frontiers | Frontiers in Physics

TYPE Original Research
PUBLISHED 06 May 2025
DOI 10.3389/fphy.2025.1569291

Check for updates

OPEN ACCESS

EDITED BY
Ji-Huan He,
Soochow University, China

REVIEWED BY
Sheng Zhang,
Bohai University, China
Che Haziqah Che Hussin,
Preparatory Centre for Science and Technology Universiti Malaysia Sabah, Malaysia

*CORRESPONDENCE
Jiao Zhang,
zhangjiaofc@163.com

RECEIVED 31 January 2025
ACCEPTED 08 April 2025
PUBLISHED 06 May 2025

CITATION
Zhang J and You F (2025) Extended $(\frac{G'}{G})$-expansion method for solving the coupled KdV equations with two arbitrary constants and its application to MEMS system. *Front. Phys.* 13:1569291.
doi: 10.3389/fphy.2025.1569291

Extended $(\frac{G'}{G})$-expansion method for solving the coupled KdV equations with two arbitrary constants and its application to MEMS system

Jiao Zhang* and Fucai You

College of Science, Shenyang Institute of Engineering, Shenyang, China

The coupled Korteweg-de Vries (cKdV) equations with two arbitrary constants hold significant importance in the field of micro-electro-mechanical systems (MEMS). These equations describe the behavior of nonlinear waves in MEMS devices. In MEMS applications, the cKdV equations can be used to analyze the dynamics of microstructures such as cantilevers, membranes, and resonators. By solving these equations, researchers can predict the behavior of MEMS devices under different operating conditions. In this paper, the $(\frac{G'}{G})$-expansion method is extended to seek more general travelling solutions of the cKdV equations with two arbitrary constants. The two arbitrary constants offer flexibility in modeling different physical phenomena and boundary conditions. As a result, many new and more general exact travelling wave solutions are obtained including soliton solutions, hyperbolic function solutions, trigonometric function solutions and rational solutions. They help in understanding the complex interactions between mechanical and electrical properties. Additionally, the study of these equations provides insights into the nonlinear behavior of MEMS systems, which is crucial for improving their performance and reliability. Overall, the cKdV equations with two arbitrary constants play a vital role in advancing the design and understanding of MEMS applications.

KEYWORDS

extended (G'/G)-expansion method, nonlinear evolution equations, coupled KdV equations, micro-electro-mechanical systems, computerized mechanization

1 Introduction

In the fields of physics and other disciplines, numerous phenomena are often described by nonlinear evolution equations (NLEEs). To gain a deep understanding of the physical mechanisms behind natural phenomena represented by the NLEEs, it is crucial to study their exact solutions. Many methods have been developed to obtain exact solutions for NLEES, such as the inverse scattering transform [1], the Darboux transformation [2], Bäcklund transformation [3], the Hirota method [4], the Wronskian technique [5], homogeneous balance method [6, 7], truncated Painlevé expansion method [8, 9], symmetry method [10], F-expansion method [11, 12], the generalized auxiliary equation method [13]. Among the numerous types of NLEEs, the cKdV equations hold a special place. The cKdV equations are widely used to model the interaction of multiple waves in different physical scenarios. For instance, in the field of micro-electro-mechanical systems (MEMS) [14], they can describe the behavior of nonlinear waves in MEMS devices. The dynamics of MEMS frequently exhibit nonlinear characteristics arising from large deformations, material nonlinearity, or electrostatic coupling effects. Such nonlinear behaviors are typically modeled using NLEEs. Soliton is stable, localized waves with inherent waveform preservation. This property proves advantageous for enhancing signal transmission efficiency in MEMS resonators and communication components. However, solving cKdV equations is a challenging task due to their inherent nonlinearity and complexity.

In recent years, the generalized $(\frac{G'}{G})$-expansion method [15–17] have emerged as a promising and powerful technique for obtaining exact solutions of NLEEs. This method offers several advantages over traditional methods. It is more flexible and can be applied to a broader class of equations. By using the generalized $(\frac{G'}{G})$-expansion method, we can obtain a variety of solutions, including solitary wave solutions, periodic wave solutions, rational function solutions, and more. These solutions can provide a more comprehensive understanding of the behavior of the physical systems described by the equations.

In this paper, we aim to extend the generalized $(\frac{G'}{G})$-expansion method. Subsequently, we apply the extended $(\frac{G'}{G})$-expansion method to solve the cKdV equations with two arbitrary constants. Our objective is to obtain numerous novel and more general travelling wave solutions, which can provide valuable insights into the wave-wave interactions described by the cKdV equations. Additionally, we will explore the application of these solutions to the MEMS field.

