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# Search for $\eta$ -mesic nuclei: a review of experimental and theoretical advances

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In the mid-1980s, theoreticians predicted possible exotic states of an eta ( $\eta$ ) meson and a nucleus bound by the strong interaction, so-called *mesic nucleus*. This led to extensive experimental efforts aimed at discovering such unstable states as well as understanding the underlying interactions via  $\eta$  meson production reactions. As experiments yielded new data, more and more sophisticated theoretical models were developed to understand the fundamental  $\eta$ -nucleon and  $\eta$ -nucleus interactions. After more than 35 years of research in this field, several signals have been identified as possible indications of hypothetical  $\eta$ -mesic nuclei, but there is still no general agreement or clear confirmation of their existence. In the paper, we review the study of the  $\eta$ -mesic nuclei with an emphasis on recent research activities.

## KEYWORDS

exotic nuclei,  $\eta$  meson,  $\eta$ -mesic nucleus, bound state,  $\eta$ -nucleon interaction,  $\eta$ -nucleus interaction,  $\eta$  meson production reaction

## 1 Introduction

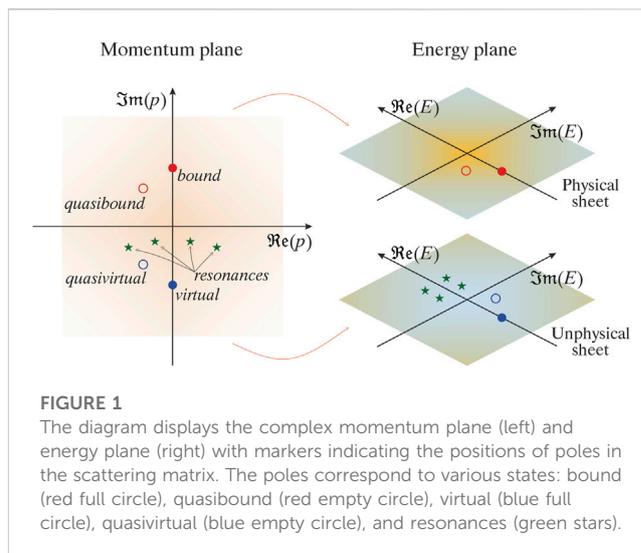
The study of exotic atoms and exotic nuclei, new kinds of subatomic matter containing exotic particles like mesons or excited baryons, is one of the hottest topics in contemporary hadron and nuclear physics. A better understanding of these systems can offer valuable insight into the nature of strong forces and hadron-nucleus interaction. An example of such an object is a mesic atom, where a negatively charged pion [1] or kaon [2] replaces an electron in an outer orbital of a standard atom. The binding in this case is primarily due to the Coulomb interaction, but as the meson undergoes transitions, it eventually reaches the range of the strong nuclear interaction, resulting in its absorption by the nucleus or loss through a nuclear reaction. Another possible formation of meson-nucleus bound states is when the strong interaction between the meson and nucleus is attractive, as is the case with the neutral eta ( $\eta$ ) meson. Its interaction with the nucleons has been found to be strong and attractive in the low-energy region (*s*-wave) [3]. Based upon this finding, Haider and Liu [4] predicted the possibility of the formation of strongly bound systems of the  $\eta$  meson and nuclei with mass number  $A \geq 12$ , and called such systems  *$\eta$ -mesic nuclei*. This conclusion was supported by similar results from other studies [5, 6], while some analyses [7–16] have even suggested that light systems like  ${}^2\text{H}$ ,  ${}^3\text{H}$ ,  ${}^3\text{He}$ , and  ${}^4\text{He}$  could potentially form a bound state with  $\eta$ .

The  $\eta$  meson belongs to the pseudoscalar nonet of mesons with zero spin and negative parity ( $J^{\pi} = 0^{-}$ ) [17]. It is an electrically neutral particle that can decay through both the strong and electromagnetic interactions. Its lifetime,  $\tau_{\eta}$ , is  $\leq 5 \times 10^{-19}$  s [18]. Additionally,

the mass of the  $\eta$  meson is  $547.862 \pm 0.017$  MeV/ $c^2$  [17], which is nearly half the mass of a nucleon. The physically observed  $\eta$  meson is actually a superposition of the flavour octet and singlet states [17]. The mixing angle between these two states has been estimated to be  $-14.1^\circ \pm 2.8^\circ$  [19]. The interaction between the  $\eta$  meson and a nucleon  $N$  close to the threshold is dominated by the excitation of the  $S_{11}$  nucleon resonance  $N^*(1,535)$  [20, 21]. This resonance has a maximum situated just about 50 MeV above the production threshold and a width of 150 MeV [17], which covers the whole low-energy region of the  $\eta N$  interaction. Since the  $N^*$  resonance can decay into  $\pi N$ ,  $\gamma N$ , and  $\pi\pi N$  channels, the coupling to these channels must be taken into account for a correct treatment of the  $\eta N$  interaction. The  $\eta N$  scattering amplitude is determined mainly phenomenologically through the analysis of these coupled channels and the fitting of the results to available experimental data [22]. Unfortunately, the extremely short lifetime of the  $\eta$  meson makes it impossible to generate  $\eta$  beams, so there are no available data on elastic  $\eta N$  scattering. As a result, the extracted scattering amplitudes and the corresponding  $\eta$ -nucleon scattering lengths,  $a_{\eta N}$ , are model dependent. The magnitude of the real part of  $a_{\eta N}$  varies from 0.18 to 1.14 fm, while the imaginary part ranges between 0.16 and 0.49 fm [18, 21, 23, 24], reflecting the sensitivity of the results to the choice of theoretical models and analyses of data on the  $\eta$  production in photon-, pion-, and proton-induced reactions. However, it is agreed upon in all calculations that the  $\eta N$  interaction is strong and attractive in the  $s$ -wave.

The attractive and strong character of the  $\eta$ -nucleon interaction led to the speculation of the existence of  $\eta$ -nucleus (quasi-)bound states. An initial theoretical prediction [4] prompted experimental searches for these unstable bound states. These searches have been conducted using various light and heavy nuclei targets and various beams such as pions [25–27], photons [28–32], protons [33–36] and deuterons [37–41]. With advancements in experimental research, new and more sophisticated models have emerged to shed light on the fundamental  $\eta N$  interaction and the multi-body systems of  $\eta$  mesons and nuclei. Despite years of investigation, there is still no clear evidence of  $\eta$ -mesic nuclei and no consensus on the strength of the  $\eta$ -nucleon and  $\eta$ -nucleus interactions.

The search for  $\eta$ -mesic nuclei aims to investigate various important issues in  $\eta$  meson physics, including the interaction of the  $\eta$  meson with nucleons within nuclear matter. The discovery of such objects would allow for a more accurate determination of the poorly known  $\eta N$  scattering length [22]. Furthermore, the examination of  $\eta$ -mesic nuclei would open up opportunities to study the structure of the  $\eta$  meson. The binding energy of the meson-nucleus system, as shown by Bass and Thomas [42], is sensitive to the flavor-singlet component and glue content of the  $\eta$  meson quark-gluon wave function. In a bound state, the wave function of the  $\eta$  meson overlaps with that of the nucleus, allowing for the observation of in-medium effects on meson properties. In particular, studies of  $\eta$ -mesic nuclei can provide precise information on the  $\eta$  meson mass shift, which is significant in understanding the dynamics of the axial  $U_A(1)$  symmetry [42–44]. Additionally, the detection of  $\eta$ -mesic nuclei would also provide information regarding the properties of the  $N^*(1,535)$  resonance in nuclear medium, useful in verifying different theoretical models related to the structure of the resonance [45–49].



The purpose of the article is to provide a comprehensive overview of the current status of research on  $\eta$ -mesic bound states. It begins with a discussion of theoretical approaches and predictions, followed by a review of experimental efforts aimed at detecting the  $\eta$ -mesic nuclei. The paper focuses on summarizing recent investigations and highlighting areas for future research.

## 2 Searching for evidence of $\eta$ -mesic bound states: theoretical and experimental efforts

### 2.1 Theoretical considerations on the $\eta$ -mesic nuclei

The study of the  $\eta$ -nucleus bound states is related to the  $\eta$  meson interactions with nucleons and nuclei. Recent reviews on this topic can be found in Refs. [18, 22, 43, 50, 51, 52]. These interactions can only be probed through the study of reactions that produce  $\eta$  mesons in the final state. The knowledge of the reaction mechanisms plays a significant role in both understanding this interaction and predicting the existence of  $\eta$ -mesic unstable nuclear states. The strong effects of the  $\eta$ -nucleus interaction are particularly pronounced close to the threshold of the  $\eta$  production reactions, where off-shell rescattering becomes relevant [53]. Since the  $\eta$  meson has an extremely short lifetime, data on elastic scattering of the  $\eta$ -nucleon or  $\eta$ -nucleus are not available. Instead, a more efficient approach in a theoretical search for  $\eta$ -mesic nuclei involves using information on the  $\eta N$  interaction obtained from models that fit the  $\eta$  meson production reactions data to construct a complex  $\eta$ -nucleus elastic scattering matrix ( $S$ -matrix) [54–56].

