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Iterative solution of the fractional Wu-Zhang equation under Caputo derivative operator

Humaira Yasmin[1]*, A. A. Alderremy[2], Rasool Shah[3]*,
Abdul Hamid Ganie[4] and Shaban Aly[5]

[1]Department of Basic Sciences, Preparatory Year, King Faisal University, Al-Ahsa, Saudi Arabia, [2]Department of Mathematics, Faculty of Science, King Khalid University, Abha, Saudi Arabia, [3]Department of Computer Science and Mathematics, Lebanese American University, Beirut, Lebanon, [4]Basic Science Department, College of Science and Theoretical Studies, Saudi Electronic University, Riyadh, Saudi Arabia, [5]Department of Mathematics, Faculty of Science, AL-Azhar University, Assiut, Egypt
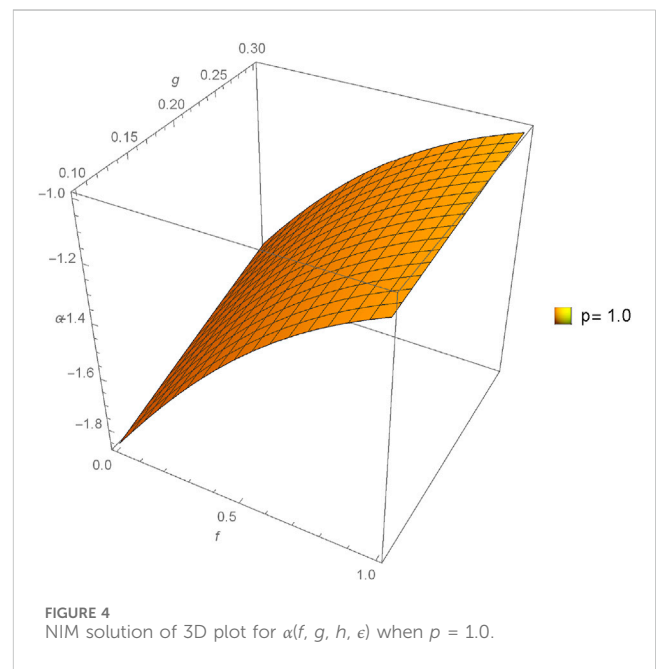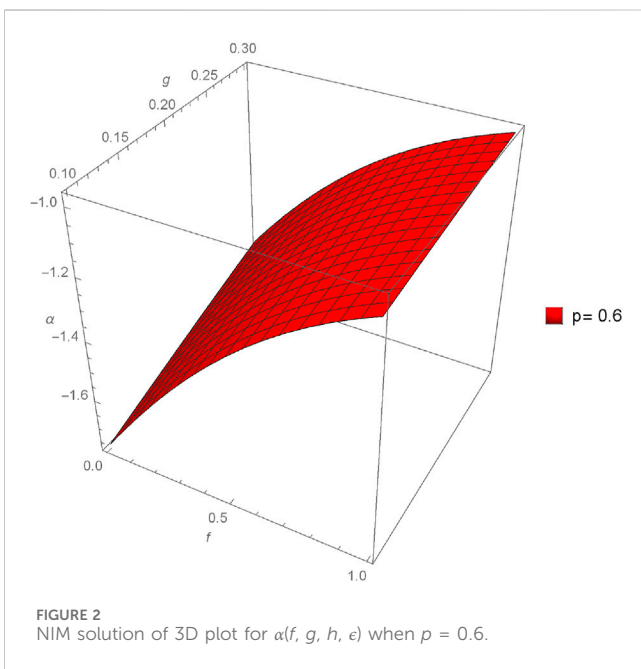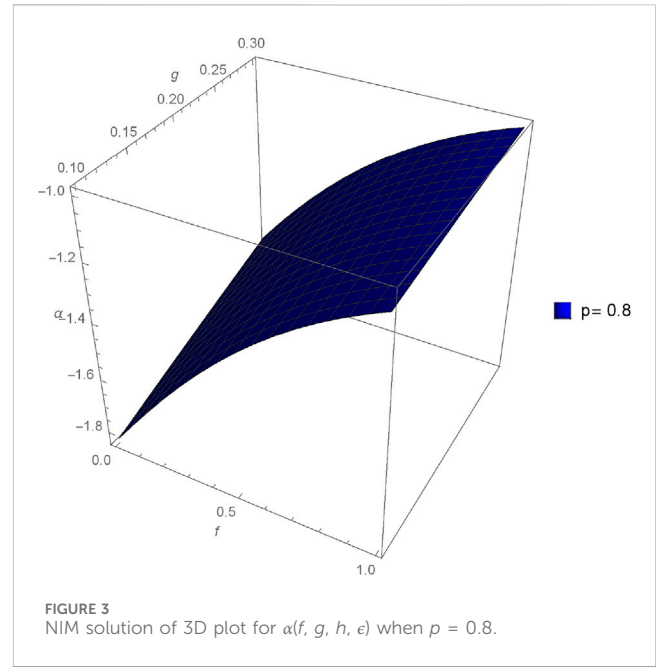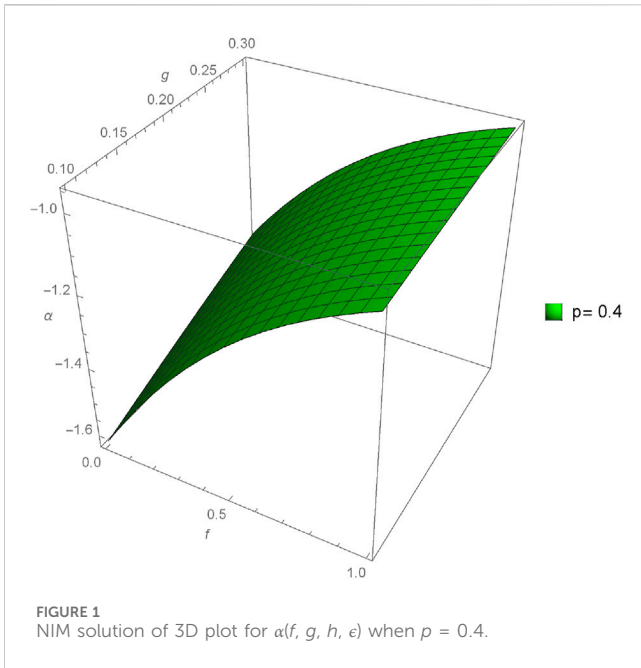
In this study, we employ the effective iterative method to address the fractional Wu-Zhang Equation within the framework of the Caputo Derivative. The effective iterative method offers a practical approach to obtaining approximate solutions for fractional differential equations. We seek to provide insights into its solution and behavior by applying this method to the Wu-Zhang Equation. Through numerical analysis and the presentation of relevant tables and Figures, we demonstrate the accuracy and efficiency of this method in solving the fractional Wu-Zhang Equation. This research contributes to the understanding and solution of fractional-order differential equations and their applications in various scientific and engineering domains.

KEYWORDS

effective iterative method, Wu-Zhang equation, fractional calculus, fractional partial difference equation, Caputo derivative
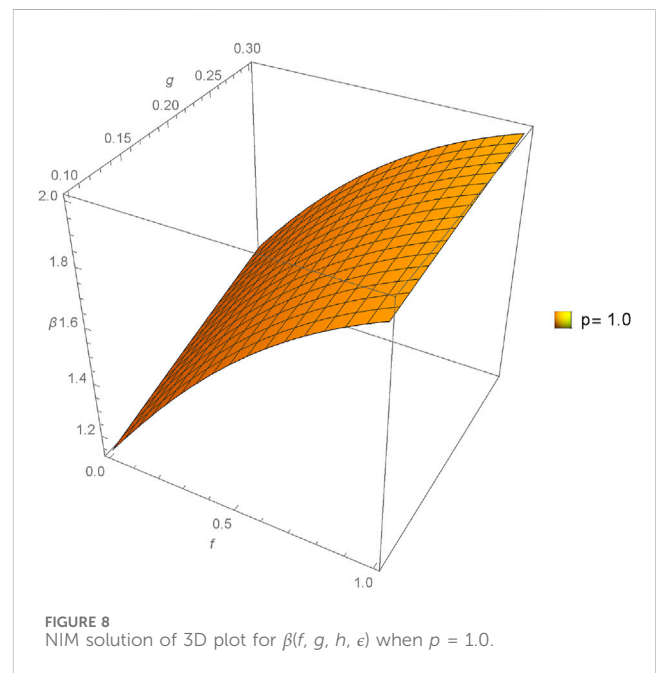
1 Introduction

