisms, $\sum_k \sigma_{jk}$, constrain the free energy consumption, $d_t (-\Delta\mu_{jk} + i\Delta Q_{jk})$. Then, the change

$$\begin{aligned} \frac{d}{dt} \frac{1}{k_B T} \sum_k (-\Delta\mu_{jk} + i\Delta Q_{jk}) &= \frac{dN_j}{dt} \frac{d}{dN_j} \frac{1}{k_B T} \sum_k (-\Delta\mu_{jk} + i\Delta Q_{jk}) \\ &\approx \sum_k \sigma_{jk} \Rightarrow \frac{dN_j}{dt} = \sum_k \sigma_{jk} N_j, \end{aligned} \quad (7)$$

is approximately exponential. For example, the weak gravity of dilute gas curtails the initial star formation; however, soon, the increasing gravity of aggregating gas accelerates further aggregation. It should be noted that, in Eq. 7, the change reduces to $d\mu_j/dN_j = d(G_j + k_B T \ln N_j)/dN_j = k_B T/N_j$ because μ_k , Q_j , and Q_k do not depend explicitly but stoichiometrically on N_j . Conversely, the final phase dies out almost exponentially when the free energy has been nearly exhausted. For example, the star formation decreases as free gas is exhausted.

The bulk distribution between a process's initial and final phases follows a power law [32]. This becomes apparent using the atomistic axiom, $N_j = \prod_k \phi_k = \alpha_j N_1^{j_1}$. It should be noted that the j -entities, in numbers N_j , assemble from the elemental constituents, N_1 , through various mn -transformations, amassed in $\alpha_j = \prod_{mn} \exp[\sum_j N_j (-\Delta\mu_{mn} + i\Delta Q_{mn})/k_B T]$. Thus, the change

$$\frac{dN_j}{dt} = j \alpha_j N_1^{j-1} \frac{dN_1}{dt} = j \frac{N_j}{N_1} \frac{dN_1}{dt} \Rightarrow \frac{dN_j}{N_j} = j \frac{dN_1}{N_1}, \quad (8)$$

when integrated, follows a power law, $\ln N_j = j \ln N_1 + \text{constant}$. The form is familiar from various self-organizing processes [51,52]. Also, Newton's second law (Eq. 6), when divided with momentum, p , and integrated, delivers the power law, $d \ln p = d \ln v + d \ln m$. Thus, it is not surprising that power laws are ubiquitous, including gravitational phenomena.

When the system evolves gradually, the change in energy is small compared with the average energy, i.e., $|\Delta\mu_{jk} + i\Delta Q_{jk}|/k_B T \ll 1$. As the variation, n , is small, i.e., $n \ll j$, around the average factor, ϕ_j , the distribution of factors,

$$\ln \phi_{j-n \dots j+n} = \ln \phi_j + \sum_n n \ln \phi_1, \quad (9)$$

can be given in terms, $\ln \phi_j = j \ln \phi_1$, of the elemental factor, ϕ_1 . By the central limit theorem, data distribute around the typical form, j , in a skewed, approximately lognormal manner [23,53,54]. For example, all red giants look like red giants and not white dwarfs—the red giant distribution peaks on the typical giant and the white dwarf distribution on the typical dwarf. Logarithmic spirals are also approximately lognormal distributions in polar coordinates, i.e., energetically optimal forms [32].

Equations 7–9 show that the skewed distributions sum up along sigmoid curves [55]. Thus, data follow closely, but not precisely, the power law, deviating at low and high ends [56]. Moreover, many data display a series of power laws whose slope changes when a mechanism of transformations changes to another. For example, in the main sequence, the most massive stars are the most effective at releasing energy through nuclear reactions, whereas the stars with the lowest mass are the least effective. Likewise, the cosmic ray flux turns from one slope to another when one mechanism of dissipation changes to another [57,58].

3 Distribution of quanta in the vacuum

Bose worked out the quantum statistics of photons that underlies Planck's law [59]. However, the vacuum structure that dictates the distribution was not explicitly spelt out. Therefore, we rederive the Bose–Einstein distribution for the quanta that embody the vacuum.

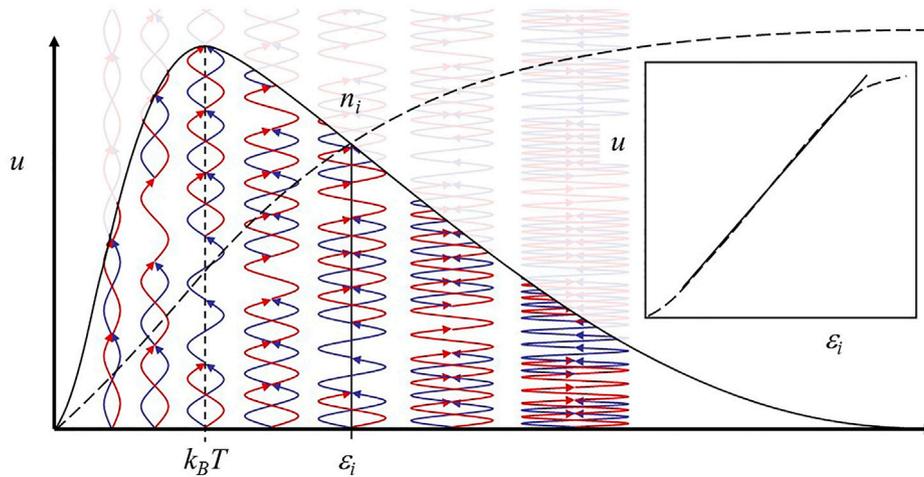


FIGURE 2 Vacuum spectral density, u , sums up from numerous rays of photons (blue-red waves) with a spectrum of energies, ϵ_i , about the average energy, $k_B T$. The paired photons cannot be seen as light but are sensed as inertia and gravitation through their coupling to matter [63]. In contrast, the odd quanta (blue or red), distributed in-phase or antiphase among the paired rays, are seen as light and manifest as electromagnetism. **Inset:** The cumulative spectral density vs. energy (dashed line) mostly follows a power law, i.e., a straight line on the log–log plot.

To regard the vacuum as physical as matter is motivated because the vacuum holds the energy of all gravitation equal to the energy of all matter [39]. Moreover, similar to matter, voids also distribute in a power-law manner [11,60], suggesting thermodynamic balance between the two. Geometrically speaking, the Ricci curvature and metric tensor equate to the stress–energy tensor in the Einstein field equations. Also, in quantum field theory, particles equate to the excitations of fields that permeate the vacuum.

While the readily detectable radiation accounts only for a tiny fraction ($\sim 10^{-4}$) of the gross energy density of space [61], it still implies that the vacuum has a physical structure, namely, the cosmic microwave background spectrum, the black body spectrum at temperature, $T = 2.7$ K, by having a distinct form rather than being formless, discloses that photons do not distribute at random but according to the structure of space.

The correspondence between the speed of light, c , and the vacuum permittivity, ϵ_0 , and permeability, μ_0 , by $c^2 = 1/\epsilon_0\mu_0$ [62], and the gravitation of the whole universe of mass, M , and radius, R , by $c^2 = GM/R$ [39], suggests that the vacuum comprises quanta of light. Indeed, the Michelson–Morley experiment did not rule out a light-embodying medium; it only rejected a light-carrying medium.

According to the atomistic axiom, two destructively interfering photons do not vanish into vacuum [63]. Instead, only their electromagnetic effects are negated. Thus, the vacuum comprising photons in such pairs is a transparent, relativistic substance without net electromagnetic force, yet with gravitational energy density.

3.1 The radiation law

The background spectrum of free space can be derived from the atomistic axiom by considering how photons of energy, ϵ_i , relative to the average, $k_B T$, distribute in numbers, n_i , among rays of paired photons (Figure 2). The number of ways, in-phase and antiphase, the photons populate the numerous rays intersecting in degenerate directions, g_i , is the product of n_i combinations of the sets with $n_i + 2g_i - 1$ elements.

$$W = \prod_i \frac{(n_i + 2g_i - 1)!}{n_i! (2g_i - 1)!} \exp(-n_i \epsilon_i / k_B T) \tag{10}$$

$$\approx \prod_i \frac{(n_i + 2g_i)!}{n_i! 2g_i!} \exp(-n_i \epsilon_i / k_B T).$$

Taking logarithm and using Stirling’s approximation for the factorials yields, we obtain the following:

$$\ln W = \sum_i (n_i + 2g_i) \ln (n_i + 2g_i) - n_i \ln n_i - 2g_i \ln 2g_i - n_i \epsilon_i / k_B T. \tag{11}$$

The most probable state is the stationary state:

$$\frac{d \ln W}{dt} = \sum_i \frac{d \ln W}{dn_i} \frac{dn_i}{dt} = \sum_i \frac{dn_i}{dt} \left(\ln \frac{n_i + 2g_i}{n_i} - \frac{\epsilon_i}{k_B T} \right) = 0, \tag{12}$$

where quanta distribute according to the Bose–Einstein distribution,

$$n_i = \frac{2g_i}{e^{\epsilon_i / k_B T} - 1}, \tag{13}$$

having two states for each photon, in-phase and antiphase, per locus in the paired ray (Figure 2) [59]. Since the spectral density of the paired-photon vacuum has precisely the form of the black body spectrum, we consider that the void is basically as physical as matter comprising quanta.

