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## EDITED BY

Chengyi Xia,  
Tiangong University, China

## REVIEWED BY

Hai-Feng Zhang,  
Anhui University, China  
Xiaoke Xu,  
Dalian Nationalities University, China

## \*CORRESPONDENCE

Linfeng Zhong,  
✉ googlezlf@163.com

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# Identifying key nodes in complex networks based on an improved gravity model

Linfeng Zhong<sup>1,2\*</sup>, Xiangying Gao<sup>1</sup>, Liang Zhao<sup>3</sup>, Lei Zhang<sup>1</sup>,  
Pengfei Chen<sup>1</sup>, Hao Yang<sup>1</sup>, Jin Huang<sup>1</sup> and Weijun Pan<sup>1</sup>

<sup>1</sup>School of Air Traffic Management, Civil Aviation Flight University of China, Guanghan, China, <sup>2</sup>Chengdu GoldTel Industry Group Co., Ltd., Chengdu, China, <sup>3</sup>Operation Management Center of ATMB, Civil Aviation Administration of China, Beijing, China

The identification of key nodes in complex networks is a hot topic. Therefore, it attracts increasing attention from different fields, like airline networks and social networks. To identify the key nodes in complex network, we suggest an improved gravity model method that takes propagation features into account. Relevant experiments were carried out in four actual airline networks based on the Susceptible Infected Recovered (SIR) model. First, we analyze the correlation between the proposed method and other benchmark methods. Then, Kendall's correlation coefficient and the imprecision function were used as evaluation metrics to analyze and validate the proposed method. Empirical results reveal that the suggested method outperforms previous benchmark methods in terms of precision and effectiveness for identifying key nodes, especially in the US air network, where Kendall's tau achieves a 107% improvement compared to the gravity centrality method.

## KEYWORDS

complex network, key nodes, gravity model, propagation feature, SIR

## 1 Introduction

Research on complex networks has gained significant traction in recent years [1, 2], and related work has been progressing in fields including airline networks [3, 4], social networks [5], and other networks [6]. Numerous scholars have devoted themselves to the study of identifying key nodes in networks in addition to analyzing and exploring some fundamental topologies of networks [7–9]. Theoretically and practically, it is critical to accurately identify key nodes in networks [10], because key nodes frequently play a more important role on the structure and function of the entire network than other nodes. The research of key nodes identification has a wide range of applications, including viral marketing, the identification of significant opinion leaders in social networks, the location of key node airports in airline networks, and so on.

Theoretically, a variety of methods have been proposed to effectively identify key nodes in complex networks, including Degree Centrality (DC) [11], *k*-shell decomposition (KS) [12], and its variants [13, 14]. However, it frequently appears that many nodes are in the same ranking when utilizing the aforementioned traditional ways to rank the importance of network nodes. Over the years, numerous innovative methods have been proposed in an effort to successfully address these issues and obtain accurate node identification [9, 15–19]. Ma et al. introduced a novel method called Gravity Centrality (GC) [20], which takes into account both the path information and the local attribute. Li et al. [21] suggested a ranking algorithm by treating node degree values as masses while taking neighboring nodes' ranges

into account. Li et al. [22] proposed an improved gravity model method that integrates the gravity model as well as the degree and  $k$ -shell values of nodes. In fact, in addition to local and global features and path-based information, the propagation feature is one of the variables that affects the propagation range of nodes in the network [23–25]. According to this idea, Zhong et al. [26] provide an enhanced information entropy [27] approach that takes into account the effects of both node neighbors and propagation features.

Inspired by the principles of GC and its modifications as well as node propagation features, this research proposed a unique method for identifying key nodes in complicated airline networks based on the gravity model and propagation feature (IGP). In four real airline networks, we evaluated the accuracy of our method, and IGP performed better on Kendall’s tau [28] and the imprecision function than six other benchmark methods. Our method can accurately identify key nodes, which not only contributes to enhancing the effectiveness and quality of the airline network but also offers theoretical support for network improvement.

The following sections make up the present paper. It begins with a description and justification of the proposed method. Second, a detailed analysis of the results of the experiment is provided. Finally, an overview of this research project is provided.