2 Introduction of the extended $(\frac{G'}{G})$-expansion method

For a given NLEEs with variable $x = (t, x_1, x_2, \ldots, x_m)$ and $u(x)$

$$P\left(u, u_t, u_{x_1}, u_{x_2}, \ldots, u_{x_m}, u_{tt}, u_{x_1x_1}, u_{x_1t}, u_{x_2x_2}, \cdots\right) = 0, \tag{2.1}$$

through the application of the travelling wave transformation $u(x) = u(\xi), \xi = k(x_1 + l_1x_2 + l_2x_3 + \cdots + l_{m-1}x_m + Vt)$(where k, V and $l_i(i = 1, 2, \ldots, m-1)$ are all constants.), Equation 2.1 can be reduced to an ordinary differential equation (ODE):

$$Q\left(u, u', u'', \cdots\right) = 0, \tag{2.2}$$

we work towards getting its solutions in a more general form:

$$u = \sum_{i=-n}^{n} a_i \left(\frac{G'}{G}\right)^i, \tag{2.3}$$

in which $G = G(\xi)$ complies with the ODE

$$G'' + \lambda G' + \mu G = 0. \tag{2.4}$$

Since Equation 2.3 contains $2n$ arbitrary constants, the solutions derived from the extended $(\frac{G'}{G})$-expansion method are more general in scope compared to those obtained through traditional approaches. To optimize the utilization of the extended $(\frac{G'}{G})$-expansion method, we list its main steps as follows:

Step 1. Determine the integer value of n. Substitute Equation 2.3 along with Equation 2.4 into Equation 2.2. By balancing the highest-order derivative term with the nonlinear terms in Equation 2.2, we are able to derive the algebraic equation related to n.

Step 2. Derive an algebraic equation system. Substituting Equation 2.3 and Equation 2.4 into Equation 2.2 with the n value from step 1. Collecting the coefficients of $(\frac{G'}{G})^p(p = 0, \pm1, \pm2, \cdots)$, then setting each coefficient to zero, we can get a set of over-determined algebraic equations for $a_i(i = 0, \pm1, \pm2, \ldots, \pm n), k, l_i(i = 1, 2, \ldots, m-1), V, \mu$ and λ.

Step 3. Solve the algebraic equation system. Employ Maple to solve the algebraic equation system and obtain the explicit expressions for $a_i(i = 0, \pm1, \pm2, \ldots, \pm n), k, l_i(i = 1, 2, \ldots, m-1), V, \mu$ and λ.

Step 4. Get the exact solutions. By substituting the outcomes from the previous steps, we are able to obtain a series of travelling solutions of Equation 2.2 which rely on the fundamental solution G of Equation 2.4.

3 Solutions of the cKdV equations

In this part, we intend to utilize our method to acquire new and more general exact travelling solutions for the cKdV equations [18].

$$u_t = a\left(u_{xxx} + 6uu_x\right) + 2bvv_x, \tag{3.1}$$

$$v_t = -v_{xxx} - 3uv_x. \tag{3.2}$$

Suppose that

$$\begin{aligned} &u = u(x,t) = u(\xi),\\ &v = v(x,t) = v(\xi),\\ &\xi = x - Vt, \end{aligned} \tag{3.3}$$

then, upon inserting Equation 3.3 into Equations 3.1, 3.2 separately, we get.

$$-Vu' - 6auu' - au''' - 2bvv' = 0, \tag{3.4}$$

$$-Vv' + 3uv' + v''' = 0, \tag{3.5}$$

by integrating Equation 3.4 with respect to ξ for one time, we get

$$-Vu - 3au^2 - au'' - bv^2 + C = 0, \tag{3.6}$$

Based on step 1, we find that $n = 2$ for u and v. We postulate that Equations 3.5, 3.6 possess the following formal solutions

$$u = \sum_{i=-2}^{2} c_i \left(\frac{G'}{G}\right)^i, \tag{3.7}$$

$$v = \sum_{i=-2}^{2} d_i \left(\frac{G'}{G}\right)^i, \tag{3.8}$$

where c_i and $d_i(i = 0, \pm1, \pm2)$ are all constants to be determined.

Upon substituting Equation 3.7 and Equation 3.8 along with Equation 2.4 into Equations 3.5, 3.6, the following results are achieved.

Case 1:

$$c_1 = c_2 = d_1 = d_2 = d_{-2} = 0, c_{-2} = -2\mu^2, c_{-1} = -2\lambda\mu, d_0 = \frac{\lambda d_{-1}}{2\mu},$$

$$c_0 = -\frac{16a\mu^3 + 2a\mu^2\lambda^2 - bd_{-1}^2 + 2\mu^2\lambda^2 + 4\mu^3}{6\mu^2(1+2a)},$$