In agreement with Planck's law of radiation, the photon pairs open up with increasing temperature and pair up with decreasing temperature. Moreover, the paired-photon vacuum responds to an accelerating charge by unpairing and radiating photons. Thus, the photons do not appear out of vacuum nor disappear into it but comprise the vacuum.

The proposed substance of space also complies with electromagnetism. Explicitly, the paired-photon vacuum, embodying the four-potential of the electric scalar potential, ϕ , and magnetic vector potential, \mathbf{A} , satisfies the Lorenz gauge [64],

$$\nabla\phi + \frac{\partial\mathbf{A}}{\partial t} = \frac{1}{c} \frac{\partial\phi}{\partial t} + c\nabla \cdot \mathbf{A} = 0, \quad (14)$$

as a seemingly continuous and indestructible substance whose wave nature gives rise to interference phenomena. For example, in the Aharonov-Bohm experiment [65], a phase difference

$$\Delta\phi = \phi_1 - \phi_2 = \frac{e}{\hbar} \int \mathbf{A}_1 \cdot d\mathbf{x}_1 - \frac{e}{\hbar} \int \mathbf{A}_2 \cdot d\mathbf{x}_2, \quad (15)$$

develops between waves when charges, e , traverse \mathbf{A}_1 and \mathbf{A}_2 along paths \mathbf{x}_1 and \mathbf{x}_2 , respectively

Gradients in the vacuum are fields [66–68]. The electric field,

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}, \quad (16)$$

for example, around an electron is a gradient in the paired-photon rays, whose winding tallies the electron winding number, a topological quantum number [69]. Therefore, it takes the full 4π , rather than only 2π , rotation of the electron, e^- , to return the vacuum to its original state [70,71]. This $SU(2)_L$ symmetry is disclosed by monodromy of the vacuum around the electron, $e^- = \int \rho dV = -\epsilon_0 \int \nabla \cdot \nabla \phi dV = -\epsilon_0 \int \nabla \phi dS$, enclosed in a volume, V , or, as by Gauss' law, bordered by an area, S .

The vacuum polarization is similar to the polarization of a dielectric material; the paired-photon rays wind up, i.e., twist, to counteract introduced charges. Conversely, when charges neutralize, the unwinding of the paired-photon rays gives rise to a time-varying electric field, i.e., displacement current, $J_D = \epsilon_0 \partial_t \mathbf{E}$.

The magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad (17)$$

is a vortex of the twists in the paired-photon rays, for example, encircling a line of moving charges (Figure 3).

The electromagnetic fields (Eqs. 16 and 17) give rise to the Lorentz force,

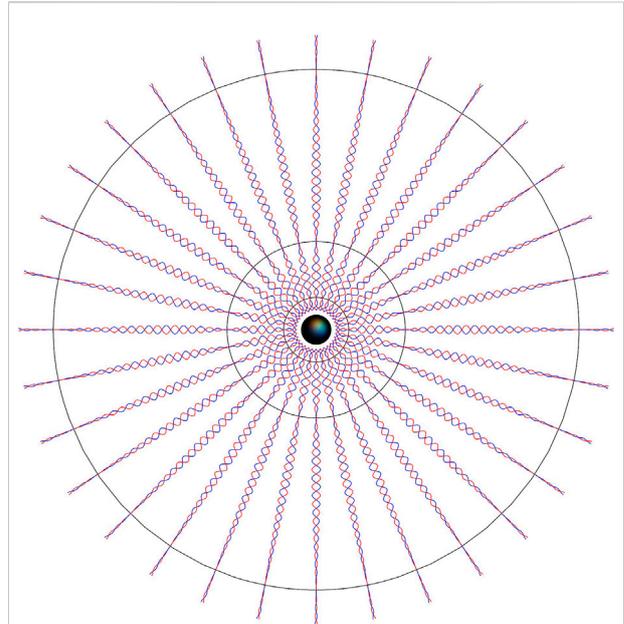


FIGURE 3

Electric potential diverges from a current-carrying wire (black, pointing straight at) as paired-photon rays (sketched as red and blue waves) whose winding matches the winding number of the charge density along the wire and decays inversely with distance, r . The magnetic field lines (thin concentric circles) correspond to the lines encircling the wire at a distance where the winding is the same.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (18)$$

experienced by a charge, q , moving with velocity, \mathbf{v} . Thus, from the proposed perspective, it is not that charges themselves would attract or repel each other; instead, the charges move as they couple with the physical vacuum that is leveling off its gradients.

The proposed substance of space also complies with gravitation. The gradient in the gravitational four-potential of the scalar, ϕ_g , and vector, \mathbf{A}_g , components is the gravitational field [72–76], i.e., acceleration.

$$\mathbf{E}_g = -\nabla\phi_g - \frac{\partial\mathbf{A}_g}{\partial t}. \quad (19)$$

For example, at a distance, r , from a body of mass, M_o , the density gradient of the paired-photon potential, $\phi = GM_o/r$, tallies the geodesic curvature, the Euler characteristic, χ , proportional to M_o (Figure 4) [77]. The density gradient is observed as the bending of light rays, a gravitational time delay, and as a gravitational frequency shift, $f_e/f_o = 1 + z = (1 - GM_o/c^2 r_e)^{-1/2}$, between emission, f_e , at the radius, r_e , and absorption, f_o , at the detection [73,78]. In turn, the leveling of a density gradient is detected as the Doppler shift of light emitted by the moving body coupled to the moving vacuum.

Similar to $\mathbf{B} = \nabla \times \mathbf{A}$ (Eq. 17), the rotational field,

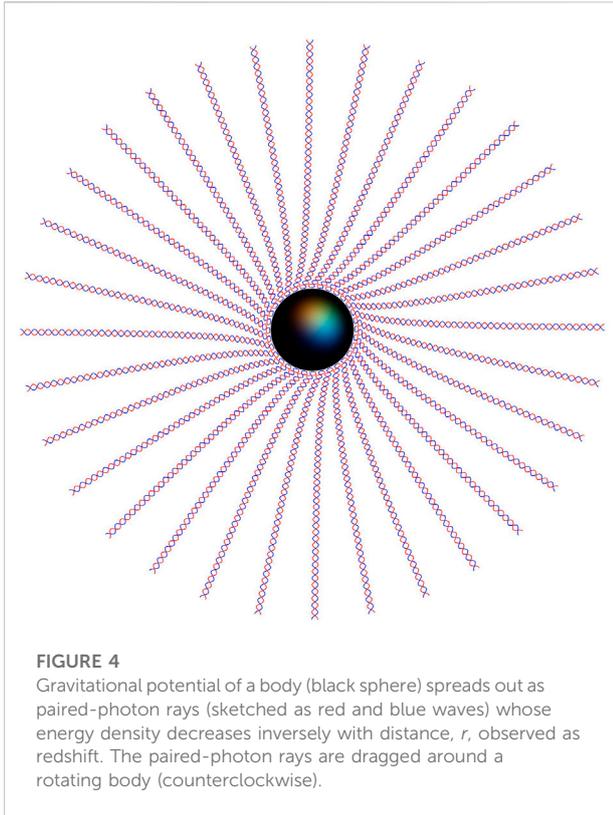


FIGURE 4
Gravitational potential of a body (black sphere) spreads out as paired-photon rays (sketched as red and blue waves) whose energy density decreases inversely with distance, r , observed as redshift. The paired-photon rays are dragged around a rotating body (counterclockwise).

$$\mathbf{B}_g = \nabla \times \mathbf{A}_g. \quad (20)$$

for example, at a distance, r , from a body with inertia, I , rotating with the angular velocity, ω , revolves at the rate, $\Omega = GI/c^2 r^3 \nabla \times (\omega \times \mathbf{r})$ [79]. This vortex in the paired-photon density displays itself as frame-dragging precession [73] (Figure 4) that was examined, for example, with Gravity Probe B [80,81].

Moreover, similar to Gauss' law, the divergence of the density gradient in the vacuum potential relates to the mass density, $\rho_M = -(1/4\pi G)\nabla^2 \phi_g$. Explicitly, $\nabla \cdot \nabla \phi_g = \nabla \cdot \mathbf{a}_R = \partial_t GM/R^2 c = -1/t^2$, of the all-embracing gravitational potential, ϕ_g , due to all mass, M , of the expanding universe of radius, $R = ct$, relates inversely to the age, t , squared using the mass–energy equivalence, $Mc^2 = GM^2/R$. In other words, the average mass density of the universe, $\rho_M = 1/4\pi Gt^2$, decreasing with time, t , is the source of space embodied by the paired photons.