## 2 Model

Given a complex network  $G$ , which is undirected and unweighted, we make  $G = (N, E)$ , which means that the graph  $G$  consists of  $N$  nodes and  $E$  edges.  $A = \{a_{ij}\}$  is the adjacency matrix of  $G$ ;  $a_{ij} = 0$  indicates that node  $i$  and  $j$  are not directly connected;  $a_{ij} = 1$  indicates that they are. Airports represent nodes in an airline network, and routes between airports substitute edges, which are abstracted into an undirected airline network. Similar to this,  $a_{ij} = 1$  if there are direct connections connecting two airports; otherwise,  $a_{ij} = 0$ . Besides,  $A$  can also be used to describe the basic topology of  $G$ .

To begin identifying the initial effect of nodes in the network, we first compute the gravity value— $G$  based on the gravity centrality.

$$G_i = \sum_{j \in \psi(i)} \frac{(k_i + k_{si})(k_j + k_{sj})}{d_{ij}^2} \tag{1}$$

Where  $G_i$  refers to the importance of node  $i$ ;  $j \in \psi(i)$  denotes that node  $j$  is a neighbor of node  $i$ . In this paper, we will consider the direct neighbors. The  $k_i, k_{si}$  represent the degree value,  $k$ -shell value of node  $i$ , respectively. The  $d_{ij}$  denotes the shortest distance from node  $i$  to node  $j$ . And  $k_i, k_j$  are expressed using Eq. 2.

$$k_i = \sum_{j \in G} a_{ij} \tag{2}$$

The influence of same-order neighbors is typically seen as being equal in networks that preserve a certain propagation rate. Using node propagation features, it is possible to further identify the degree of relevance of nodes with the same influence (nodes with the same centrality value) in the real propagation process, as well as to distinguish the degree of influence of a node on its neighbors. In

**TABLE 1** Basic topology information of US air, CA, CZ, and MU networks. The names of the four networks are placed in the first column under network. The  $N$  and  $E$  denote the total number of nodes and edges, respectively. The  $\langle k \rangle$  and  $\beta_{th}$  represent the average degree and epidemic threshold. The  $r, C$  and  $\langle d \rangle$  are assortative coefficient, clustering coefficient and the average distance, respectively.

Network	$N$	$E$	$\langle k \rangle$	$\beta_{th}$	$r$	$C$	$\langle d \rangle$
US air	332	2,126	12.8	0.023	−0.208	0.625	2.738
CA	237	855	7.2	0.026	−0.356	0.447	2.358
CZ	281	1,138	8.1	0.025	−0.367	0.380	2.519
MU	293	1,247	8.4	0.025	−0.361	0.435	2.563

order to further differentiate, we therefore successfully combine the propagation rate and node degree. We will use  $C_j$  to represent the influence effect of node  $j$  when the propagation rate is  $\beta$ .

$$C_j = 1 - (1 - \beta)^{k_j} \tag{3}$$

Finally, the gravity model and the propagation feature are considered simultaneously to analyze the influence of the nodes in the network, as described by the following equation.

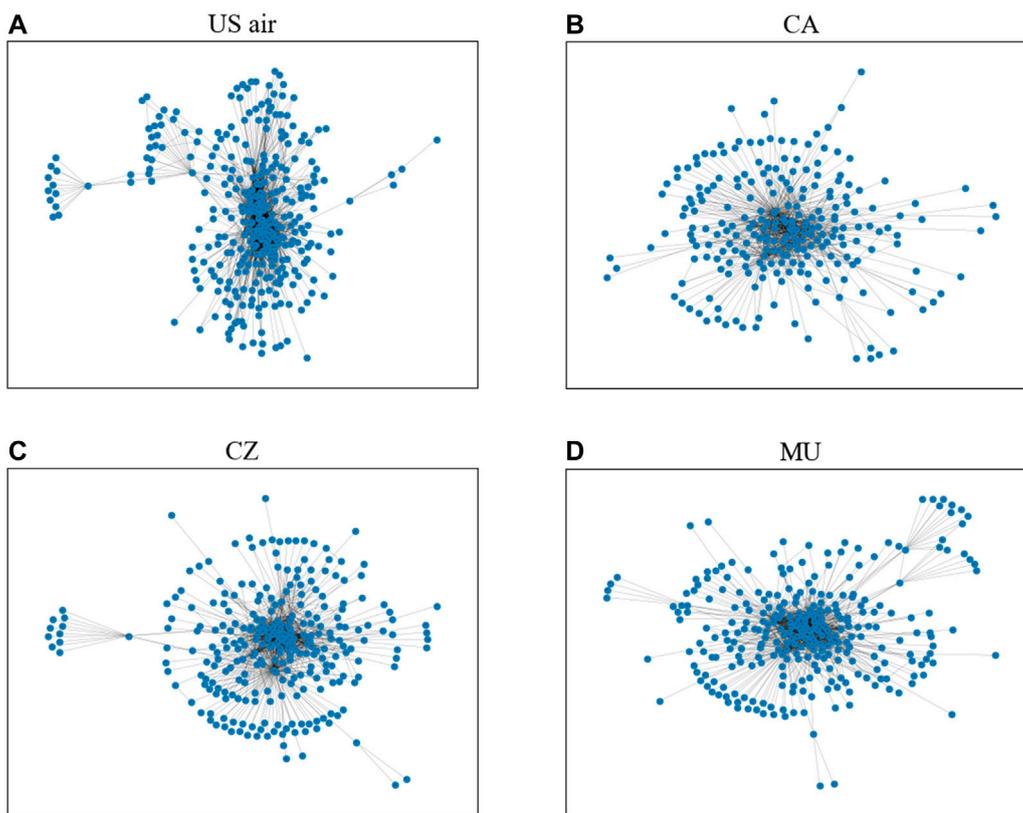
$$IGP(i) = \sum_{j \in \psi(i)} C_j * G_i = \sum_{j \in \psi(i)} [1 - (1 - \beta)^{k_j}] \frac{(k_i + k_{si})(k_j + k_{sj})}{d_{ij}^2} \tag{4}$$