The scattering matrix elements are treated as functions of complex incident momentum  $p$ . The analytical properties of matrix elements are determined by the interaction potential, specifically its asymptotic behavior. If the potential decreases exponentially at large distances  $r$  and is analytic for  $\Re(r) > 0$ , then the matrix is considered analytic over the whole complex plain except for a finite number of singularities (poles). The  $S$ -matrix

elements can also be expressed in terms of complex energy using formula  $E = p^2/2\mu$ , where  $\mu$  is the reduced mass of the  $\eta$ -nucleus system. In the complex energy plane, the scattering matrix is defined on a Riemann surface with two sheets: physical (corresponding to  $\Im \mathbf{m}(p) > 0$ ) and unphysical ( $\Im \mathbf{m}(p) < 0$ ). The occurrence of various states can be attributed to the poles in the complex momentum or energy plane of this matrix, as shown in Figure 1.

The poles located on the physical sheet ( $\Im \mathbf{m}(p) > 0$ ) with negative real energy ( $\Re(E) < 0$ ) indicate either bound or quasibound states, represented by a red full circle or a red empty circle in Figure 1, respectively. The pole representing a bound state lies on the positive imaginary  $p$  axis and is characterized by a real interaction potential (i.e., no absorption channels). If the interaction potential includes an imaginary part, this pole moves into the second quadrant of the complex momentum plane and is referred to as a quasibound state. On the other hand, the pole located on the unphysical sheet ( $\Im \mathbf{m}(p) < 0$ ) with negative real energy ( $\Re(E) < 0$ ) is referred to as virtual or antibound state (represented by a blue full circle), and is located on the negative imaginary  $p$  axis. In the case of a complex interaction potential, this pole shifts into the third quadrant and is considered a quasivirtual state (indicated by a blue empty circle). Finally, poles located near the real axis of momentum on the unphysical sheet with positive real energy ( $\Re(E) > 0$ ) correspond to resonances (represented by green stars). The width of the corresponding resonance is given by the imaginary part of energy,  $\Im \mathbf{m}(E)$ . In the context of this discussion, the  $\eta$ -mesic nucleus, if it exists, is considered a quasibound state by definition.

The possible binding energy of the meson-nucleus state is determined by both the  $\eta$ -nucleon optical potential and the value of the  $\eta$ -nucleon scattering length  $a_{\eta N}$  [57]. The real part of  $a_{\eta N}$  has a broad range of phenomenological estimates that vary by up to six times depending on the underlying model assumptions [18, 21, 23, 24]. Consequently, this significant uncertainty in the value of  $a_{\eta N}$  results in different predictions regarding the potential existence of  $\eta$ -mesic nuclei.

### 2.1.1 The $\eta$ -nucleus optical potential

One of the commonly used theoretical approaches to study the interaction between the  $\eta$  meson and atomic nuclei involves constructing an appropriate optical potential  $U_{\eta\text{-nucleus}} = V(r) + iW(r)$  based on the  $\eta$ -nucleon scattering length  $a_{\eta N}$ , and then using it to solve the wave equation. A negative real part of the optical potential ( $V = \Re(U_{\eta\text{-nucleus}}) < 0$ ) indicates an attractive interaction [21], which is a criterion for the creation of an  $\eta$ -mesic nucleus. Haider and Liu used this formalism to calculate the binding energies and half-widths for bound states of the  $\eta$  meson with  $^{12}\text{C}$  and several heavier nuclei [4, 58], which led to the hypothesis of the existence of  $\eta$ -mesic nuclei with  $A \geq 12$ . In their later study [59], they investigated a microscopic optical potential, that took into account the effects of off-shell  $\eta N$  interaction, and compared these results with calculations within a factorization approximation (FA), which included a downward shift of the  $\eta N$  interaction energy. The authors found that the in-medium  $\eta N$  interaction for bound-state formation is about 30 MeV below the free-space  $\eta N$  threshold. This shift causes a reduction in the  $\eta N$  attraction inside the nucleus, which means that models based upon using free-space  $\eta N$  interaction may provide incorrect estimates of the  $\eta$ -nucleus binding energy.

The binding energies and widths of possible  $\eta$ -mesic states in  $^{12}\text{C}$ ,  $^{40}\text{Ca}$ , and  $^{208}\text{Pb}$  nuclei were also calculated using the self-energies of the  $\eta$  meson in the nuclear medium [60]. The results obtained by applying the local density approximation (LDA) showed that the optical potential generates bound states with a very large width, which makes it unlikely to experimentally observe narrow peaks corresponding to  $\eta$ -mesic states in these nuclei. Another study [6], which used a chiral unitary approach [47] to evaluate the self-energy of the  $\eta$  meson, similarly reported that the resulting bound states have a half width larger than the separation of the levels.

An alternative method is the quark-meson coupling (QMC) model, based on quantum chromodynamics. Within this model, the  $\eta$  meson is considered to be embedded in the nuclear medium and couples to quarks, mixing with its heavier partner,  $\eta'$  meson [42, 61, 62]. The in-medium mass of the  $\eta$  meson  $m_{\eta}^*$ , which differs from its mass  $m_{\eta}$  in free space, can be determined by solving the mean-field equations. The optical potential of the  $\eta$ -nucleus system, taking into account the  $\eta - \eta'$  mixing effect, can then be expressed as  $U_{\eta\text{-nucleus}}(r) = m_{\eta}^*(r) - m_{\eta}(r)$  [22]. To obtain a more realistic estimate of the meson-nucleus bound state, it is necessary to take into account the width of the  $\eta$  meson in the nucleus, since the QMC model neglects the imaginary part of the potential [61]. This potential has been used to solve the Klein-Gordon equation for several closed-shell nuclei ( $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{90}\text{Zr}$ ,  $^{208}\text{Pb}$ ) as well as for  $^6\text{He}$ ,  $^{11}\text{B}$ , and  $^{26}\text{Mg}$  [61, 62]. The results suggest that bound states should be expected in all of these nuclei.

### 2.1.2 Theoretical prospects on unstable $\eta$ -mesic states of light nuclei

The feasibility of forming  $\eta$ -mesic bound state in light nuclei is closely related to the constraints on the complex  $\eta$ -nucleus scattering length  $a_{\eta\text{-nucleus}}$ . Specifically, to create such a state, the imaginary part of  $a_{\eta\text{-nucleus}}$  must be positive, i.e.,  $\Im \mathbf{m}(a_{\eta\text{-nucleus}}) > 0$  [34], and the magnitude of its real part is required to be greater than the magnitude of the imaginary part corresponding to the  $\eta$  meson absorption:  $|\Re(a_{\eta\text{-nucleus}})| > |\Im \mathbf{m}(a_{\eta\text{-nucleus}})|$  [59]. Moreover, to ensure that the pole of the scattering matrix lies on the bound- rather than the virtual-state plane, the real part of the scattering length should be negative ( $\Re(a_{\eta\text{-nucleus}}) < 0$ ) [34].

Using the optical potential approach, Wilkin [8] has investigated the  $^3\text{He}-\eta$  system, which is the final state of the  $pd \rightarrow ^3\text{He}\eta$  reaction. The analysis yielded the  $\eta$ -nucleus scattering length of  $a_{\eta\text{-}^3\text{He}} = -2.31 + i2.57$  fm, indicating the presence of strong  $\eta$ -nucleus interaction (due to the large negative real part of the scattering length), and suggesting the possible existence of an  $^3\text{He}-\eta$  bound state near the threshold. However, according to the above criteria, this scattering length does not satisfy the requirements for the existence of a bound state, due to its large imaginary component.

Theoretical models based on few-body equations for systems consisting of  $\eta$  meson and two, three, or four nucleons can be useful for studying light  $\eta$ -mesic nuclei. One of the initial calculations considered the  $\eta NN-\pi NN$  coupled system in a three-body equation and predicted a quasibound  $^2\text{H}-\eta$  state near the threshold with a mass of 2,430 MeV and a width of 10–20 MeV [63]. Few years later, further investigations were conducted within the framework of the finite-rank approximation (FRA) to explore the possibility of forming  $^2\text{H}-\eta$ ,  $^3\text{H}-\eta$ ,  $^3\text{He}-\eta$ , and  $^4\text{He}-\eta$  bound states [7, 64]. The

analysis suggested that  $\eta$ -nucleus quasibound states are possible for  $A \geq 2$  when the value of  $\Re(a_{\eta N})$  is within the range of (0.27, 0.98) fm [7]. Another study [65] using three-body equations indicated that a quasibound or virtual state could exist in the  ${}^2\text{H}-\eta$  system for a value of  $\Re(a_{\eta N})$  about 0.8 fm, which could be detected through the observation of final-state interactions. In a separate analysis [66], the Alt-Grassberger-Sandhas (AGS) formalism was applied to investigate elastic  $\eta$ -deuteron scattering for various values of  $a_{\eta N}$  as input parameters. The results showed the existence of a resonance or quasibound state close to the  ${}^2\text{H}-\eta$  threshold. In a later study [67], a three-body resonant state was found close to the  ${}^2\text{H}-\eta$  threshold. The position of this resonance shifts towards the threshold with increasing  $\Re(a_{\eta N})$ , leading to its turn into a quasibound state at 0.7–0.8 fm (which varies with changes in  $\Im m(a_{\eta N})$ ).