$$V = \frac{-8a\mu^3 + 2a\mu^2\lambda^2 + bd_{-1}^2}{2\mu^2(1+2a)},$$

$$\begin{aligned} C = &\big(8a^2\mu^2\lambda^2 bd_{-1}^2 - 32a^2\mu^3 bd_{-1}^2 + 8a\mu^2\lambda^2 bd_{-1}^2 + b^2 d_{-1}^4 - 32a\mu^3 bd_{-1}^2 \\ &+ 64a^2\mu^6 - 32a^2\mu^5\lambda^2 + bd_{-1}^2\mu^2\lambda^2 - 4bd_{-1}^2\mu^3 + 64a^3\mu^6 + 4a^2\mu^4\lambda^4 \\ &+ ab^2 d_{-1}^4 + 4a^3\mu^4\lambda^4 - 32a^3\mu^5\lambda^2\big)/12\mu^4(1+2a)^2, \end{aligned} \tag{3.9}$$

where λ, μ and d_{-1} are arbitrary constants.
Case 2:

$$c_1 = c_2 = d_1 = d_2 = 0, c_{-1} = -4\lambda\mu, c_{-2} = -4\mu^2,$$

$$d_{-1} = \pm 2\lambda\mu\sqrt{\frac{-6a}{b}}, d_{-2} = \pm 2\mu^2\sqrt{\frac{-6a}{b}},$$

$$c_0 = \frac{\mp d_0 b\sqrt{\frac{-6a}{b}} + (1+a)(\lambda^2 + 8\mu)}{1+2a},$$

$$V = \frac{a\lambda^2 + 8a\mu \pm d_0 b\sqrt{\frac{-6a}{b}}}{1+2a},$$

$$\begin{aligned} C = &\Big(\mp\big(2a^2\lambda^2 + 16a\mu + \lambda^2 + 8\mu + 16a^2\mu\big) d_0 b\sqrt{\frac{-6a}{b}} - 2a\lambda^2 d_0 b \\ &+ a^3\lambda^4 - 24a\mu^2 - 32a^3\lambda^2\mu - 32a^3\mu^2 + 3bd_0^2 + a^2\lambda^4 - 32a^2\mu^2 \\ &- 32a^2\lambda^2\mu - 12a\mu\lambda^2 + 6abd_0^2 + 6a^2bd_0^2\Big)/3(1+2a)^2, \end{aligned} \tag{3.10}$$

where λ, μ and d_0 are arbitrary constants.
Case 3:

$$c_{-2} = c_{-1} = d_{-2} = d_{-1} = d_2 = 0, c_1 = -2\lambda, c_2 = -2, V = \lambda^2 + 2\mu + 3c_0,$$

$$d_1 = \pm\sqrt{\frac{2a\lambda^2 + 12ac_0 + 16a\mu + 6c_0 + 2\lambda^2 + 4\mu}{b}},$$

$$d_0 = \pm\frac{\lambda}{2}\sqrt{\frac{2a\lambda^2 + 12ac_0 + 16a\mu + 6c_0 + 2\lambda^2 + 4\mu}{b}},$$

$$C = \frac{a\lambda^4}{2} + 3ac_0\lambda^2 + 2a\mu\lambda^2 + 3ac_0^2 - 4a\mu^2 + \frac{\lambda^4}{2} + \lambda^2\mu + \frac{5c_0\lambda^2}{2} + 2c_0\mu + 3c_0^2,$$

where λ, μ and c_0 are arbitrary constants.
Case 4:

$$d_{-2} = d_2 = 0, c_2 = -2, c_1 = -2\lambda,$$

$$d_{-1} = d_1\mu, c_{-1} = -2\lambda\mu, c_{-2} = -2\mu^2,$$

$$V = \frac{2a\lambda^2 + 16a\mu + bd_1^2}{2(1+2a)},$$

$$c_0 = -\frac{2a\lambda^2 + 16a\mu + 2\lambda^2 + 16\mu - bd_1^2}{6(1+2a)},$$

$$d_0 = \frac{\lambda(24a\mu - bd_1^2)}{2bd_1},$$

$$\begin{aligned} C = &\big(1728a^2\lambda^2\mu^2 + 8b^2d_1^4\mu + b^2d_1^4\lambda^2 + 448bd_1^2a^2\lambda^2\mu + 96bd_1^2a\lambda^2\mu \\ &+ 6912a^4\lambda^2\mu^2 + 64b^2d_1^4a\mu + 8d_1^4a\lambda^2b^2 + 6912a^3\lambda^2\mu^2 + b^3d_1^6 \\ &+ 4bd_1^2a^2\lambda^4 + 1024bd_1^2a^2\mu^2 + 192bd_1^2a\mu^2 + 4a^3bd_1^2\lambda^4 + ab^3d_1^6 \\ &+ 1024a^3bd_1^2\mu^2 + 8a^2b^2d_1^4\lambda^2 + 64a^2b^2d_1^4\mu \\ &+ 448a^3bd_1^2\lambda^2\mu\big)/12bd_1^2(1+2a)^2, \end{aligned} \tag{3.11}$$