## 3 Results

### 3.1 Data description

The accuracy of the IGP method will be evaluated in this experiment using four real airline networks. The US air airline network is a network of American airlines [29]. The real airline networks of China’s three domestic carriers are denoted, respectively, by CA (the airline network of Air China), CZ (the airline network of China Southern), and MU (the airline network of China Eastern). Each node in the airline network represents a separate airport, and the edges indicate the routes that connect those airports. Table 1 displays some of the topological properties of the four airline networks.

According to Table 1, it is clear that the US air network covers the most airports and routes among the four airline networks. By comparing the average degree of each airline network, it is possible to compare network sparsity. The epidemic threshold identified for each airline network using the SIR model is indicated by the  $\beta_{th}$ . The assortative coefficients for four airline networks are negative, and the values are near to each other, indicating that the clustering effect of each airline network is quite close. The US air network’s clustering coefficient has a maximum value of 0.625, the CZ has the lowest value. This shows that some nodes (airports) in four airline networks have a tendency to be connected with preferred large degree values. The average distance between all four airline networks is kept between 2 and 3.



**FIGURE 1** It is a topology depiction of the four airline network diagrams for US air, CA, CZ, and MU, respectively.

**TABLE 2** The top—10 nodes ranked by different methods in the US air network.

Rank	SIR	BC	CC	DC	KS	GC	KSGC	IGP
1	118	118	118	118	67	118	118	118
2	261	<b>8</b>	261	261	<b>94</b>	261	261	261
3	255	261	67	255	<b>109</b>	255	255	255
4	152	<b>201</b>	255	152	112	182	182	182
5	182	<b>47</b>	201	182	118	152	152	152
6	230	182	<b>182</b>	230	<b>131</b>	230	166	230
7	166	255	<b>47</b>	166	<b>146</b>	112	230	166
8	112	152	166	67	147	166	67	67
9	67	<b>313</b>	<b>248</b>	112	<b>150</b>	67	112	112
10	147	<b>13</b>	112	<b>201</b>	152	147	147	147

The top-ranked node numbers from the other identification methods that do not fall within the range of the top-ranked node numbers of the SIR are noted with a bold value.

### 3.2 Measurement

This study will use the node propagation range of the SIR model to describe the influence of each node [30]. The nodes in the SIR model have three discrete states: susceptible, infected and recovered,

**TABLE 3** The top—15 nodes ranked by different methods in the CA network.

Rank	SIR	BC	CC	DC	KS	GC	KSGC	IGP
1	148	148	148	148	<b>14</b>	148	148	148
2	33	33	33	33	19	33	33	33
3	153	153	153	153	22	153	153	153
4	179	<b>136</b>	179	179	<b>23</b>	179	179	179
5	27	179	27	27	27	27	27	27
6	71	<b>56</b>	71	71	33	71	71	71
7	149	27	41	149	41	149	149	41
8	41	41	197	41	69	197	41	197
9	197	71	171	197	71	41	197	171
10	171	22	69	69	<b>80</b>	171	171	149
11	69	149	22	171	<b>87</b>	204	69	69
12	204	171	204	22	<b>106</b>	69	204	204
13	113	<b>159</b>	113	204	113	113	22	113
14	22	2	19	113	<b>115</b>	22	113	19
15	19	106	<b>56</b>	19	<b>120</b>	<b>195</b>	19	22

The top-ranked node numbers from the other identification methods that do not fall within the range of the top-ranked node numbers of the SIR are noted with a bold value.