The Wigner's time delay and dwell time delay concepts have been used as an alternative approach to search for light  $\eta$ -mesic nuclei [10, 11, 68]. The delay times for  $\eta$ -nucleus elastic scattering were calculated to locate quasibound states in  ${}^2\text{H}-\eta$ ,  ${}^3\text{He}-\eta$ , and  ${}^4\text{He}-\eta$  systems. By varying the strength of the  $\eta N$  interaction, quasibound states were found to be more favorable for small  $\eta N$  scattering lengths, while higher values corresponded to resonances [68].

In Ref. [12], the stochastic variational method (SVM) was used to study the  $NNN-\eta$  and  $NNNN-\eta$  systems within a pionless effective field theory. Based on the  $\eta$ -nucleon scattering lengths, the authors found that the formation of a bound state of the  $\eta$  meson and  ${}^3\text{He}$  nucleus requires a value of  $\Re(a_{\eta N})$  greater than 1 fm, while in the case of  ${}^4\text{He}$  nucleus, it must exceed 0.7 fm. These results were later confirmed in SVM calculations of few-body systems, where semi-realistic  $NN$  interaction potentials and energy-dependent  $\eta N$  subthreshold interaction potentials were used [13].

A recent theoretical analysis of the  $\eta$  meson interaction with the  ${}^3\text{He}$  nucleus considered the total cross sections and asymmetry parameter for the  $pd \rightarrow {}^3\text{He}\eta$  reaction near the threshold [14]. Based on this data, the authors calculated the  ${}^3\text{He}-\eta$  optical potential and predicted the existence of a weakly bound  ${}^3\text{He}-\eta$  state with a binding energy of 0.3 MeV and a width of 3 MeV.

The theoretical model presented by Ikeno *et al.* in Ref. [15] describes the formation of  ${}^4\text{He}-\eta$  bound states in the  $dd$  fusion reaction. The model takes into account the experimental data on the production of the  $\eta$  meson near the  $dd \rightarrow {}^4\text{He}\eta$  reaction threshold and predicts that strong attractive and low absorptive  $\eta$ -nucleus interactions are required to observe the  $\eta$ -mesic bound state. Based on a phenomenological calculations, the authors for the first time reproduced the shapes of the spectral structure and cross section values for the  $dd \rightarrow ({}^4\text{He}-\eta)_{\text{bound}} \rightarrow {}^3\text{He}N\pi$  process below the threshold, considering a wide range of  ${}^4\text{He}-\eta$  optical potential parameters. Another analysis [16] was conducted to search for poles of the  ${}^4\text{He}-\eta$  scattering matrix. The results indicated the existence of the poles corresponding to bound states, unbound states, or no poles at all, depending on the  ${}^4\text{He}-\eta$  potential parameters. Using an approximation of the scattering amplitude for the two-body process, Wycech and Krzemiński [69] estimated the cross section for the  $dd \rightarrow ({}^3\text{He}-\eta)_{\text{bound}} \rightarrow {}^3\text{He}p\pi^-$  reaction to be about 4.5 nb.

In contrast, Fix and Kolesnikov [70] used a non-relativistic five-body problem to investigate the  ${}^4\text{He}-\eta$  interaction and solved the corresponding AGS equations. Their analysis showed that the predicted parameters of the  $\eta N$  interaction do not provide enough attraction in the  $NNNN-\eta$  system to support the

existence of the  ${}^4\text{He}-\eta$  bound state. The authors subsequently expanded their analysis to include the  ${}^2\text{H}-\eta$  and  ${}^3\text{He}-\eta$  systems [71]. Their new calculations once again did not confirm the hypothesis of an  $\eta$ -mesic bound state in  ${}^4\text{He}$ , while the status of the  ${}^3\text{He}-\eta$  system remains ambiguous. Significantly, the analysis revealed that the interaction between the  $\eta$  meson and  ${}^4\text{He}$  is unexpectedly less attractive compared to the  ${}^3\text{He}-\eta$  case. In turn, the weaker attraction in the case of deuteron was attributed to the smaller number of nucleons.

In summary, while some theoretical analyses suggest that the formation of certain  $\eta$ -nucleus bound states is unlikely, many others predict their existence. The significant variation in input parameters used in different calculations results in a wide range of predicted outcomes regarding the potential existence or non-existence of  $\eta$ -mesic nuclei.

## 2.2 Challenges and outcomes of experimental searches for $\eta$ -mesic nuclei

The concept of mesic nuclei, first introduced by Haider and Liu in 1986 [4], has garnered significant attention and has prompted numerous experimental and theoretical investigations aimed at discovering both light and heavy mesic nuclei. The experimental searches for this exotic form of nuclear matter involve the production of  $\eta$  mesons, analyzing their interaction with nuclei, and detecting  $\eta$ -mesic states through their possible decay modes. This section provides an overview of previous measurements in this area.

### 2.2.1 Investigating $\eta$ meson binding in heavy atomic nuclei

The initial theoretical predictions indicated that  $\eta$  mesons could be bound in nuclei with a mass number greater than 12 [4, 5], leading to initial experimental searches for these bound states being conducted in heavy nuclei region.

The first proposal of an experiment devoted to the search for  $\eta$ -mesic nuclei was introduced by Liu and Haider in Ref. [58]. They suggested using a positively charged pion beam to induce the reaction  $\pi^+ + {}^A\text{X} \rightarrow p + {}^{A-1}\text{X}-\eta$ . The goal was to form  $\eta$ -mesic nuclei under "recoilless" kinematic conditions, when the  $\eta$  meson is produced with zero kinetic energy in the laboratory frame. According to the theory, if the  $\eta$ -nucleus was formed in the reaction, a narrow peak (with a width of  $\Gamma_p = 9$  MeV) would appear in the proton spectrum, with a scattering angle of  $\theta_p = 15^\circ$  and energy of  $E_p = 240$  MeV. Two independent experiments were carried out in 1988 to test this hypothesis, one by a research group at the Brookhaven National Laboratory (BNL) [25] and the other at the Los Alamos Meson Physics Facility (LAMPF) [26]. The experiments used a  $\pi^+$  meson beam with energy slightly above the recoilless kinematics threshold. The targets included lithium, carbon, oxygen, and aluminium, where the lithium target ( $A = 7$ ) serving as a control since its total nuclear potential was believed to be too weak to form an  $\eta$ -nucleus. However, the Brookhaven experiment did not observe any peaks in the proton spectrum at the expected angle for any of the targets, while the Los Alamos experiment was not completed. The lack of positive results from these experiments was attributed to various reasons, such as large

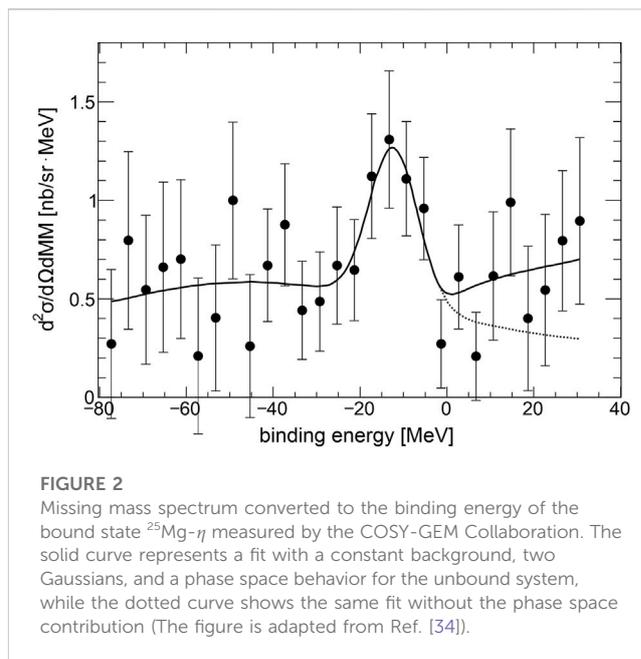
background (that could not be suppressed by detecting only one particle) and a lower cross section of the reaction than expected. Moreover, for the angle of  $\theta_p = 15^\circ$  the momentum transfer exceeds 200 MeV/c [49], making it impossible to satisfy the recoilless condition, which significantly decreases the probability of possible bound state formation in the  $(\pi, N)$  process.

An alternative proposal to investigate the  $\eta$  production induced by pions on lithium and carbon targets was presented at the Japan Proton Accelerator Research Complex (J-PARC) [72, 73]. In contrast to previous experiments [25, 26], this proposed study would focus on the  $(\pi^-, n)$  reaction, with the aim of optimizing the conditions for recoilless kinematics using a pion beam momentum in the range of 0.7–1.0 GeV/c. Additionally, the cross-sections of the  $\pi^+d \rightarrow pp\eta$  reaction were also proposed to be studied, in order to obtain an exclusive measurement and estimate background from  $\pi p$  pairs [72]. Unfortunately, this experiment has not been carried out.

