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Commentary: Mini-review on periodic properties of MEMS oscillators

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A Commentary on
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1 Introduction

In the landscape of micro-electromechanical system (MEMS) research, the exploration of system behavior has been a topic of great interest. Tian and Shao [1] previously investigated the periodic properties of MEMS systems. However, their model is too simplistic to be applicable in practical scenarios.

Their model

$$\ddot{x} + \omega^2 x - \frac{K}{1-x} = 0, \tag{1}$$

is only valid under ideal working conditions. Equation 1 is widely studied for its periodic property [2–4]. In actual applications, MEMS systems are exposed to various noise sources. These noises can have a profound impact on the performance and characteristics of MEMS devices, highlighting the need for more comprehensive models.

The micro-electromechanical system influenced by multiplicative and additive cyclic noise can be represented as follows Equation 2:

$$\ddot{x} - \left(-\varepsilon + \alpha_1 x^2 - \alpha_2 x^4 - \alpha_3 x^6 - \alpha_4 x^8\right)\dot{x} + \omega^2 x - \frac{k}{1-x} + {}_0^c D^p x = \eta_1(t) + x(t)\eta_2(t), \tag{2}$$

where ε is the coefficient of linear damping, K is a stiffness coefficient, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are nonlinear damping coefficients, ω is the frequency, $\eta_1(t)$ and $\eta_2(t)$ are independent recycling noises, i.e.,

$$D_1 \neq D_2, \eta_i(t) = \xi_i(t) + K\xi_i(t-\tau), (i = 1, 2).$$

The power spectral density (PSD) of recycling noise is obtained as:

$$S_i(\omega) = 2D_i\left[1 + k^2 + 2k\cos(\omega\tau)\right], (i = 1, 2), \tag{3}$$

${}_{0}^{c}D^{p}[x(t)]$ is the Caputo fractional derivative [1, 2] of $p(0 \le p \le 1)$ order about $x(t)$ defined as Equation 4:

$$ {}_{0}^{c}D^{p}[x(t)] = \frac{1}{\Gamma(m-p)} \int_{0}^{t} \frac{x^{(m)}(u)}{(t-u)^{1+p-m}} du \, (m-1 < p \le m, m \in N). \tag{4} $$

2 Model analysis

2.1 Influence of noise on system dynamics

The presence of multiplicative and additive recycling noises, denoted as $\eta_1(t)$ and $\eta_2(t)$ respectively, in the MEMS model significantly alters its dynamic behavior by introducing fundamental physical uncertainties inherent in micro-scale systems. These noise terms represent random fluctuations originating from various physical sources, such as thermal-mechanical (Brownian) noise, electronic noise in sensing/actuation circuits, or even fluctuations in environmental parameters like pressure or temperature. This inherent randomness perturbs the system's state variables (e.g., displacement, velocity, voltage, or current), leading to deviations from the ideal, noise-free response trajectory. For instance, in a MEMS oscillator, these noises directly manifest as frequency jitter (random variations in oscillation period) and amplitude variations (random fluctuations in the oscillation peak magnitude), degrading the device's precision and stability [5–7].

The PSD of the recycling noises, as described by Equation 3, provides crucial insights into the distribution of noise energy across different frequencies. Physically, this PSD quantifies how the intensity of these random fluctuations varies with frequency. A higher power spectral density at certain frequencies implies that the noise components within those specific frequency bands have a more significant impact on the system because they can more effectively couple into the device's natural mechanical resonant modes or its control/sensing bandwidth. This spectral information is essential for understanding the susceptibility of the system across different operational frequency ranges and pinpointing the critical bands where noise suppression is most vital. Consequently, this knowledge is fundamental to designing effective noise-filtering or noise-shaping mechanisms tailored to the specific spectral characteristics of the dominant noise sources affecting the MEMS device.

2.2 Role of fractional-order derivative

The Caputo fractional-order derivative ${}_{0}^{c}D^{p}[x(t)]$ in Equation 1 adds an extra degree of complexity to the model. Fractional-order derivatives can capture non-local and memory-dependent effects in the system. In MEMS, these effects might be related to the material properties, such as viscoelasticity, or the interaction between different components.