**TABLE 4 The top—10 nodes ranked by different methods in the CZ network.**

Rank	SIR	BC	CC	DC	KS	GC	KSGC	IGP
1	27	27	27	27	27	27	27	27
2	215	233	245	215	32	215	215	215
3	245	215	233	245	<b>33</b>	245	245	245
4	39	245	39	233	39	39	39	39
5	233	39	32	39	44	44	233	233
6	32	<b>109</b>	215	32	<b>46</b>	32	32	32
7	44	32	44	44	<b>79</b>	233	44	44
8	189	<b>57</b>	206	206	<b>85</b>	137	206	189
9	206	<b>95</b>	189	57	<b>95</b>	206	189	206
10	137	<b>83</b>	<b>193</b>	189	<b>131</b>	189	137	<b>193</b>

The top-ranked node numbers from the other identification methods that do not fall within the range of the top-ranked node numbers of the SIR are noted with a bold value.

**TABLE 5 The top—15 nodes ranked by different methods in the MU network.**

Rank	SIR	BC	CC	DC	KS	GC	KSGC	IGP
1	208	208	208	208	<b>25</b>	208	208	208
2	137	267	137	137	<b>32</b>	137	137	137
3	267	137	267	267	<b>38</b>	267	267	267
4	191	<b>115</b>	191	191	<b>44</b>	191	191	191
5	223	191	237	223	46	223	259	259
6	237	223	223	259	<b>58</b>	259	223	223
7	259	204	259	237	<b>64</b>	237	237	237
8	201	<b>149</b>	201	201	<b>82</b>	46	46	46
9	46	259	89	46	89	248	201	201
10	248	<b>269</b>	46	89	<b>90</b>	89	89	89
11	89	237	280	204	<b>103</b>	201	204	248
12	280	46	204	248	<b>112</b>	187	248	204
13	204	201	153	280	<b>132</b>	280	280	280
14	187	<b>283</b>	187	187	137	204	187	153
15	153	89	248	153	<b>147</b>	153	153	187

The top-ranked node numbers from the other identification methods that do not fall within the range of the top-ranked node numbers of the SIR are noted with a bold value.

respectively. A certain infection step, propagation rate, and recovery rate are first determined prior to the experiment. The susceptible neighbors of every infected node will, at random and with a predetermined probability  $\beta$ , get infected at each time step of the SIR model. At the same time, at a certain recovery rate  $\mu$ , certain infected nodes will return to their pre-infection state and not become infected again. When there are no infected nodes left in the overall network, the propagation process comes to an end. To quantify the SIR propagation process, we recorded the average number of infected nodes for each node in this experiment after

5,000 experimental propagation processes. According to the SIR model, a node is more significant and plays a more critical function when there are more nodes infected by it.

To evaluate whether there is a correlation between two sequences, one uses Kendall’s tau coefficient [28]. The value of Kendall’s coefficient ranges from 0 to 1. The closer the calculated Kendall’s tau correlation coefficient is to 1, the stronger the correlation between the two sequences. The correlation coefficient can be defined as Eq. 5.

$$\tau = \frac{2}{N(N-1)} \sum_{i < j} \text{sgn}[(v_i - v_j)(w_i - w_j)] \tag{5}$$

Given two sequences with the same length, name the two sequences as  $V$  and  $W$ , their  $i_{th}$  values are denoted by  $v_i$  and  $w_i$ , respectively. There are three relationships between  $(v_i, w_i)$  and  $(v_j, w_j)$ :  $v_i > v_j$  and  $w_i > w_j$  or  $v_i < v_j$  and  $w_i < w_j$ ;  $v_i > v_j$  and  $w_i < w_j$  or  $v_i < v_j$  and  $w_i > w_j$ ;  $v_i = v_j$  and  $w_i = w_j$ , denoted as concordant, discordant, and neither concordant nor discordant, respectively. The Kendall’s tau coefficient will be used to analyze the correlation between network ranking lists produced by various node ranking algorithms and SIR model in this paper. If the obtained  $\tau$  value is nearer 1, it indicates that the ranking list produced by the ranking method is more likely to reflect the ranking in the actual propagation process, indicating that the method is more efficient.