While all photons are bound to the universe they constitute, a paired photon is not a bound state because the vacuum's energy spectrum is continuous. Also, the difference in the photon phases may change continuously, for example, in response to a moving charge. Despite its dynamics, the paired-photon ground-state vacuum does not break apart easily because the electromagnetic force is huge, e.g., compared with gravity.

The vacuum moves toward thermodynamic balance so that gradients in the vacuum energy, e.g., electromagnetic and

gravitational fields, decrease, which is seen as charges attracting or repelling, magnets reorienting, and bodies moving.

Since light cannot but propagate at the speed of light, c , in the relativistic medium comprising photons, the proposed physical vacuum makes sense of the classical experiments carried out by Arago [82], Fizeau [83], Michelson and Morley [84], Trouton and Noble [85], and Sagnac [86], and modern measurements named after Casimir [87] and Aharonov and Bohm [65,88]. Moreover, the dynamic Casimir effect yields photons out of the vacuum, showing two by two at a time [89].

In agreement with measurements [61] and theory [39], the vacuum and matter comprising quanta are in thermodynamic balance $Mc^2 = GM^2/R$, where the energy of all mass, M , in the universe of radius, $R = ct$, and age, t , tallies with all gravitational energy. This is in line with general relativity. The vacuum quanta embodying the Ricci curvature tensor and metric tensor are in balance with the quanta bound in the sources of gravitation denoted by the stress–energy tensor.

On the largest scale, gravitation is geometrized by the cosmological principle so that the metric, $ds^2 = a(t)^2 dx^2 - c^2 dt^2$, sums up by Pythagoras' theorem the infinitesimal distance, ds , from the spatial, dx , and temporal, dt , coordinates, which correspond to the photon wavelength and period. While the scale factor, $a(t)$, parameterizes the expansion, the thermodynamic balance between all matter and space implies that the universe expands physically rather than [para]metrically. As the quanta of matter transform into the paired photons of the vacuum, the energy density decreases. Redshifts from distant sources document this density change from the past to the present [7,10]. In geometric terms, the curvature of spacetime not only corresponds to but also spacetime itself emerges from the stress–energy tensor [90].

Also, consistent with quantum field theory, the physical vacuum comprising quanta of light, both paired and single, in balance with matter can be modeled with the mean-energy density photon wave function,

$$\psi(\mathbf{r}, t) = \int \frac{\sqrt{E} d\mathbf{p}}{h^{3/2}} \gamma(\mathbf{p}, t) e^{i\mathbf{p}\cdot\mathbf{r}/h}, \quad (21)$$

as an integral over the photon probability amplitudes, γ , of momentum, \mathbf{p} , and energy, E , in the unit volume, h^3 . The dependence on the position, \mathbf{r} , is formal because the massless photon does not localize [91,92].

Accordingly, a particle perturbing the vacuum can be presented with a wave function. This wave packet of the vacuum wave functions follows the Schrödinger equation and disperses as the particle with mass couples to the vacuum. In other words, mass is the measure of the coupling, i.e., inertia. In contrast, a photon as part of the vacuum (Fig. 2) abides by the dispersionless wave equation. Without the physical vacuum, particles would not have mass just as without the Higgs field. While, in this sense, the proposed paired-photon boson

resembles the Higgs boson, the massless photon does not decay, whereas the massive Higgs boson is short-lived.

3.2 The graviton

Assuming that the paired-photon strings or filaments form the fabric of space, their density gradients are gravitational fields [93–97]. Since the substance of space seeks thermodynamic balance in the least time, the bodies move coupled to the moving vacuum along the least-time paths, i.e., geodesics. Conversely, orbits are closed at a stationary state of the vacuum, where the influxes and effluxes of quanta tally. Then, the kinetic energy, $2K = mv^2$, of an orbiter of mass, m , and velocity, v , balances gravitational potential energy, $U = GmM/r$, at a distance, r , due to a central mass, M , as according to Kepler's third law.

The photon qualifies as the ground-state substance because it does not decay. In our judgment, the paired photon, a massless spin-2 particle, is indistinguishable from the theorized graviton, the carrier of gravitation [73]. Correspondingly, the photon, a massless spin-1 particle, is the carrier of electromagnetism. Moreover, gravitation and electromagnetism cannot but share the same $1/r$ -form when the photon-embodied vacuum mediates both forces [63], as was thought already early on [66].

4 Dark matter

According to concordance cosmology, orbital velocities of stars and gas clouds in galaxies, radial velocities of galaxies in clusters, and gravitational lensing by galaxies imply the presence of large amounts of hypothetical dark matter. Dark matter exceeds ordinary matter five-fold when the standard Λ CDM model is tuned to match the data.

In contrast, we argue that the excess gravitation attributed to dark matter, in fact, displays the aforementioned thermodynamic distribution of vacuum quanta across the expanding universe in balance with quanta bound in all ordinary matter. We motivate our conclusion empirically. Sky surveys show that galaxies and voids distribute in a power-law manner [11,60]. Newton's universal law of gravitation equates gravitational force to the distribution of matter [98], and the Einstein field equations equate the distribution of matter, in the broadest sense of energy, to the curvature of spacetime [99].

We arrive at our conclusion from the understanding that the universe is expanding. Thus, the distant early universe is dense, and the nearby present is sparse. In line with Eq. 6, as by the universal law of gravitation, this isotropic gradient in all mass, M , across the radius, $R \equiv ct$, about 13.8 billion light years causes acceleration, $a_R = GM/R^2 = c/t$, of the order of 10^{-10} m/s². The universal density gradient due to the expansion is most

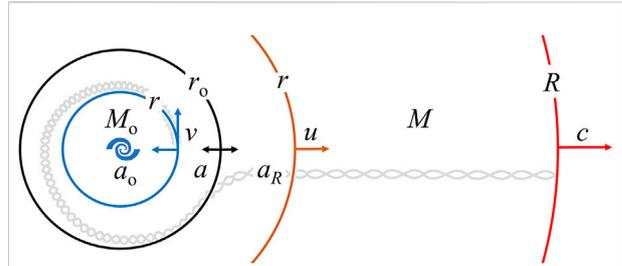


FIGURE 5

Within the radius, r_o , confining a local mass, M_o , the paired-photon graviton efflux from local processes exceeds the influx from distant sources. Thus, a body spirals inward until v^2/r balances the sum of the universal, a , and local, a_o , acceleration. Since the circumference, $2\pi r$, shortens, graviton by graviton, as much as the distant space lengthens along with radius, r , the inward and outward acceleration relate as $a = a_R/2\pi$. Beyond r_o , the graviton influx from numerous processes transforming matter into space exceeds the efflux. Thus, the body recedes at speed, $u < c$. At R , enclosing all mass, M , the total graviton flux from all processes generates the expansion at the speed of light, c .

discernible from gravitational redshifts of light originating from the most distant sources [7,78].

The universal gravitation builds up with distance, r , from numerous sources, i.e., galaxies, as their number $\propto r^2$ overpowers the $1/r$ -potential [100,101]. Quantitatively speaking, the vacuum energy density, $\rho = c^2/4\pi Gt^2 = GM^2/4\pi R^4 \approx 10^{-9}$ J/m³, gauged by WMAP [61], integrates to the gravitational potential, $U = GM^2/R$, over all mass, M , equaling the total energy, Mc^2 . This universal balance holds to an excellent approximation since, at any given moment, the dissipated energy is much smaller than the energy bound in matter, i.e., $Q \ll U$ (Eq. 6). Thus, we employ the virial theorem,

$$2K + U = 0 \Leftrightarrow Mc^2 = \frac{GM^2}{R} \Leftrightarrow a_R = \frac{GM}{R^2} = \frac{c^2}{R} = \frac{c}{t} \equiv cH, \quad (22)$$

to infer the acceleration, $a_R \approx 6.87 \cdot 10^{-10}$ m/s², of the expansion, happening per definition at the Hubble rate, $H \equiv 1/t$, decreasing $d_t H = -1/t^2$ as the universe ages with time, t . The rate of expansion is not free from the flow of time, assuming quanta of space emerge from quanta of matter because energy, E , and [period of] time, t , are physical properties of the quanta [49,50].

The density gradient across the cosmos from the beginning to the present manifests itself in the outward acceleration, a_R , known as the Hubble flow, as well as in the inward acceleration per orbital cycle, $a \approx a_R/2\pi$, also deemed as modified gravity (Figure 5) [102,103].