In mathematics, fractional calculation is a generalization of classical calculus. During the last few decades, researchers have paid much more attention to fractional calculus. Several fields have benefited from the application of fractional calculus, including physics, engineering, biology, medicine, hydrology, economics and finance [1–8]. In differential equations, linear models can be solved by different methods and do not require excessive effort to obtain their exact solutions. Non-linear models are more difficult to solve. It's hard to solve most problems in the real world because they're non-linear. There is no exact solution to the majority of non-linear problems. To solve these problems, researchers use a variety of approaches [9–15]. In recent years, the exploration of advanced mathematical methods and novel approaches has significantly impacted the domain of fractional calculus. A series of publications by notable researchers Tao, He, Anjum, Yang, and others have introduced pioneering concepts, unveiling the potential of transformative methodologies in the field. Among these, the Aboodh transformation-based homotopy perturbation method, highlighted in the work by [16] in Frontiers in Physics, presents a new ray of hope for the application of fractional calculus. He, Anjum, and others also, in their 2023 publication in Therm. Sci. has illuminated the challenges and prospects beyond Laplace and Fourier transforms, setting the stage for a broader understanding of mathematical transformations [17]. Exploring applications, Anjum, Ain, Din, and their team have undertaken an insightful analysis of Caputo fractional order dynamics, particularly in the context of the Middle East Lungs Coronavirus (MERS-CoV) model, as revealed in their

FIGURE 1
NIM solution of 3D plot for $\alpha(f, g, h, \epsilon)$ when $p = 0.4$.

FIGURE 3
NIM solution of 3D plot for $\alpha(f, g, h, \epsilon)$ when $p = 0.8$.

FIGURE 2
NIM solution of 3D plot for $\alpha(f, g, h, \epsilon)$ when $p = 0.6$.

FIGURE 4
NIM solution of 3D plot for $\alpha(f, g, h, \epsilon)$ when $p = 1.0$.

2022 [18]. Furthermore, the work of Anjum, He, He, and collaborators (2021) in Fractals introduces the intriguing concept of two-scale fractal theory in population dynamics [19]. In contrast, Anjum, Ain, Li, and others (2021) present a groundbreaking two-scale mathematical model for tsunami waves in the GEM-International Journal on Geo mathematics [20].

Coastal and harbor design is often influenced by the Wu-Zhang system of equations, which describes the non-linear water wave availability in the ocean. To obtain the exact solution, this is one of the most critical topics in mathematical physics. Several authors

have recently used various numerical and analytical approaches to find the numerical and analytical solution to the WZ equations, for example, using the first integral method [21], the modified Adomian decomposition method [22], the homotopy perturbation method [23], the extended Tanh method and the exp-function method [24], the exponential rational function method [25], the successive approximation method [26], the modified variation iteration method [27], the extended trial equation method [28] and the dynamic system method [29]. In addition, more solitonic solutions were extracted using the mapping method. According

FIGURE 5
NIM solution of 3D plot for $\beta(f, g, h, \epsilon)$ when $p = 0.4$.

FIGURE 7
NIM solution of 3D plot for $\beta(f, g, h, \epsilon)$ when $p = 0.8$.

FIGURE 6
NIM solution of 3D plot for $\beta(f, g, h, \epsilon)$ when $p = 0.6$.

FIGURE 8
NIM solution of 3D plot for $\beta(f, g, h, \epsilon)$ when $p = 1.0$.
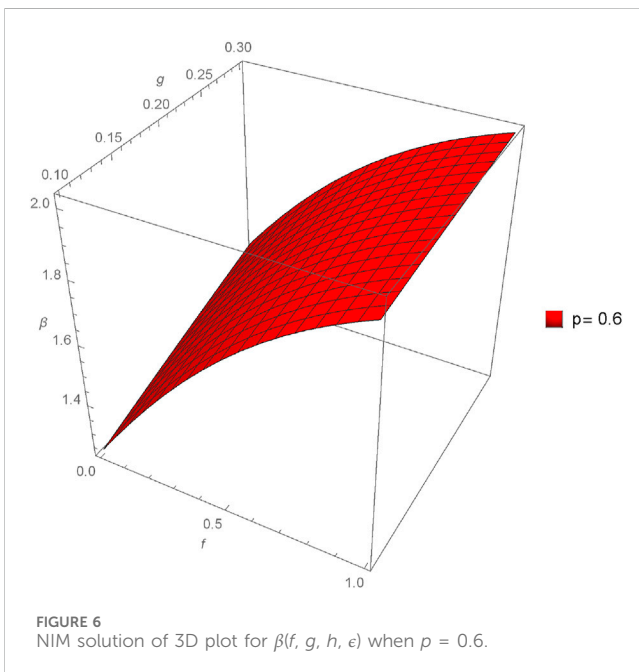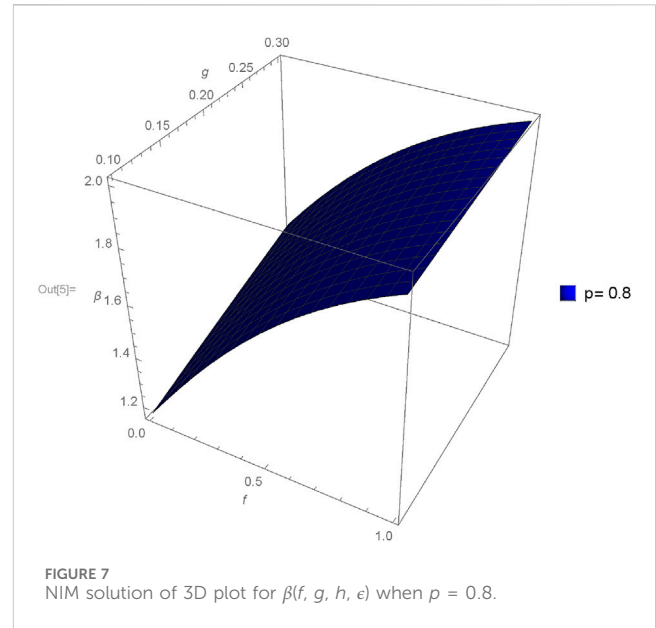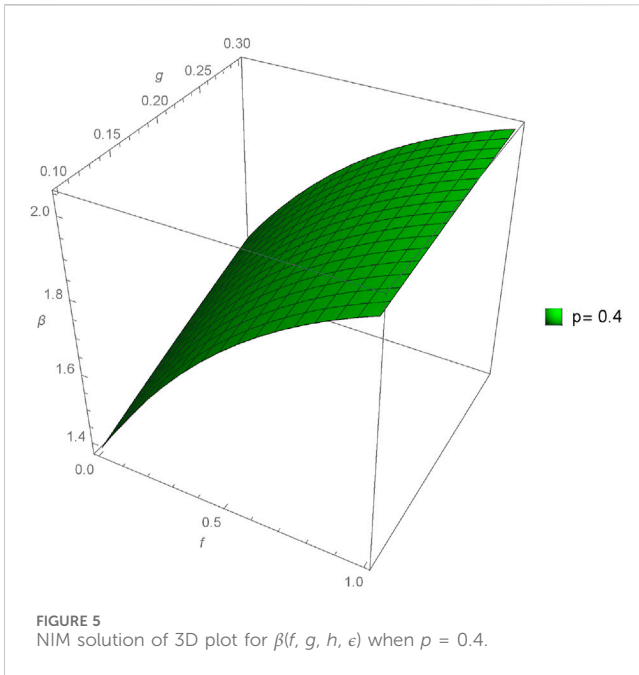
to [30], the extended trial equation method, Lie symmetry analysis, and mapping method are used to obtain solutions for solitary, shock, and singular solitary waves. According to [31], the 3-component Wu-Zhang equation is solved using the ansatz method, followed by a one-solution solution. Several methods have been used recently to enhance these equations, including Backlund transformations [32], Darboux transformations [33–35], asymptotic analysis methods [36], Painleve analyses [37] and extended Painleve expansions [38].

The new iterative method (NIM) was introduced in 2006 by Daftardar-Gejji and Jafari for solving linear and non-linear

equations [39]. A straightforward method is proposed for handling linear and non-linear equations. In contrast to ADM and numerical methods, NIM doesn't require the calculation of tedious Adomian polynomials in non-linear terms like ADM, a Lagrange multiplier like VIM, or discretization like numerical methods. A significant advantage of this method is that it does not require small parameter assumptions, unlike regular perturbation methods. Its primary disadvantage lies in the fact that NIM is an iterative method requiring an initial condition. An iterative method was developed in [40] to solve linear and non-linear fractional diffusion-wave equations on finite domains with Dirichlet boundary conditions. In this study, the New Iterative Method [41] has been applied to linear and non-linear fractional

FIGURE 9
NIM solution of 3D plot for $\gamma(f, g, h, \epsilon)$ when $p = 0.4$.