where λ, μ and d_1 are arbitrary constants.
Case 5:

$$c_{-1} = c_{-2} = d_{-1} = d_{-2} = 0, c_1 = -4\lambda, c_2 = -4,$$

$$d_1 = \pm 2\lambda\sqrt{\frac{-6a}{b}}, d_2 = \pm 2\sqrt{\frac{-6a}{b}},$$

$$V = \lambda^2 + 8\mu + 3c_0,$$

$$d_0 = \pm\frac{\lambda^2 + a\lambda^2 + 8a\mu + 6ac_0 + 8\mu + 3c_0}{\sqrt{-6ab}},$$

$$\begin{aligned} C = -&\big(128a\mu^2 + 9c_0^2 + \lambda^4 + 96ac_0\mu + 6c_0\lambda^2 + a^2\lambda^4 \\ &+ 18a^2c_0^2 + 112a^2\mu^2 + 2a\lambda^4 + 12ac_0\lambda^2 \\ &+ 40a^2\mu\lambda^2 + 12ac_0\lambda^2 + 96a^2c_0\mu + 64\mu^2 \\ &+ 48c_0\mu + 16\lambda^2\mu + 18ac_0^2 + 32a\lambda^2\mu\big)/6a, \end{aligned} \tag{3.12}$$

where λ, μ and c_0 are arbitrary constants.

Substituting Equations 3.9–3.12 into Equations 3.7, 3.8 respectively, we have five kinds of formal solutions of Equations 3.1, 3.2:

$$u_1 = -2\mu^2\left(\frac{G'}{G}\right)^{-2} - 2\lambda\mu\left(\frac{G'}{G}\right)^{-1} - \frac{16a\mu^3 + 2a\mu^2\lambda^2 - bd_{-1}^2 + 2\mu^2\lambda^2 + 4\mu^3}{6\mu^2(1+2a)}, \tag{3.13}$$

$$v_1 = d_{-1}\frac{G}{G'} + \frac{d_{-1}\lambda}{2\mu}, \tag{3.14}$$

where $\xi = x - \frac{-8a\mu^3 + 2a\mu^2\lambda^2 + bd_{-1}^2}{2\mu^2(1+2a)}t$.

$$u_2 = -4\mu^2\left(\frac{G'}{G}\right)^{-2} - 4\lambda\mu\left(\frac{G'}{G}\right)^{-1} + \frac{a\lambda^2 \mp d_0 b\sqrt{\frac{-6a}{b}} + \lambda^2 + 8\mu + 8a\mu}{1+2a}, \tag{3.15}$$

$$v_2 = \pm 2\mu^2\sqrt{\frac{-6a}{b}}\left(\frac{G'}{G}\right)^{-2} \pm 2\lambda\mu\sqrt{\frac{-6a}{b}}\left(\frac{G'}{G}\right)^{-1} + d_0, \tag{3.16}$$

where $\xi = x - \frac{a\lambda^2 + 8a\mu \pm d_0 b\sqrt{\frac{-6a}{b}}}{1+2a}t$.

$$u_3 = -2\left(\frac{G'}{G}\right)^2 - 2\lambda\frac{G'}{G} + c_0, \tag{3.17}$$

$$v_3 = \pm\sqrt{\frac{2a\lambda^2 + 12ac_0 + 16a\mu + 6c_0 + 2\lambda^2 + 4\mu}{b}}\left(\frac{G'}{G} + \frac{\lambda}{2}\right). \tag{3.18}$$

where $\xi = x - (\lambda^2 + 2\mu + 2c_0)t$.

$$\begin{aligned} u_4 = &-2\mu^2\left(\frac{G'}{G}\right)^{-2} - 2\lambda\mu\left(\frac{G'}{G}\right)^{-1} - 2\lambda\frac{G'}{G} - 2\left(\frac{G'}{G}\right)^2 \\ &- \frac{2a\lambda^2 + 16a\mu + 2\lambda^2 + 16\mu - bd_1^2}{6+12a}, \end{aligned} \tag{3.19}$$

$$v_4 = d_1\mu\left(\frac{G'}{G}\right)^{-1} + d_1\frac{G'}{G} - \frac{\lambda\left(24a\mu - bd_1^2\right)}{2bd_1}, \tag{3.20}$$

where $\xi = x - \frac{2a\lambda^2+16a\mu+bd_1^2}{2(1+2a)}t$.

$$u_5 = -4\left(\frac{G'}{G}\right)^2 - 4\lambda\frac{G'}{G} + c_0, \tag{3.21}$$

$$v_5 = \pm 2\sqrt{\frac{-6a}{b}}\left(\frac{G'}{G}\right)^2 \pm 2\lambda\sqrt{\frac{-6a}{b}}\frac{G'}{G} \pm \frac{\lambda^2 + a\lambda^2 + 8a\mu + 6ac_0 + 8\mu + 3c_0}{\sqrt{-6ab}}, \tag{3.22}$$

where $\xi = x - (\lambda^2 + 8\mu + 3c_0)t$.