The outward and inward fluxes of quantized space (Figures 2–5), originating from local and remote processes, balance each other at a distance of about 4 million light years. This zero-velocity radius, $r_o = GM_o/c^2$, relates to $R = GM/c^2$ as M_o of the local group relates to M of the universe (Figure 5) [58,104,105]. Within r_o , the Milky Way, Andromeda, and other nearby galaxies

approach one another because the efflux of space emerging from processes within the local group to the distant universe exceeds the influx therefrom (Figure 5).

The thermodynamic balance between all matter and space free of matter (Eq. 6) [39] suggests that the cosmos is not ballooning through intrinsic metric expansion [106] without any cause. Instead of being a cosmic coincidence, the flatness, $\Lambda \equiv H^2/c^2 = 4\pi G\rho_c/c^4$, follows from matter transforming into the physical space in stars, black holes, etc. These high-energy reactions, similar to annihilation, power the expansion by $P = Fc = Ma_Rc = c^5/G$, generating pressure, $p = F/A = c^4/4\pi R^2G = \rho_c$, and dispersing distant galaxies apart with velocity asymptote, $c^4 = a_RGM$.

In agreement with empirical evidence, it follows from the atomistic axiom that space emerges from matter. This old tenet [107,108] explains why the energy density, ρ , is precisely the critical ρ_c . There is neither a need nor room to adjust the cosmological constant, $\Lambda \approx 1/R^2$, to the observed flatness [109,110]. Since the scale-free data emerge from the least-time consumption of free energy [32], there is no point in modeling the expansion with scale-free cosmology [111].

Backed up by this reasoning, we focus on motions of the relativistic void, the paired-photon substance coupled to bodies, instead of assuming bodies themselves would attract one another.

4.1 Galaxy rotation

To an excellent approximation, a spiral galaxy is a stationary system. At the dynamic steady state, integrated over its characteristic orbital period, t ,

$$\int (\mathbf{v} \cdot d_t \mathbf{p} + \mathbf{v} \cdot \nabla U) dt = 0, \quad (23)$$

momentum, \mathbf{p} , and acceleration, \mathbf{a} , are orthogonal. The orbital velocity, v , at a distance, r , balances by v^2/r the galactic acceleration, $a_o = GM_o/r^2$, due to the central mass, M_o , within the circumference, $2\pi r$, and the universal acceleration, $a = GM/2\pi R^2$, due to all mass, M , within the radius, R , of the expanding universe [112],

$$\frac{v^2}{r} = a + a_o = a_o \left(1 + \frac{a}{a_o} \right) = \frac{GM_o}{r^2} \left(1 + \frac{1}{2\pi} \frac{Mr^2}{M_o R^2} \right), \quad (24)$$

near the galactic center, $a \ll a_o$. Therefore, the feeble universal acceleration is hardly detectable in motions of strongly bound bodies, such as stars in clusters [113].

Conversely, far away from the luminous edge, $a \gg a_o$, where $v^2 a_o/r \approx aGM_o/r^2$, Eq. 24 limits to the Tully–Fisher relation, $v^4 = aGM_o$, for example, orbital velocities of dwarf galaxy profile Eq. 24 due to their low amounts of baryonic rather than high amounts of dark matter [114,115]. The flat tail of the orbital velocity curve signifies that distantly, the universal curvature, $1/R$, dominates the curvature, $1/r = a/v^2$, of a bound geodesic. A

closer match with a given observed rotation curve beyond the point-mass approximation (Eq. 24) would employ a detailed mass distribution, $M_o(r)$, instead of an interpolation [116,117], from the dense galactic surroundings to the sparse universal voids.

Moreover, satellites of the Milky Way, Andromeda Galaxy, and other spirals can be understood to home in on the galactic plane [118] under the central force of the expansion because space, the substance of gravitation, zeros in on the free energy minimum of the least curvature. So, a galaxy punched by another realigns for the same reason as a poked top reorients. The substance of space also shapes bodies into spheroids.

4.2 Galaxy velocity dispersion

Just as stars in galaxies orbit around, galaxies also move about in clusters to balance the local gravity and the universal gravitation [16,119]. The Faber–Jackson relation, ranging from tiny dwarf satellite galaxies through giant spiral galaxies to rich clusters of galaxies [120], even encompassing the whole expanding universe, displays the universal acceleration, $a_R = c^2/R = u^2/r = 2\pi a = 2\pi v^2/r$, in terms of speed of light, c , radial, u , and orbital, v , velocity. Since $u^2/r = 2\pi v^2/r$, the mass vs. radial velocity line on a log–log plot, relevant to galaxy velocity distribution, is offset by the factor $\sqrt{2\pi} \approx 2.5$ from the mass vs. orbital velocity line, relevant to galaxy rotational curve [8]. So, there is no need to extend MOND [121] or invoke the dark matter hypothesis to account for the velocity dispersion of galaxies in clusters.

While the universal acceleration due to the expansion shares the mathematical form (Eq. 24) of MOND [116], the thermodynamic tenet is intrinsically a relativistic formulation. The paired-photon void is a relativistic medium complying with the Bose–Einstein statistics (Eq. 13).

4.3 Bending of light

According to concordance cosmology, gravitational lensing provides yet another evidence of dark matter. This belief is based on images of background galaxies lensed through foreground galaxy gravitation because the images are more distorted than expected from the lensing masses of visible, ordinary matter [122].

We follow Einstein, who stated early on that light going through a gravitational potential, such as through an optical medium, gets deflected and delayed [93]. The stronger the gravity, the more curved the geodesic, i.e., the least-time path. A local gravitational potential, GM_o/r , at a distance, r , from a mass, M_o , is part of the vacuum's total potential due to M within the universe of radius, R . Thus, near a celestial body of mass, M_o , the refractive index, $n^2 = c^2/v^2 = (1 - GM_o/c^2r)^{-1} \approx 1 + GM_o/c^2r$ [81,94,95].

According to Eq. 22, the space curving around a body, for example, the Sun, causes an angular acceleration, α . It integrates over time, t , to an angle of precession, φ ,

$$\begin{aligned} \frac{dL}{dt} &= I\alpha = I \frac{d}{dt} \frac{d\varphi}{dt} = mr^2\omega^2 = (2\pi)^2 mv^2 = (2\pi)^2 \frac{GmM_\odot}{r} \\ \Rightarrow \varphi &= \iint (2\pi)^2 \frac{GM_\odot}{r^3} dt dt = 2\pi^2 \frac{GM_\odot}{c^2 r}, \end{aligned} \quad (25)$$

where L is the angular momentum, G gravitational constant, $r = ct$ the distance to the center of the Sun of mass, $M_\odot = 1.99 \cdot 10^{30}$ kg, and the mass, m , corresponds to light's energy, hf , per the speed of light squared, c^2 . Eq. 25 gives $\varphi = 8.65''$ for the ray that tangents the Sun's surface at $r_\odot \approx 695,700$ km. Accordingly, it takes an excess time, $\Delta t = 2r_\odot\varphi/c = 196 \mu\text{s}$, for the light to make a round trip along the geodetic line than along the night-sky straight line, in agreement with measurements [78].

The least-time principle reproduces bending and delay consistently with the same equation (Eq. 25), as the two phenomena are identical. In contrast, derivations from the Einstein field equations deliver the delay and bending in terms of different equations, which leads to a discrepancy. By Eq. 25, light passing a galaxy deflects $2\pi^2/4 \approx 5$ times more than by general relativity. So, there is neither a need nor room for dark matter to account for the galaxy image distortions due to gravitational lensing [78].

The amount of bending due to the Sun is determined by the difference between the line of sight to a star during an eclipse and the night-sky line 6 months later. Although obvious, it is essential to note that the ray ends up at a different point on the ground when it has passed through the gravitational lens than when coming directly from the night sky. So, to trace the same ray, the telescope should be displaced by the distance corresponding to the change in direction.

It is easy to grasp the relation between the change in direction and the telescope's displacement. For example, looking at your finger with one eye and then switching to the other seems to shift it relative to the background. If the parallax is to be averted, the position of the eye, like the location of the telescope, has to be displaced to match the angle from which the object is seen. Unfortunately, as Eddington did not consider this, the bending appeared smaller than it was [78]. Thus, based on gravitational lensing, there are no compelling arguments to purport dark matter.

Since the gravitational time delay, Δt , relates to the local mass, M_\odot , as the age of the universe, t , relates to the mass of the universe, M , i.e., $\Delta t/t = (2\pi)^2 M_\odot/M$ [123], Δt between two images of a variable signal [124] can be used to determine the Hubble parameter, $H = 1/t = (\Delta\theta)2d_s d_l/d_s R \Delta t$, from the image separation, $\Delta\theta$, the observer-source, d_s , observer-lens, d_l , and lens-source, d_s distances [125]. However, as contemporary inconsistencies in H imply, interpretations of galaxy lensing, time delay data [126], cosmic background radiation, and Type Ia supernovae data are predicated on the amount of postulated dark matter [127]. Thus,

the bending of light passing by the Sun, seemingly free of dark matter, provides the crucial experimental reference to judge the hypothesis.