Another attempt to search for such exotic states using the LAMPF accelerator [27], inspired by the prior work of Haider and Liu [74], was focused on the production of the  $\eta$ -mesic fluorine nucleus ( $^{18}\text{F}-\eta$ ) through the double charge exchange (DCX) reaction  $\pi^+ + ^{18}\text{O} \rightarrow \pi^- + ^{18}\text{Ne}$ . In such a process, interaction of a beam pion with a neutron inside the oxygen nucleus could produce the  $\eta$  meson ( $\pi^+n \rightarrow \eta p$ ), potentially resulting in the formation of the bound  $^{18}\text{F}-\eta$  state. The predicted decay of this bound state was through the absorption of the  $\eta$  meson by a neutron, followed by the emission of a negatively charged pion ( $\eta n \rightarrow \pi^- p$ ). Although the obtained DCX excitation curves showed a narrow resonance structure with a width of  $\Gamma \sim 7$  MeV, the poor statistics of the experiment did not provide sufficient evidence to confirm the presence of such a state near the  $\eta$  threshold. The observed peak was likely a statistical fluctuation in the background.

The research group at the Lebedev Physical Institute (LPI) reported the first observation of  $\eta$ -mesic nuclei formed in the photoproduction process induced by bremsstrahlung  $\gamma$  rays from the electron synchrotron [28, 29]. In the experiment, a beam of photons with energy of 650–850 MeV was used together with a carbon target. The study was performed through the reaction  $\gamma + ^{12}\text{C} \rightarrow p(n) + ^{11}\text{B}-\eta$  (or  $^{11}\text{C}-\eta$ )  $\rightarrow \pi^+ + n + X$ , where  $X$  denotes all unregistered particles. The incoming photon produced a fast nucleon ( $p$  or  $n$ ) which escaped the nucleus, and a slow  $\eta$  meson that could be captured by the remaining nucleons, forming a bound  $^{11}\text{B}-\eta$  (or  $^{11}\text{C}-\eta$ ) state. The predicted bound state would then decay via the excitation of the  $N^*(1,535)$  resonance followed by its deexcitation, emitting a pion and nucleon in opposite directions. The experiment was designed to detect the correlated  $\pi^+n$  pairs using a two-arm Time-of-Flight (TOF) spectrometer. The analysis of the obtained invariant mass distribution of the  $\pi^+n$  pairs revealed a narrow peak, shifted by  $90 \pm 15$  MeV/ $c^2$  relative to the position of the  $N^*$  resonance maximum. This peak was interpreted as evidence of the binding of the  $\eta$  meson in the boron or carbon nucleus [29].

A similar experiment on the photoproduction of  $\eta$ -mesic nuclei was conducted at the LPI via the observation of the two-nucleon decay that results from the two-nucleon absorption of the captured  $\eta$  meson in the nucleus [30]. An experimental setup consisting of two TOF spectrometers was used to detect correlated  $\pi^+n$  and  $pn$  pairs from the annihilation of  $\eta$  mesons that were stopped in the nuclear matter, and the velocity distributions of these particles were



measured. For a photon energy of  $E_\gamma = 850$  MeV (above the  $\eta$  meson photoproduction threshold), the velocity of protons was found to peak in the  $\beta_C \sim 0.6$ – $0.7$  region. This observation was interpreted as evidence of the production of low-energy  $\eta$  mesons followed by their two-nucleon absorption ( $\eta NN \rightarrow NN$ ). In contrast, no such effect was observed in standard photoproduction with a photon energy of 650 MeV, when high-momentum particles are not produced. Based on the assumption that the  $\pi^+n$  and  $pn$  pairs were primarily produced through the formation and decay of  $\eta$ -mesic nuclei with  $A = 11$ , the upper limit of the total cross section of the studied reaction was estimated to be  $10 \mu\text{b}$ . It is worth noting, however, that the energy resolution of the measurement was not sufficient to definitively confirm the position and width of a possible bound state, so the results of this experiment are not conclusive.

The formation of  $\eta$ -mesic nuclei in a reaction induced by deuterons was investigated at the Joint Institute for Nuclear Research (JINR) [37, 75]. The experiment was performed using an internal deuteron beam at the NUCLOTRON accelerator with a primary beam energy ranging from 1.5 to 2.2 GeV/nucleon. The  $d + C \rightarrow \pi^- + p + X$  reaction was studied by measuring the velocities, masses, and angles of emitted particles. The results of the experiment showed clear back-to-back  $\pi^-p$  correlation and a resonance-like peak below the  $\eta$  production threshold. This finding was interpreted as a signature of the decay of the  $N^*$  resonance, which is related to the formation of an  $\eta$ -mesic nucleus. However, further investigation required to confirm these results needs more intense beam and higher spectrometer acceptance.

Investigating the existence of  $\eta$ -mesic nuclei can also be done through transfer reaction experiments. In these reactions, the entire beam momentum is transferred to one or multiple nucleons, leaving a remaining system at rest and increasing the chance of the produced  $\eta$  meson binding to the nucleus. The experiment of this type was conducted at the GSI Helmholtz Centre for Heavy Ion Research in Darmstadt [38]. The study used a recoil-free transfer reaction ( $d, ^3\text{He}$ ) and was performed on  $^7\text{Li}$  and  $^{12}\text{C}$  targets with the aid

of the GSI Fragment Separator System (FRS). The preliminary results showed clear identification of  ${}^3\text{He}$  through time-of-flight measurement and the suppression of proton background. The position measurement was confirmed with the use quasi mono-energetic  ${}^3\text{He}$  emitted in the  $d p \rightarrow {}^3\text{He}\pi^0$  reaction on a  $(\text{CH}_2)_n$  target. The final results of the experiment, however, have not been confirmed or published.

The COSY-GEM Collaboration at the Forschungszentrum Jülich conducted an experiment examining the transfer reaction  $p + {}^{27}\text{Al} \rightarrow {}^3\text{He} + X$  under quasi-free conditions, where the beam momentum was transferred by the  ${}^3\text{He}$  ion [33, 34]. System  $X$  was believed to be the bound state of  ${}^{25}\text{Mg}-\eta$  at rest. The experiment was designed to investigate the possibility of disintegration of  ${}^{25}\text{Mg}-\eta$  state via the excitation of the  $N^*(1,535)$  and its subsequent decay into a  $\pi^- p$  pair ( $\eta n \rightarrow N^* \rightarrow \pi^- p$ ). The momenta of the outgoing  ${}^3\text{He}$  ions were measured by the BIG KARL spectrometer, while the ENSTAR detector was used to detect  $\pi^- p$  pairs. The cross sections of the reaction are shown in Figure 2. The data revealed an enhancement around  $-13$  MeV below the  $\eta$  creation threshold, which is in agreement with prior theoretical predictions [6] and is considered evidence of  ${}^{25}\text{Mg}-\eta$  production with an estimated upper limit on the total cross section at  $0.46$  nb [33]. However, more data is required to establish a clear understanding of the nature of the spectrum, as the current statistics are insufficient. Additionally, the analysis methods used by the COSY-GEM Collaboration were questioned [76], as the observed peak-like structure in the data could be caused by the interference of two different reaction processes, either with the creation of an intermediate  ${}^{25}\text{Mg}-\eta$  bound state or without forming such a state. This interference results in a shift of the peak position towards a stronger energy compared to the actual  ${}^{25}\text{Mg}-\eta$  binding energy.

The COSY-GEM group has also investigated the  $p + {}^6\text{Li} \rightarrow \eta + {}^7\text{Be}$  reaction using a lighter  ${}^6\text{Li}$  nucleus as a target [77]. In particular, they focused on the  $\eta$  meson interaction with the  ${}^7\text{Be}$  nucleus in the final state. The experimental results were compared to model calculations [78], which treated the target and residual nuclei as consisting of  ${}^4\text{He}-d$  and  ${}^4\text{He}-{}^3\text{He}$  clusters, respectively, and assumed an  $\eta N$  interaction with a large scattering length (consistent with a bound state). Only two possible final states were found at an excess energy of approximately  $11$  MeV above the threshold. However, it was not possible to determine the size of the FSI. More data, especially near the threshold, is needed to solve this issue.

## 2.2.2 Probing the existence of $\eta$ -mesic states in light nuclei

As previously mentioned, studies on the production of  $\eta$  mesons in various processes have resulted in a wide range of possible values for the  $\eta$ -nucleon scattering lengths ( $a_{\eta N}$ ) [18, 21, 23, 24]. This opens the possibility for the formation of  $\eta$ -mesic states in light nuclei like helium ( ${}^3\text{He}$ ,  ${}^4\text{He}$ ) [8, 9] or hydrogen isotopes ( ${}^2\text{H}$ ,  ${}^3\text{H}$ ) [7, 65]. The smaller absorption of  $\eta$  mesons in the light nuclei causes an increase in their lifetime in the nuclear medium, resulting in the formation of narrower bound states [52] that may be easier to observe compared to those in heavy nuclei. Additionally, there are no complications in interpreting results related to excitation of higher nuclear levels.