The imprecision function  $\epsilon(p)$  calculates the mean propagation capacity of the top- $M$  nodes in the ranking list generated by various key node detection methods, it can assess the performance of the ranking method. The imprecision function  $\epsilon(p)$  is defined as Eq. 6.

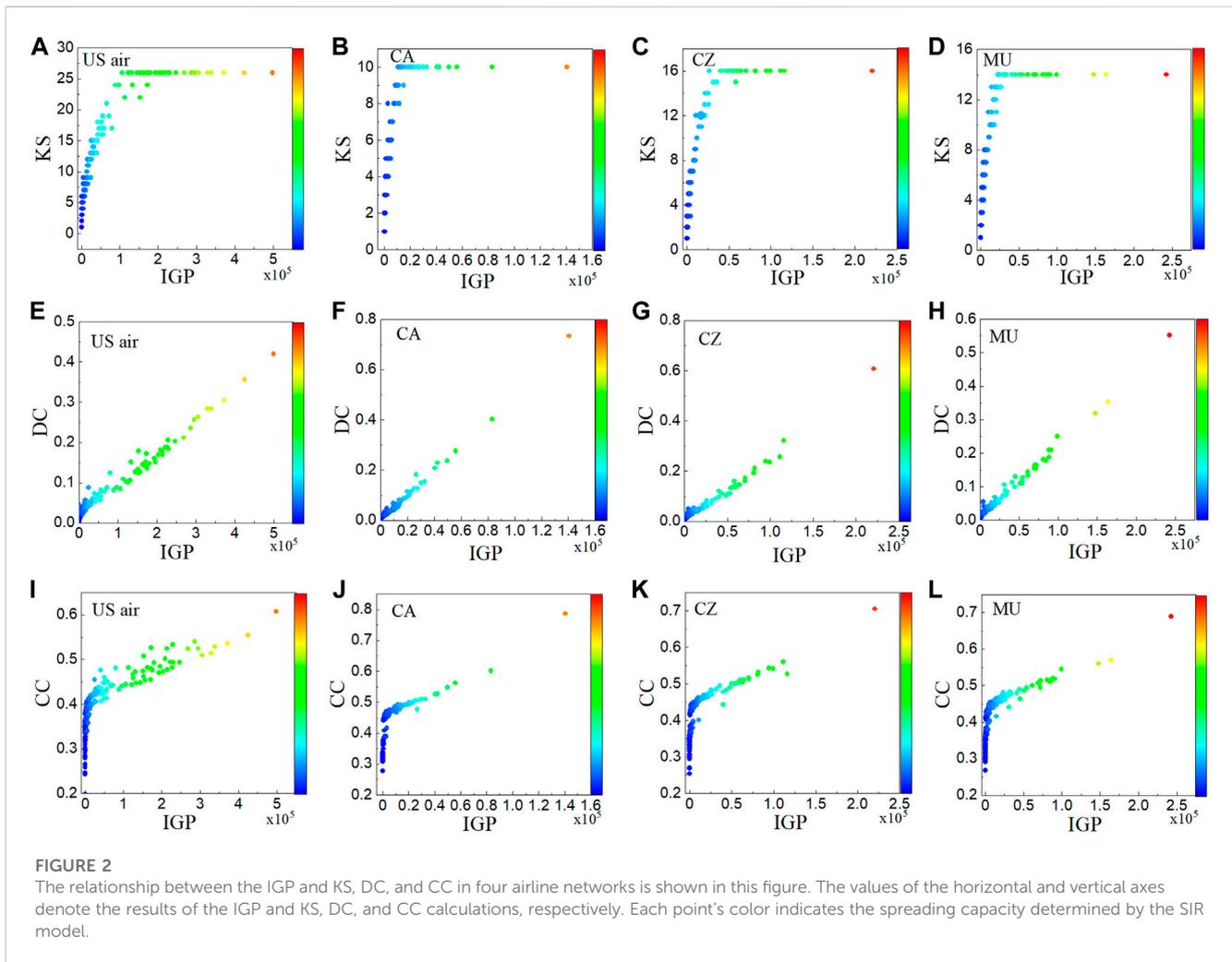
$$\epsilon(p) = 1 - \frac{M(p)}{M_{eff}(p)} \tag{6}$$