5 Dark energy

According to concordance cosmology, the universe is expanding at an accelerating rather than decelerating rate because distant Type Ia supernovae are slightly fainter, i.e., further away than projected from their redshifts. Dark energy is hypothesized to account for the difference between observations and calculations. When the Λ CDM model is adjusted to match the data, the rate of expansion, the Hubble parameter, H , is mostly a function of dark energy, less of dark matter and even less of ordinary matter, and minimally of radiation and curvature of the universe.

However, we argue that the standard candle data attributed to dark energy, in fact, display the distribution of vacuum quanta in thermodynamic balance with matter quanta, extending from the early dense universe to the present sparse surroundings. We arrive at our conclusion by examining light's propagation from a supernova across the expanding universe as a physical process rather than modeling the supernovae data through metric expansion.

5.1 The rate of expansion

Considering a photon emitted with the standard-candle energy, hf_e , the photon shifts red on its least-time path from the dense past to the sparse present to balance the decreasing universal gravitation. Otherwise, the photon would gain energy out of nothing [128] relative to the surrounding photon distribution, diluting, and cooling due to the expansion (Eq. 13).

The optical distance, D_L , to the supernova is deduced from the flux, $F = L/4\pi D_L^2$, i.e., the observed luminosity, $L = hf_e f_o$, that scales down as light spreads across an ever-larger area, $4\pi D_L^2$. For calibration, an explosion at a known distance, e.g., a nearby one without a marked shift in the observed frequency, i.e., $f_e \cong f_o$, serves as a reference flux, F_r , [78]. Thus,

$$\begin{aligned} F &= \frac{L}{4\pi D_L^2} = F_r \frac{L/4\pi D_L^2}{L_r/4\pi D_r^2} \cong F_r \frac{hf_e f_o r_r^2}{hf_e f_o D_L^2} \\ &= F_r \frac{1}{1+z} \frac{r_r^2}{R^2} \frac{(1+z)^2}{z^2} = F_r \frac{r_r^2}{R^2} \frac{1+z}{z^2}, \end{aligned} \quad (26)$$

where frequencies are in terms of the redshift, $z = (f_e - f_o)/f_o$, and the optical distance, $D_L = Rz/(1+z)$, relative to the radius of the universe, R .

On its way, light shifts to red due to the supernova's recession velocity and the expanding universe's decreasing gravitational energy density. More specifically, the gravitational frequency shift [73,129] is not the hypothetical tired light [130].

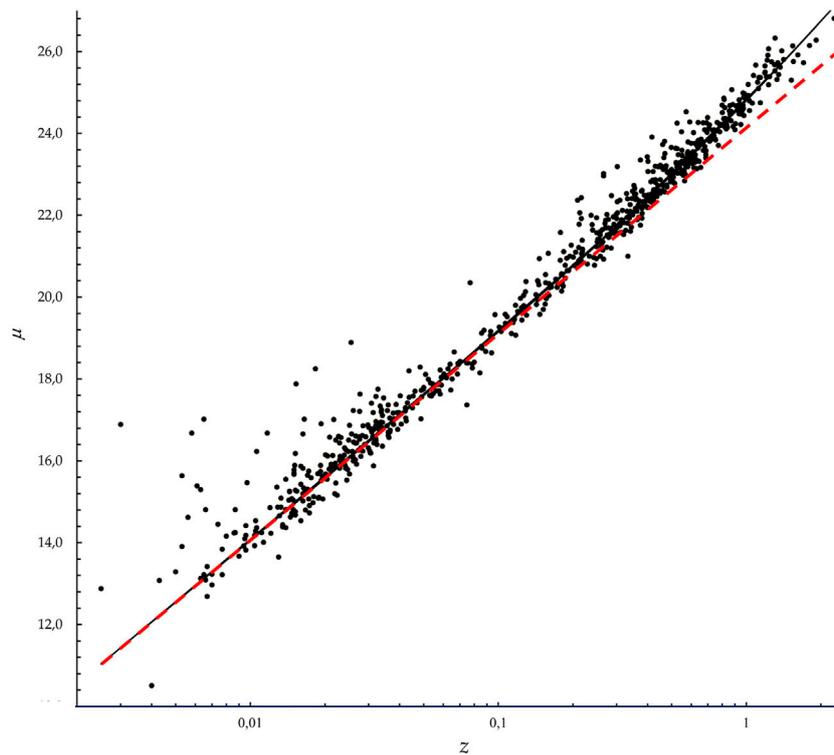


FIGURE 6

Curve calculated by the least-time principle (Eq. 27) (solid line) closely follows the brightness of Type Ia supernovae data, the distance modulus, μ , vs. redshift, z , rather than the straight line of metric expansion without dark energy [7,9,10].

However, if the physical process is parametrized with the scale factor, as it is in standard cosmology, instead of using Eq. 26, the distance seems longer than it is. Then, the Type Ia supernova data give a false impression of the universe expanding at an accelerating rate [78].

Due to the same Λ CDM parametrization, the angular size redshift relation is non-monotonous. When $z > 1.5$, objects ever further away appear peculiarly, i.e., larger, contrary to empirical evidence [131]. In contrast, as quanta bound in matter transform into the vacuum quanta, the total energy density, $\rho = c^2/4\pi G t^2$, decreases with time, t , and the angular size, $\theta \propto 1/t$ [123].

On the logarithm scale, the brightness of a star (Eq. 26), the distance modulus,

$$\begin{aligned} \mu &= -2.5 \log\left(\frac{L/4\pi D_L^2}{L_r/4\pi D_r^2}\right) + K \\ &= -2.5 \log\left(\frac{R}{r_r}\right)^2 + 2.5 \log\left(\frac{z^2}{1+z}\right) + K, \end{aligned} \quad (27)$$

is a function of two terms, $5\log(z)$ and $-2.5\log(1+z)$. So, $\mu(z)$ does not follow a straight line but curves at $z \approx 1$. The instrument factor, $K \approx 5\log(1+z)$, corrects for sensitivity, i.e., the more light shifts to red, the less is detected [131,132].

The calculated moduli (Eq. 27) follow the measured ones closely (Figure 6) [78], the difference mostly being within the scatter (Figure 7). Thus, the least-time propagation of light explains the supernovae data. There is no room for purporting dark energy. In other words, the expansion rate does not increase. On the contrary, it decreases at the rate, $-d_t H = 1/t^2 = 4\pi G \rho_M$, with decreasing density, ρ_M , as quanta in matter transform into quanta of space.

5.2 The uniformity of the horizon

The uniformity of the cosmic microwave background temperature and the evenness of the distant galaxy distribution are customarily ascribed to a hypothetical inflationary scenario, a nascent exponential expansion [133].

However, by the thermodynamic theory, the correlation beyond causal connection follows naturally and necessarily from the least-time quest for balance (Eq. 5), namely, Newton's second law of motion states that the bigger a force, the faster the change in motion (Eq. 6), which means, for instance, that the higher the temperature difference, the faster the rate of cooling. Thus, there will be minute temperature

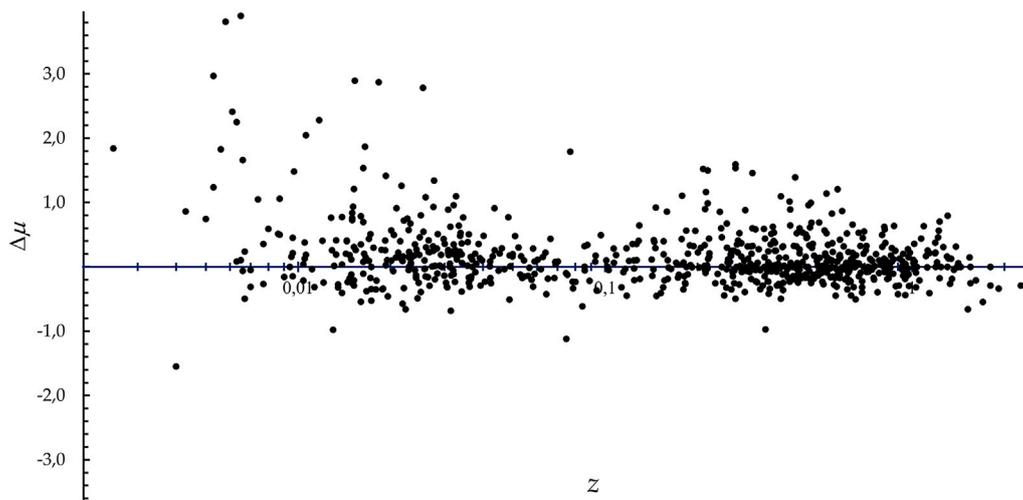


FIGURE 7

Difference, $\Delta\mu$ vs. z , between the measured supernova brightness vs. redshift and the values calculated by the least-time principle (Eq. 27).

differences in due course, regardless of how massive the differences were initially.