The existence of  $\eta$ -mesic bound states in helium is supported by several indirect experimental observations. In particular, the studies

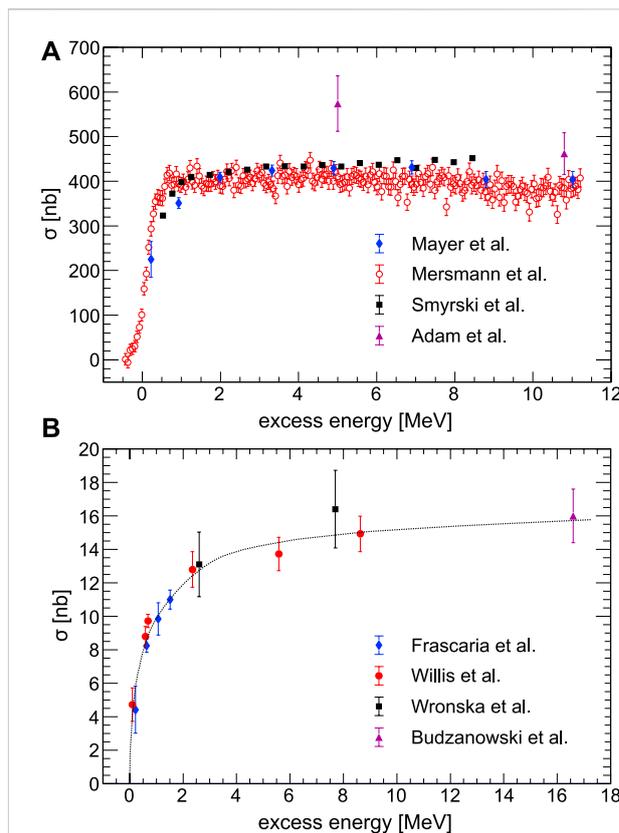
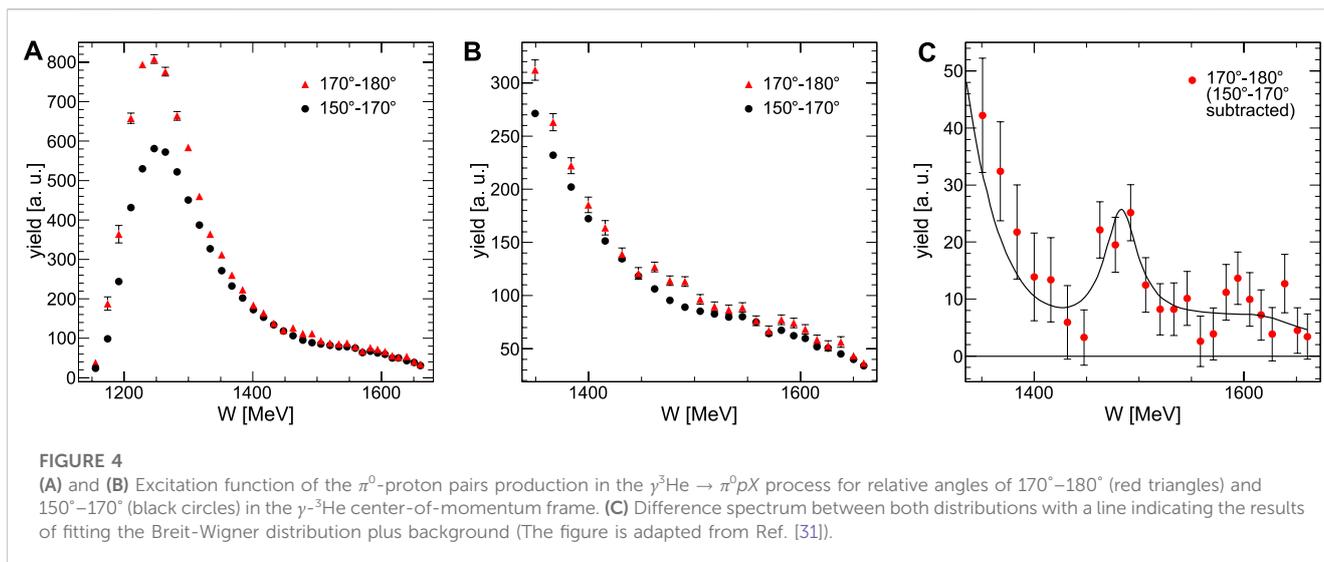


FIGURE 3

The close-to-threshold total cross section for the  $\eta$  meson production reactions plotted as a function of excess energy. (A) Data for the  $dp(pd) \rightarrow {}^3\text{He}\eta$  reaction taken from Mayer et al. [79], Mersmann et al. [80], Smyski et al. [81] and Adam et al. [82]. (B) Data for the  $dd \rightarrow {}^4\text{He}\eta$  reaction taken from Frascaria et al. [83], Willis et al. [84], Wronska et al. [85] and Budzanowski et al. [86]. The black dotted curve is added for better visualization of the trend.

of the  $dp(pd) \rightarrow {}^3\text{He}\eta$  [79–82] and  $dd \rightarrow {}^4\text{He}\eta$  [83–86] reactions have revealed a sharp rise in the total cross section at  $\eta$  production threshold, as shown in Figures 3A,B. Additionally, the photoproduction reaction  $\gamma^3\text{He} \rightarrow {}^3\text{He}\eta$  [31, 32] also shows similar behavior in the excitation function. The steep structure of the total cross section in both hadronic and photoproduction processes near threshold may be a clear manifestation of the strong final-state interaction in  ${}^3\text{He}-\eta$  system, which indicates the possible existence of quasibound states in such a system. Moreover, the extremely rapid rise in the cross section in the close-to-threshold region, followed by a plateau, suggests that the FSI, and thus the  $\eta$ -nucleus interaction, is stronger in the case of  ${}^3\text{He}$ —so it is more likely for the  $\eta$  meson to bind to  ${}^3\text{He}$  than to  ${}^4\text{He}$  (compare the A and B panels of Figure 3). Further support for the existence of strong  ${}^3\text{He}-\eta$  FSI comes from a small, energy-independent value of the tensor analysing power  $T_{20}$  in the  $\vec{d}p \rightarrow {}^3\text{He}\eta$  process, which has been measured up to  $11$  MeV excess energy at the COSY-ANKE facility [87]. Experimental data [80, 81] also shows a very strong variation with energy of both the phase and magnitude of the  $s$ -wave production amplitude, suggesting the presence of a pole in the  ${}^3\text{He}-\eta$  scattering matrix near the threshold [88]. However, whether this pole corresponds to a bound or virtual state is still uncertain.



The first experiment to directly search for a bound state of the  $\eta$  meson with a light nucleus was conducted using the Two-Arm Photon Spectrometer (TAPS) at the Mainz Microtron (MAMI) accelerator facility [31]. The experiment studied the  $\gamma^3\text{He} \rightarrow \pi^0 p X$  reaction for photon energies ranging from threshold to 820 MeV. To detect the signature from the decay of the  $\eta$ -mesic nucleus, the  $\pi^0$  mesons and protons were measured for two ranges of the relative angle between them. A resonance-like structure was found just above the  $\eta$  production threshold in the spectrum, which represented the difference between the excitation curves for the opening angles of  $170^\circ\text{--}180^\circ$  and  $150^\circ\text{--}170^\circ$  in the center-of-mass frame (see Figure 4). This enhancement was interpreted as a possible signature of the  $^3\text{He}\text{--}\eta$  bound state, where the  $\eta$  meson, captured by one of the nucleons inside helium, forms an intermediate  $N^*(1,535)$  resonance that decays into a pion-nucleon pair. The binding energy and width for the expected  $\eta$ -mesic bound state were estimated from the fit of the Breit-Wigner distribution function to the experimental points, resulting in  $(-4.4 \pm 4.2)$  MeV and  $(25.6 \pm 6.1)$  MeV, respectively. However, as Hanhart pointed out in Ref. [89], the limited statistics of the data obtained in the TAPS experiment were more compatible with the formation of a virtual  $^3\text{He}\text{--}\eta$  state rather than a bound state. Consequently, it was then concluded [90] that improved statistics would allow for finer energy binning, leading to a more precise interpretation of the results. The experiment was subsequently repeated using the upgraded Crystal Ball/TAPS combined detection setup [32], which benefited from almost  $4\pi$  acceptance and much higher statistics. In addition, the experiment measured the  $\eta \rightarrow 2\gamma$  and  $\eta \rightarrow 3\pi^0 \rightarrow 6\gamma$  decay channels to obtain better control over systematic uncertainties. Regardless, this investigation did not lead to more conclusive results, as the structure observed in the  $\pi^0 p$  excitation function was explained as an artifact from the quasi-free pion production background.