FIGURE 11
NIM solution of 3D plot for $\gamma(f, g, h, \epsilon)$ when $p = 0.8$.

FIGURE 10
NIM solution of 3D plot for $\gamma(f, g, h, \epsilon)$ when $p = 0.6$.

FIGURE 12
NIM solution of 3D plot for $\gamma(f, g, h, \epsilon)$ when $p = 1.0$.
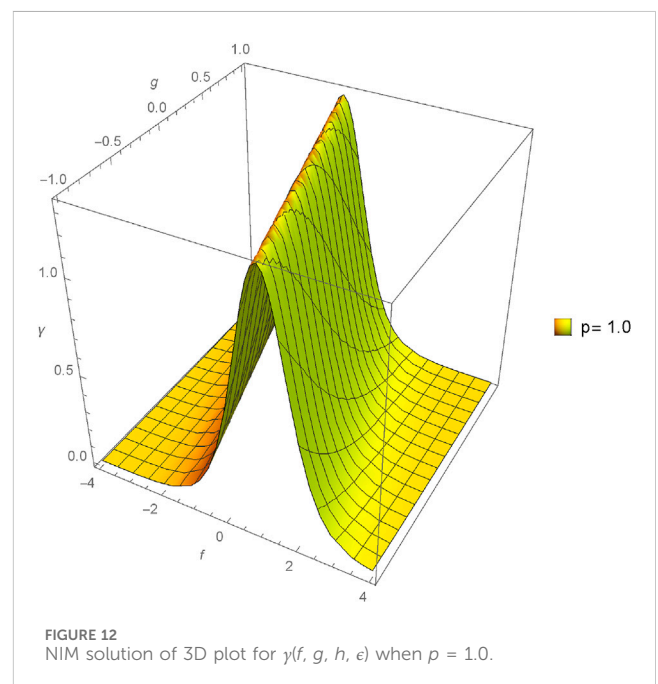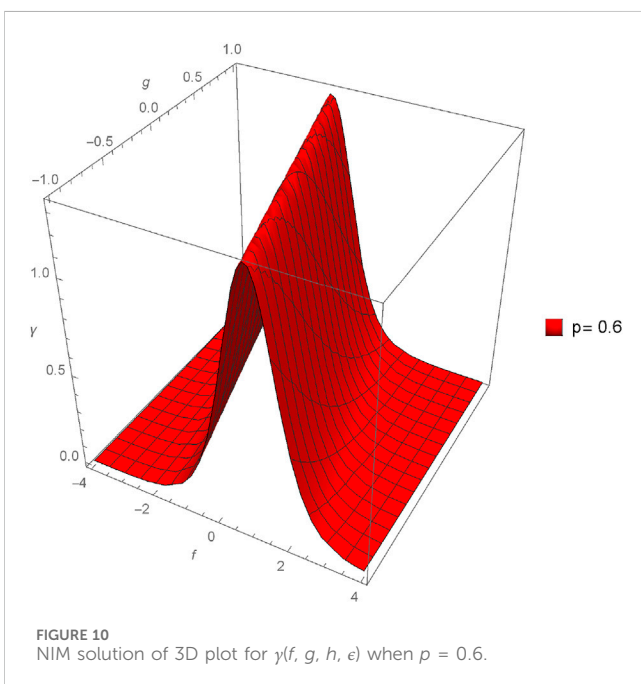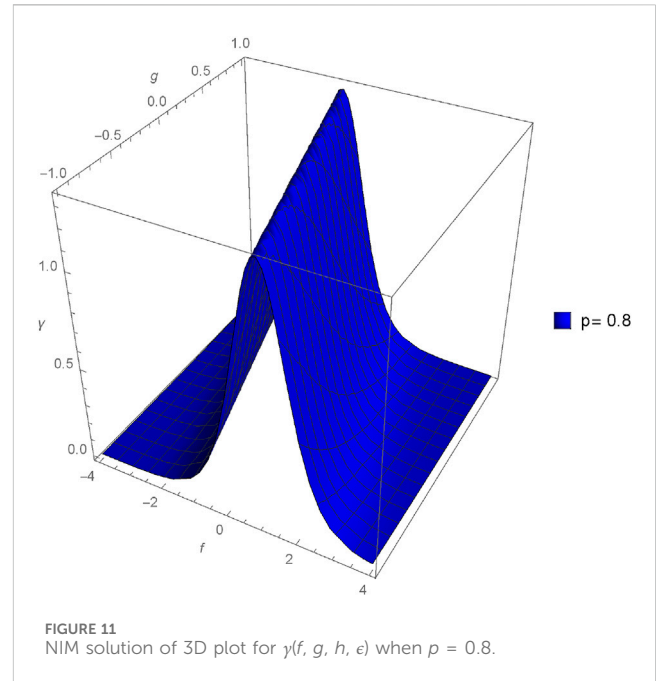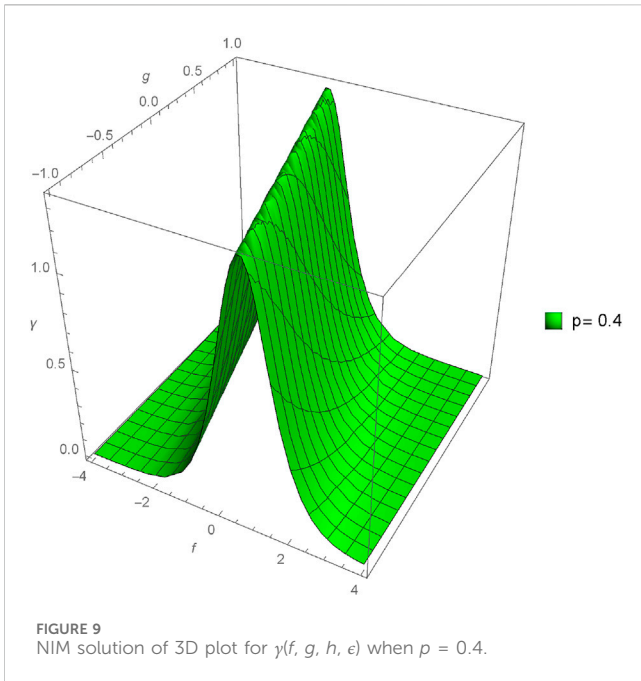
diffusion-wave equations. It has been reported in [42] that new iterative methods were used to study a fractional version of a logistic equation. The scientific landscape in recent publications showcases diverse research spanning various disciplines. Among these, significant contributions emerge in physics, fluid dynamics, material science, and mathematical engineering [43–47]. Notably, [48] have made strides in particle physics by establishing the first hidden-charm pentaquark with strangeness. Concurrently, [49] delve into the characteristics of cavity dynamics in water entries with paired spheres, contributing