Although the early differences in temperature and density have by now nearly evened out, the process is still ongoing. A large difference in energy decreases rapidly and a small one evens out slowly. For example, the most massive stars shine the brightest and for the shortest time, well below 100 million years; however, small stars glow over 100 billion years. Thus, there will be only small differences over time, as is observed. The derived distributions of quanta in matter (Eq. 2) and space (Eq. 13) imply that eventually, as $T \rightarrow 0$, everything will become even, regardless of how uneven the earliest universe was.

6 Discussion

Massive amounts of astronomical data obtained through sky surveys have disclosed scale-free patterns. Observables distribute nearly in a power-law or lognormal manner and hence trail mostly straight lines on log-log plots over many orders of magnitude.

This universality in data has long been suspected to reflect the unity of nature. However, the lead pointed out by Boltzmann and Gibbs has hardly been pursued beyond statistical mechanics of stationary-state systems. Yet, the same atomistic axiom also serves as the foundation for statistical mechanics of open, evolving systems. Assuming that all entities ultimately comprise the same basic constituents, motions of quanta from one state to another toward balance can be derived. This universal quest for

thermodynamic balance between energy-dense matter and sparse space drives expansion and gives rise to the gradient of gravitational energy across the universe from the past to the present. As trivial as it might be, this universal gradient in the gravitation of all ordinary matter makes sense of galaxy rotation, velocity dispersion, and the Type 1a supernovae data without unsubstantiated hypotheses of dark matter and dark energy.

As is well known, when deriving general relativity from postulates, Einstein conformed to the tradition of theorizing, but departed by adding the cosmological constant. The mistake was insignificant in mistaking the universe as static but degrading the proper theory to an effective theory amenable for further fine-tuning rather than falsification [134].

Without question, the standard model of cosmology splendidly reproduces many local observations. Still, it falls short universally by assuming intrinsic metric expansion rather than recognizing that quanta in matter transform into quanta of space. Consequently, despite filling in the discrepancy between data and expectations, the dark matter hypothesis disregards the gravitation of all ordinary matter. Likewise, despite fulfilling galaxy rotation curves remarkably well [135], the modified Newtonian dynamics [136] misses identifying the universal acceleration with the gravitation of the expanding universe. Moreover, while fitting data, the dark energy model disregards the redshift due to the gravitational gradient of all ordinary matter across the universe.

So, despite a model matching data, its parameters need not relate to reality, whereas any data mismatching a theory founded on an axiom falsify that theory.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary Material; further inquiries can be directed to the corresponding author.

Author contributions

All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

Acknowledgments

We thank Pekka Teerikorpi for comments and corrections.

References

- Rubin VC, Ford J, Kent W. Rotation of the andromeda nebula from a spectroscopic survey of emission regions. *Astrophys J* (1970) 159:379. doi:10.1086/150317
- Zwicky F. On the redshift of spectral lines through interstellar space. *Proc Natl Acad Sci U S A* (1929) 15:773–9. doi:10.1073/pnas.15.10.773
- Walsh D, Carswell RF, Weymann RJ. 0957 + 561 a, b: Twin quasistellar objects or gravitational lens? *Nature* (1979) 279:381–4. doi:10.1038/279381a0
- Kaiser N. Weak gravitational lensing of distant galaxies. *Astrophys J* (1992) 388:272. doi:10.1086/171151
- Sanders R. *The dark matter problem: A historical perspective*. Cambridge, UK: Cambridge University Press (2010).
- Ellis RS. Gravitational lensing: A unique probe of dark matter and dark energy. *Phil Trans R Soc A* (2010) 368:967–87. doi:10.1098/rsta.2009.0209
- Riess AG, Filippenko AV, Challis P, Clocchiatti A, Diercks A, Garnavich PM, et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *Astron J* (1998) 116:1009–38. doi:10.1086/300499
- McGaugh SS. A tale of two paradigms: The mutual incommensurability of Λ CDM and MOND. *Can J Phys* (2014) 93:250–9. doi:10.1139/cjp-2014-0203
- Suzuki N, Rubin D, Lidman C, Aldering G, Amanullah R, Barbary K, et al. The hubble space telescope cluster supernova survey. v. improving the dark-energy constraint above $z > 1$ and building an early-type-hosted supernova sample. *Astrophys J* (2012) 746:85. doi:10.1088/0004-637x/746/1/85
- Riess AG, Rodney SA, Scolnic DM, Shafer DL, Strolger LG, Ferguson HC, et al. Type Ia supernova distances at redshift > 1.5 from the *Hubble Space Telescope* Multi-cycle treasury programs: The early expansion rate. *Astrophys J* (2018) 853:126. doi:10.3847/1538-4357/aaa5a9
- Pycke JR, Russell E. A new statistical perspective to the cosmic void distribution. *Astrophys J* (2016) 821:110. doi:10.3847/0004-637x/821/2/110
- Mihalas D, Routly P. *Galactic astronomy*. San Francisco, CA, USA: Freeman (1968).
- Hertzsprung E. Über die Sterne der Unterabteilungen c und ac nach der Spektralklassifikation von Antonia C. Maury. *Astr Nachr ; AN* (1908) 179:373–80. doi:10.1002/asna.19081792402
- Tully RB, Fisher JR. A new method of determining distance to galaxies. *Astron Astrophysics* (1977) 54:661–73.
- McGaugh SS. The baryonic tully-Fisher relation of galaxies with extended rotation curves and the stellar mass of rotating galaxies. *Astrophys J* (2005) 632:859–71. doi:10.1086/432968
- Faber SM, Jackson RE. Velocity dispersions and mass-to-light ratios for elliptical galaxies. *Astrophys J* (1976) 204:668. doi:10.1086/154215
- Gebhardt K, Bender R, Bower G, Dressler A, Faber SM, Filippenko AV, et al. A relationship between nuclear black hole mass and galaxy velocity dispersion. *Astrophys J* (2000) 539:L13–6. doi:10.1086/312840
- Schneider P. *Extragalactic astronomy and cosmology: An introduction*. Berlin, Heidelberg: Springer Berlin Heidelberg (2007).
- Salpeter EE. The luminosity function and stellar evolution. *Astrophys J* (1955) 121:161. doi:10.1086/145971
- Ferrarese L, Merritt D. A fundamental relation between supermassive black holes and their host galaxies. *Astrophys J* (2000) 539:L9–L12. doi:10.1086/312838
- Hague JD, Becker BR, Gold MS, Matthews JAJ. Power laws and the cosmic ray energy spectrum. *Astroparticle Phys* (2007) 27:455–64. doi:10.1016/j.astropartphys.2007.02.002
- Galilei G. *Mathematical discourses concerning two new Sciences*. London: J. Hooke, at the Flower-de-Luce, over-against St. Dunstan's Church in Fleet-street (1730).
- Kapteyn JC. Skew frequency curves in biology and statistics. *Recueil des travaux botaniques néerlandais* (1916) 13:105.
- Thompson DW. *On growth and form*. Cambridge, UK: Cambridge University Press (1917).
- Baryshev Y, Teerikorpi P. *Discovery of cosmic fractals*. Singapore: World Scientific Publishing Co. Pte Ltd (2002).
- Newman MEJ. Power laws, pareto distributions and zipf's law. *Contemp Phys* (2005) 46:323–51. doi:10.1080/00107510500052444
- Bashkirov AG, Vityazev AV. Information entropy and power-law distributions for chaotic systems. *Physica A: Stat Mech its Appl* (2000) 277:136–45. doi:10.1016/S0378-4371(99)00449-5
- Visser M. Zipf's law, power laws and maximum entropy. *New J Phys* (2013) 15:043021. doi:10.1088/1367-2630/15/4/043021
- Leitner M, Leubner MP, Vörös Z. Creating kappa-like distributions from a galton board. *Physica A: Stat Mech its Appl* (2011) 390:1248–57. doi:10.1016/j.physa.2010.11.044
- Dover Y. A short account of a connection of power laws to the information entropy. *Physica A: Stat Mech its Appl* (2004) 334:591–9. doi:10.1016/j.physa.2003.09.029
- Sharma V, Annala A. Natural process – natural selection. *Biophys Chem* (2007) 127:123–8. doi:10.1016/j.bpc.2007.01.005
- Mäkelä T, Annala A. Natural patterns of energy dispersal. *Phys Life Rev* (2010) 7:477–98. doi:10.1016/j.plev.2010.10.001
- Annala A. Natural thermodynamics. *Physica A: Stat Mech its Appl* (2016) 444:843–52. doi:10.1016/j.physa.2015.10.105
- Annala A. Statistical physics of evolving systems. *Entropy* (2021) 23:1590. doi:10.3390/e23121590
- Corda C. Interferometric detection of gravitational waves: The definitive test for general relativity. *Int J Mod Phys D* (2009) 18:2275–82. doi:10.1142/S0218271809015904

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Publisher's note

All claims expressed in this article are solely those of the authors and do not necessarily represent those of their affiliated organizations, or those of the publisher, the editors, and the reviewers. Any product that may be evaluated in this article, or claim that may be made by its manufacturer, is not guaranteed or endorsed by the publisher.