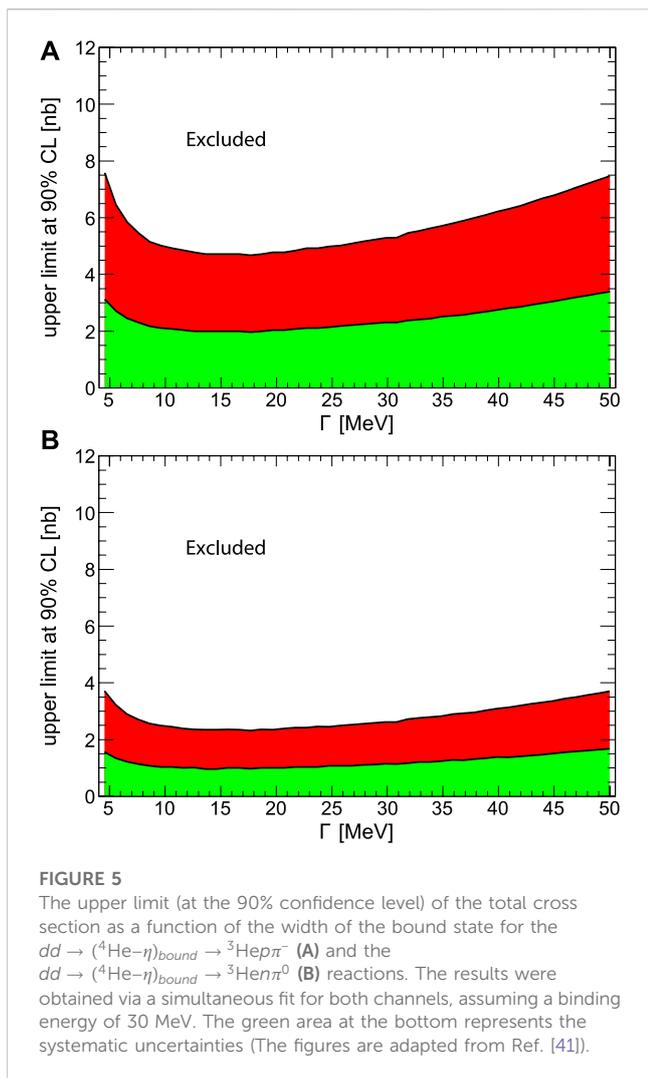
The search for  $\eta$ -mesic helium nuclei was also carried out using the internal deuteron beam of the Cooler Synchrotron (COSY) [51] at the Forschungszentrum Jülich. During the experiment [39, 91–94], the deuterons were scattered on a proton target of the cluster-jet type, and the resulting charged products of the reaction

were detected by the COSY-11 facility [95]. The beam momentum was continuously ramped within each acceleration cycle, crossing the threshold for the  $dp \rightarrow ^3\text{He}\eta$  reaction, resulting in excess energies ranging from  $-10$  to  $9$  MeV. Excitation functions were determined for the  $dp \rightarrow ^3\text{He}\pi^0$  and  $dp \rightarrow ppp\pi^-$  reactions as potential decay channels of the  $\eta$ -mesic  $^3\text{He}$  nucleus. The excitation function for the first process did not reveal any structure that could originate from a decay of  $^3\text{He}\text{--}\eta$  bound state. An upper limit for the cross-section of the  $dp \rightarrow (^3\text{He}\text{--}\eta)_{\text{bound}} \rightarrow ^3\text{He}\pi^0$  reaction chain was estimated to be  $70$  nb [92]. The  $ppp\pi^-$  decay channel of the  $^3\text{He}\text{--}\eta$  bound state was considered favorable, because it corresponds to a one-step process of the  $\eta$  meson absorption by a neutron inside the  $^3\text{He}$  nucleus, followed by excitation of the neutron to the  $N^*(1,535)$  resonance state and its subsequent decay into the proton-pion pair ( $\eta n \rightarrow N^* \rightarrow p\pi^-$ ). However, only 9 events that could originate from the decay of the  $^3\text{He}\text{--}\eta$  bound state were observed. An upper limit of  $270$  nb [93] was derived for the  $dp \rightarrow (^3\text{He}\text{--}\eta)_{\text{bound}} \rightarrow ppp\pi^-$  reaction chain.

### 2.2.2.1 Search for $\eta$ -mesic helium with WASA-at-COSY detector

In more recent experiments, the search for hypothetical  $\eta$ -bound states in helium nuclei has been carried out employing the Wide Angle Shower Apparatus (WASA) [96] at the COSY accelerator. Three high-statistics, exclusive measurements were performed in 2008, 2010, and 2014, using a deuteron pellet target and deuteron and proton beams to study the possible formation of  $^4\text{He}\text{--}\eta$  and  $^3\text{He}\text{--}\eta$  bound states, respectively [97]. During each experiment, data were collected as the beam was slowly and continuously accelerated in each cycle, crossing the kinematic threshold for the production of the  $\eta$  meson (corresponding to the  $\eta$ -helium excess energy range  $Q \in (-70, 30)$  MeV).

The study of  $\eta$ -mesic nuclei involves investigating the physical processes in which a virtual  $\eta$  meson is produced and then forms a bound state with the nucleus, which eventually decays. In the WASA-at-COSY experiments, two mechanisms for  $\eta$ -mesic helium decay were tested. The first hypothesis involves the absorption of the  $\eta$  meson by a nucleon inside helium, exciting it to the  $N^*(1,535)$  resonance, which

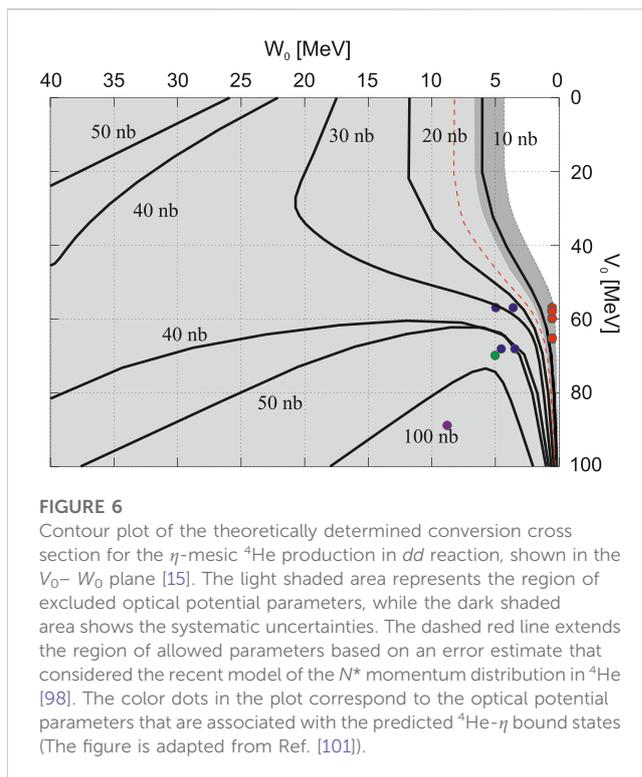


**FIGURE 5**  
The upper limit (at the 90% confidence level) of the total cross section as a function of the width of the bound state for the  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}p\pi^-$  (A) and the  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}n\pi^0$  (B) reactions. The results were obtained via a simultaneous fit for both channels, assuming a binding energy of 30 MeV. The green area at the bottom represents the systematic uncertainties (The figures are adapted from Ref. [41]).

subsequently decays into a nucleon-pion pair, causing the disintegration of the mesic nucleus. Three reactions were analyzed to test this mechanism:  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow N^*-^3\text{He} \rightarrow ^3\text{He}p\pi^-$  [40, 41],  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow N^*-^3\text{He} \rightarrow ^3\text{He}n\pi^0$  [41], and  $pd \rightarrow (^3\text{He}-\eta)_{\text{bound}} \rightarrow N^*-NN \rightarrow dp\pi^0$  [35]. Alternatively, in the second considered scenario, the  $\eta$  meson decays directly while still “orbiting” around the nucleus, leaving it intact. To investigate this mechanism, the reaction chains  $pd \rightarrow (^3\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}2\gamma$  and  $pd \rightarrow (^3\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}3\pi^0 \rightarrow ^3\text{He}6\gamma$  were examined [36].

In the reactions that proceed via the excitation of the  $N^*$  resonance, the accuracy of determining the cross section depends on the momentum distributions of the  $N^*$  inside the mesic nuclei, which determine the kinematics of the particles in the final state. Calculations, based on elementary  $NN^* \rightarrow NN^*$  amplitudes within a  $\pi$  plus  $\eta$  meson exchange model, allowed to determine the momentum distributions of  $N^*$  in the  $N^*-^3\text{He}$  and  $N^*-NN$  systems [98–100]. These distributions were used in the analysis of the experimental data.