Here  $p$  denotes the ratio of selected nodes  $M$  to the total number of nodes  $N$  ( $p = M/N$ ), which ranges from 0 to 1. The  $M(p)$  and  $M_{eff}(p)$ , respectively, represent the average spreading influence of the top  $pN$  nodes in the ranked list produced by different methods and the SIR model. The better it is for identifying key nodes in networks, the smaller the difference between  $M(p)$  and  $M_{eff}(p)$ , the smaller the  $\epsilon(p)$ , and the more accurate the ranking of the network nodes computed by a particular method.

### 3.3 Simulation results

To evaluate the effectiveness of our proposed method (IGP), we compare it with six ranking methods, including BC (Betweenness Centrality) [31], CC (Closeness Centrality) [32], DC (Degree Centrality) [11], KS ( $k$ -shell decomposition) [12], GC (Gravity Centrality) [20], and KSGC (the  $k$ -shell based on gravity centrality) [33]. Besides, we set propagation rate  $\beta$  in the SIR model for different networks.

We organized and evaluated the data from the four airline networks to produce the structure map of the network. After that, we plot the combined routes of the four airline networks to get Figure 1, from which we may roughly infer the distribution of each network’s nodes (the total number of airports) and connections (the total number of routes connecting those airports). As can be observed in Figure 1, the nodes in the four airline networks have



highly similar topologies, and each of the four networks exhibits the characteristics of high core node connectivity density and low edge node connectivity.

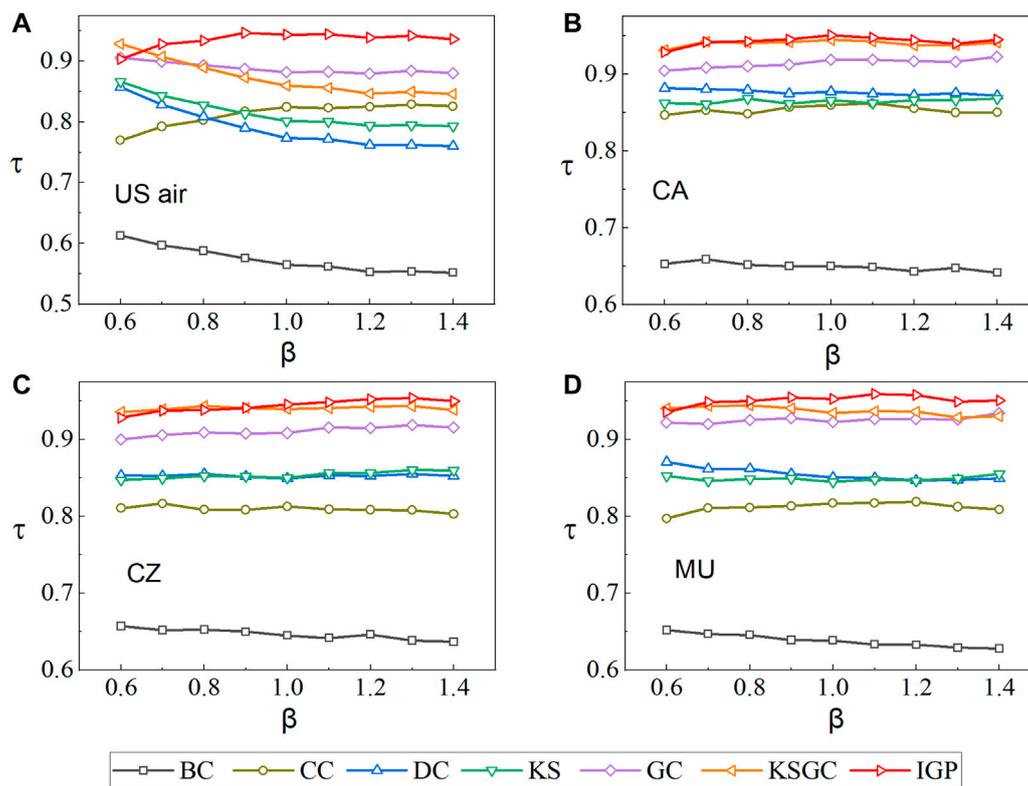
We compared the top 10 or 15 key nodes obtained by the IGP approach to those found by other benchmark methods. In our experiments, we ranked the nodes in each of the four airline networks using the IGP method before choosing the top-10 or top-15 of the ranked nodes. These nodes were then ranked for selection using the other six benchmark methods in the same way. Tables 2–5 displays the results of node ranking.