36. Corda C (2012). Dark energy and dark matter like intrinsic curvature in extended gravity. viability through gravitational waves. doi:10.48550/ARXIV.1211.1373
37. Boltzmann L. *The Second Law of Thermodynamics, reprinted in Ludwig Boltzmann, Theoretical physics and philosophical problem (Original work published 1886)*. Boston: Reidel (1974).
38. Gibbs JW (1902). *Elementary principles in statistical mechanics: Developed with especial reference to the rational foundation of thermodynamics* (New York: Charles Scribner's Sons). doi:10.1017/CBO9780511686948
39. Feynman R, Morinigo F, Wagner W, Hatfield B. *Feynman lectures on gravitation*. Reading, Massachusetts: Addison-Wesley (1995).
40. Newton I. *Opticks: Or, A treatise of the reflections, refractions, inflections and colours of light*. at the West End of St. Paul's: William and John Innys (1721).
41. Griffiths D. *Introduction to elementary particles*. New York, NY: Wiley (2020).
42. Noether E. Invariante variationsprobleme. *Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse* (1918) 1918: 235–57.
43. Lewis GN. The conservation of photons. *Nature* (1926) 118:874–5. doi:10.1038/118874a0
44. Annala A. All in action. *Entropy* (2010) 12:2333–58. doi:10.3390/e12112333
45. Annala A, Salthé S. Physical foundations of evolutionary theory. *J Non-Equilibrium Thermodynamics* (2010) 35:301–21. doi:10.1515/jnetdy.2010.019
46. Griffiths D. *Introduction to quantum mechanics*. Upper Saddle River, New Jersey: Prentice-Hall (1995).
47. Kondepudi D, Prigogine I. *Modern thermodynamics: From heat engines to dissipative structures*. New York: John Wiley & Sons (1999).
48. Maupertuis PLMD. Les lois du mouvement et du repos déduites d'un principe métaphysique. *Histoire de l'Académie Royale des Sci des Belles Lettres* (1746) 1746: 267–94.
49. Tuisku P, Pernu TK, Annala A. In the light of time. *Proc R Soc A* (2009) 465: 1173–98. doi:10.1098/rspa.2008.0494
50. Annala A. The matter of time. *Entropy* (2021) 23:943. doi:10.3390/e23080943
51. Aschwanden M. *Self-Organized criticality in astrophysics: The statistics of nonlinear processes in the universe*. Berlin Heidelberg: Springer (2011).
52. Bak P. *How nature works: The science of self-organized criticality*. New York: Springer (2013).
53. Gaddum JH. Lognormal distributions. *Nature* (1945) 156:463–6. doi:10.1038/156463a0
54. Aitchison J, Brown J. *The lognormal distribution with special reference to its uses in economics*. Cambridge, UK: The University Press (1963).
55. Simon HA. On a class of skew distribution functions. *Biometrika* (1955) 42: 425–40. doi:10.2307/2333389
56. Koen C, Kondlo L. Fitting power-law distributions to data with measurement errors. *Mon Not R Astron Soc* (2009) 397:495–505. doi:10.1111/j.1365-2966.2009.14956.x
57. Spiering C. *Astroparticle physics in europe: Status and perspectives*. New York, NY: John Wiley & Sons (2008). p. 375–405. chap. 16. doi:10.1002/9783527622993.ch16
58. Annala A. Cosmic rays report from the structure of space. *Adv Astron* (2015) 2015:1–11. doi:10.1155/2015/135025
59. Bose SN. Plancks Gesetz und Lichtquantenhypothese. *Z Physik* (1924) 26: 178–81. doi:10.1007/BF01327326
60. Peebles PJE, Ratra B. The cosmological constant and dark energy. *Rev Mod Phys* (2003) 75:559–606. doi:10.1103/RevModPhys.75.559
61. Unsöld A, Baschek B. *The new cosmos: An introduction to astronomy and astrophysics*. Berlin: Springer (2001).
62. Maxwell JC. On physical lines of force. *Philos Mag* (1861) 90:11–23. doi:10.1080/14786431003659180
63. Grah P, Annala A, Kolehmainen E. On the carrier of inertia. *AIP Adv* (2018) 8:035028. doi:10.1063/1.5020240
64. Lorenz L. XXXVIII on the identity of the vibrations of light with electrical currents. *Lond Edinb Dublin Philos Mag J Sci* (1867) 34:287–301. doi:10.1080/14786446708639882
65. Aharonov Y, Bohm D. Significance of electromagnetic potentials in the quantum theory. *Phys Rev* (1959) 115:485–91. doi:10.1103/PhysRev.115.485
66. Faraday M. On the possible relation of gravity to electricity. *Phil Trans R Soc A* (1851) 1:141.
67. Faraday M. Experimental researches in electricity.—twenty-eighth series. *Philos Trans R Soc Lond* (1852) 142:25–56.
68. Faraday M. LVIII. On the physical character of the lines of magnetic force. *Lond Edinb Dublin Philos Mag J Sci* (1852) 3:401–28. doi:10.1080/14786445208647033
69. Thouless DJ. Topological quantum numbers in nonrelativistic physics. *Int J Mod Phys B* (1997) 11:3319–27. doi:10.1142/S0217979297001623
70. Staley M. Understanding quaternions and the Dirac belt trick. *Eur J Phys* (2010) 31:467–78. doi:10.1088/0143-0807/31/3/004
71. Schiller C. Testing a conjecture on the origin of the standard model. *Eur Phys J Plus* (2021) 136:79. doi:10.1140/epjp/s13360-020-01046-8
72. Heaviside O. A gravitational and electromagnetic analogy, Part I. *The Electrician* (1893) 31:281–2.
73. Misner C, Thorne K, Wheeler J. *Gravitation*. San Francisco: W. H. Freeman (1973).
74. Thorne KS. Gravitomagnetism, jets in quasars, and the stanford gyroscope experiment. In: JD Fairbank, BS Deaver, CWF Everitt, PF Michelson, editors. *Near zero: New Frontiers of Physics*. New York: W. H. Freeman and Company (1988). p. 573–86.
75. Clark SJ, Tucker RW. Gauge symmetry and gravito-electromagnetism. *Class Quan Gravity* (2000) 17:4125–57. doi:10.1088/0264-9381/17/19/311
76. Iorio L, Corda C. Gravitomagnetism and gravitational waves. *Open Astron J* (2011) 4:84–97. doi:10.2174/1874381101004010084
77. Do Carmo M. *Differential geometry of curves and surface*. Hoboken, NJ, USA: Prentice-Hall (1976).
78. Annala A. Least-time paths of light. *Mon Not R Astron Soc* (2011) 416:2944–8. doi:10.1111/j.1365-2966.2011.19242.x
79. Ciufolini I, Wheeler JA. *Gravitation and inertia, vol. 101*. Princeton, NJ, USA: Princeton university press (1995).
80. Everitt CWF, DeBra DB, Parkinson BW, Turneaure JP, Conklin JW, Heifetz MI, et al. Gravity probe b: Final results of a space experiment to test general relativity. *Phys Rev Lett* (2011) 106:221101. doi:10.1103/PhysRevLett.106.221101
81. Annala A. Probing mach's principle. *Mon Not R Astron Soc* (2012) 423: 1973–7. doi:10.1111/j.1365-2966.2012.12022.x
82. Arago DFJ. M'emoire sur la vitesse de la lumiere, lu 'a la premi'ere classe de l'institut, le 10 d'ecembre 1810. *Comptes Rendus de l'Acad'emie des Sci* (1853) XXXVI:38–49.
83. Fizeau H. Sur les hypothèses relatives à l'éther lumineux. *Comptes Rendus* (1851) 33:349–55.
84. Michelson AA, Morley EW. On the relative motion of the Earth and the luminiferous ether. *Am J Sci* (1887) 34:333–45. doi:10.2475/ajs.s3-34.203.333
85. Trouton FT, Noble HR. The forces acting on a charged electric condenser moving through space. *Philosophical Trans R Soc Lond Ser A, Containing Pap a Math or Phys Character* (1904) 202:165–81. doi:10.1098/rsta.1904.0005
86. Sagnac G. L'éther lumineux démontré par l'effet du vent relatif d'éther dans un interféromètre en rotation uniforme. *Comptes Rendus* (1913) 157:708–10.
87. Casimir HBG. On the attraction between two perfectly conducting plates. *Proc Koninklijke Nederlandse Akademie van Wetenschappen* (1948) 51:793–5.
88. Overstreet C, Asenbaum P, Curti J, Kim M, Kasevich MA (2022). Observation of a gravitational aharonov-bohm effect. *Science* 375, 226–9. doi:10.1126/science.abl7152
89. Wilson CM, Johansson G, Pourkabirian A, Simoen M, Johansson JR, Duty T, et al. Observation of the dynamical casimir effect in a superconducting circuit. *Nature* (2011) 479:376–9. doi:10.1038/nature10561
90. Kastner RE, Kauffman S. Are dark energy and dark matter different aspects of the same physical process? *Front Phys* (2018) 6. doi:10.3389/fphy.2018.00071
91. Sipe JE. Photon wave functions. *Phys Rev A (Coll Park)* (1995) 52:1875–83. doi:10.1103/PhysRevA.52.1875
92. Smith BJ, Raymer MG. Photon wave functions, wave-packet quantization of light, and coherence theory. *New J Phys* (2007) 9:414. doi:10.1088/1367-2630/9/11/414
93. Einstein A. Relativität und gravitation. erwidern auf eine bemerkung von m. abraham. *Ann Phys* (1912) 38:1059–64. doi:10.1002/andp.19123431014
94. Wilson H. An electromagnetic theory of gravitation. *Phys Rev* (1921) 17:54–9. doi:10.1103/PhysRev.17.54
95. Dicke RH. Gravitation without a principle of equivalence. *Rev Mod Phys* (1957) 29:363–76. doi:10.1103/RevModPhys.29.363