insights into fluid behavior. Yang and Kai [50] explore the dynamical properties and chaotic behaviors of non-linear coupled Schrodinger equations in fiber Bragg gratings, while [51] propose a Dilatancy Equation for Geomaterials based on property-dependent plastic potential theory. Furthermore, [52] present an Iterative Threshold Algorithm for Sparse Problems, and [53] focus on aerial target threat assessment using improved methods in mathematical biosciences and engineering. These diverse studies underscore the multidisciplinary nature and broad scope of recent scientific endeavors [54–56].
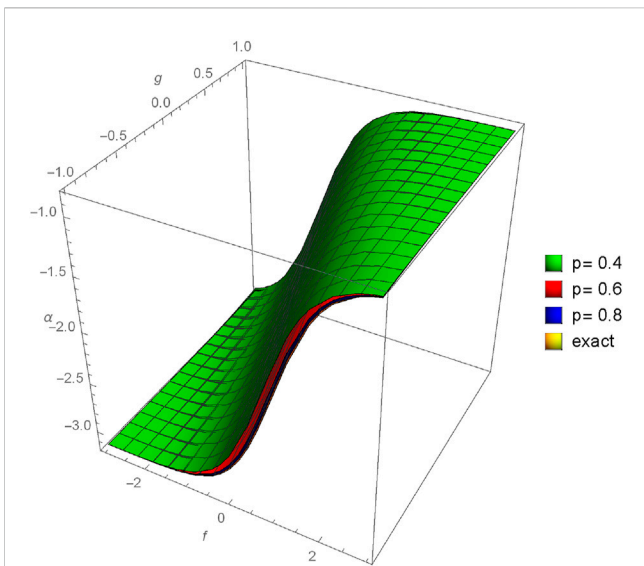
FIGURE 13
Comparison between exact and NIM solution for 3D plot $\alpha(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.
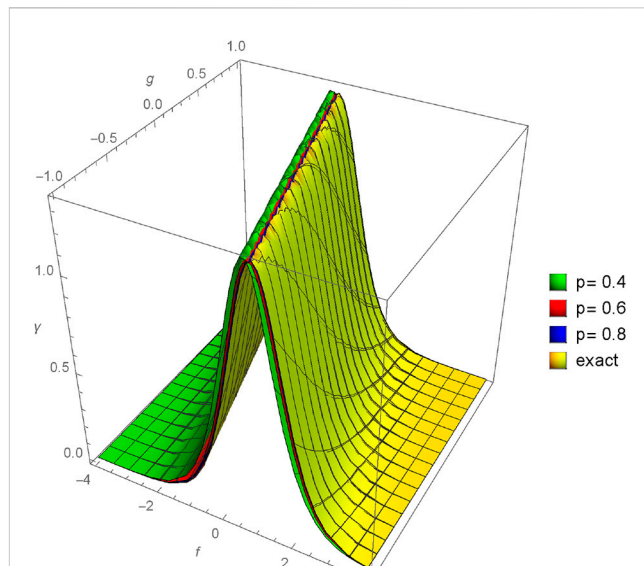
FIGURE 15
Comparison between exact and NIM solutions for a 3D plot $\gamma(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.
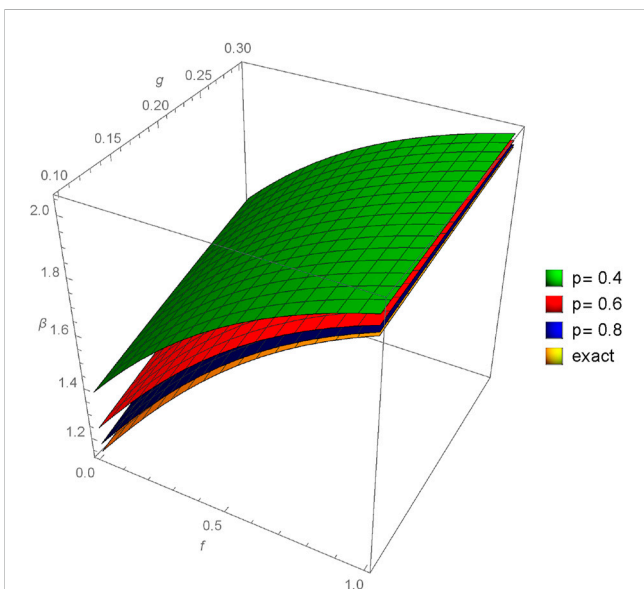
FIGURE 14
Comparison between exact and NIM solution for 3D plot $\beta(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.
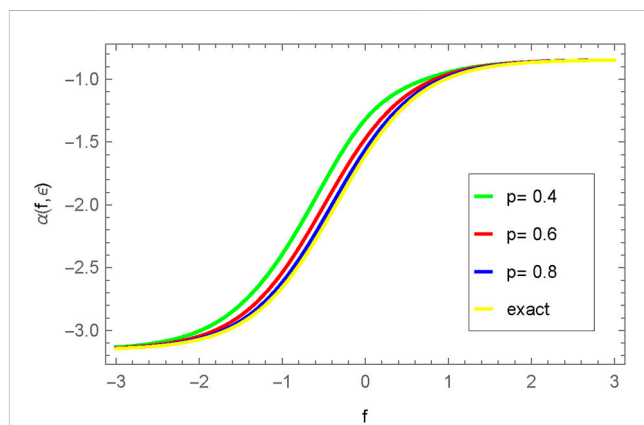
FIGURE 16
Comparison between exact and NIM solution for 2D plot $\gamma(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.

The Wu-Zhang equation is a nonlinear partial differential equation that describes the $(1 + 1)$-dimensional dispersive long wave in two horizontal directions on shallow waters. It has been applied in various fields, including engineering and coastal design. The equation has been extended to include time-fractional and space-fractional derivatives, leading to the time-fractional $(2 + 1)$-dimensional Wu-Zhang system and the space-fractional $(2 + 1)$-dimensional Wu-Zhang system. The Wu-Zhang equation and its generalizations have found applications in various fields, such as ocean engineering, coastal design, and the study of nonlinear waves.

The equation has also been used to model nonlinear and dispersive waves in other contexts, such as conformable time-fractional systems and time-fractional $(2 + 1)$-dimensional systems. Throughout the paper, a fractional approach will be used to solve the Wu-Zhang equation and analyze the behavior that lies behind the phenomenon in a fractional model. This research aims to extend the implementation of NIM to solve time-fractional partial differential equations, including systems of two or more partial differential equations, which are applied in engineering and science. Moreover, the study explores various fractional order values across three distinct situations, examining 3D and contour plots to determine the physical characteristics of the solution. A comparison between the approximate and exact solution is done to ensure the model is accurate. Physical problems with fractional orders can be solved

FIGURE 17
Comparison between exact and NIM solution for 2D plot $\gamma(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.

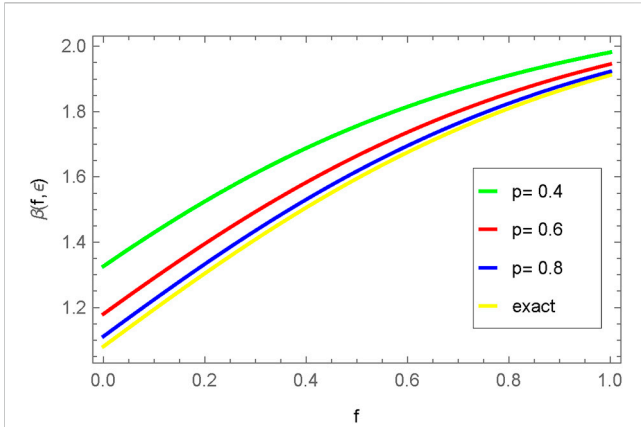**TABLE 2** Using the NIM solution of $\beta(f, g, \epsilon)$, compare the exact solution and the absolute error.

f	$\beta(f,g,\epsilon)_{NIM}$	$\beta(f,g,\epsilon)_{Exact}$	Absoluteerror
0.1	1.12674	1.12674	4.45355×10^{-12}
0.2	1.23921	1.23921	5.26512×10^{-12}
0.3	1.34712	1.34712	5.57865×10^{-12}
0.4	1.44877	1.44877	5.41389×10^{-12}
0.5	1.54283	1.54283	4.86677×10^{-12}
0.6	1.62847	1.62847	4.07652×10^{-12}
0.7	1.70529	1.70529	3.18545×10^{-12}
0.8	1.7733	1.7733	2.31193×10^{-12}
0.9	1.8328	1.8328	1.53522×10^{-12}
1	1.88432	1.88432	8.9595×10^{-13}
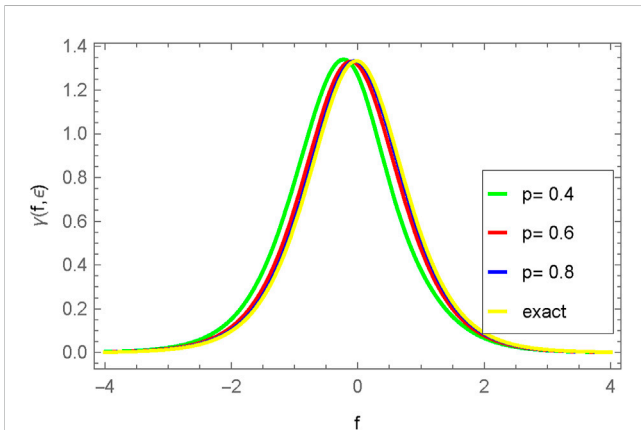
FIGURE 18
Comparison between exact and NIM solution for 2D plot $\gamma(f, g, h, \epsilon)$ for different fractional order values of $p = 0.4, 0.6, 0.8, 1.0$.