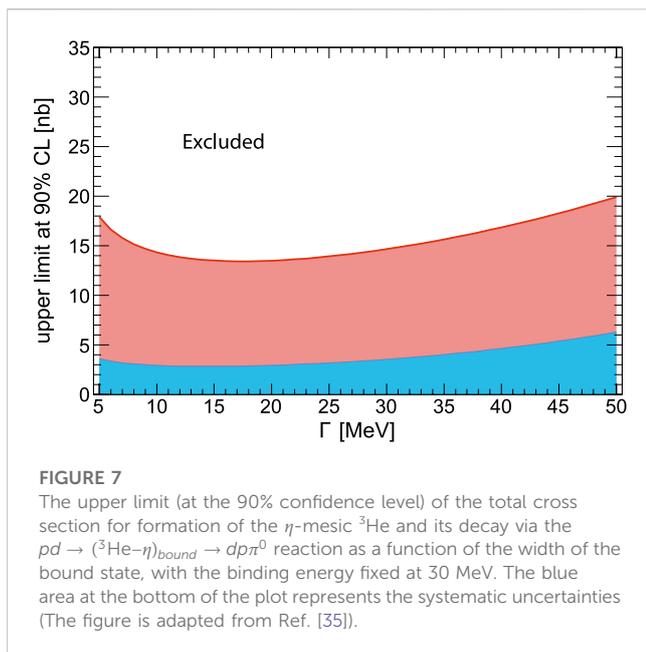
To search for  $^4\text{He}-\eta$  bound states, the WASA-at-COSY Collaboration measured the excitation functions for the  $dd \rightarrow ^3\text{He}p\pi^-$  [40, 41] (2008 and 2010 data sets) and  $dd \rightarrow ^3\text{He}n\pi^0$  [41] (2010) reactions near the  $\eta$  production



**FIGURE 6**  
Contour plot of the theoretically determined conversion cross section for the  $\eta$ -mesic  $^4\text{He}$  production in the  $dd$  reaction, shown in the  $V_0 - W_0$  plane [15]. The light shaded area represents the region of excluded optical potential parameters, while the dark shaded area shows the systematic uncertainties. The dashed red line extends the region of allowed parameters based on an error estimate that considered the recent model of the  $N^*$  momentum distribution in  $^4\text{He}$  [98]. The color dots in the plot correspond to the optical potential parameters that are associated with the predicted  $^4\text{He}-\eta$  bound states (The figure is adapted from Ref. [101]).

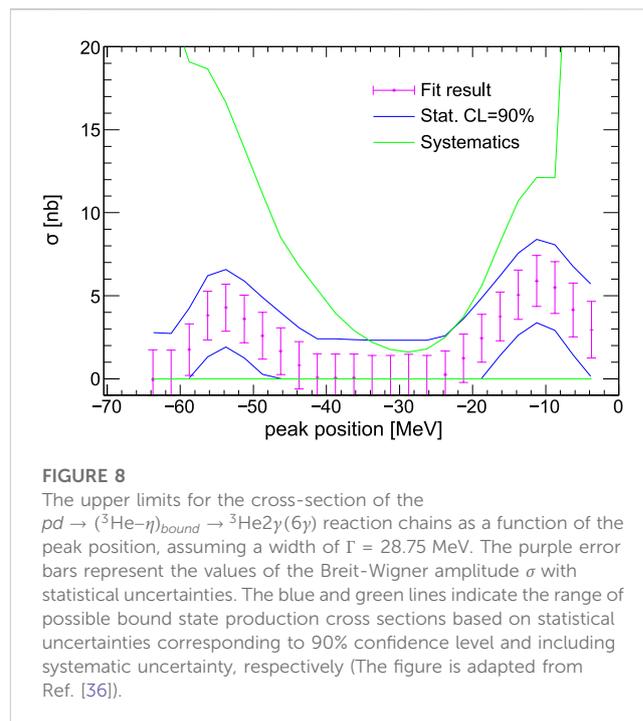
threshold. It was expected that the formation of a bound state would manifest itself as a resonance-like structure below the threshold. However, no such structure was observed in the obtained excitation curves. Therefore, the upper limits of the total cross section for the production and decay of  $^4\text{He}-\eta$  bound states in both studied channels were determined at a 90% confidence level. To evaluate the upper limits quantitatively, the excitation curves were fitted using a function being a combination of Breit-Wigner distribution (representing the signal) and a second-order polynomial (describing the background). The sensitivities of the cross section achieved for the  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}p\pi^-$  and  $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}n\pi^0$  reactions are  $\sim 6$  nb and  $\sim 3$  nb, respectively. The upper limits for both processes, plotted as a function of bound state width, are presented in Figure 5.

A recently developed phenomenological model describing subthreshold cross sections for the  $dd \rightarrow ^4\text{He}\eta$  reaction [15] was applied to fit theoretical spectra with various optical potential parameters ( $V_0$ ,  $W_0$ ) to experimental excitation functions [101]. This method led to an improved determination of the upper limit of the total cross section for the production of  $\eta$ -mesic  $^4\text{He}$  in the  $dd \rightarrow ^3\text{He}N\pi$  process, which varies from 5.2 to 7.5 nb. After accounting for the systematic uncertainties of the order of 44% estimated in Ref. [41], a wide range of possible parameters for the  $^4\text{He}-\eta$  optical potential, which result in a cross section larger than 10.7 nb, were excluded. This exclusion region is represented by a light shaded area in Figure 6, with the systematic error represented by a dark shaded area. In addition, consideration of a model describing the momentum distribution of the  $N^*$  resonance inside  $^4\text{He}$  nucleus [98] led to the extension of the allowed region in the  $V_0 - W_0$  plane, as shown by the dashed red line. The colored dots shown in the figure represent the results of some optical model calculations.



In particular, Figure 6 shows predictions based on a few-body formalism and an optical model [13] that uses the  $\eta$ -nucleon scattering amplitude obtained from two different models described in Ref. [102] (purple dot) and [103] (green dot). Results for a class of potentials, which include Gaussian, exponential and Hulthen potentials [104], are shown as blue dots. The analysis [101] suggested that the  $\eta$ -mesic helium states within these models are excluded. Finally, the study also predicted very narrow and weakly bound states of  ${}^4\text{He}-\eta$  with binding energies and widths of  $\sim 2 - 230$  keV and  $\sim 8 - 64$  keV, respectively, represented by red dots at the edge of the allowed parameter region and obtained by solving the Klein-Gordon equation as in Ref. [15].

The final experimental campaign conducted by WASA-at-COSY Collaboration in 2014 aimed to search for  $\eta$ -mesic  ${}^3\text{He}$  state in the proton-deuteron nuclear fusion [35, 36, 105–108]. This run resulted in the world's largest data sample collected thus far [105]. As previously mentioned, phenomenological calculations indicate the potential existence of a weakly bound  ${}^3\text{He}-\eta$  state with an estimated binding energy of approximately 0.3 MeV and a width of approximately 3 MeV [14]. Analysis of this data, assuming the  ${}^3\text{He}-\eta$  bound state decays through the creation of the  $N^*$  resonance, allowed for the establishment of an excitation function for the  $pd \rightarrow dp\pi^0$  reaction in the region of the  $\eta$  meson production thresholds. However, no narrow resonance-like structure associated with an  $\eta$ -mesic helium was observed. Therefore, an upper limit for the total cross section of the  $pd \rightarrow ({}^3\text{He}-\eta)_{\text{bound}} \rightarrow dp\pi^0$  process was estimated with values ranging from 13 to 24 nb [35]. The upper limit primarily depends on the width of the bound state rather than its binding energy. Figure 7 illustrates the results for the binding energy  $B_s = -30$  MeV. It is worth noting that bound states predicted with  $\eta$ -nucleon scattering lengths of  $\sim 1$  fm [12, 13] remain a possibility within the determined limits.



The data obtained from this experiment also allowed the investigation of the second proposed mechanism for the decay of  ${}^3\text{He}-\eta$  bound state, which involves the direct decay of the  $\eta$  meson. This mechanism was studied for the first time by analysing the  $pd \rightarrow {}^3\text{He}2\gamma$  and  $pd \rightarrow {}^3\text{He}3\pi^0 \rightarrow {}^3\text{He}6\gamma$  reactions [36]. For this purpose, the Fermi momentum distribution of a bound  $\eta$  meson orbiting around the  ${}^3\text{He}$  nucleus as well as the in-medium branching ratios of  $\eta \rightarrow 2\gamma$  and  $\eta \rightarrow 3\pi^0$  processes were determined for various combinations of the  ${}^3\text{He}-\eta$  optical potential parameters [109]. The analysis revealed a weak signal from the bound state with a width greater than 20 MeV and a binding energy within the range of 0–15 MeV. However, this signal was covered by the systematic error, and thus, no conclusions could be drawn about the existence of the bound state in the considered mechanism. Finally, the estimated upper limit for the formation of the  $\eta$ -mesic  ${}^3\text{He}$  at a 90% confidence level, obtained by fitting simultaneously the excitation functions for both channels, varied between 2 and 15 nb depending on the bound state parameters [36]. Figure 8 shows the results for the bound state width  $\Gamma = 28.75$  MeV.