96. Sarazin X, Couchot F, Djannati-Ataï A, Urban M. Can the apparent expansion of the universe be attributed to an increasing vacuum refractive index? *Eur Phys J C* (2018) 78:444. doi:10.1140/epjc/s10052-018-5932-8
97. Edwards MR. Optical gravity in a graviton spacetime. *Optik* (2022) 260:169059. doi:10.1016/j.jleo.2022.169059
98. Newton I. *Principia: The mathematical principles of natural philosophy*. 3rd ed. New York: Daniel Adee (1846).
99. Einstein A. Die grundlage der allgemeinen relativitätstheorie. *Ann Phys* (1916) 354:769–822. doi:10.1002/andp.19163540702
100. Mach E. *The science of mechanics: A critical and historical exposition of its principles (1893)*. La Salle: Open Court (1960).
101. Sciamia DW. On the origin of inertia. *Monthly Notices R Astronomical Soc* (1953) 113:34–42. doi:10.1093/mnras/113.1.34
102. Sanders RH. Cosmology with modified Newtonian dynamics (mond). *Mon Not R Astron Soc* (1998) 296:1009–18. doi:10.1046/j.1365-8711.1998.01459.x
103. Kroupa P, Pawlowski M, Milgrom M. The failures of the standard model of cosmology require a new paradigm. *Int J Mod Phys D* (2012) 21:1230003. doi:10.1142/S0218271812300030
104. Sandage A. The redshift-distance relation. ix. perturbation of the very nearby velocity field by the mass of the local group. *Astrophys J* (1986) 307:1–19. doi:10.1086/164387
105. Karachentsev ID, Kashibadze OG, Makarov DI, Tully RB. The hubble flow around the local group. *Mon Not R Astron Soc* (2009) 393:1265–74. doi:10.1111/j.1365-2966.2008.14300.x
106. Weinberg S. *Cosmology*. Oxford: Oxford University Press (2008).
107. Yarkovsky IO. *Hypothese Cinétique de la Gravitation Universelle et Connexion avec la Formation des Elements Chimiques*. Moscow: Moscow (1888).
108. Riemann B. *Neue mathematische prinzipien der Naturphilosophie*. Leipzig: B. G. Teubner (1892).
109. Barrow JD, Tipler FJ. *The anthropic cosmological principle*. Oxford: Clarendon Press (1986).
110. Collins CB, Hawking S. Why is the universe isotropic? *Astrophys J* (1973) 180:317–34. doi:10.1086/151965
111. Zhu ZH, Hu M, Alcaniz JS, Liu YX. Testing power-law cosmology with galaxy clusters. *Astron Astrophys* (2008) 483:15–8. doi:10.1051/0004-6361:20077797
112. Annala A. Rotation of galaxies within gravity of the universe. *Entropy* (2016) 18:191–205. doi:10.3390/e18050191
113. Ibata R, Nipoti C, Sollima A, Bellazzini M, Chapman SC, Dalessandro E. Do globular clusters possess dark matter haloes? A case study in ngc 2419. *Mon Not R Astron Soc* (2013) 428:3648–59. doi:10.1093/mnras/sts302
114. Klessen RS, Zhao H. Are dwarf spheroidal galaxies dark matter dominated or remnants of disrupted larger satellite galaxies? A possible test. *Astrophys J* (2002) 566:838–44. doi:10.1086/338222
115. Simon JD, Geha M. The kinematics of the ultra-faint milky way satellites: Solving the missing satellite problem. *Astrophys J* (2007) 670:313–31. doi:10.1086/521816
116. Milgrom M. A modification of the Newtonian dynamics - implications for galaxies. *Astrophys J* (1983) 270:371. doi:10.1086/161131
117. Milgrom M. A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis. *Astrophys J* (1983) 270:365. doi:10.1086/161130
118. Pawlowski MS. The planes of satellite galaxies problem, suggested solutions, and open questions. *Mod Phys Lett A* (2018) 33:1830004. doi:10.1142/S0217732318300045
119. van den Bergh S. The local group of galaxies. *Astron Astrophysics Rev* (1999) 9:273–318. doi:10.1007/s001590050019
120. Sanders RH. The universal faber–jackson relation. *Mon Not R Astron Soc* (2010) 407:1128–34. doi:10.1111/j.1365-2966.2010.16957.x
121. Hodson AO, Zhao H. Generalizing mond to explain the missing mass in galaxy clusters. *Astron Astrophys* (2017) 598:A127. doi:10.1051/0004-6361/201629358
122. Refregier A. Weak gravitational lensing by large-scale structure. *Annu Rev Astron Astrophys* (2003) 41:645–68. doi:10.1146/annurev.astro.41.111302.102207
123. Koskela M, Annala A. Least-action perihelion precession. *Mon Not R Astron Soc* (2011) 417:1742–6. doi:10.1111/j.1365-2966.2011.19364.x
124. Hewitt JN. Gravitational lensing of active galactic nuclei. *Proc Natl Acad Sci U S A* (1995) 92:11434–8. doi:10.1073/pnas.92.25.11434
125. Refsdal S. On the possibility of determining hubble's parameter and the masses of galaxies from the gravitational lens effect. *Monthly Notices R Astronomical Soc* (1964) 128:307–10. doi:10.1093/mnras/128.4.307
126. Paraficz D, Hjorth J. The hubble constant inferred from 18 time-delay lenses. *Astrophys J* (2010) 712:1378–84. doi:10.1088/0004-637x/712/2/1378
127. Di Valentino E, Mena O, Pan S, Visinelli L, Yang W, Melchiorri A, et al. In the realm of the hubble tension—A review of solutions. *Class Quan Gravity* (2021) 38:153001. doi:10.1088/1361-6382/ac086d
128. Pound RV, Rebka GA. Gravitational red-shift in nuclear resonance. *Phys Rev Lett* (1959) 3:439–41. doi:10.1103/PhysRevLett.3.439
129. Berry MV. *Principles of cosmology and gravitation*. Cambridge, UK: Cambridge University Press (1976).
130. Seife C. 'Tired-Light' hypothesis gets Re-tired. *Science* (2001) 292:2414. doi:10.1126/science.292.5526.2414a
131. Narlikar J. *An introduction to cosmology*. Cambridge: Cambridge University Press (2002).
132. Hogg DW, Baldry IK, Blanton M, Eisenstein DJ. *The k correction* (2002). arXiv:astro-ph/0210394.
133. Guth AH. *The inflationary universe*. Reading, Massachusetts: Perseus Books (1997).
134. Merritt D. Cosmology and convention. *Stud Hist Philos Sci B: Stud Hist Philos Mod Phys* (2017) 57:41–52. doi:10.1016/j.shpsb.2016.12.002
135. Banik I, Zhao H. From galactic bars to the hubble tension: Weighing up the astrophysical evidence for milgromian gravity. *Symmetry* (2022) 14:1331. doi:10.3390/sym14071331
136. Milgrom M. Mond vs. dark matter in light of historical parallels. *Stud Hist Philos Sci Part B: Stud Hist Philos Mod Phys* (2020) 71:170–95. doi:10.1016/j.shpsb.2020.02.004