TABLE 3 Using the NIM solution of $\gamma(f, g, \epsilon)$, compare the exact solution and the absolute error.

f	$\gamma(f,g,\epsilon)_{NIM}$	$\gamma(f,g,\epsilon)_{Exact}$	Absoluteerror
0.1	1.31727	1.31727	1.20437×10^{-11}
0.2	1.27611	1.27611	6.54965×10^{-12}
0.3	1.21284	1.21284	7.3519×10^{-13}
0.4	1.13194	1.13194	4.35252×10^{-12}
0.5	1.03867	1.03867	8.00382×10^{-12}
0.6	0.938363	0.938363	9.96481×10^{-12}
0.7	0.835894	0.835894	1.03821×10^{-11}
0.8	0.735335	0.735335	9.64584×10^{-12}
0.9	0.639773	0.639773	8.21843×10^{-12}
1	0.551309	0.551309	6.5119×10^{-12}

TABLE 1 Using the NIM solution of $\alpha(f, g, \epsilon)$, compare the exact solution and the absolute error.

f	$\alpha(f,g,\epsilon)_{NIM}$	$\alpha(f,g,\epsilon)_{Exact}$	Absoluteerror
0.1	−1.87326	−1.87326	4.45355×10^{-12}
0.2	−1.76079	−1.76079	5.26512×10^{-12}
0.3	−1.65288	−1.65288	5.57865×10^{-12}
0.4	−1.55123	−1.55123	5.41389×10^{-12}
0.5	−1.45717	−1.45717	4.86677×10^{-12}
0.6	−1.37153	−1.37153	4.07652×10^{-12}
0.7	−1.29471	−1.29471	3.18545×10^{-12}
0.8	−1.2267	−1.2267	2.31193×10^{-12}
0.9	−1.1672	−1.1672	1.53522×10^{-12}
1	−1.11568	−1.11568	8.9595×10^{-13}

TABLE 4 Numerical values of $\alpha(f, g, \epsilon)$ using the NIM solution for different values of fractional order of p.

f	$NIM_{p=0.4}$	$NIM_{p=0.6}$	$NIM_{p=0.8}$	$NIM_{p=1.0}$
0.1	−1.18334	−1.39435	−1.54218	−1.6488
0.2	−1.08479	−1.30431	−1.44618	−1.54642
0.3	−1.01175	−1.22827	−1.36007	−1.45208
0.4	−0.96415	−1.16602	−1.28417	−1.36659
0.5	−0.93839	−1.11612	−1.21816	−1.29027
0.6	−0.92871	−1.07639	−1.16129	−1.22299
0.7	−0.92883	−1.04448	−1.11255	−1.16431
0.8	−0.93327	−1.01826	−1.07091	−1.11359
0.9	−0.93809	−0.99606	−1.03539	−1.07009
1	−0.94097	−0.97676	−1.00512	−1.03302

TABLE 5 Numerical values of $\beta(f, g, \epsilon)$ using the NIM solution for different values of fractional order of p.

f	$NIM_{p=0.4}$	$NIM_{p=0.6}$	$NIM_{p=0.8}$	$NIM_{p=1.0}$
0.1	1.81666	1.60565	1.45782	1.3512
0.2	1.91521	1.69569	1.55382	1.45358
0.3	1.98825	1.77173	1.63993	1.54792
0.4	2.03585	1.83398	1.71583	1.63341
0.5	2.06161	1.88388	1.78184	1.70973
0.6	2.07129	1.92361	1.83871	1.77701
0.7	2.07117	1.95552	1.88745	1.83569
0.8	2.06673	1.98174	1.92909	1.88641
0.9	2.06191	2.00394	1.96461	1.92991
1	2.05903	2.02324	1.99488	1.96698

TABLE 6 Numerical values of $\gamma(f, g, \epsilon)$ using the NIM solution for different values of fractional order of v.

f	$NIM_{p=0.4}$	$NIM_{p=0.6}$	$NIM_{p=0.8}$	$NIM_{p=1.0}$
0.1	1.2735	1.11691	1.16217	1.22387
0.2	0.994528	0.959897	1.05273	1.1379
0.3	0.692635	0.796728	0.935302	1.03929
0.4	0.413849	0.643804	0.818174	0.934323
0.5	0.192384	0.512861	0.707707	0.828522
0.6	0.043514	0.409143	0.607749	0.726182
0.7	−0.03555	0.33198	0.519809	0.630253
0.8	−0.05949	0.276869	0.443702	0.542476
0.9	−0.04746	0.237847	0.378313	0.463644
1	−0.0173	0.209282	0.3222	0.393877

TABLE 7 Comparison of the present solution (NIM) with the HPM solution [58] for the Wu-Zhang equation.

g	$\lvert\alpha_{Exact} - \alpha_{HPM}\rvert$	$\lvert\alpha_{Exact} - \alpha_{NIM}\rvert$	$\lvert\beta_{Exact} - \beta_{HPM}\rvert$	$\lvert\beta_{Exact} - \beta_{NIM}\rvert$
−40	0.340×10^{-7}	2.22×10^{-16}	0.800×10^{-10}	1.66×10^{-16}
−30	0.400×10^{-8}	4.81×10^{-14}	0.400×10^{-8}	4.79×10^{-14}
−20	0.750×10^{-7}	1.93×10^{-11}	0.500×10^{-11}	1.93×10^{-11}
−10	0.270×10^{-7}	7.70×10^{-9}	0.330×10^{-9}	7.70×10^{-9}
0	0.100×10^{-8}	0.39×10^{-8}	0.100×10^{-9}	0.79×10^{-8}
10	0.100×10^{-8}	7.44×10^{-9}	0.100×10^{-9}	7.44×10^{-9}
20	0.710×10^{-8}	1.89×10^{-11}	0.200×10^{-9}	1.89×10^{-11}
30	0.810×10^{-7}	4.70×10^{-14}	0.700×10^{-9}	4.70×10^{-14}

using the New Iterative Method (NIM), which can be applied to both linear and non-linear fractional order problems. Three main sections in the article are arranged as follows: Section 2 represents some definitions, and Section 3 shows the methodology of the New Iterative Method (NIM). Section 4 discusses the problem's solution. Section 5 provides the numerical results and discussions and the last Section 6 shows the short conclusion.

2 Preliminaries

In this section, we will discuss several basic definitions and conclusions relating to the Caputo fractional derivative.

Definition 1. *The formula for the Riemann fractional integral is as follows* [57]:

$$J_t^\sigma \omega(x,t) = \frac{1}{\Gamma(\sigma)} \int_0^t (t-r)^{\sigma-1} \omega(x,r) dr$$

Definition 2. *The fractional derivative of f according to the Caputo formula is defined as* [57]:

$$^C D_t^\sigma \omega(x,t) = \frac{1}{\Gamma(m-\sigma)} \int_0^t (t-r)^{m-\sigma-1} \omega(x,r) dr, \quad m-1 < \sigma \le m, \ t > 0.$$

Lemma 1. *For* $n - 1 < \sigma \le n$, $p > -1$, $t \ge 0$ *and* $\lambda \in R$, *we have:*

1. $D_t^\sigma t^p = \frac{\Gamma(\sigma+1)}{\Gamma(p-\sigma+1)} t^{p-\sigma}$
2. $D_t^\sigma \lambda = 0$
3. $D_t^\sigma I_t^\sigma \omega(x,t) = \omega(x,t)$
4. $I_t^\sigma = \omega(x,t) - \sum_{i=0}^{n-1} \partial^i \omega(x,0) \frac{t^i}{i!}$

3 General procedure for the proposed methods

3.1 General procedure of new iterative method

For the basic idea of the new iterative method, we consider the general functional equation:

$$\alpha(\omega) = K(\omega) + N(\alpha(\omega)), \tag{1}$$

where N is non linear operator and K is unknown function. We have been looking for a solution of Eq. 1 having the series form

$$\alpha(\omega) = \sum_{i=0}^\infty \alpha_i(\omega),$$

The nonlinear term can be decomposed as

$$N\left(\sum_{i=0}^\infty \alpha_i(\omega)\right) = N(\alpha_0) + \sum_{i=0}^\infty \left[N\left(\sum_{j=0}^i \alpha_j(\omega)\right) - N\left(\sum_{j=0}^{r-1} \nu_j(\omega)\right)\right]$$

From 4 to 5, 3 is equivalent to

$$\sum_{r=0}^{\infty} \alpha_r(\omega) = K + N(\alpha_0) + \sum_{r=0}^{\infty}\left[N\left(\sum_{j=0}^{r}\alpha_j(\omega)\right) - N\left(\sum_{j=0}^{r-1}\alpha_j(\omega)\right)\right]$$

We define the following recurrence relation:

$$\omega_0 = K,$$
$$\omega_1 = N(\alpha_0),$$
$$\omega_2 = N(\alpha_0 + \alpha_1) - N(\alpha_0),$$
$$\omega_{n+1} = N(\alpha_0 + \alpha_1 + \cdots\alpha_n) - N(\alpha_0 + \alpha_1 + \cdots\alpha_{n-1}), \quad n = 1, 2, 3\cdots,$$