Then, by substituting the solutions of Equation 2.4 into Equations 3.13, 3.14, we derive three types travelling solutions of the cKdV equations as follows:

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_{1_1} = &\frac{-2\mu^2}{\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}\right)^2} \\ &- \frac{2\lambda\mu}{-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}} \\ &- \frac{16a\mu^3 + 2a\mu^2\lambda^2 - bd_{-1}^2 + 2\mu^2\lambda^2 + 4\mu^3}{6\mu^2(1+2a)}, \end{aligned} \tag{3.23}$$

$$v_{1_1} = \frac{d_{-1}}{-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}} + \frac{d_{-1}\lambda}{2\mu}. \tag{3.24}$$

When $\lambda^2 - 4\mu < 0$, we obtain

$$\begin{aligned} u_{1_2} = &\frac{-2\mu^2}{\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2} \\ &- \frac{2\lambda\mu}{-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}} \\ &- \frac{16a\mu^3 + 2a\mu^2\lambda^2 - bd_{-1}^2 + 2\mu^2\lambda^2 + 4\mu^3}{6\mu^2(1+2a)}, \end{aligned} \tag{3.25}$$

$$v_{1_2} = \frac{d_{-1}}{-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}} + \frac{d_{-1}\lambda}{2\mu}. \tag{3.26}$$

When $\lambda^2 - 4\mu = 0$,

$$\begin{aligned} u_{1_3} = &\frac{-2\mu^2}{\left(-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}\right)^2} - \frac{2\lambda\mu}{-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}} \\ &- \frac{16a\mu^3 + 2a\mu^2\lambda^2 - bd_{-1}^2 + 2\mu^2\lambda^2 + 4\mu^3}{6\mu^2(1+2a)}, \end{aligned} \tag{3.27}$$

$$v_{1_3} = \frac{d_{-1}}{-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}} + \frac{d_{-1}\lambda}{2\mu}. \tag{3.28}$$

By substituting the solutions of Equation 2.4 into Equations 3.15, 3.16, We possess three types of travelling solutions of the cKdV equations in the following:

When $\lambda^2 - 4\mu > 0$,

$$\begin{aligned} u_{2_1} = &\frac{-4\mu^2}{\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}\right)^2} \\ &- \frac{4\lambda\mu}{-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}} \\ &+ \frac{a\lambda^2 \mp d_0 b\sqrt{\frac{-6a}{b}} + \lambda^2 + 8\mu + 8a\mu}{1+2a}, \end{aligned} \tag{3.29}$$

$$\begin{aligned} v_{2_1} = &\frac{\pm 2\mu^2\sqrt{\frac{-6a}{b}}}{\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}\right)^2} + d_0 \\ &\pm \frac{2\lambda\mu\sqrt{\frac{-6a}{b}}}{-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}}. \end{aligned} \tag{3.30}$$

When $\lambda^2 - 4\mu < 0$,

$$\begin{aligned} u_{2_2} = &\frac{-4\mu^2}{\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2} \\ &- \frac{4\lambda\mu}{-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}} \\ &+ \frac{a\lambda^2 \mp d_0 b\sqrt{\frac{-6a}{b}} + \lambda^2 + 8\mu + 8a\mu}{1+2a}, \end{aligned} \tag{3.31}$$

$$\begin{aligned} v_{2_2} = &\frac{\pm 2\mu^2\sqrt{\frac{-6a}{b}}}{\left(-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2} + d_0 \\ &\pm \frac{2\lambda\mu\sqrt{\frac{-6a}{b}}}{-\frac{\lambda}{2} + \frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}}. \end{aligned} \tag{3.32}$$

When $\lambda^2 - 4\mu = 0$,

$$u_{2_3} = \frac{-4\mu^2}{\left(-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}\right)^2} - \frac{4\lambda\mu}{-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}} + \frac{a\lambda^2 \mp d_0 b\sqrt{\frac{-6a}{b}} + \lambda^2 + 8\mu + 8a\mu}{1+2a}, \tag{3.33}$$

$$v_{2_3} = \frac{\pm 2\mu^2\sqrt{\frac{-6a}{b}}}{\left(-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}\right)^2} \pm \frac{2\lambda\mu\sqrt{\frac{-6a}{b}}}{-\frac{\lambda}{2} + \frac{C_2}{C_1+C_2\xi}} + d_0. \tag{3.34}$$

Upon substituting the general solutions of Equation 2.4 into Equations 3.17, 3.18, here are three types of travelling solutions of the cKdV equations.