When $p = 1$, the fractional-order derivative reduces to the ordinary first-order derivative, representing a more traditional, local-behavior-based model. However, for $0 < p < 1$, the system exhibits memory-like characteristics. This means that the current state of the system depends not only on its immediate past but also on events that occurred further back in time. This characteristic is particularly crucial for simulating energy dissipation mechanisms in MEMS, such as anomalous viscoelastic relaxation in polymeric components, damping effects in rarefied gases, and interfacial interactions. Furthermore, fractional-order derivatives provide a more accurate means of describing phenomena like non-exponential decay and frequency-dependent phase lag. Thus, they offer a more precise modeling tool for enhancing the accuracy, reliability, and dynamic performance of MEMS devices [7–10].

2.3 Nonlinear damping and stiffness effects

The nonlinear damping terms (α^{+} and α^{2}) and the stiffness coefficient K play important roles in determining the system's stability and response. Nonlinear damping can act as a self-regulating mechanism. At high amplitudes, the nonlinear damping terms can increase, dissipating more energy and preventing the system from reaching extreme states [11, 12].

The stiffness coefficient K affects the natural frequency of system. A higher stiffness leads to a higher natural frequency, making the system more responsive to external stimuli. However, in the presence of noise, a high-stiffness system might also be more sensitive to noise-induced vibrations, which could potentially lead to instability or inaccurate measurements in MEMS applications. Therefore, investigating the interactions among nonlinear damping coefficients, stiffness coefficients, and noise is of great significance for optimizing the design of MEMS systems and improving their precision and stability.

3 Conclusion

This study presents a generalized MEMS model that incorporates multiplicative and additive recycling noises, providing a more realistic representation of MEMS systems in practical applications. Through the analysis of the model, it is evident that the noises, fractional-order derivative, and nonlinear terms all have significant impacts on the system's dynamics.

Recycling noises critically undermine MEMS precision and stability through frequency jitter and amplitude variations, where Power Spectral Density (PSD) analysis is vital for identifying noise-susceptible frequency bands to guide targeted mitigation strategies. Simultaneously, the Caputo fractional-order derivative ${}_{0}^{c}D^{p}[x(t)]$ (for $0 < p < 1$) introduces essential memory effects and non-local behavior, enabling accurate modeling of complex energy dissipation mechanisms, such as anomalous viscoelastic relaxation, that are unattainable with traditional integer-order descriptions. Nonlinear damping further provides crucial amplitude-limiting stabilization, whereas the stiffness coefficient K dictates resonant frequency while significantly amplifying sensitivity to noise-induced vibrations, potentially compromising stability and measurement fidelity. Collectively, the intricate interplay between stochastic noise, fractional-order memory dynamics, and nonlinear stiffness/damping governs MEMS behavior, making understanding these coupled effects fundamental for optimizing device design toward enhanced precision, stability, and reliability.

The establishment of this model opens a pioneering and highly promising frontier for future MEMS research. Subsequent investigations should focus on refining the model to enhance its predictive accuracy. This can be achieved by more precisely quantifying noise parameters and fractional-order derivative exponents, thereby enabling better simulation of the dynamic behavior exhibited by actual MEMS devices operating in complex environments. Concurrently, a deeper exploration of the intrinsic mechanisms governing the interactions between nonlinear damping coefficients, stiffness coefficients, and noise holds significant potential to reveal novel physical properties. Such insights are expected to provide crucial theoretical underpinnings for designing high-performance, high-reliability MEMS devices.

Furthermore, integrating advanced numerical computation methods with sophisticated experimental techniques will be essential to validate and refine the model. This integrated approach will propel its practical application in MEMS system design, control strategies, and fault diagnosis methodologies. Ultimately, these advancements will advance the continuous innovation of MEMS technology at the micro/nano scale, enabling it to meet the increasingly demanding requirements for precision, stability, and adaptability in micro-electromechanical systems.

Author contributions

HC: Validation, Writing – review and editing, Methodology, Writing – original draft, Software. J-GZ: Supervision, Writing – review and editing, Investigation, Writing – original draft, Project administration.

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Conflict of interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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