When using different methods to identify key nodes for the same network, the results are frequently not exactly the same. We consider that it is related to the accuracy of each method and also has a certain relationship with the network's structure. For each method in the table, the more the number of bolded numbers, the less accurate the method is. The SIR model here is with  $\beta = \beta_{th}$  for different networks. As is shown in the US air network, the IGP, GC, KSGC and SIR all have the same node numbers. Comparable circumstances are represented in the MU network as well. Besides, the DC and CC are method that perform better in the CA and MU networks, the poorest effective method is KS since there are multiple nodes scattered across the same shell. The BC method also showed

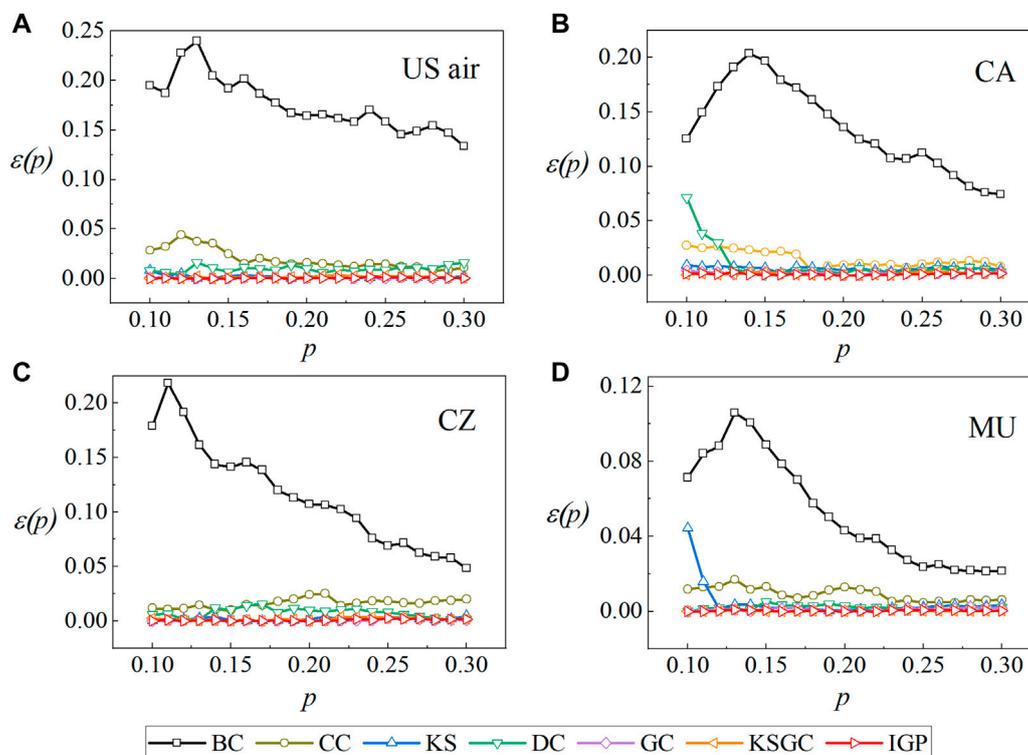
poor performance in the four networks. After that, we'll conduct more experiments to further evaluate these methods' effectiveness.

We obtain Figure 2 to further examine the correlations between the IGP and other benchmark methods. The KS, DC and CC are picked as comparison methods. Each node in the network is represented by a point on the graph. In this experiment, we set propagation rate  $\beta = \beta_{th}$  and the infection step  $F(t) = 10$ . Each point's color indicates the spreading capacity determined by the SIR model. The closer a point's color is to red represents its larger relevance, according to the color bar.

As shown in Figure 2, we can see that IGP and KS have a positive correlation from Figures 2A–D. The CC and IGP also have a positive correlation from Figures 2I–L, but neither of the two methods mentioned above has as strong of a correlation as the one between DC and IGP. From Figures 2E–H, we recognize that DC and IGP have a strong positive correlation, meaning that nodes with larger DC values will also have larger IGP values. For this reason, we believe that the DC has a stronger correlation with the IGP compared to both the CC and KS methods, likely because the degree value of the node is taken into account when considering the influence of the propagation feature on the node. In conclusion, the IGP method has varying degrees of positive correlation with



**FIGURE 3** In the four airline networks, the Kendall's tau  $\tau$  is obtained using seven methods. The range of  $\beta$  is from  $0.6\beta_{th}$  to  $1.4\beta_{th}$ . The results are the average of more than 100 independent experiments conducted at different propagation rates.