When $\lambda^2 - 4\mu > 0$,

$$u_{3_1} = \left(2\mu - \frac{\lambda^2}{2}\right)\left(\frac{C_1\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2} + C_2\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1\sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2} + C_2\cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}\right)^2 + \frac{\lambda^2}{2} + c_0, \tag{3.35}$$

$$v_{3_1} = \pm\sqrt{\frac{2(\lambda^2-4\mu)(a\lambda^2+6ac_0+8a\mu+3c_0+\lambda^2+2\mu)}{b}} \times \frac{C_1 cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2 sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}{C_1 sinh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}+C_2 cosh\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}. \tag{3.36}$$

Taking $c_0 = -\frac{a\lambda^2+8a\mu+\lambda^2+2\mu}{3+6a}$ in Equation 3.36, i.e., $v_{3_1}=0$, then Equations 3.1, 3.2 become the KdV equation

$$u_t = a(u_{xxx}+6uu_x), \tag{3.37}$$

from Equation 3.35, the solutions of Equation 3.37 can be rewritten as

$$u_{3_1} = \left(2\mu-\frac{\lambda^2}{2}\right)\frac{2C_1e^{\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}+2C_1e^{-\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}+C_2e^{\sqrt{\lambda^2-4\mu}\xi}+C_2e^{-\sqrt{\lambda^2-4\mu}\xi}+2C_2}{2C_1e^{\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}-2C_1e^{-\frac{\sqrt{\lambda^2-4\mu}\xi}{2}}+C_2e^{\sqrt{\lambda^2-4\mu}\xi}+C_2e^{-\sqrt{\lambda^2-4\mu}\xi}+2C_2} + (4a\lambda^2-4\mu-16a\mu+\lambda^2)/(6+12a), \tag{3.38}$$

if we set $C_1=0$, Equation 3.38 becomes

$$u_{3_1} = \frac{4\mu-a\lambda^2-\lambda^2+4a\mu}{3+6a} - \frac{2C_2(4\mu-\lambda^2)}{C_2e^{\sqrt{\lambda^2-4\mu}\xi}+C_2e^{-\sqrt{\lambda^2-4\mu}\xi}+2C_2}. \tag{3.39}$$

Comparing our results in Equation 3.39 with other results by Exp-function method in [19], then it can be seen that the forms are similar.

When $\lambda^2-4\mu<0$,

$$u_{3_2} = \left(\frac{\lambda^2}{2}-2\mu\right)\left(\frac{C_1cos\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi-C_2sin\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sin\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cos\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2+\frac{\lambda^2}{2}+c_0, \tag{3.40}$$

$$v_{3_2} = \mp\sqrt{\frac{2(4\mu-\lambda^2)(a\lambda^2+6ac_0+8a\mu+3c_0+\lambda^2+2\mu)}{b}} \times \frac{C_1 cos\frac{\sqrt{4\mu-\lambda^2}\xi}{2}-C_2 sin\frac{\sqrt{4\mu-\lambda^2}\xi}{2}}{C_1 sin\frac{\sqrt{4\mu-\lambda^2-}\xi}{2}+C_2 cos\frac{\sqrt{4\mu-\lambda^2}\xi}{2}}. \tag{3.41}$$

When $\lambda^2-4\mu=0$,

$$u_{3_3} = \frac{C_2^2(2c_0+\lambda)\xi^2+2C_1C_2(2c_0+\lambda^2)\xi+2c_0C_1^2-4C_2^2+C_1^2\lambda^2}{2(C_1+C_2\xi)^2}, \tag{3.42}$$

$$v_{3_3} = \pm\sqrt{\frac{2a\lambda^2+12ac_0+16a\mu+6c_0+2\lambda^2+4\mu}{b}}\frac{C_2}{C_1+C_2\xi}. \tag{3.43}$$

Puting the general solutions of Equation 2.4 into Equations 3.19, 3.20, three types of travelling solutions of the cKdV equations are given in the following:

When $\lambda^2-4\mu>0$,

$$u_{4_1} = \frac{-2\mu^2}{\left(-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2} - \frac{2\lambda\mu}{-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}} - \frac{\lambda^2-4\mu}{2} \times \left(\frac{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2 - \frac{16a\mu-4a\lambda^2-\lambda^2+16\mu-bd_1^2}{6+12a}, \tag{3.44}$$

$$v_{4_1} = \frac{d_1\mu}{-\frac{\lambda}{2}+\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}} + \frac{\sqrt{\lambda^2-4\mu}}{2}\frac{C_1cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi} - \frac{12\lambda a\mu}{bd_1}. \tag{3.45}$$

When $\lambda^2-4\mu<0$,

$$u_{4_2} = \frac{-2\mu^2}{\left(-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1sin\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2} - \frac{2\lambda\mu}{-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2sin\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}} + \frac{\lambda^2-4\mu}{2}\left(\frac{C_1cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2 - \frac{16a\mu-4a\lambda^2-\lambda^2+16\mu-bd_1^2}{6+12a}, \tag{3.46}$$

$$v_{4_2} = \frac{d_1\mu}{-\frac{\lambda}{2}+\frac{\sqrt{4\mu-\lambda^2}}{2}\frac{C_1cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2sin\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}} + \frac{d_1\sqrt{4\mu-\lambda^2}}{2}\frac{C_1cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi-C_2sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2cos\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi} - \frac{12\lambda a\mu}{bd_1}. \tag{3.47}$$