Then

$$(\alpha_0 + \alpha_1 + \cdots\alpha_n) = N(\alpha_0 + \alpha_1 + \cdots\vartheta_n), \quad n = 1, 2, 3\cdots,$$
$$\alpha = \sum_{r=0}^{\infty}\alpha_r(\omega) = K + N\left(\sum_{r=0}^{\infty}\alpha_r(\omega)\right)$$

3.1.1 Basic road map of NIM

In this section, we discuss basic idea for solving fractional-order nonlinear PDE using the NIM. Consider the following fractional-order PDE:

$$D_\epsilon^\mu \alpha(\zeta, \epsilon) = Y(\alpha, \partial\alpha) + Z(\omega, \epsilon), \quad m - 1 < \mu \le m, \ m \in W \quad (2)$$

$$\frac{\partial^t}{\partial\epsilon^t}\alpha(\omega, 0) = S_t(\omega), \quad t = 0, 1, 2, 3\cdots m - 1 \quad (3)$$

where A is non linear function of ν and $\partial\nu$ (partial derivative of ν with respect to ν) and B is the source function. In view of the new iterative method, the intimal value problem Eqs 2, 3 is equivalent to the integral equation

$$\alpha(\omega, \epsilon) = \sum_{t=0}^{m-1} s_t(\omega)\frac{\epsilon^t}{t!} + I_\epsilon^\rho(Y) + I_\epsilon^\rho(Z) = K + N(\omega)$$

where

$$K = \sum_{t=0}^{m-1} s_t(\omega)\frac{\epsilon^t}{t!} + I_\epsilon^\rho(Z)$$
$$N(\alpha) = I_\epsilon^\rho(Y)$$

4 Solution of Wu-Zhang (WZ) equation by NIM

In this section, we apply NIM to understand the anomalous behavior of the fractional-order Wuâ"Zhang (WZ) equation, which is given by

$$\frac{\partial^p \alpha}{\partial\epsilon^p} + \alpha\alpha_f + \beta\alpha_g + \gamma_f = 0$$
$$\frac{\partial^p \beta}{\partial\epsilon^p} + \alpha\beta_f + \beta\beta_g + \gamma_g = 0 \quad (4)$$
$$\frac{\partial^p \gamma}{\partial\epsilon^p} + (\alpha\gamma)_f + (\beta\gamma)_g + \frac{1}{3}\left(\alpha_{fff} + \alpha_{fgg} + \beta_{ffg} + \beta_{ggg}\right) = 0$$

Subject to initial condition

$$\alpha(f, g, 0) = -\frac{a_3 + a_2 a_0}{a_1} + \frac{2\sqrt{3}}{3}a_1 \tanh(a_1 f + a_2 g)$$
$$\beta(f, g, 0) = a_0 + \frac{2\sqrt{3}}{3}a_2 \tanh(a_1 f + a_2 g) \quad (5)$$
$$\gamma(f, g, 0) = \frac{2}{3}(a_1^2 + a_2^2)\text{sech}^2(a_1 f + a_2 g)$$

in Eq. 4, $\gamma(f, g, \epsilon)$ represent the elevation of the water wave, $\alpha(f, g, \epsilon)$ represent the surface velocity of water along the x-axis, and $\beta(f, g, \epsilon)$ represents the surface velocity of water along the y-axis. In Eq. 5 a_0, a_1, a_2, and a_3 are arbitrary constant.

Now using the NIM procedure, we get the following few terms for $\alpha(f, g, \epsilon)$

$$\alpha_0(f, g, \epsilon) = \frac{1}{3}(2\sqrt{3})a_1 \tanh(a_1 f + a_2 g) - \frac{a_0 a_2 + a_3}{a_1}$$

$$\alpha_1(f, g, \epsilon) = \frac{2a_1 a_3 e^p \text{sech}^2(a_1 f + a_2 g)}{\sqrt{3}p\Gamma(p)}$$

$$\alpha_2(f, g, \epsilon) = \frac{1}{3\sqrt{3}p\Gamma(p)}(2a_1 \epsilon^p(2\sqrt{3}(a_1^2 + a_2^2)\tanh(a_1 f + a_2 g)$$
$$- 3a_3)\text{sech}^2(a_1 f + a_2 g)) + \frac{1}{9p^2\Gamma(p)^2\Gamma(3p)\Gamma\left(p + \frac{1}{2}\right)}$$
$$(a_1 2^{1-2p}\epsilon^p \text{sech}^2(a_1 f + a_2 g)(2a_3^2 \epsilon^p \tanh(a_1 f + a_2 g)$$
$$\left(a_1^2 4^{p+1}\epsilon^p\Gamma(2p)\Gamma\left(p + \frac{1}{2}\right)\text{sech}^2(a_1 f + a_2 g)\right.$$
$$+ a_2^2 4^{p+1}\epsilon^p\Gamma(2p)\Gamma\left(p + \frac{1}{2}\right)\text{sech}^2(a_1 f + a_2 g)$$
$$- 3\sqrt{3\pi}\,p\Gamma(p)\Gamma(3p)) - 3(a_1^2 + a_2^2)2^{2p+1}p\Gamma(p)\Gamma(3p)\Gamma$$
$$\left(p + \frac{1}{2}\right)\tanh(a_1 f + a_2 g) + 3\sqrt{3}\,a_3 4^p p\Gamma(p)\Gamma(3p)\Gamma \ \left(p + \frac{1}{2}\right)\right)\right)$$

Now for $\beta(f, g, \epsilon)$ the zeroth, first and second order approximation is

$$\beta_0(f, g, \epsilon) = \frac{1}{3}(2\sqrt{3})a_2 \tanh(a_1 f + a_2 g) + a_0$$

$$\beta_1(f, g, \epsilon) = \frac{2a_2 a_3 \epsilon^p \text{sech}^2(a_1 f + a_2 g)}{\sqrt{3}p\Gamma(p)}$$

$$\beta_2(f, g, \epsilon) = \frac{1}{3\sqrt{3}p\Gamma(p)}(2a_2 \epsilon^p(2\sqrt{3}(a_1^2 + a_2^2)\tanh(a_1 f + a_2 g)$$
$$- 3a_3)\text{sech}^2(a_1 f + a_2 g)) + \frac{1}{9p^2\Gamma(p)^2\Gamma(3p)\Gamma\left(p + \frac{1}{2}\right)}$$
$$(a_2 2^{1-2p}\epsilon^p \text{sech}^2(a_1 f + a_2 g)(2a_3^2 \epsilon^p \tanh(a_1 f + a_2 g)$$
$$\left(a_1^2 4^{p+1}\epsilon^p\Gamma(2p)\Gamma\left(p + \frac{1}{2}\right)\ \text{sech}^2(a_1 f + a_2 g)\right.$$
$$+ a_2^2 4^{p+1}\epsilon^p\Gamma(2p)\Gamma\left(p + \frac{1}{2}\right)\ \text{sech}^2(a_1 f + a_2 g)$$
$$- 3\sqrt{3\pi}\,p\Gamma(p)\Gamma(3p)) - 3(a_1^2 + a_2^2)2^{2p+1}p\Gamma(p)\Gamma(3p)\Gamma$$
$$\left(p + \frac{1}{2}\right)\tanh(a_1 f + a_2 g) + 3\sqrt{3}\,a_3 4^p p\Gamma(p)\Gamma(3p)\Gamma \ \left(p + \frac{1}{2}\right)\right)\right)$$

Similarly the same procedure for $\gamma(f, g, \epsilon)$ we will get

$$\gamma_0(f,g,\epsilon) = \frac{2}{3}(a_1^2 + a_2^2)sech^2(a_1 f + a_2 g)$$

$$\gamma_1(f,g,\epsilon) = -\frac{4(a_1^2 + a_2^2)a_3\epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3\Gamma(p+1)}$$

$$\gamma_2(f,g,\epsilon) = -\frac{1}{9p\Gamma(p)}\left(2(a_1^2 + a_2^2)\epsilon^p sech^4(a_1 f + a_2 g)\right.$$

$$(-3a_3 \sinh(2a_1 f + 2a_2 g) + 2\sqrt{3}((a_1^2 + a_2^2)$$

$$\cosh(2a_1 f + 2a_2 g) - 2a_1^2) - 4\sqrt{3}a_2^2))$$

$$+ \frac{1}{3\Gamma(p)\Gamma\left(p+\frac{1}{2}\right)}\left(\sqrt{\frac{\pi}{3}}(a_1^2 + a_2^2)^2 a_3 2^{3-2p}\epsilon^{2p}\right.$$