**FIGURE 4** The imprecision function  $\varepsilon(p)$  is obtained by different methods in the four airline networks. The  $p$  ranges from 0.1 to 0.3. We set  $\beta = 0.05$  and  $\mu = 1$ .

several previous centrality methods, and the infectivity of the nodes with higher IGP values is larger.

By calculating Kendall's tau coefficient, we are able to compare the efficacy of various methods. We adjusted the propagation rate range in the SIR model, and the propagation rate varies from  $0.6\beta_{th}$  to  $1.4\beta_{th}$  for each network in order to further examine the impact of propagation rate on the IGP method. Figure 3 shows Kendall's tau, which was calculated using various methods in the four airline networks.

As shown in Figure 3, the IGP method performs well in the US air network and also performs better in the MU and CA networks, with the corresponding value almost always coming in first. The superiority of our proposed method is visible in the US air network, where IGP has a more stable largest Kendall's tau coefficient and has an advantage of up to 107% over the GC method. There is a similar phenomenon in MU network, with the exception of the propagation rate of  $0.6\beta_{th}$ , where the Kendall's tau values obtained by the IGP method remain at their maximum values for all propagation rate cases.

In the MU network, the Kendall's coefficients obtained by the IGP method increase as the propagation rate increases. We can find that our suggested method and the KSGC method are somewhat competitive in both CA and CZ networks. For both networks, the two methods are nearly equally effective when the propagation rate is from  $0.6\beta_{th}$  to  $0.9\beta_{th}$ . When the propagation rate exceeds  $0.9\beta_{th}$ , the IGP method obtains a larger value of Kendall's tau, demonstrating the greater validity and accuracy of IGP.

After calculating Kendall's tau coefficient, we learn that the IGP method outperforms other methods. To further back up this conclusion, we will then employ the imprecision function. The accuracy of the method increases as the value of the imprecision function decreases. We calculate the  $\epsilon(p)$  achieved using the IGP method and the other six benchmark methods, respectively, in four airline networks. Figure 4 shows the experimental results. The  $p$ -value range was defined to be between 0.1 and 0.3. The figure shows that the  $\epsilon(p)$  acquired by the IGP method is in a relatively low value range for each airline network, and the overall experimental findings are encouraging, demonstrating the accuracy and effectiveness of our method.

## 4 Discussions and conclusion

In order to identify key nodes in complex networks, we present the IGP method, an improved gravity model based on propagation features. First, we compare the proposed method to the top node rankings obtained by other benchmark methods. Next, we analyze the correlation with the DC, CC, and KS methods. Finally, we evaluate the performance of the proposed method using Kendall's correlation coefficient and imprecision function as evaluation criteria. Experimental simulations in four airline networks show that the proposed method outperforms other benchmark methods. The IGP method produces larger Kendall's coefficient than other methods, particularly in the US air network, with a clear advantage. Additionally, each network has lower imprecision function values related to the IGP method, indicating that the IGP method can

identify key nodes more accurately than benchmark methods. The results of our experiments demonstrate that our method has accuracy as well as practicality, and that the IGP method can theoretically assist in the accurate identification of key node airports in the airline network.

Accurately and effectively identifying key nodes in complex networks is increasingly important as network topologies become more complex in the real world. In addition to the complex airline networks, the approach we propose can be utilized to locate key nodes in other complicated networks. Furthermore, we will improve our method by applying it to multilayer network topologies or temporal networks, in conjunction with later practical advances and modifications, in order to achieve more precise identification of critical nodes.

## Data availability statement

The original contributions presented in the study are included in the article/Supplementary material, further inquiries can be directed to the corresponding author.

## Author contributions

LnZ and XG wrote the original draft of this opinion manuscript. LaZ, LiZ, LnZ, HY, WP, and JH provided feedback on the manuscript. LnZ and XG edited the final manuscript. All authors contributed to the article and approved the submitted version.

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## Conflict of interest

LnZ was employed by Chengdu GoldTel Industry Group Co., Ltd.

The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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