When $\lambda^2-4\mu=0$,

$$u_{4_3} = \frac{-2\mu^2}{\left(\frac{-\lambda}{2}+\frac{C_2}{C_1+C_2\xi}\right)^2} - \frac{2\lambda\mu}{\frac{-\lambda}{2}+\frac{C_2}{C_1+C_2\xi}} - \frac{2C_2^2}{(C_1+C_2\xi)^2} - \frac{16a\mu-4a\lambda^2-\lambda^2+16\mu-bd_1^2}{6+12a}, \tag{3.48}$$

$$v_{4_3} = \frac{d_1\mu}{\frac{-\lambda}{2}+\frac{C_2}{C_1+C_2\xi}} + \frac{d_1\sqrt{4\mu-\lambda^2}}{2}\frac{C_2}{C_1+C_2\xi} - \frac{12\lambda a\mu}{bd_1}. \tag{3.49}$$

Substituting the general solutions of Equation 2.4 into Equations 3.21, 3.22, we have three types travelling solutions of the cKdV equations in the following:

When $\lambda^2 - 4\mu > 0$,

$$u_{5_1} = -(\lambda^2 - 4\mu)\left(\frac{C_1 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2 + \lambda^2 + c_0, \quad (3.50)$$

$$v_{5_1} = \pm\frac{(\lambda^2 - 4\mu)\sqrt{\frac{-6a}{b}}}{2}\left(\frac{C_1 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}{C_1 sinh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + C_2 cosh\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi}\right)^2 \pm \frac{\lambda^2 + 4a\lambda^2 + 8a\mu + 6ac_0 + 8\mu + 6c_0}{\sqrt{-6ab}}. \quad (3.51)$$

If $C_2 > 0, C_1^2 < C_2^2$, then from Equations 3.23, 3.24, we can obtain bell soliton solutions

$$u_{5_2} = (\lambda^2 - 4\mu)\, sech^2\left(\frac{1}{2}\sqrt{\lambda^2 - 4\mu}\xi + \xi_0\right) + 4\mu + c_0, \quad (3.52)$$

$$v_{5_2} = \pm\sqrt{\frac{-6a}{b}}\frac{(\lambda^2 - 4\mu)}{2} sech^2\left(\frac{\sqrt{\lambda^2 - 4\mu}\xi}{2} + \xi_0\right) \pm \frac{\lambda^2 + a\lambda^2 + 20a\mu + 6ac_0 + 8\mu + 3c_0}{\sqrt{-6ab}}, \quad (3.53)$$

where $\xi_0 = tanh^{-1}\frac{C_1}{C_2}$.

When $\lambda^2 - 4\mu < 0$,

$$u_{5_3} = (\lambda^2 - 4\mu)\left(\frac{C_1 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi - C_2 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2 + \lambda^2 + c_0, \quad (3.54)$$

$$v_{5_3} = \pm\frac{(4\mu - \lambda^2)\sqrt{\frac{-6a}{b}}}{2}\left(\frac{C_1 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi - C_2 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}{C_1 sin\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi + C_2 cos\frac{1}{2}\sqrt{4\mu - \lambda^2}\xi}\right)^2 \pm \frac{\lambda^2 + 4a\lambda^2 + 8a\mu + 6ac_0 + 8\mu + 3c_0}{\sqrt{-6ab}}. \quad (3.55)$$

When $\lambda^2 - 4\mu = 0$,

$$u_{6_1} = \frac{-4C_2^2}{(C_1 + C_2\xi)^2} + \lambda^2 + c_0, \quad (3.56)$$

$$v_{6_1} = \pm\sqrt{\frac{-6a}{b}}\frac{2C_2^2}{(C_1 + C_2\xi)^2} \pm \frac{\lambda^2 + 4a\lambda^2 + 8a\mu + 6ac_0 + 8\mu + 3c_0}{\sqrt{-6ab}}. \quad (3.57)$$

4 Conclusion

In summary, the extended $(\frac{G'}{G})$-expansion method has been proposed and applied to construct exact solutions of the cKdV equations. With the aid of Maple, we have obtained many new and more general exact travelling wave solutions, presented as Equations 3.32–3.36 and 3.40–3.57. These solutions span a wide spectrum, including soliton solutions, hyperbolic function solutions, trigonometric function solutions, as well as rational solutions. By applying the solutions obtained from the cKdV equations to MEMS systems, the presence of the two arbitrary constants allows for customization of the model to fit specific experimental data or design requirements. This enables more accurate predictions and optimization of MEMS devices. Additionally, the study of these equations provides insights into the nonlinear behavior of MEMS systems, which is crucial for improving their performance and reliability. Overall, the cKdV equations with two arbitrary constants play a vital role in advancing the design and understanding of MEMS applications. We hope to contribute to the development of more efficient and reliable MEMS devices.

Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author.

Author contributions

JZ: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Writing – original draft, Writing – review and editing. FY: Software, Supervision, Validation, Visualization, Writing – review and editing.

Funding

The author(s) declare that financial support was received for the research and/or publication of this article. This work was supported by Natural Science Foundation of LiaoningProvince (2023-MSLH-229, 2024-MSLH-336) and Scientific Research Fund of Liaoning Provincial Education Department (LJ212411632012, LJ222411632015).

Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Generative AI statement

The author(s) declare that no Generative AI was used in the creation of this manuscript.

References

1. Ablowitz MJ, Segur H. *Solitons and the inverse scattering transform*. Philadelphia: SIAM (1981).

2. Matveev VB, Salle MA. *Darboux transformations and solitons*. Berlin: Springer-Verlag (1991).

3. Nimmo JJC. A bilinear Bäcklund transformation for the nonlinear Schrödinger equation. *Phys Lett A* (1983) 99:279–80. doi:10.1016/0375-9601(83)90884-8

4. Hirota R. *The direct method in soliton theory (in English)*. Cambridge University Press (2004).

5. Freeman NC, Nimmo JJC. Soliton solutions of the Korteweg-de Vries and Kadomtsev-Petviashvili equations: the wronskian technique. *Phys Lett* (1983) 95:1–3. doi:10.1016/0375-9601(83)90764-8

6. Wang ML. Exact solutions for a compound KdV-Burgers equation. *Phys Lett* (1996) 213:279–87. doi:10.1016/0375-9601(96)00103-x

7. Fan EG, Zhang HQ. A note on the homogeneous balance method. *Phys Lett* (1998) 246:403–6. doi:10.1016/s0375-9601(98)00547-7

8. Bo T, Gao YT. Truncated Painlevé expansion and a wide-ranging type of generalized variable-coefficient Kadomtsev-Petviashvili equations. *Phys Lett* (1995) 209:297–304. doi:10.1016/0375-9601(95)00836-5

9. Choudhury SR. Painlevé analysis of nonlinear evolution equations-an algorithmic method. *Chaos Solitons and Fractals* (2006) 27:139–52. doi:10.1016/j.chaos.2005.02.043

10. Moussa MHM, Rehab M, Shikh E. Similarity Reduction and similarity solutions of Zabolotskay CKhoklov equation with a dissipative term via symmetry method. *Phys A* (2006) 371:325–35. doi:10.1016/j.physa.2006.04.044

11. Zhou YB, Wang ML, Wang YM. Periodic wave solutions to a coupled KdV equations with variable coefficients. *Phys Lett A* (2003) 308:31–6. doi:10.1016/s0375-9601(02)01775-9

12. Abdou MA. The extended F-expansion method and its application for a class of nonlinear evolution equations. *Chaos Solitons Fractals* (2007) 31:95–104. doi:10.1016/j.chaos.2005.09.030

13. Zhang J, Zhang J, Bo LL. Abundant travelling wave solutions for KdV-Sawada-Kotera equation with symbolic computation. *Appl Math Comput* (2008) 203:233–7. doi:10.1016/j.amc.2008.04.035

14. Sun CP, Sang JQ, Zheng DM, Liu BL, Li XD. Sensitivity analysis for dual-membrane capacitive MEMS microphone. *IEEE SENSORS JOURNAL* (2024) 24:24015–22. doi:10.1109/JSEN.2024.3417384

15. Zhang J, Wei XL, Hou JC. Symbolic computation of exact solutions for the compound KdV-Sawada-Kotera equation. *Int J Computer Mathematics* (2010) 87:94–102. doi:10.1080/00207160801965289

16. Zhang J, Wei XL, Lu YJ. A generalized -expansion method and its applications. *Phys Lett* (2008) 372:3653–8. doi:10.1016/j.physleta.2008.02.027

17. Zhang J, Jiao FL, Zhao XY. An improved expansion method for solving nonlinear evolution equations. *Int J Computer Mathematics* (2010) 87:1716–25. doi:10.1080/00207160802450166

18. Kaya D, Inan IE. Exact and numerical traveling wave solutions for nonlinear coupled equations using symbolic computation. *Appl Math Comput* (2004) 151:775–87. doi:10.1016/s0096-3003(03)00535-6

19. Ebaid A. Exact solitary wave solutions for some nonlinear evolution equations via Exp-function method. *Phys Lett* (2007) 365:213–9. doi:10.1016/j.physleta.2007.01.009