$$(\cosh(2a_1 f + 2a_2 g) - 5)\tanh(a_1 f + a_2 g)sech^4$$

$$(a_1 f + a_2 g)) - \left(\frac{1}{\Gamma(p)}\right)\left(\frac{1}{3\sqrt{3}p}(4a_1^2(a_1^2 + a_2^2)\epsilon^p sech^4\right.$$

$$(a_1 f + a_2 g)) + \frac{4a_2^2(a_1^2 + a_2^2)\epsilon^p sech^4(a_1 f + a_2 g)}{3\sqrt{3}p}$$

$$- \frac{4a_0 a_2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma(p)sech^4(a_1 f + a_2 g)}{3\Gamma(2p+1)}$$

$$- \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}\left(a_1^2(a_1^2 + a_2^2)a_3^2 2^{2p+3}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\right.$$

$$sech^6(a_1 f + a_2 g)) - \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}$$

$$(a_2^2(a_1^2 + a_2^2)a_3^2 2^{2p+3}e^{3p}\Gamma\left(p+\frac{1}{2}\right)sech^6(a_1 f + a_2 g))$$

$$+ \frac{4(a_1^2 + a_2^2)a_3(a_0 a_2 + a_3)\epsilon^{2p}\Gamma(p)sech^4(a_1 f + a_2 g)}{3\Gamma(2p+1)} -$$

$$\frac{4a_0 a_2(a_1^2 + a_2^2)\epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3p}$$

$$- \frac{1}{3\sqrt{3}\Gamma(2p+1)}(32a_1^2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma(p)$$

$$\tanh(a_1 f + a_2 g)sech^4(a_1 f + a_2 g)) - \frac{1}{3\sqrt{3}\Gamma(2p+1)}$$

$$(32a_2^2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma(p)\tanh(a_1 f + a_2 g)sech^4(a_1 f + a_2 g))$$

$$+ \frac{4(a_1^2 + a_2^2)(a_0 a_2 + a_3)\epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3p}$$

$$- \frac{1}{3\sqrt{3}p}(8a_1^2(a_1^2 + a_2^2)\epsilon^p \tanh^2(a_1 f + a_2 g)sech^2$$

$$(a_1 f + a_2 g)) - \frac{1}{3\sqrt{3}p}(8a_2^2(a_1^2 + a_2^2)\epsilon^p \tanh^2(a_1 f + a_2 g)$$

$$sech^2(a_1 f + a_2 g)) + \frac{1}{3\Gamma(2p+1)}(8a_0 a_2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma$$

$$(p)\tanh^2(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)) + \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}$$

$$\left(a_1^2(a_1^2 + a_2^2)a_3^2 2^{2p+5}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\tanh^2(a_1 f + a_2 g)\right.$$

$$sech^4(a_1 f + a_2 g)) + \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}(a_2^2(a_1^2 + a_2^2)$$

$$a_3^2 2^{2p+5}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\tanh^2(a_1 f + a_2 g)sech^4(a_1 f + a_2 g))$$

$$- \frac{1}{3\Gamma(2p+1)}(8(a_1^2 + a_2^2)a_3(a_0 a_2 + a_3)\epsilon^{2p}\Gamma(p)\tanh^2$$

$$(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)) + \frac{1}{3\sqrt{3}\Gamma(2p+1)}$$

$$(16a_1^2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma(p)\tanh^3(a_1 f + a_2 g)sech^2(a_1 f + a_2 g))$$

$$+ \frac{16a_2^2(a_1^2 + a_2^2)a_3\epsilon^{2p}\Gamma(p)\tanh^3(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3\sqrt{3}\Gamma(2p+1)}\right)$$

According to Nim procedure we will get solution of the system is

$$\alpha(f,g,\epsilon) = \alpha_0(f,g,\epsilon) + \alpha_1(f,g,\epsilon) + \alpha_2(f,g,\epsilon)$$
$$\beta(f,g,\epsilon) = \beta_0(f,g,\epsilon) + \beta_1(f,g,\epsilon) + \beta_2(f,g,\epsilon)$$
$$\gamma(f,g,\epsilon) = \gamma_0(f,g,\epsilon) + \gamma_1(f,g,\epsilon) + \gamma_2(f,g,\epsilon)$$

Solution for $\alpha(f, g, \epsilon)$

$$\alpha(f,g,\epsilon) = \frac{1}{3}(2\sqrt{3})a_1 \tanh(a_1 f + a_2 g) - \frac{a_0 a_2 + a_3}{a_1}$$

$$+ \frac{2a_1 a_3\epsilon^p sech^2(a_1 f + a_2 g)}{\sqrt{3}p\Gamma(p)} + \frac{1}{3\sqrt{3}p\Gamma(p)}$$

$$(2a_1\epsilon^p(2\sqrt{3}(a_1^2 + a_2^2)\tanh(a_1 f + a_2 g) - 3a_3)sech^2$$

$$(a_1 f + a_2 g)) + \frac{1}{9p^2\Gamma(p)^2\Gamma(3p)\Gamma\left(p+\frac{1}{2}\right)}$$

$$(a_1 2^{1-2p}\epsilon^p sech^2(a_1 f + a_2 g)(2a_3^2 e^p \tanh(a_1 f + a_2 g)$$

$$\left(a_1^2 4^{p+1}\epsilon^p \Gamma(2p)\Gamma\left(p+\frac{1}{2}\right)sech^2(a_1 f + a_2 g)\right.$$

$$+ a_2^2 4^{p+1}\epsilon^p \Gamma(2p)\Gamma\left(p+\frac{1}{2}\right)sech^2(a_1 f + a_2 g)$$

$$- 3\sqrt{3\pi}p\Gamma(p)\Gamma(3p)) - 3(a_1^2 + a_2^2)2^{2p+1}p\Gamma(p)\Gamma(3p)\Gamma$$

$$\left(p+\frac{1}{2}\right)\tanh(a_1 f + a_2 g) + 3\sqrt{3}a_3 4^p p\Gamma(p)\Gamma(3p)\Gamma$$

$$\left.\left(p+\frac{1}{2}\right)\right))$$

Solution for $\beta(f, g, \epsilon)$

$$\beta(f,g,\epsilon) = \frac{1}{3}(2\sqrt{3})a_2 \tanh(a_1 f + a_2 g) + a_0$$

$$+ \frac{2a_2 a_3\epsilon^p sech^2(a_1 f + a_2 g)}{\sqrt{3}p\Gamma(p)} + \frac{1}{3\sqrt{3}p\Gamma(p)}$$

$$(2a_2\epsilon^p(2\sqrt{3}(a_1^2 + a_2^2)\tanh(a_1 f + a_2 g) - 3a_3)sech^2$$

$$(a_1 f + a_2 g)) + \frac{1}{9p^2\Gamma(p)^2\Gamma(3p)\Gamma\left(p+\frac{1}{2}\right)}$$

$$(a_2 2^{1-2p}\epsilon^p sech^2(a_1 f + a_2 g)(2a_3^2 e^p \tanh(a_1 f + a_2 g)$$

$$\left(a_1^2 4^{p+1}\epsilon^p \Gamma(2p)\Gamma\left(p+\frac{1}{2}\right)sech^2(a_1 f + a_2 g)\right.$$

$$+ a_2^2 4^{p+1}\epsilon^p \Gamma(2p)\Gamma\left(p+\frac{1}{2}\right)sech^2(a_1 f + a_2 g)$$

$$- 3\sqrt{3\pi}p\Gamma(p)\Gamma(3p)) - 3(a_1^2 + a_2^2)2^{2p+1}p\Gamma(p)\Gamma(3p)\Gamma$$

$$\left(p+\frac{1}{2}\right)\tanh(a_1 f + a_2 g) + 3\sqrt{3}a_3 4^p p\Gamma(p)\Gamma(3p)\Gamma$$

$$\left.\left(p+\frac{1}{2}\right)\right))$$

Similarly solution for $\gamma(f, g, \epsilon)$

$$\gamma(f,g,\epsilon) = \frac{2}{3}(a_1^2 + a_2^2)sech^2(a_1 f + a_2 g)$$

$$- \frac{4(a_1^2 + a_2^2)a_3 \epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3\Gamma(p+1)}$$

$$- \frac{1}{9p\Gamma(p)}\left(2(a_1^2 + a_2^2)\epsilon^p sech^4(a_1 f + a_2 g)\right.$$

$$\left.(-3a_3 \sinh(2a_1 f + 2a_2 g) + 2\sqrt{3}\left((a_1^2 + a_2^2)\right.\right.$$

$$\left.\left.\cosh(2a_1 f + 2a_2 g) - 2a_1^2\right) - 4\sqrt{3}a_2^2\right)\right) + \frac{1}{3p\Gamma(p)\Gamma\left(p+\frac{1}{2}\right)}$$

