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# Editorial: Advances in information geometry: beyond the conventional approach

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### Editorial on the Research Topic

Advances in information geometry: beyond the conventional approach

### 1 Introduction

Information Geometry (IG) is an active interdisciplinary field, employing that employs the tools of differential geometry to explore the structure of classical and quantum statistical models of probability distributions and quantum states, respectively. IG has provided powerful insights into the geometrical aspects of classical and quantum statistical inference, machine learning, signal processing, and neural networks, p. This is primarily accomplished by endowing statistical models with geometric structures like such as Riemannian metrics (e.g., Fisher-Rao metric, Bures-Helstrom metric) and affine connections (e.g., Amari-Chentsov  $\alpha$ -connections). This geometric perspective allows for a deeper understanding of parameter estimation, model selection, and learning dynamics.

The Research Topic Advances in Information Geometry: Beyond the Conventional Approach was conceived to capture the ever-evolving spirit of innovation of the investigation in the field of IG. It aims to showcase some of the most innovative research that pushes the boundaries of traditional IG, whether through novel theoretical extensions, the exploration of new application domains, or theor development of methodologies that challenge standard assumptions. The Research Topic of articles within in this Research Topic offers a glimpse into these exciting advancements, e. Each study contributing contributes a unique perspective on how IG can address contemporary challenges and open new avenues of investigation moving "beyond the conventional approach."

# 2 Thermodynamics modeling of deep learning systems for a temperature-based filter pruning technique

The work Lapenna et al. presents a novel link between information geometry-related concepts, thermodynamics, and the rapidly advancing field of deep learning. The authors

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propose a thermodynamic analogy to analyze the dynamics of parameters in Convolutional Neural Networks (CNNs) that allows, allowing for a rigorous definition of "temperature" for convolutional filters. Their research demonstrates that high-temperature filters have a minimal impact on model performance when removed, whereas removing low-temperature filters significantly affects accuracy and loss decay. This insight leads to a practical, temperature-based filter pruning technique.

This application of thermodynamic and statistical mechanical thinking, which shares deep conceptual roots with information geometry (e.g., geometrization of state spaces), to the architecture and optimization of deep learning models is a significant step beyond conventional applications of IG, showcasing its potential utility in understanding and refining complex neural networks.

### 3 Exponential arcs in manifolds of quantum states

The article Naudts et al. investigates the foundations of quantum IG. The conventional approach often relies on quantum statistical models for finite-dimensional quantum systems. However, tThe author, however, considers manifolds of faithful normal states on a sigma-finite von Neumann algebra in standard form. The paper study generalizes the concept of an exponential arc connecting quantum states, using a given relative entropy (divergence function).

A key finding is the uniqueness (up to an additive constant) of the generator of such an arc. Specifically, when using Araki's relative entropy, any self-adjoint element of the von Neumann algebra can generate an exponential arc. This formulation demonstrates that the metric derived from Araki's relative entropy aligns with the Kubo-Mori metric, which is crucial in linear response theory. These submanifolds, formed by states connected via these exponential arcs, represent a quantum generalization of dually flat statistical manifolds. This contribution outlines a promising approach to developing IG for more general and potentially infinite-dimensional quantum systems, which moves beyond the standard density matrix formalism.

# 4 Geometric properties of noninformative priors based on the chi-square divergence

The article Tanaka et al. discusses the foundational aspects of Bayesian statistics concerning noninformative priors. While the Jeffreys prior, derived from the Fisher information metric, is a cornerstone in this area, derived from the Fisher information metric, the author explores an interesting alternative: a noninformative prior based on the  $\chi^2$ -divergence. This prior, an extension of Bernardo's reference prior, offers a different geometric interpretation.

The paper study elucidates that this prior corresponds to a parallel volume element within the framework of information geometry. Furthermore, in the context of flat model manifolds, it can be expressed as a power of the Jeffreys prior. This work contribution moves beyond conventional reliance on the Jeffreys prior by investigating the geometric underpinnings and properties of priors derived from alternative divergence measures, thus broadening the toolkit for objective Bayesian inference.

### 5 Information geometry of markov kernels: a survey

The article Wolfer et al. offers a comprehensive overview of the Information Geometry of Markov Kernels. Traditional information geometry focuses primarily focuses on static manifolds of probability distributions. This surveycontribution, however, explores the information geometry of Markov kernels, which represent conditional probabilities or stochastic transitions, thereby extending IG to dynamical processes. The authors present a self-contained treatment of the foundational concepts, including information projections and Nagaoka's construction of Fisher metrics and dual affine connections on sets of irreducible stochastic matrices.

The survey article also discusses recent advancements, such as geometric structures arising that arise from time reversibility, the lumpability of Markov chains, and tree models. Applications in parameter estimation, hypothesis testing, large deviation theory, and the maximum entropy principle are highlighted. This work study clearly highlights demonstrates how information geometry can be extended beyond static distributions to characterize the structure of transformations and evolving systems.

### 6 Conclusion

The journey "beyond the conventional approach" of Information Geometry is an ongoing one. The articles collected in this Research Topic, while diverse in their specific focus, collectively highlights highlight the broad scope of Information Geometry. They illustrate demonstrate that the field is not static, but is actively incorporating new mathematical tools, addressing more complex systems, and finding relevance in an increasingly diverse range of scientific and technological disciplines. The insights presented here are expected to push inspire further research, inspiring leading to new theoretical developments and practical applications that will continue to redefine the field of Information Geometry.

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