In summary, while some research efforts suggests the existence of  $\eta$ -mesic nuclei, none of these findings have been independently confirmed. Some of the experiments mentioned above require higher precision or improved detection systems. Conducting new experiments with better statistics and less background uncertainty would provide more conclusive evidence for the existence of  $\eta$ -mesic nuclei in the future.

### 3 Discussion of the results and outlook

Mesic nuclei would represent a novel form of nuclear matter in which a meson is bound with a nucleus solely through the strong

interaction, without the influence of electromagnetic Coulomb effects. The  $\eta$  meson is a particularly promising candidate for exploring such bound states, due to its strong attractive interaction with nucleons [3]. Mesic nuclei would serve as a unique laboratory for investigating the behavior of mesons and their possible modifications in the nuclear medium, offering valuable insights for hadron physics research.

As previously mentioned, the value of  $\eta N$  scattering length is significantly dependent on the chosen model [18], resulting in a wide range of predictions for the possible  $\eta$ -nuclear states. Therefore, distinguishing between different theoretical approaches can be challenging. Some models suggest the existence of narrow, easily observable bound states, while others predict widths that are too large or too weak to generate bound states. Despite numerous experiments aimed at discovering  $\eta$ -mesic nuclei using pion, photon, proton, and deuteron beams, conclusive results have not been obtained. The absence of a clear positive signal may be due to significant non- $\eta$  background. The uncertainty related to the predictions of possible  $\eta$ -nuclear states from different theoretical models and inconclusive experimental results leave the question of the existence of hypothetical  $\eta$ -mesic nuclei unanswered. Therefore, new strategies and measurements with higher statistics are necessary to advance in this field.

Transfer reactions have been identified as a particularly intriguing topic for further investigation due to current evidence that supports the existence of the  $^{25}\text{Mg-}\eta$  mesic nucleus [33]. Moreover, it is possible that mesic nuclei could be discovered in lighter systems such as  $^3\text{He}$ ,  $^4\text{He}$ , and  $^{11}\text{B}$  [21]. Analysis of the excitation function of the pion DCX reaction  $\pi^+ + ^{18}\text{O} \rightarrow \pi^- + ^{18}\text{Ne}$  has revealed the presence of a resonance structure at energies corresponding to the  $\eta$  threshold [27], although the statistical significance of this observation was negligible. Hence, repeating the experiment with significantly improved precision and statistics, as well as conducting DCX excitation function measurements on other nuclei, would be beneficial. Research groups at the LPI [28–30] and JINR [37, 75] have reported promising results with photoproduction and deuteron-induced reactions, respectively. Nevertheless, these findings need to be verified and repeated with greater accuracy and statistical significance. Experiments on the production of  $\eta$  mesons and the searching for the  $\eta$ -mesic nuclei carried out at the Jülich Research Center have also made significant contributions to the field. A particularly interesting result was obtained for the  $pd \rightarrow ^3\text{He}2\gamma(6\gamma)$  reactions using the WASA-at-COSY detector [36]. The development of a new experiment to confirm this result would be a good direction for further investigation. Unfortunately, the research program at the COSY accelerator in Jülich ended in 2015 [51].

Research on hypothetical mesic nuclei involves also investigating the formation of bound states of atomic nuclei and the  $\eta'$  meson, which is the heavier partner of the  $\eta$  meson and has a mass of 957.78 MeV [17]. The  $\eta'$  meson is a promising candidate for mesic nuclei searches due to its strong and attractive interactions with nucleons [18, 110]. Ongoing experimental [111, 112] and theoretical [44, 113–115] studies are actively pursuing this research direction. In 2014, the pioneering experiment aimed at detecting  $\eta'$ -mesic bound states was conducted using the  $^{12}\text{C}(p, d)$  reaction at the Fragment Separator (FRS) facility located at the GSI [111]. Despite the excellent statistics, no narrow structure was

observed, and an upper limit for the formation cross-section of  $\eta'$ -mesic nuclei of  $\sim 20$  nb/(sr-MeV) was established. Recently, a unique experimental setup was developed by integrating the central part of the WASA detector into the FRS [111]. The first series of experiments using this setup were successfully conducted in 2022, and analysis of the obtained data is currently ongoing.

## 4 Summary

A detailed account of the various theoretical and experimental efforts for the search for bound  $\eta$  meson in nuclei was given in this review.

Initially, based on the assumption of a relatively small  $\eta$ -nucleon scattering length  $a_{\eta N}$ , theorists estimated that the lightest nucleus on which the  $\eta$  meson might bind is  $^{12}\text{C}$  [4, 5, 58]. The larger value of  $a_{\eta N}$  [18, 21, 23, 24] suggested later indicates that  $\eta$  could tightly bind with heavy nuclei, resulting in large and overlapping widths, making it difficult to detect, but also raises the possibility of binding even in lighter systems, such as helium [9–16]. Numerous experimental attempts and theoretical investigations have been conducted since the first prediction, but most searches for signals of  $\eta$ -mesic nuclei have been unsuccessful.

Due to limited knowledge about the  $\eta$ -nucleon and  $\eta$ -nucleus interactions, diverse theoretical predictions of mesic nuclei have been proposed, using various assumptions and considerations. Binding energies and widths of several heavy  $\eta$ -mesic nuclei have been calculated using techniques based on different approaches, such as  $\eta$ -nucleus optical potentials [4, 58, 59], self-energies of the  $\eta$  meson in the nuclear medium [60], or unitarized chiral perturbation theory [6]. The QMC approach has also been applied to study the behavior of  $\eta$ -mesons in the nuclear medium, where it mixes with the  $\eta'$  meson [42, 61, 62], which allowed to predict of bound states in several closed-shell nuclei. Exploring the existence of  $\eta$  binding in light nuclei typically involves identifying poles in the scattering matrix and calculating corresponding  $\eta$ -nucleus scattering lengths. A bound state requires a large, negative real component of the scattering length [59]. Calculations based on few-body equations have been performed for the  $^2\text{H-}\eta$ ,  $^3\text{H-}\eta$ ,  $^3\text{He-}\eta$ , and  $^4\text{He-}\eta$  systems [7, 13, 63]. The time delay concept has also been applied to locate these bound states [10, 11, 68]. More recent studies based on optical potential [14] and pionless effective field theory [12] suggest the possibility of bound  $\eta$ -mesic states in  $^3\text{He}$  and  $^4\text{He}$ . However, some works [70, 71] indicate that the attraction in these systems is very weak, which makes their detection unlikely.

Early experiments with low statistics using pion [25–27], photon [28–32], proton [33, 34], or deuteron [37–39] beams provided some indications of potential  $\eta$ -mesic bound states, but no definitive evidence was found due to limited sensitivity. Recent searches with using the COSY [51] accelerator have concentrated on the possibility of  $\eta$ -mesic bound states in  $^3\text{He}$  and  $^4\text{He}$  isotopes. Evidence of possible mesic helium nuclei was indicated by a sharp rise in cross sections near threshold observed in photoproduction [31, 32], proton-deuteron ( $pd$ ) [79–82], and deuteron-deuteron ( $dd$ ) [83–86] experiments. These hypothetical states in helium require a  $\eta$ -nucleon scattering length with a real part larger than 0.7 to 1 fm [12, 13]. Experimental efforts to search for the  $\eta$ -mesic  $^3\text{He}$  have been conducted by the COSY-11 Collaboration [39, 91–94] and resulted in the upper limits of the cross section of 270 nb

and 70 nb for the  $pd \rightarrow (^3\text{He}-\eta)_{\text{bound}} \rightarrow ppp\pi^-$  and  $pd \rightarrow (^3\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{He}\pi^0$  reactions, respectively. Recent experiments conducted by the WASA-at-COSY Collaboration [35, 36, 40, 41] aimed to observe the formation of  $\eta$ -mesic nuclei in  $^4\text{He}$  and  $^3\text{He}$ . Two decay mechanisms were investigated: absorption of the  $\eta$  meson by a nucleon, followed by decay into an  $N\pi$  pair, and direct decay of the  $\eta$  meson. The obtained excitation functions showed no clear structure indicating the formation of the expected bound states. The upper limits for the cross sections of the production and decay of  $\eta$ -mesic nuclei were found to be 3–6 nb for  $dd \rightarrow ^3\text{He}n\pi^0$  and  $dd \rightarrow ^3\text{He}p\pi^-$ , 13–24 nb for  $pd \rightarrow d\pi^0$ , and 2–15 nb for  $pd \rightarrow ^3\text{He}2\gamma(6\gamma)$  channels.

In conclusion, despite more than 35 years of theoretical and experimental efforts, the search for unambiguous evidence of  $\eta$ -mesic states has proven to be a challenging task. The lack of a conclusive signal highlights the difficulty of this field of research. Therefore, the development of novel experimental strategies with higher statistics measurements is crucial to make significant progress in this direction.

## Author contributions

The manuscript was initially drafted by AK, and later edited and contributed to by MS and PM. All authors contributed to the final editing and proofreading of the manuscript and approved it for submission. All authors listed have made a substantial, direct, and intellectual contribution to the work and approved it for publication.

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## Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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