$$\left(\sqrt{\frac{\pi}{3}}(a_1^2 + a_2^2)^2 a_3 2^{3-2p}\epsilon^{2p}(\cosh(2a_1 f + 2a_2 g) - 5)\right.$$

$$\left.\tanh(a_1 f + a_2 g)sech^4(a_1 f + a_2 g)\right) - \left(\frac{1}{\Gamma(p)}\right)$$

$$\left(\frac{1}{3\sqrt{3}p}\left(4a_1^2(a_1^2 + a_2^2)\epsilon^p sech^4(a_1 f + a_2 g)\right)\right.$$

$$+ \frac{4a_2^2(a_1^2 + a_2^2)\epsilon^p sech^4(a_1 f + a_2 g)}{3\sqrt{3}p}$$

$$- \frac{4a_0 a_2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)sech^4(a_1 f + a_2 g)}{3\Gamma(2p+1)}$$

$$- \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}\left(a_1^2(a_1^2 + a_2^2)a_3^2 2^{2p+3}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\right.$$

$$\left.sech^6(a_1 f + a_2 g)\right) - \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}\left(a_2^2(a_1^2 + a_2^2)\right.$$

$$\left.a_3^2 2^{2p+3}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)sech^6(a_1 f + a_2 g)\right)$$

$$+ \frac{4(a_1^2 + a_2^2)a_3(a_0 a_2 + a_3)\epsilon^{2p}\Gamma(p)sech^4(a_1 f + a_2 g)}{3\Gamma(2p+1)}$$

$$- \frac{4a_0 a_2(a_1^2 + a_2^2)\epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3p}$$

$$- \frac{1}{3\sqrt{3}\Gamma(2p+1)}\left(32a_1^2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)\right.$$

$$\left.\tanh(a_1 f + a_2 g)sech^4(a_1 f + a_2 g)\right)$$

$$- \frac{1}{3\sqrt{3}\Gamma(2p+1)}\left(32a_2^2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)\right.$$

$$\left.\tanh(a_1 f + a_2 g)sech^4(a_1 f + a_2 g)\right)$$

$$+ \frac{4(a_1^2 + a_2^2)(a_0 a_2 + a_3)\epsilon^p \tanh(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3p}$$

$$- \frac{1}{3\sqrt{3}p}\left(8a_1^2(a_1^2 + a_2^2)\epsilon^p \tanh^2(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)\right)$$

$$- \frac{1}{3\sqrt{3}p}\left(8a_2^2(a_1^2 + a_2^2)\epsilon^p \tanh^2(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)\right)$$

$$+ \frac{1}{3\Gamma(2p+1)}\left(8a_0 a_2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)\tanh^2(a_1 f + a_2 g)\right.$$

$$\left.sech^2(a_1 f + a_2 g)\right) + \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}$$

$$\left(a_1^2(a_1^2 + a_2^2)a_3^2 2^{2p+5}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\tanh^2(a_1 f + a_2 g)\right.$$

$$\left.sech^4(a_1 f + a_2 g)\right) + \frac{1}{9\sqrt{3\pi}p^2\Gamma(3p)}$$

$$\left(a_2^2(a_1^2 + a_2^2)a_3^2 2^{2p+5}\epsilon^{3p}\Gamma\left(p+\frac{1}{2}\right)\tanh^2(a_1 f + a_2 g)\right.$$

$$\left.sech^4(a_1 f + a_2 g)\right) - \frac{1}{3\Gamma(2p+1)}\left(8(a_1^2 + a_2^2)a_3(a_0 a_2 + a_3)\right.$$

$$\left.\epsilon^{2p}\Gamma(p)\tanh^2(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)\right) + \frac{1}{3\sqrt{3}\Gamma(2p+1)}$$

$$\left(16a_1^2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)\tanh^3(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)\right)$$

$$+ \frac{16a_2^2(a_1^2 + a_2^2)a_3 \epsilon^{2p}\Gamma(p)\tanh^3(a_1 f + a_2 g)sech^2(a_1 f + a_2 g)}{3\sqrt{3}\Gamma(2p+1)}\right)$$

5 Results and discussions

The graphical analysis presented in this section offers a comparative study between the approximate and exact solutions derived through the proposed method, shedding light on the method's accuracy and practical utility. Figures 1–4 showcase three-dimensional plots depicting the approximate solutions $\alpha(f, g, h, \epsilon)$ across different values of fractional orders (p). Notably, an observable trend emerges as the fractional orders (p) increase, corresponding to an increment in the plotted graphs, implying a relationship between the solutions and the variation in fractional orders. Furthermore, Figures 5–12 present three-dimensional plots of the approximate solutions $\beta(f, g, h, \epsilon)$ and $\gamma(f, g, h, \epsilon)$ concerning varied fractional orders (p) at a fixed value of $\epsilon = 0.02$. These visual representations offer insights into the influence of fractional orders on the solutions, revealing potential patterns or dependencies within the system at constant values of other parameters.

Figures 13–15 provide a comparison between the 3D plots of $\alpha(f, g, h, \epsilon)$, $\beta(f, g, h, \epsilon)$, and $\gamma(f, g, h, \epsilon)$, respectively, illustrating the discrepancies and agreements between the approximate solutions and the exact solution across different fractional orders (p). Similarly, Figures 16–18 focus on the comparison of two-dimensional plots for $\alpha(f, g, h, \epsilon)$, $\beta(f, g, h, \epsilon)$, and $\gamma(f, g, h, \epsilon)$, utilizing the New Iterative Method (NIM). These comparisons aim to highlight the closeness or divergence between the approximate solutions obtained through the proposed method and the exact solutions, offering a comprehensive understanding of the method's efficacy in capturing the intricate dynamics of the system under varied fractional orders.

The tables provided in this discussion serve to compare the numerical solutions derived from the New Iterative Method (NIM) with the exact solution of the fractional-order nonlinear Wu-Zhang equation, demonstrating the method's efficacy in solving nonlinear partial differential equations plagued by scaling issues. Tables 1–3 exhibit the obtained solutions and compare them to the exact solutions while evaluating their absolute error for a fractional integer order (p = 1). These tables offer a quantitative assessment of the accuracy and closeness of the NIM-derived solutions to the exact solutions, providing insights into the method's performance under specific fractional orders.

On the other hand, Tables 4–6 provide a comparative analysis of $\alpha(f, g, h, \epsilon)$, $\beta(f, g, h, \epsilon)$, and $\gamma(f, g, h, \epsilon)$, respectively, for varying fractional orders (p = 0.4, 0.6, 0.8, 1.0) at a fixed value of $\epsilon = 0.2$. These tables facilitate a comprehensive examination of the solutions obtained through the NIM method across different fractional orders, enabling researchers to discern any patterns or variations in the solutions concerning changes in the fractional parameters. By comparing the solutions across various fractional orders, the tables offer insights into how the method performs under different degrees of fractional derivatives, aiding in understanding the behavior and dependency of the solutions on fractional order variations.

Additionally, the visualization of the solutions in 3D and 2D plots for various fractional order values complements the tables, allowing for a comprehensive assessment of the method's performance through

both numerical data and graphical representations. In Table 7, comparison of the present solution (NIM) with the HPM solution for the Wu-Zhang equation. Together, these tables and plots serve as invaluable tools in evaluating the NIM's efficiency in handling fractional-order nonlinear equations, providing a detailed understanding of the method's accuracy and performance under different fractional orders and parameter settings.

6 Conclusion

In conclusion, we have successfully applied the New Iterative Method to solve the fractional Wu-Zhang Equation within the Caputo Derivative framework. The method has demonstrated its efficacy in providing approximate solutions to this complex fractional differential equation. Through our numerical analysis and the presentation of pertinent tables and figures, we have showcased the accuracy and reliability of the method in addressing the Wu-Zhang Equation. This research highlights the significance of the New Iterative Method as a valuable tool for solving fractional-order differential equations, contributing to the broader field of mathematics and its applications. It opens up opportunities for further exploration and application in various scientific and engineering disciplines.

Data availability statement

The original contributions presented in the study are included in the article/Supplementary material, further inquiries can be directed to the corresponding authors.

Author contributions

HY: Writing–original draft, Formal Analysis, Investigation, Methodology, Visualization. AA: Project administration, Conceptualization, Methodology, Software, Data curation, Writing–review and editing. RS: Investigation, Writing–review and editing, Formal Analysis, Funding acquisition, Methodology, Supervision. AH: Conceptualization, Resources, Software, Supervision, Validation, Writing–original draft. SA: Methodology, Software, Writing–review and editing.

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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